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# DISCRETE METHODS FOR SOIL MODELING

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## Abstract

A common problem of excavation machinery based on mechanical actions is the unknown interaction of the cutting tools with various types of rock and more in general with geological settings. The problem is very important for the industry: interaction of the cutting tools with different soils determines a different wear, and consequently different economical costs for the excavation. Due to the involved non-linearities, the numerical analysis of such phenomena is very complex. To overcome this aspects some authors proposed to model the soil as a collection of spheres.

In this paper we apply a strategy for soil modelling which is based on discretization of the soil with rigid spheres and suitable contact models. The basic idea is to concentrate at the contact level the real mechanical behavior of the soil. The goal is achieved by extending the general concept of contact as a unilateral constraint condition, through a suitable constitutive law: constitutive

laws have been implemented in the node-to-segment contact formulation within the framework of the penalty method.

The final aim of this work is to study the behavior of some soils under different loading conditions, and to develop contact constitutive laws suitable for the discrete model. In order to carry out the proposed strategy a “macro” and a “micro” level are established, and macromechanical and micromechanical models are developed. In the micromechanical model the mutual contact interaction between two spheres is studied. In the macromechanical model the behavior of an array of spheres is described. The framework for the plastic behavior consists of a failure criterion; a one-dimensional, rate-independent elasto-plastic flow rule for normal and tangential force; two specific yield surfaces, and a hardening or softening law. In this paper we have focused our attention on the micromechanical model: a new constitutive contact law is developed and applied for the simulation of different soils with different testing condition (such as uniaxial and shear tests).

L'indeterminatezza dell'interazione fra gli attrezzi per lo scavo e il terreno costituisce un notevole problema nelle macchine per il movimento della terra.

La soluzione di tale problema presenta interessanti aspetti applicativi anche in ambito industriale: l'interazione dei mezzi meccanici con i diversi tipi di terreno determina una differente usura degli stessi e, per conseguenza, un diverso costo economico per l'opera di scavo. Per semplificare le nonlinearità correlate alle modellazioni numeriche del fenomeno è stata sviluppata una strategia di modellazione del terreno basata sulla discretizzazione dello stesso attraverso sfere rigide e modelli di contatto specifici. L'idea di base è quella di concentrare a livello delle zone di contatto il reale comportamento del suolo. Alcuni legami costitutivi sono stati quindi implementati nella classica formulazione di contatto “node-to-segment”, attraverso il metodo di risoluzione penalty.

Lo scopo finale di questa ricerca riguarda lo studio del comportamento di alcuni tipi di suolo sottoposti a varie condizioni di carico, e lo sviluppo di legami costitutivi di contatto specifici per il modello discreto. Per poter sviluppare detta strategia, sono stati adottati due livelli di studio: uno micromeccanico e l'altro macromeccanico. Nel modello micromeccanico è stata studiata la mutua interazione tra due sfere. Nel modello macromeccanico si è indagato il comportamento di un insieme di sfere. La formulazione elastoplastica consiste in un criterio di rottura, un modello monodimensionale di tipo “rate-independent” elastoplastico per le azioni di tipo normale e tangenziale, e un criterio di snervamento di tipo nonlineare.

## 1. INTRODUCTION

In many civil engineering projects the earth surface has to be excavated, prepared or removed for needs of specific constructions. A common problem of excavation machinery based on mechanical actions is the unknown interaction of the cutting tools with various types of rock and more in general with geological settings.

The problem is very important for the industry. In fact the soil characteristics play an important role in the wear of the rock cutting tools: the

interaction of the cutting tools with different soils determines a different wear and consequently different economical costs for the excavation.

For this reason many rock-cutting theories have been developed in the past decades. In most of the cases these theories were applied to tunneling. Due to the involved non-linearities, the numerical analysis of such phenomena is very complex. The classical continuum approach in fact, presents several drawbacks, especially when large strains and crack propagations take place. Alternative strategies were developed in the past to overcome the defects of the continuum approach. To simplify the non-linear aspects some authors proposed to model the soil as a collection of spheres [1], [2], [3], [4].

In this paper we enhance a strategy for soil modelling [5], which is based on a discretization of the soil with rigid spheres and suitable contact models. The basic idea is to concentrate at the contact level the real mechanical behavior of the soil. The goal is achieved by extending the general concept of contact as a unilateral constraint condition, through a suitable constitutive law. For this purpose constitutive laws have been implemented in the node-to-segment contact formulation within the framework of the penalty method [6]. In this case the penalty parameter is not simply a constant based on numerical necessities (e.g. to avoid ill conditioning of the stiffness matrix), but it is transformed into a non-linear function through the constitutive law itself.

In such a way any tool penetration, crack propagation or change of shape of the soil will be transformed into a series of contact openings and/or slidings. The main advantage of this approach lies in the fact that classical remeshing and crack propagation problems of a continuum discretization are avoided. The final aim of this research is to study the behavior of some soils (with particular attention to stiff soils and rock) under different loading conditions, and to develop contact constitutive laws suitable for the discrete model.

The resulting constitutive laws can be very complex, due to the discretization strategy and to the intrinsic characteristics of the soil such as composition, strength, texture, hardness etc.

In order to carry out the proposed strategy a “macro” and a “micro” level is established, and macromechanical and micromechanical models are developed. In the micromechanical model the mutual contact interaction between two spheres is studied. In the macromechanical model the behavior of a random array of spheres will be described. This modelling approach requires that, by linking the micro and macro levels, the most relevant parameters of the micromechanical model, which are the unknowns, have to be tuned to obtain a macroscopic behaviour coinciding with the experimental data, see Figure 1.

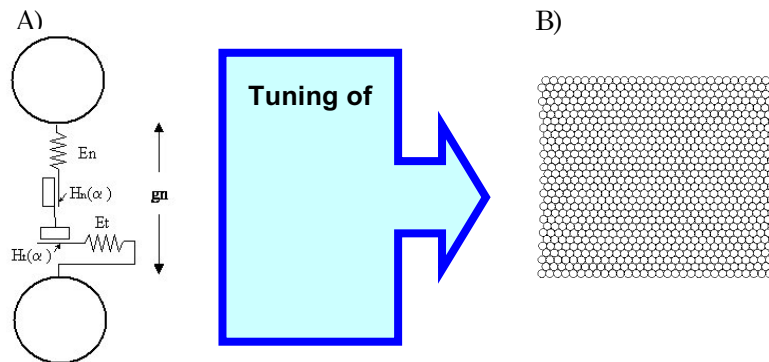


Figure 1. Strategy to discretize the soil as random collection of spheres: micromechanical and macromechanical models.

The relationship between the variables of the models can be expressed in two ways [7]:

- The “averaging” relationship: the macroscale variables are expressed as a function of the microscale variables.
- The “tracking” relationship: the microscale variables are expressed as a function of the macroscale variables

We take here two neighboring disks of such a chain as representative elements of the macromechanical model as shown in [8], i.e. the macromechanical model coincides with the microscopical one. Based on this simple model the constitutive behaviour of frictional materials such as sand, clay and rocks is investigated.

The contact models are based on the classical elasticity and plasticity theories, with suitable modifications. The framework for the plastic behavior consists of a failure criterion; a one-dimensional, rate-independent elasto-plastic flow rule for normal and tangential forces; two specific yield surfaces, and a hardening or softening law (according to the real behaviour of the examined material). A new constitutive law was also developed. Finally the application of the model to different soils and the behaviour of an array of disks will be studied in detail, and the interaction between soil and the excavating tools will be developed.

In the following we summarize first some basic considerations about contact mechanics and contact constitutive laws. Then we discuss the formulation of constitutive laws to model the normal and tangential behavior of the soil, with particular emphasis on the yield function and the failure criterion.

## **1. THE MICROSCOPICAL MODEL**

### **1.1.Contact constitutive law**

We propose a constitutive law that can be applied directly to the contact formulation and to the adopted strategy of discretization of the soils. Two different yield criteria are considered: one for the normal and another one for the tangential stress-strain relation. This involves a greater number of parameters; it permits however to model in a better way the real behavior of the soil. The obtained law is adaptable to different types of soil just by changing and tuning some specific parameters. This choice was basically suggested by the geometrical formulation of the contact element; in fact starting from the definition of the node-to-segment contact it is obvious that the two components of the forces (normal and tangential) have to be formulated separately, and then linked together through an arbitrary physical parameter. Here we develop a constitutive model where the two laws are related by the failure criterion formulation.

### **1.2. Elastic stress-strain relationships**

We assume that the total strain increments (also experimentally observed) in a material can be divided into elastic and plastic components

$$d\epsilon_{ij} = d\epsilon_{ij}^e + d\epsilon_{ij}^p \quad (1)$$

These strain increments are then calculated separately: the plastic strains by a suitable plastic stress-strain law, and the elastic strains by the Hooke's law. However, based on a more recently developed model for the non-linear variation of shear modulus with the stress state, another formulation of the elastic strain increments has been developed.

The developed strategy needs to define the elastic and shear moduli at macroscopical and microscopical level. At contact level the elastic modulus, shear modulus and the Poisson ratio are not linked through the classical relation. In fact the continuum is discretised with spheres, and the normal and tangential behaviour are treated separately. In the contact law the Young's modulus is supposed constant and the expression for the shear modulus is derived from experimental observations, and it depends directly on the normal applied stress as follows

$$G = (\sqrt[3]{F_N} - R)m \quad (2)$$

where  $F_N$  is the applied normal contact force;  $R$  is a constant which depends on the peak stress value, and  $m$  is a constant parameter, both derived from experimental tests, such as the triaxial ones, the direct shear tests or ring shear tests. According to these assumptions the elastic part of the normal stress-strain relation is linear. This approximation doesn't affect strongly the results of the model. In fact some experimental tests with soils (especially simple compression tests) have shown that the elastic part of the real stress-strain relationship can be assumed as linear. The elastic stress-strain relationship, which is a first component of the considered constitutive law and is valid both for tension and compression, can be expressed as

$$\sigma_i(F_i) = E_i \cdot (\epsilon_i(g) - \epsilon_i^p(g^p)), \quad i = n, t \quad (3)$$

where  $\sigma$  is the normal or tangential stress;  $\epsilon$  is the total strain;  $\epsilon^p$  is the plastic component of the total strain;  $F$  is the contact force;  $g$  is the total relative displacement (in the FEM formulation);  $g^p$  is the plastic component of the relative displacement (in the FEM formulation);  $n, t$  are, respectively, the normal and tangential direction;  $E_i$  are the elastic parameters which characterize the elastic properties of the soil. The parameter  $E$  is usually related to the bulk modulus,  $K$ , (or the elastic modulus) for the normal direction and to the shear modulus for the tangential direction.

### 1.3. Yield criterion for normal law

The yield function selected is a general function, which was developed from the classical plasticity theory [9]. The basic one-dimensional plasticity function is

$$f(\sigma, \alpha) = |\sigma| - (\sigma\gamma + K\alpha) \quad (4)$$

where  $\sigma_Y > 0$  delimits the plastic stress;  $K$  is the plastic hardening (or softening) modulus;  $\alpha$  is the so called internal hardening variable, i.e. a non-negative function of the amount of plastic flow, which is function of the time

$$\alpha: [0, T] \rightarrow \mathbb{R}_+ \quad (5)$$

This criterion defines a general law, which separately limits the elastic and the plastic part of the material behavior. The first term of the equation expresses the elastic range of the material given by (3).

This formulation of the yield condition is very useful also to study fracture mechanics problems, and particularly the fracture propagation in materials that present softening behavior, which is one of the goals of this research. From this general criterion it is possible to obtain, as a particularization, some well-known criteria such as the Huber-von Mises one, which can be formulated in terms of Kirchhoff stress tensor as

$$f(\sigma, \alpha) = |\text{dev}[\sigma]| - \sqrt{\frac{2}{3}}(\sigma_Y + K\alpha) \quad (6)$$

Starting from equation (4), in this work we have formulated a more general non-linear expression of the elasto-plasticity for normal stress yield criterion. The plastic hardening (or softening) parameter is not a constant, but a general function of the internal hardening variable. This permits to build different yield functions for different materials. The mathematical formulation of the yield surfaces is then

$$f(\sigma, \alpha) = |\sigma| - [\sigma_Y + H(\alpha)] \quad (7)$$

where  $H(\alpha)$  is the plastic function depending on the softening or hardening variable,  $\alpha$ . The variation of  $H(\alpha)$  permits to change significantly the shape of the yield surface. For this reason this model can be applied to different soils.

#### 1.4. Yield criterion for tangential law

The variability of the tangential stress due to the influence of the normal one is expressed by means of a two-dimensional failure criterion. At each step the numerical value of the normal stress is fixed and a two-dimensional yield surface is used to establish the elastic range of the tangential stress, as shown in Figure 2.

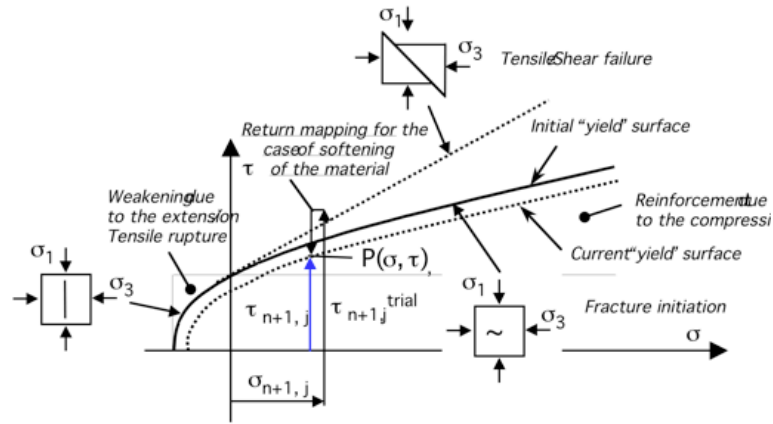


Figure 2. Schematization of the 2D yield surface for one-dimensional constitutive law.

The shape of the yield surface depends directly on the applied criterion, and the elastic range for the tangential force depends on the yield surface and on the intrinsic characteristics of the plasticity function.

Here we adopt for the tangential yield function the same shape used for the normal one. Obviously, it is characterized by different parameters such as the shear modulus and the tangential plastic function.

This formulation can describe the critical state of a wide class of different geomaterials. Moreover this approach permits to consider many standard parameters and fracture parameters of the material such as: cohesion, internal friction coefficient, compressive and tensile strength, fracture energy etc...

The relation of the normal and tangential force is also depicted in Figure 2, together with the varying influence of the stresses in the material behaviour. The weakening in shear strength due to traction in the normal direction, and the opposite effect when a normal compressive stress is applied is also shown.

## 1.5. Flow rule

The plastic strain increments for both formulations (i.e. the normal and tangential strain) are calculated with the classical flow rule

$$\dot{\epsilon}^P : [0, T] \rightarrow \mathfrak{R} ; \dot{\epsilon}^P = \frac{\partial}{\partial t} \epsilon^P \quad (8)$$

where  $\dot{\epsilon}^P$  is the time derivative of the plastic strain (normal or tangential) and  $T$  is the time parameter. The plastic increment can be completely described for any admissible state of stress  $\sigma \in E_\sigma$  by

$$\dot{\epsilon}^P = \gamma \cdot \text{sign}(\sigma) \quad (9)$$



where  $\gamma \geq 0$  is the absolute value of the slip rate,  $\sigma$  is the stress in the normal or tangential direction.  $E_\sigma$  is the domain of the admissible stresses and is defined as

$$E_\sigma = \left\{ (\sigma, \alpha) \in \mathbb{R} \times \mathbb{R}_+ \mid f(\sigma, \alpha) \leq 0 \right\} \quad (10)$$

In the non-linear formulation of plasticity, the variation of  $\gamma$  is non-linear and it is calculated through the consistency parameter  $\Delta\gamma$ .

### Failure criterion

The basic hypothesis of the new law is the existence of two separate yield functions for the normal and tangential direction. These functions are linked through the failure criterion, which relates the tangential behaviour of the material with the normal one.

An original two-parameters model of failure has been developed for soils. This formulation is based on experimental test observations and on the basic assumptions of the classical strength criteria such as the Von Mises, Hoek-Brown and Mohr-Coulomb one [10].

In this criterion the slip between two surfaces is assumed to occur when the shear stress,  $\tau$ , on any plane at a point in the soil reaches a critical value. This value depends non-linearly upon the normal stress in the same plane. The starting point of the strength criterion is that the shear strength of a rock material is made up of two parts varying with the normal stress: an elastic “cohesive” part and a frictional plastic one.

The shear strength which develops on a generic plane of a continuum (see Figure 3), can be expressed as follows

$$\tau = f(\varepsilon_t, \varepsilon_t^p) + f^1(\sigma_n, H) \quad (11)$$

where  $\tau$  is the tangential critical stress;  $\sigma_n$  is the applied normal stress;  $\varepsilon_t$  and  $\varepsilon_t^p$  are, respectively, the total tangential strain and the plastic tangential strain;  $H$  is a general non-linear function of plasticity, as described in the yield criterion. The cohesive part of the formulation is non-linear elastic, as far as the shear modulus is concerned: in fact it was experimentally noticed that for some soils the slope of the shear-normal stresses relationship changes when the normal applied stress increases or decreases.

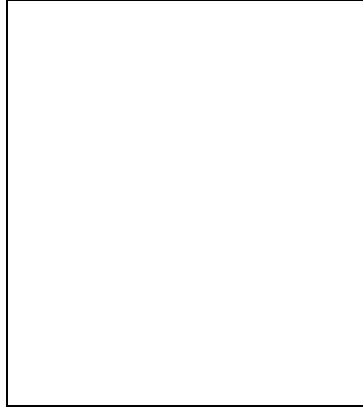


Figure 3. Typical shear failure plane for biaxial test.

The plastic softening (or hardening) frictional behavior starts after that  $\tau$  has reached the limit yield stress value, which is a function of the normal applied stress, see Figure 1. As experimentally observed for some soils, if the normal applied stress increases, the tangential peak stress increases also. According to all previous considerations equation (11) can be more specifically rewritten as follows

$$\tau = G(\sigma_n) \cdot (\varepsilon_t - \varepsilon_t^p) - (f(\sigma_n) + H^1(\alpha)) \quad (12)$$

where  $f(\sigma_n)$  is a function which delimits the elastic range  $E_v \cap H(\alpha) = const$ . The function  $f(\sigma_n)$  depends on the normal stress and it is expressed as

$$f(\sigma_n) = \eta \cdot (\sigma_n + c) \quad (13)$$

This formulation reproduces the characteristic behavior of a wide range of cohesive soils, from sand to some soft rocks.

The material parameters  $\eta$  and  $c$  are constant during the loading process:

- $\eta$  is the parameter that gives a contribution to the frictional behavior of the material. This parameter characterizes also the angle of the shear failure plane for a typical biaxial test: when the numerical value of  $\eta$  increases, this angle increases too.
- $c$  is a constant which is correlated with the cohesive properties of the material.

## 2. APPLICATIONS

### 2.1. Micromechanical model

A numerical simulation of the uniaxial traction and compression test for Tage tuff using the micromechanical model was carried out. All the main parameters

of the contact law were tuned by trial and error to approximate as well as possible the real behaviour of the soil. All these parameters are listed in the following. The comparisons between numerical and experimental results are resumed in Figure 4 and Figure 5.

**Parameters for normal contact law**

Young's modulus  $E = 3.6 \times 10^9 \text{ N/m}^2$

Plastic function  $H_n(\alpha) = b_1 \alpha_n^2 + b_2 \alpha_n + b_3$ ;

parameters for compression

$b_1 = 0.0$ ;  $b_2 = -1.216 \times 10^3$ ;  $b_3 = 0.0$ ;  $\sigma_c = 1.13 \times 10^6 \text{ N/m}^2$

parameters for tension

$b_1 = 1.5 \times 10^3$ ;  $b_2 = -2.0 \times 10^3$ ;  $b_3 = 0.0$ ;  $\sigma_t = 1.17 \times 10^6 \text{ N/m}^2$ ;  $g_n = 0.0002 \text{ fm}$

**Parameters for tangential contact law**

Shear modulus  $G = 1.0 \times 10^9 \text{ N/m}^2$

Plastic function  $H_t(\alpha) = b_1 \alpha_t^2 + b_2 \alpha_t + b_3$

Parameters

$b_1 = 1.5 \times 10^3$ ;  $b_2 = -2.8 \times 10^3$ ;  $b_3 = 0.0$ ;  $g_t = 0.005 \text{ fm}$

Yield stress function for tangential contact  $r(\sigma_n)$

$c = 1.0$ ;  $\eta = 0.7$ ;  $r(\sigma_n) = \eta \cdot (c + \sigma_n)$

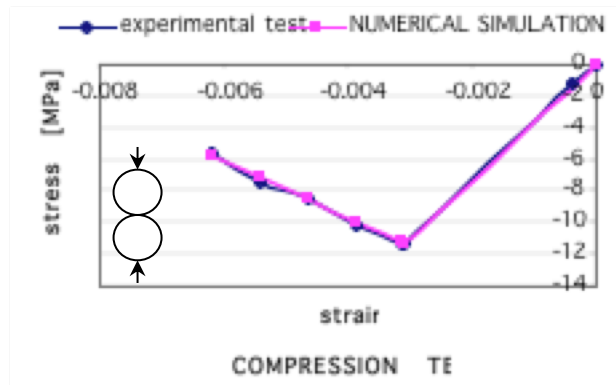
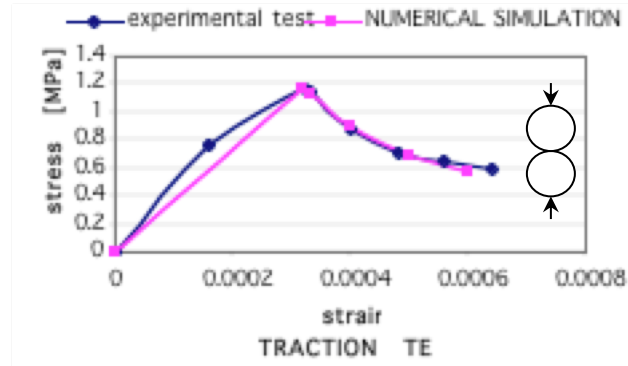


Figure 4. Uniaxial compression tests for Tuge tuff: comparison between the experimental test and numerical simulation



*Figure 5. Uniaxial compression tests for Tuge tuff: comparison between the experimental test and numerical simulation and tension*

The test was conducted under displacement control, to capture the softening behavior of the material after the peak load in the descendent branch. The experimental stress-strain curve of the pure compression test of the tuff is reproduced in Figure 4. The ascendant branch is almost linear elastic and just near the peak load it is possible to recognize a non-linear behaviour. This validates the hypothesis to approximate in the numerical simulation the ascendant branch as linear elastic. In the post-failure region (softening branch of curve) the loading capacity decreases gradually, and it is possible to note a large residual strength. In fact this residual is almost 50% of the peak strength. The comparisons between experimental results and numerical simulations show a good correspondence in capturing the real behaviour of the soil.

## **2.2. MACROMECHANICAL MODEL (1250 disks)**

A specimen composed of a collection of 1250 disks (Figure 6) was built to reproduce the described experimental tests. The radius of the disks, which was supposed constant, is equal to  $0.5 \text{ mm}$ . In this simulation the original dimension of the real specimen has been reproduced.

In this case the number of contacts for each column of disks is 49 and for each row is 24: the tuning procedure take into account this change.

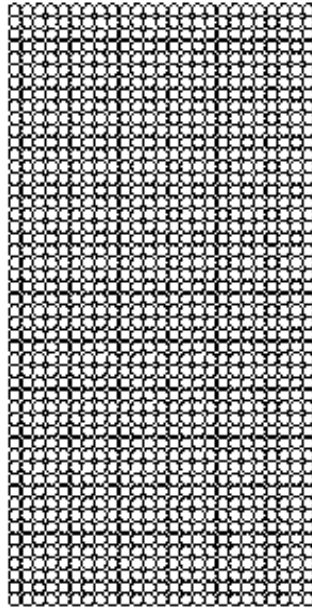


Figure 6. Mesh discretization: micromechanical model, macromechanical model with 1250 disks.

Starting from the evaluations of the paragraph §2.1. the values of the parameters at contact level were tuned as follows

**Parameters for normal contact law**

Young's modulus  $E = 1788 \times 10^9 \text{ N/m}^2$

Plastic function  $H_n(\alpha) = b_1 \alpha_n^2 + b_2 \alpha_n + b_3$ ;

parameters for compression

$b_1 = 0.0$ ;  $b_2 = -5.95841 \times 10^4$ ;  $b_3 = 0.0$ ;  $\sigma_c = 1.13 \times 10^6 \text{ N/m}^2$

parameters for tension

$b_1 = 3.60151 \times 10^4$ ;  $b_2 = -9.8 \times 10^4$ ;  $b_3 = 0.0$ ;  $\sigma_t = 1.17 \times 10^6 \text{ N/m}^2$ ;

$g_n = 0.00025 \text{ m}$

**Parameters for tangential contact law**

Shear modulus  $G = 490 \times 10^9 \text{ N/m}^2$

Plastic function  $H_t(\alpha) = b_1 \alpha_t^2 + b_2 \alpha_t + b_3$

Parameters

$b_1 = 3.60151 \times 10^4$ ;  $b_2 = -1.372 \times 10^4$ ;  $b_3 = 0.0$ ;  $g_t = 0.0058 \text{ m}$

Yield stress function for tangential contact  $r(\sigma_n)$

$c = 1.0$ ;  $\eta = 0.7$ ;  $r(\sigma_n) = \eta \cdot (c + \sigma_n)$

The “tracking” procedure permits to simulate the behaviour of ductile rocks such as the Tuge tuff, as shown in the Figure 7-8. The plastic function for this material is linear in compression and parabolic in tension.

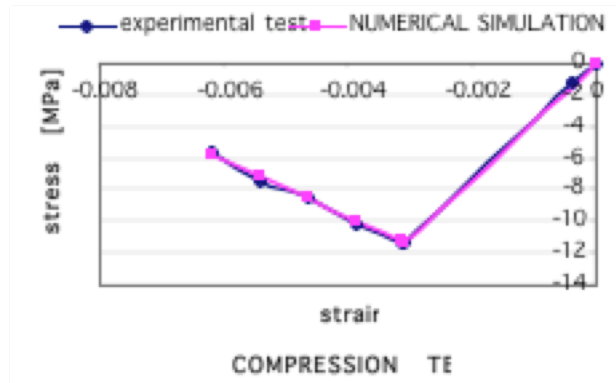


Figure 7. Uniaxial compression tests for Tage tuff: comparison between the experimental test and numerical simulation

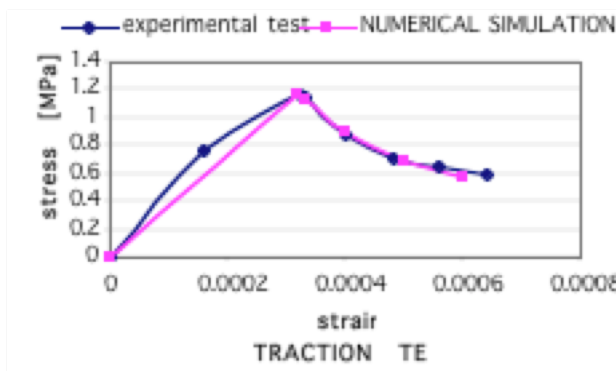


Figure 8. Uniaxial compression tests for Tage tuff: comparison between the experimental test and numerical simulation and tension

### 3. Conclusions

A quasi-static model based on a collection of spheres in contact has been enhanced by adopting sophisticated contact models. With this models we want to simulate the experimentally observed soil behaviour. In this first part we concentrate our attention on the contact formulation and on the constitutive model for normal and tangential contact. On a simple model, composed of two spheres in contact, we have shown that with this model real constitutive behaviours can be obtained. In the second part we focus our attention on collection of spheres and on parameters tracking.

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