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A FORCE CONTROL METHOD FOR CONTACT PROBLEMS WITH LARGE PENETRATIONS

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Abstract. In this paper we present a new strategy to deal with contact problems with large penetrations. The method is based on a check of the nodal contact forces to select the technique that has to be used to perform each iteration. In case the contact forces are smaller than a set limit the problem is solved in a standard way using consistent linearization and Newton's method. When contact forces exceed the limit a modified Newton method is used. This method is based on enforcement of a contact force limit and use of a simplified secant stiffness where the geometric stiffness term is disregarded. The efficiency of the procedure is illustrated by the solution of example problems in which large initial penetrations occur.

1 INTRODUCTION

In recent years attention has been given to the efficiency and consistency of algorithms used to solve non-linear problems in computational mechanics. In particular the benefits of a linearization consistent with the algorithm for the solution of any type of non-linear problem are well known. Consistent linearization in fact guarantees a quadratic rate of asymptotic convergence when the field of the unknowns is close to a solution value. It is a matter of fact however that, the bigger the step, the greater the solution point is from the starting one. This aspect is often disregarded, and standard Newton procedures are advocated from the first iteration, regardless of the fact that perhaps a different method can be more reliable and faster in converging the solution in the right direction.

In the case of contact problems, direct application of Newton's method usually produces difficulties during the first few iterations, and some iterations are needed simply to stabilize the solution before a quadratic rate of convergence is obtained. In fact one of the main difficulties encountered in solving contact problems is related to large initial penetrations which occur between contacting bodies. Contact algorithms are usually activated a-posteriori, i.e. they first let the bodies penetrate each other and the penetration is then detected as a violation of impenetrability constraint conditions. Only at this latter stage are specific strategies activated to avoid penetration. This strategy usually works in cases of small penetrations and smooth contact surfaces evolution, however for problems in which the contact area varies significatively during the transient solution, the step size is generally limited by the contact conditions.

Regardless of the strategy adopted, i.e. penalty or Lagrangian multiplier method, the current state which violates the impenetrability conditions is used to compute the virtual work contribution for contact, and this equation set is linearized, (often in an in-consistent way), to solve the non-linear problem with a Newton type method.

In case of large initial penetrations the resulting contact forces can be very big. This affects both the tangent stiffness and the residual vector, and generally produces a large local distortion of the mesh. Usually a Newton method cannot recover to a smooth deformation state; accordingly it is currently necessary to use smaller loading or time increments which limit the amount of penetration.

The most evident shortcomings of the straight application of consistent linearization from the first iteration are hence the strict limitation on the step size, and the poor convergence rate during the first few iterations. In this paper we propose a method to perform large steps in the presence of large penetrations. To do this we split the solution strategy into two phases and use different solution strategies for each of them. A smooth transition between these two phases is also presented. The first phase takes place during the first iterations of each time or loading step. Within this phase it is an easy task to check that both the full Newton strategy with consistent linearization is often useless since the contact forces can be many orders larger than the real ones. The second phase takes place when, due to the iterations performed in the first phase, the contact penetrations have been reduced significatively. The problem has hence been driven close to the solution point, and then a Newton strategy with consistent linearization guarantees the best convergence rate.

2 STRATEGY OUTLINE

The target of the present work is to set up a better strategy than a consistent full Newton linearization for the above cited phase one. What we seek is a criterion to limit contact forces and to construct the modified stiffness matrix and residual vector. In such a way we want to control local contact instabilities, and to quickly construct a solution path to an almost converged point.

As first step we avoid the introduction in the system of large physically meaningless contact forces which originate from unconstrained large penetrations. To do this we note that the range of the maximum contact force that the contacting bodies can generally experience can be easily estimated. This estimate is then used to set a bound for such forces. We remark that we are discussing a range of values, not exact values, hence the estimation is easy to do.

The employment of the upper bound plus some modifications to the standard Newton procedure permits us to perform steps of unusual size, where the limit on the step size in general still comes from the large deformations of the continuum, and not from the contact.

For a better understanding of what happen within the first iterations, we start by considering the characteristics of the consistent tangent stiffness and the residual vector for a typical contact problem. In the case where a penalty method is used, the term that is included as a discrete potential for each active contact element is:

$$W_c = \frac{1}{2} A_c \varepsilon g_N^2 \tag{1}$$

where A_c is the element contact area, ε is the penalty parameter and g_N is the local approach of the two surfaces (normal gap). The contact force F_N, is recovered as

$$F_N = \frac{\partial W_c}{\partial g_N} = A_c \varepsilon g_N \tag{2}$$

The virtual variation of the potential for active contacts hence becomes

$$\delta W_c = A_c \varepsilon g_N \delta g_N = F_N \delta g_N \tag{3}$$

where the symbol δ denotes a variation. For simplicity, the contact area is considered to be a constant in the simple example cited here.

Linearization of the contact contribution produces two terms, one related to the contact force, and a second related to the contact approach. Hence the standard Newton procedure at each iteration solves the following equation set

$$-F_N \delta g_N = \frac{\partial F_N}{\partial g_N} \Delta g_N \delta g_N + F_N \Delta(\delta g_N) \tag{4}$$

where Δ denotes a linearized increment. Rearranging virtual variations and increments of the unknowns, and then disregarding virtual variation quantities, eq. (4) can also be written in matrix form as [1]:

$$\mathbf{R} = \mathbf{K}_T \boldsymbol{\Delta} \tag{5}$$

with $\mathbf{R}$ the residual vector, $\mathbf{K}_T$ the tangent stiffness and $\boldsymbol{\Delta}$ the increments of the unknown displacement.

If now we focus on the point-to-segment geometrical formulation, we get the following well known equations [2]

$$\Delta g_N = \Delta\left[(\mathbf{x}_s - \mathbf{x}_1)\cdot\mathbf{n}\right] = (\Delta\mathbf{x}_s - \Delta\mathbf{x}_1)\cdot\mathbf{n} + (\mathbf{x}_s - \mathbf{x}_1)\cdot\Delta\mathbf{n} \qquad (6)$$

$$\Delta\delta g_N = (\delta\mathbf{x}_s - \delta\mathbf{x}_1)\cdot\Delta\mathbf{n} + (\Delta\mathbf{x}_s - \Delta\mathbf{x}_1)\cdot\delta\mathbf{n} + (\mathbf{x}_s - \mathbf{x}_1)\cdot\Delta\delta\mathbf{n} \qquad (7)$$

where the involved terms are easily recovered from Fig. 1. After some algebra eq. (6) and eq. (7) result in

$$\Delta g_N = \left[\Delta\mathbf{x}_s - (1-\xi)\Delta\mathbf{x}_1 - \xi\Delta\mathbf{x}_2\right]\cdot\mathbf{n} \qquad (8)$$

$$\begin{aligned}\Delta\delta g_N = &-\frac{1}{l}\{[-\delta\mathbf{x}_1 + \delta\mathbf{x}_2]\cdot\mathbf{n}\}\\ &\{[\Delta\mathbf{x}_s - (1-\xi)\Delta\mathbf{x}_1 - \xi\Delta\mathbf{x}_2]\cdot\mathbf{t}\} -\\ &\frac{1}{l}\{[\delta\mathbf{x}_s - (1-\xi)\delta\mathbf{x}_1 - \xi\delta\mathbf{x}_2]\cdot\mathbf{t}\}\\ &\{[-\Delta\mathbf{x}_1 + \Delta\mathbf{x}_2]\cdot\mathbf{n}\} -\\ &\frac{g_N}{l^2}\{[-\Delta\mathbf{x}_1 + \Delta\mathbf{x}_2]\cdot\mathbf{n}\}\{[-\delta\mathbf{x}_1 + \delta\mathbf{x}_2]\cdot\mathbf{n}\}\end{aligned} \qquad (9)$$

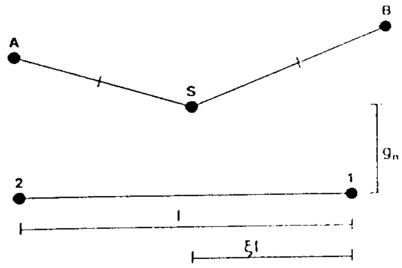

Figure 1. Geometry and variable names for point-to-segment contacts.

The matrix form for the consistent tangent stiffness and the residual may be established from eq. (8) and eq. (9) as

$$\mathbf{R} = -F_N\mathbf{N}_s \qquad (10)$$

$$\mathbf{K}_T = A_c\varepsilon\mathbf{N}_s\mathbf{N}_s^T - \frac{F_N}{l}\left(\mathbf{N}_0\mathbf{T}_s^T + \mathbf{T}_s\mathbf{N}_0^T + \frac{g_N}{l}\mathbf{N}_0\mathbf{N}_0^T\right) \qquad (11)$$

where the following definitions are used

$$\mathbf{N}_s = \begin{bmatrix}\mathbf{n}\\ -(1-\xi)\mathbf{n}\\ -\xi\mathbf{n}\end{bmatrix} \quad \mathbf{N}_0 = \begin{bmatrix}0\\ -\mathbf{n}\\ \mathbf{n}\end{bmatrix} \qquad (12)$$

$$\mathbf{T}_s = \begin{bmatrix}\mathbf{t}\\ -(1-\xi)\mathbf{t}\\ -\xi\mathbf{t}\end{bmatrix} \qquad (13)$$

To discuss the characteristics of the tangent stiffness in more detail we can explicitly write the terms as follows

$$\mathbf{N}_s\mathbf{N}_s^T = \mathbf{n}\otimes\mathbf{n}\begin{bmatrix}1 & -(1-\xi) & -\xi\\ -(1-\xi) & (1-\xi)^2 & \xi(1-\xi)\\ -\xi & \xi(1-\xi) & \xi^2\end{bmatrix} \qquad (14)$$

$$\mathbf{N}_0\mathbf{T}_s^T = \mathbf{n}\otimes\mathbf{t}\begin{bmatrix}0 & 0 & 0\\ -1 & (1-\xi) & \xi\\ 1 & -(1-\xi) & -\xi\end{bmatrix} \qquad (15)$$

$$\mathbf{T}_s\mathbf{N}_0^T = \mathbf{t}\otimes\mathbf{n}\begin{bmatrix}0 & -1 & 1\\ 0 & (1-\xi) & -(1-\xi)\\ 0 & \xi & -\xi\end{bmatrix} \qquad (16)$$

$$\mathbf{N}_0\mathbf{N}_0^T = \mathbf{n}\otimes\mathbf{n}\begin{bmatrix}0 & 0 & 0\\ 0 & 1 & -1\\ 0 & -1 & 1\end{bmatrix} \qquad (17)$$

where each term of the matrices is multiplied by the dyadic product placed outside square brackets.

The analysis of the tangent stiffness shows that the term $\mathbf{N}_s\mathbf{N}_s^T$ is independent of the amount of penetration. This term depends only on the contact area, the penalty parameter, the orientation of the master segment which determines the normal unit vector $\mathbf{n}$, and the normalized position of the projection of the node along the segment, ξ. It is easy to recognise that this term is a core part of the stiffness. On the other hand the second term depends strongly on the amount of penetration because it involves both the contact force and the geometrical approach. In the case of large penetrations it is evident that the stiffness contribution related to the contact force can be several orders of magnitude different from its values when the solution is almost reached. The situation is even worse for the term which depends on $F_N g_N$.

Eq. (7) shows that the second term is

strongly related to the change of orientation of the contact element, and in fact it vanishes in case of a constant normal vector, which results in

$$\Delta \mathbf{n} = \delta \mathbf{n} = \Delta \delta \mathbf{n} = 0 \qquad (18)$$

hence the terms related to F_N lead to the geometric stiffness contributions.

Considering the complete stiffness matrix, we note that the second (geometrical) term does not affect the diagonal term related to the slave node S. This fact is evident again when contact problems are solved. In case of large penetrations it is usually the master surface that has large distortion. In case of high contact forces the geometrical term becomes bigger and bigger, and due to a negative contribution for the penetration case the sign of the diagonals associated with the master segment can change. Many other non-diagonal terms change sign also. We also note that the geometrical term becomes more and more important with the reduction of the initial penetration. Its presence is necessary for the quadratic rate of convergence, but the main dominating term when convergence is achieved is the first term, at least for usual values of the penalty parameter. It is then clear that, in case of non realistic contact forces due to large penetration the geometric term is both useless, because the geometry is simply too far from the final one, and dangerous because it strongly affects the local properties of the stiffness matrix.

To overcome the difficulties cited above, we modify the linearization during the first phase disregarding the geometrical term, which results in

$$\mathbf{K}_I = \frac{F_N}{g_N} \mathbf{N}_s \mathbf{N}_s^t \qquad (19)$$

In the above, the penalty parameter and area are replaced using eq. (2).

The second modification for phase one of the solution is related to the contact force which goes into the residual. For large penetration, the contact forces computed from eq. (2) are grossly in error. Redistribution of these forces to the nodes by the vector $\mathbf{N}_s$ is the second element of instability of the solution. To limit this force we propose to modify the linear relationship by using a cut-off with a maximum value independent of the penetration (see Fig. 2). In this way the addition of unrealistic forces into the system is prevented.

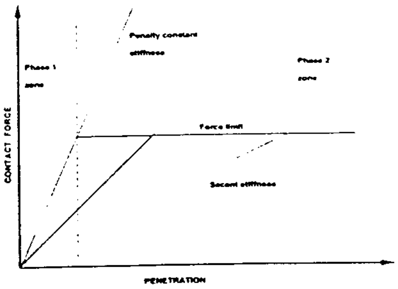

Figure 2. Contact force versus penetration with zone distinction.

It has to be remarked that the cut-off alone is not sufficient to perform large steps. In fact if consistent linearization is used we get a zero derivative when the cut-off limit is reached, and hence no contact stiffness is associated to the residual, even if penetration persists. The contact forces are then applied without any contact resistance, i.e. they are totally applied to the continuum. Once again this will have dangerous effects, because the resulting displacements for most cases leads to release, and then a new instability often takes place with part of the contacting surfaces changing from an open to a closed status during one iteration and vice versa for the next. Also, the contact stiffness defined by the penalty parameter is too big and results in a sort of locking of the penetration, which requires then many iterations to achieve near convergence.

We have achieved very good performance for phase one by using the secant stiffness. In most cases the secant stiffness is able to keep the gap closed and rapidly relax the contact conditions to achieve a penetration to values where consistent linearization can then be employed. The secant stiffness is related to the amount of penetration and to the maximum contact force by

$$K_s = \frac{F_{MAX}}{g_N} \qquad (20)$$

and is used directly in eq. (19). Notice that, due to the constant force limit, the secant stiffness depends only on the penetration and increases with a reduction of the penetration until it reaches the standard penalty value. Subsequently the contact solution procedure shifts from phase one to phase two, where standard consistent linearization is performed, see also Fig. 2.

In cases where the maximum contact force has been underestimated it is necessary to add one more feature to increase the maximum force limit within the iterations. This is necessary to achieve convergence to the correct solution. The increment of such limit however should be done very carefully, because it presents similar aspects as the increase of contact stiffness within augmentations [3], [4]. Since the cut-off limit for the residual coincides with the introduction of the geometrical term into the tangent stiffness a too rapid increase of the cut-off can result in a too early shift into phase two. The evolution of the contact state has hence to be monitored to decide when and if the increment can take place. An easy criterion is a check of the contact force evolution. The ratio between new and old values of the penetration can be monitored to decide if increase of the contact force limit can be applied. The increment is not performed if the penetration is still rapidly changing within the iterations, i.e. if the ratio if far from 1. When the increment ratio satisfies the imposed limit a linear increment of the initial force can be applied.

Our experience has shown that the proposed strategy permits convergence to be achieved in few iterations even for very large steps, which are well in excess of reasonable limits for converging the continuum part of the problem.

3 EXAMPLES

The examples given below demonstrate the effectiveness of the proposed contact algorithm for large penetrations. Each continuum is discretized with a simple 4 node large deformation elastic plane-strain element. All the examples involve contact between two or more deformable bodies. The stiffer material has an elastic modulus $E = 25000$ and a Poisson ratio $\nu = 0.25$. The chosen ratio between the stiffer and the softer material is equal to 10, and the contact penalty parameter is $\varepsilon = 100000$. The material model is characterized by an extension of small strain linear elasticity to finite deformations as described by Simo [5].

The first example deals with a parabolic indentor pressed into flat plate. For the initial penetration, depicted in Fig. 3, convergence is achieved in 15 iterations, see Table 1. The outline of the geometry at the second and the final iteration is depicted in Fig. 4.

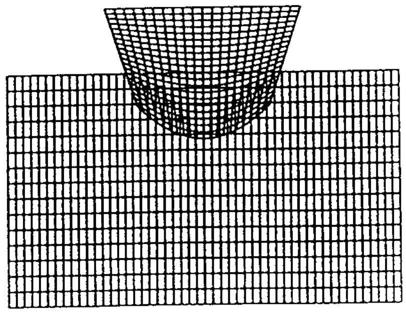

Figure 3. Parabolid indentor initial penetration.

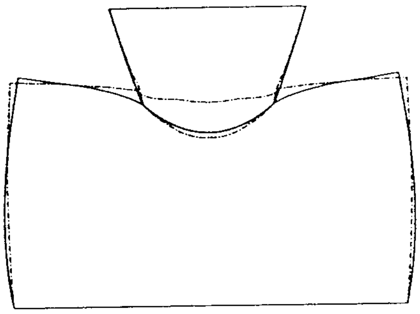

Figure 4. Parabolid indentor geometry for the second iteration and at convergence.

The problem has been solved also with a coarse mesh. In this case the large dimension of the continuum elements prevents local distortions which occur for smaller elements, and the initial step can be larger. The initial penetration that we are able to apply in this case involve the complete initial penetration of the indentor into the plate in a single step. The solution for the step requires 15 iterations, and for 9 of them the cut-off of the contact forces takes place as reported in Table 2, Example 6. The deformed geometry at convergence is shown in Fig. 5.

The proposed strategy deals also for problems with multiple contact surfaces, as depicted in Fig. 6 where the applied penetration at the first iteration is depicted. In this example 6 contact surfaces are defined. Also in this case the solution is achieved in a single step with 14 iterations. The final geometry is depicted in Fig. 7.

TEST 1 fine mesh		TEST 2 coarse mesh		TEST 3 multiple contact	
Residual	lim.	Residual	lim.	Residual	lim.
8.64 E+5	-	8.93 E+5	-	6.00 E+3	-
5.75 E+3	19	3.34E+4	29	2.68 E+3	32
4.29 E+3	15	2.60 E+4	29	2.44 E+3	36
2.78 E+3	11	2.06 E+4	25	1.93 E+3	30
6.87 E+3	13	9.08 E+3	15	1.58 E+3	24
4.46 E+3	9	7.68 E+3	13	1.22 E+3	20
3.60 E+3	11	5.97 E+3	9	1.10 E+3	14
2.26 E+3	6	6.94 E+3	10	6.64 E+2	7
1.22 E+3	2	3.03 E+3	4	6.95 E+2	6
3.50 E+2	-	3.37 E+3	4	5.11 E+2	5
4.24 E+1	-	1.21 E+3	-	2.15 E+2	-
1.49 E+0	-	3.64 E+1	-	4.92 E-1	-
2.14 E-3	-	2.24 E-1	-	2.58 E-5	-
3.87 E-9	-	4.81 E-5	-	4.03 E-11	-
3.12 E-10	-	3.70 E-10	-		

Table 1. Residuals and number of elements for which force cut-off takes place:

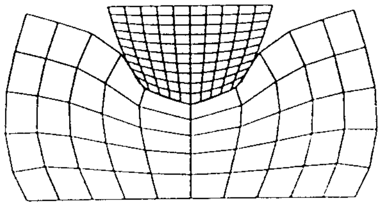

Figure 5. Geometry at convergence of the first step.

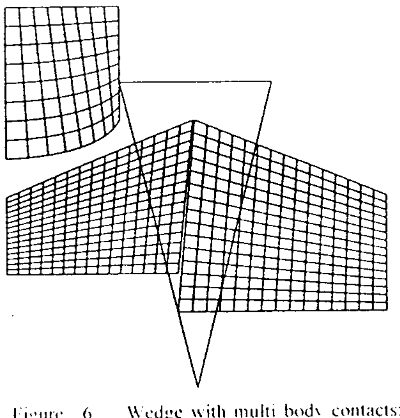

Figure 6. Wedge with multi body contacts: penetration after the first iteration.

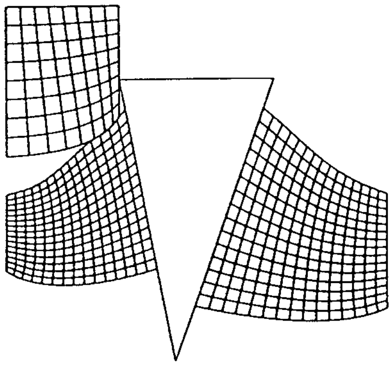

Figure 7. Geometry at convergence.

4 CONCLUSIONS

The strategy described in the paper has shown a good capability to deal with contact problems subjected to large initial penetrations. The load control method coupled with the employment of a secant stiffness permits one to enforce gradually the violated impenetrability condition within the iterations. The tests performed to date have shown good behavior, and the convergence is usually achieved in a limited number of iterations. The solutions are characterized by good stability and efficiency. Thus the proposed method can be used to solve problems which have large penetrations and rapidly changing contact area.

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