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Aggregation of incomplete preference rankings: Robustness analysis of the ZMII-technique / Franceschini, F., Maisano, D.A.F.. - In: JOURNAL OF MULTI-CRITERIA DECISION ANALYSIS. - ISSN 1099-1360. - STAMPA. - 27:5-6(2020), pp. 337-356. [10.1002/mcda.1721]

*Availability:*

This version is available at: 11583/2851068 since: 2020-11-04T15:32:00Z

*Publisher:*

Wiley

*Published*

DOI:10.1002/mcda.1721

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# An innovative technique for strengthening of masonry edge vaults: experiments and modeling

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**Abstract:** Masonry edge vaults are typical of southern Italy and in particular of Salento. They are constituted by four barrel webs, whose vertex points do not meet at the crown of the vault as in the cross vault but are moved backwards, leaving in the middle an empty space covered with a double-curvature shell portion. This central shell has the shape of a four-point star, for which reason the structure is also commonly termed “star vault”. This paper summarizes the main results of an experimental investigation on masonry edge vaults strengthened with fiber-reinforced polymer (FRP) composites and subjected to uniform loading with measurement of the lateral thrust. Test results and theoretical predictions are presented and discussed.

**Keywords:** edge vault, fiber-reinforced polymers, lateral thrust, star vault.

## 1. INTRODUCTION

Edge vaults are a valuable part of the architectural and cultural heritage of some regions of southern Italy, and in particular of the Salento peninsula in Puglia. They are structurally similar to cross-vaults except for the presence of a double-curvature shell portion in the middle of four barrel webs (Colaianni 1967).

Masonry vaults are usually subjected to symmetric loading, as a result of the large dead-to-live load ratio. Hence, collapse of a vault typically occurs when no tie-rods or tie-beams are adopted and the piers are unable to bear the thrust of the vault. In these cases, fiber-reinforced polymer (FRP) composites in the form of externally bonded sheets applied to the surface of vault with the wet lay-up technique have been demonstrated to be an effective solution (De Lorenzis et al. 2007). In addition to being structurally effective, FRPs present several advantages over conventional techniques: they add no extra weight to the structure, are corrosion-resistant, have minimal aesthetic impact, and can be removed by raising the temperature above the glass transition temperature of the resin matrix. The available studies on strengthening of masonry vaults with FRP composites are relatively limited. The most comprehensive test program on vaults is that in Foraboschi (2004), encompassing barrel, cross, and cloist vaults. Test results on barrel vaults have also been reported in Valluzzi et al. (2001).

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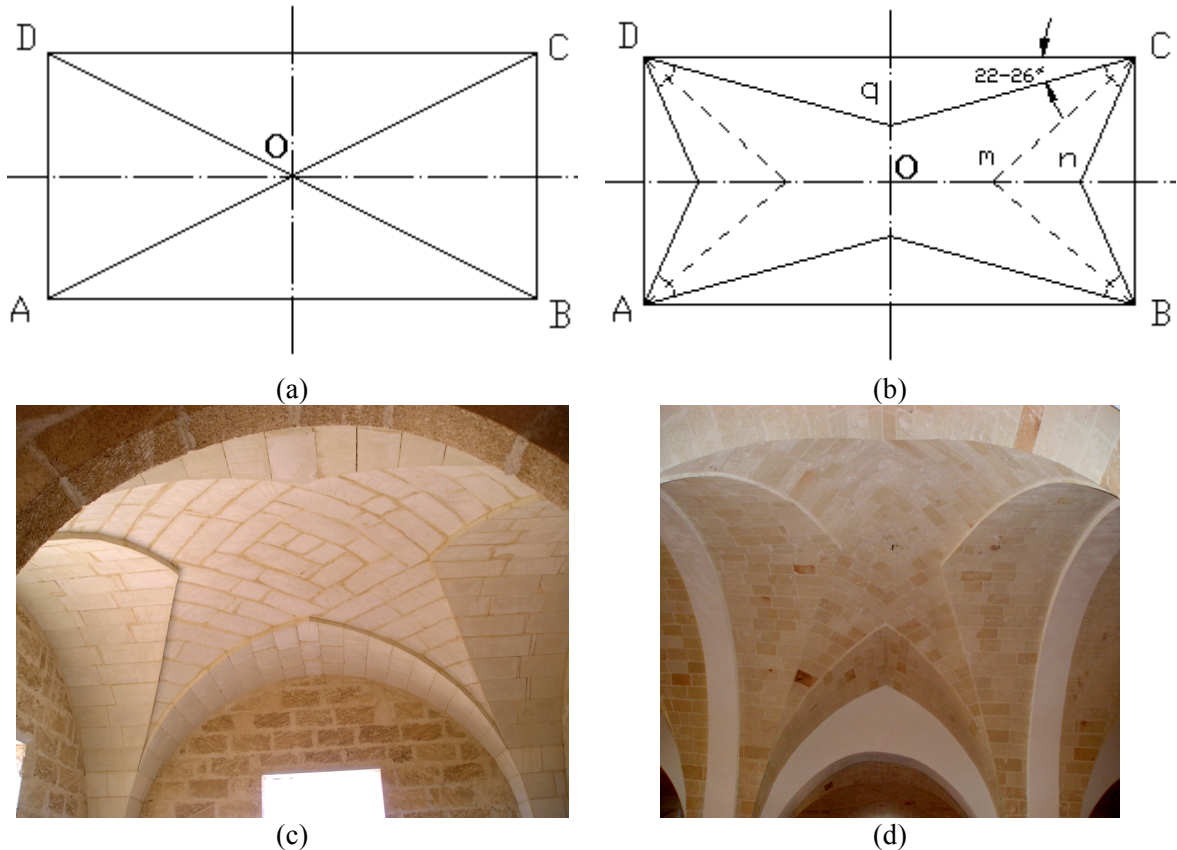
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This paper summarizes the results of an experimental investigation on masonry edge vaults strengthened with FRP composites and subjected to uniform loading with measurement of the lateral thrust. Theoretical predictions are also briefly reported, whereas full details on both the experiments and the modeling approach can be retrieved in De Lorenzis et al. (2007).

## 2. MORPHOLOGY OF MASONRY EDGE VAULTS

Masonry edge vaults are a typology of masonry vaults typical of some regions of southern Italy, and in particular of the Salento peninsula situated in Puglia and comprising the province of Lecce and part of the provinces of Brindisi and Taranto. Edge vaults are structurally similar to cross-vaults. They are constituted by four barrel webs, whose vertex points do not meet at the crown of the vault as in the cross-vault (point O in Figure 1a) but are moved backwards, leaving in the middle an empty space covered with a double-curvature shell portion (Figure 1b). This central shell has the shape of a four-point star, and for this reason the edge vault is also commonly termed “star vault”. The boundary edges between the barrel webs and the central shell for the long and for the short side of a rectangular plan (e.g. edges Cq for the long side and Cn for the short side in Figure 1b) are symmetric with respect to the bisecting lines of the 90-degree angles between the sides of the plan (i.e. in Figure 1b, Cq and Cn are symmetric with respect to the bisecting line Cm of the 90-degree angle BCD). The angle formed by these boundary edges with the corresponding sides (e.g.  $q\hat{C}D = n\hat{C}B$  in Figure 1b) is equal to 22 or 26 degrees when the width of the voussoirs equals 200 or 250 mm, respectively (Colaïanni 1967).



**Figure 1** – Geometry of edge vaults. (a) Plan of a cross-vault. (b) Plan of an edge vault. (c) An example of edge vault over square plan with semicircular webs. (d) An example of edge vault over rectangular plan with semicircular (long sides) and pointed (short sides) webs.

For vaults with rectangular plan, the shape of the directrix of the webs is usually semicircular, elliptical or pointed for the webs of the two long sides, and pointed (less commonly semicircular) for the webs of the two short sides, whereas vaults over square plans frequently have semicircular

directrix for all four webs (Figures 1c, 1d). The heights of the vault key and the height of the keys of the barrel webs differ by a small amount, known as *sovrassesto*. The initial portions of the webs (i.e. those that, for each web, encompass an angle of about 30 degrees starting from the springers on both sides) are similar to those of the cross vault with the same directrix, hence the edges of the webs coincide in plan with the bisectors of the angles. In local technical dialect these portions are called *appese*, this term will be translated as *abutments* as follows. The portions of the webs supported by the abutments are herein considered as the four lateral arches of the vault, reflecting the fact that these portions transmit the lateral thrust from the vaults to the piers. Figure 1c also shows the pattern of the ashlars, which typically follows the French system in the webs and a spiral configuration in the central shell.

Edge vaults are typically made of calcarenite tuff or similar locally available varieties of stone, and the ashlars are assembled with thin mortar joints. More details about history, geometry and construction of these vaults are available elsewhere (Colaïanni 1967, Pecoraro 2002). It is worth noting that edge vaults are similar in appearance to the better known Gothic stellar vaults. However, the structural behaviour of the two types of vaults is significantly different (Colaïanni 1967).

### **3. MINIMUM LATERAL THRUST OF MASONRY EDGE VAULTS**

Analysis of masonry arches and vaults in the framework of limit analysis assumes that i) masonry has no tensile strength and infinite compressive strength, and ii) sliding failure does not occur (Heyman 1982). The consequence of these assumptions is that failure of a masonry arch theoretically occurs by formation of a sufficient number of hinges transforming the arch into a mechanism, and stability under given loads depends essentially on the geometry of the structure. From the kinematic standpoint, the effect of the FRP composites is to inhibit the formation of the hinges. At a location where the FRP sheet is bonded, no hinge can open on the opposite side of the arch thickness. Depending on the extension and location of the strengthened portions of the arch and on the loading pattern, the formation of hinges may be either altered (i.e. hinges form at different locations than in the unstrengthened arch) or completely prevented. Therefore, the capacity of the arch may be controlled by local failure mechanisms depending on material properties, such as masonry crushing, sliding of mortar joints, and FRP debonding or rupture.

From the static standpoint, the presence of the FRP reinforcement allows the line of thrust to fall outside the thickness of the arch by introducing tension resistance. As the value of the lateral thrust is dictated by the slope of the line of thrust at the abutments, the possibility of exceeding the limits of the thickness allows a decrease in the value of the thrust which can be computed analytically in a straightforward manner (see De Lorenzis et al. 2007 for details).

The computation of the minimum lateral thrust for a vault requires appropriate simplifying assumptions to be made for account for the three-dimensional geometry. For a simplified computation of the lateral thrust transmitted from the edge vault to the piers, the loads acting on the vault were computed and appropriately distributed to the four lateral arches according to the respective “influence widths”. In turn, the lateral arches were analyzed in a two-dimensional setting under a variable distributed load  $q(x)$ , whose form was determined based on the influence widths. In accordance with the characteristics of the tested vault, the vault extrados was considered filled with inert material to the level of the vault key. The load acting on each of the four lateral arches was then taken as the sum of: self weight of the arch itself and of its spandrel fill; self weight of the portion of web and central shell “pertaining” to the arch and of the relative fill; uniformly distributed load applied on the extrados of the vault in the region which “pertains” to the arch. The mentioned load components were computed analytically based on an approximate definition of the vault mid-plane surface, see De Lorenzis et al. (2007) for full details.

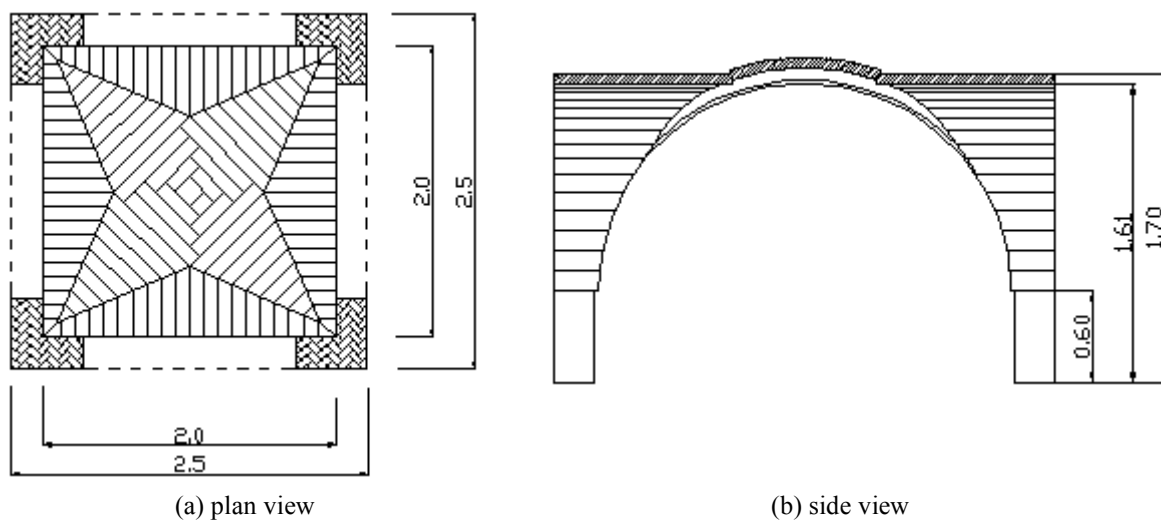
## **4. EXPERIMENTAL TESTS**

### **4.1. Test program**

Two tests were carried out to evaluate the effectiveness of FRP strengthening at the intrados to reduce the lateral thrust of the vault. The tests were conducted on a masonry edge vault with square plan and

semicircular directrix of the barrel webs, whose dimensions are illustrated in Figure 2. These dimensions are assumed to be half the dimensions of the “real” vault, of which the one subjected to test is the scaled prototype. This 1:2 scale was used to determine the dimensions of the single voussoirs, in accordance with the traditional geometric rules of the edge vault construction. The thickness of the four webs and of the central shell was equal to 90 mm. Spandrel fill made of tuff was located on the vault so as to obtain a flat top surface at the level of the crown, on which the external uniform load could be applied.

The tests reported were both conducted on the same vault. First, the unstrengthened vault was tested under uniform loading, measuring the lateral thrust by means of four steel tie-rods (as better detailed later). The vault was then unloaded and strengthened with FRP sheets, as described in a following section. A second test, under the same uniform loading of the first test, was conducted on the strengthened vault, again measuring the lateral thrust absorbed by the tie-rods.



**Figure 2** – Scaled masonry vault subjected to testing (dimensions in m).

#### 4.2. Material properties

The ashlar were made of a type of local calcarenite tuff, on which characterization tests were performed to measure compressive strength, compressive modulus of elasticity, and bending tensile strength. This particular type of tuff, upon visual observation, shows no distinguishable planes of sedimentation. Hence it was assumed that the plane of sedimentation would exert no appreciable influence on the material properties. This was confirmed by the low scatter observed in test results. The mortar used for the joints was made of lime and tuff sand. Characterization tests were performed to measure its compressive strength and bending tensile strength. For both tuff and mortar, the specimen number and geometry and the test results are reported in Table 1. The carbon FRP (CFRP) unidirectional sheet used for strengthening of the vault had an average tensile strength of 3846 MPa and an average tensile modulus of elasticity of 233.7 GPa, as obtained from previous characterization tests. The nominal thickness of one ply (referred to the fibers only) was 0.165 mm.

#### 4.3. Specimen preparation

After the first test, the vault was strengthened with one ply of FRP at the intrados of the four lateral arches, spanning an angle of about 150 degrees. Application of the sheets followed the usual steps of the wet lay-up technique, with the only addition that FRP spikes were used to prevent debonding of the sheets from the masonry substrate. As well known, application of the reinforcement at the intrados makes debonding particularly critical. In this case, equilibrium dictates that the shear bond stresses between FRP and masonry are accompanied by normal tensile stresses, which accelerate debonding failure (Foraboschi 2004, De Lorenzis and Zavarise 2009). Previous researchers proposed the use of FRP spikes to enhance bond to curved substrates and proved its effectiveness (Eshwar et al. 2005).

In this program, each spike was made of a 50-mm piece of CFRP sand-coated bar with 8-mm diameter, mechanically connected to a bundle of 100-mm long dry carbon fibers. Installation of the

spikes was carried out as follows. First, holes were drilled into the masonry substrate to a depth slightly larger than the length of the CFRP bar pieces. The holes were positioned on alternate voussoirs where the FRP had to be bonded, starting from the last voussoir of the abutment, for a total of 12 holes per lateral arch. The holes were cleaned with pressurized air. The bond surface, including the holes, was primed, then the first layer of saturant was applied on the masonry surface and the same saturant was used to partially fill the holes. The FRP sheet and the second layer of saturant were applied next. At the location of each hole, the bar extremity of the FRP spike was inserted through the sheet into the hole by locally enlarging the unidirectional fibers of the sheet (Figure 3a). The fiber bundle was then spread in circular fashion, folded and pressed over the CFRP sheet, and impregnated with a layer of saturant (Figure 3b). The strengthened vault was tested three days after the application of the FRP.

**Table 1** – Material properties.

Measured property	Specimen geometry	N. of specimens	UNI (Italian national) standard	Average value (N/mm <sup>2</sup> )	Coeff. of var. (%)
<b>STONE</b>					
Compressive strength	Cubes, 70-mm edge	10	9724-3	1.81	10.2
Bending tensile strength	Prisms, 30x20x120 mm, 100-mm net span	5	9724-5	0.93	5.5
Compressive elastic modulus	Prisms, 50x50x200 mm	4	9724-8	392.9	4.9
<b>MORTAR</b>					
Compressive strength	Prisms, 40x40x80 mm	16	EN 196-1	0.55	14.4
Bending tensile strength	Prisms, 40x40x160 mm, 100-mm net span	8	EN 196-1	0.30	16.8
Compressive elastic modulus	Prisms, 40x40x160 mm	9	EN 196-1	78.8	31



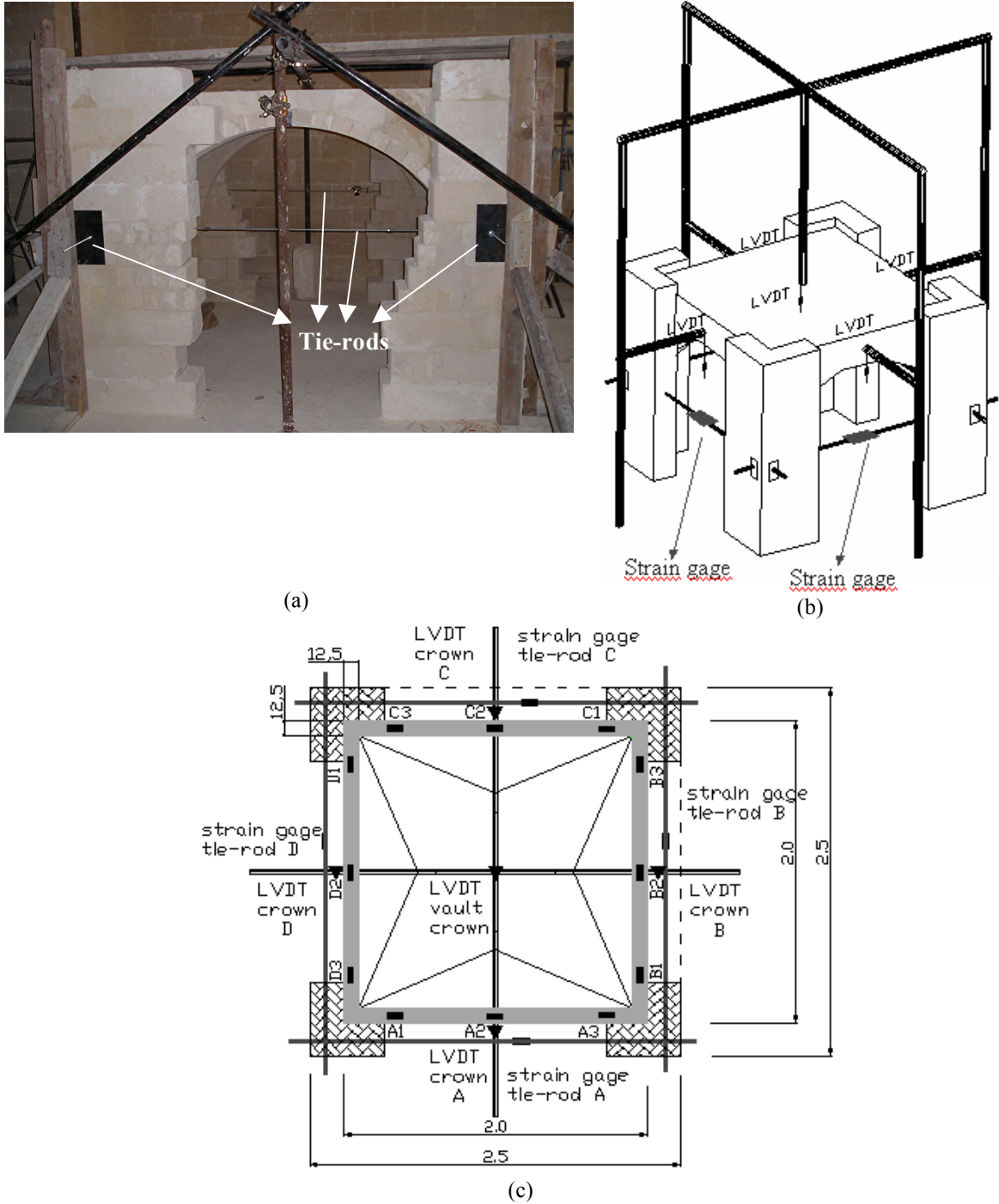
**Figure 3** – Strengthening of the vault with FRP sheet. (a) the spikes are inserted through the sheet into the holes; (b) picture of the vault after completion of strengthening.

#### 4.4. Instrumentation and test procedure

The lateral thrust of the vault was measured by means of four steel tie-rods with 10-mm diameter, applied on the four lateral arches at a height corresponding to about 30 degrees from the springers. Each rod had a smooth surface and was threaded at the ends to react the force against the external surfaces of the piers with a screw nut, a washer and a reaction plate.

The unstrengthened vault was instrumented with five LVDTs, measuring the vertical displacements of the crowns of the four lateral arches and of the vault crown, and four strain gages, measuring the

strains of the four tie-rods from which the lateral thrusts of the four lateral arches could be computed. The strengthened vault was also instrumented with twelve additional strain gages, three per lateral arch, applied on the FRP sheet parallel to the fiber direction. The three strain gages on each arch were located at approximately 50 degrees from the horizontal on both sides, and at the crown. Figure 4 illustrates the instrumentation setup. A uniform load was applied on the extrados of the vault by means of manually positioned sand bags. The load was applied in three steps, corresponding to 200, 400 and 562.5 kg/m<sup>2</sup>, respectively, for a total load of 2250 kg on the entire vault at the final step.



**Figure 4** – Test setup and instrumentation. (a) Unstrengthened vault with steel tie-rods, temporary wooden buttresses and frame of steel tubes used to fix the LVDTs. (b) Scheme of the instrumentation on the unstrengthened vault. (c) Strengthened vault with strain gages on the FRP sheet at various locations.

### 5. EXPERIMENTAL AND ANALYTICAL RESULTS

Figures 5a and 5b show the vertical displacement of the crown of three of the four lateral arches (displacement of the crown of arch A was lost due to malfunctioning of the LVDT) and of the vault crown for the unstrengthened and the strengthened vaults, respectively. In Figure 5a, the first vertical line in the graph marks the removal of the temporary wooden buttresses, used to react the thrust of the vault prior to installation of the tie-rods. In Figure 5b, the vertical line at time = 0 corresponds to starting of the test. In both figures, the subsequent vertical lines mark the instants at which one, two and three layers of bags are laid on the top surface of the vault and the applied load is correspondingly 200, 400 and 562.5 kg/m<sup>2</sup>. The unloading steps are also indicated. In both figures, the displacement of the vault crown is about twice the average displacement of the crown of the lateral arches, and they both increase as the applied load increases and decrease during unloading. A comparison of the two figures shows a notable reduction of the measured displacements as a result of application of the FRP. For the unstrengthened vault, a residual displacement is noted when the applied load is removed, due to settling phenomena in the mortar joints and in the piers. Such phenomenon is not relevant in the strengthened vault.

Figures 6a and 6b illustrate the lateral thrusts absorbed by the tie-rods on the lateral arches of the unstrengthened and strengthened vaults, respectively (one of the four strain gages did not function properly). In both figures, the lateral thrusts increase as the applied load increases, and decrease during unloading. Figure 6a shows as the thrusts build up already at the removal of the wooden buttresses, due to partial transfer to the tie-rods of the thrust generated by self weight and spandrel fill. The remaining portion is absorbed by the piers at least in the initial phase. A comparison of the two figures shows a notable reduction of the measured thrusts as a result of application of the FRP. The reduction in lateral thrust as a result of FRP strengthening results equal to 62%, 44% and 32% after the first, second, and third step of loading, respectively. This demonstrates as the application of FRP at the intrados produces significant adjustments of the line of thrust, yielding a notable reduction in the lateral thrust transmitted to the piers.

Finally, Figures 7a and 7b illustrate the axial load deduced from the strains measured on the FRP sheet at the haunches and at the crowns of the lateral arches, respectively. The axial load in the FRP is compressive at the haunches and tensile at the crown. The compressive strains are characterized by a large scatter, probably due to irregularities caused by local fiber buckling. Conversely, the tensile strains are reasonably close on all the four lateral arches.

It is worth noting that no visually detectable cracks in the vault and no sign of debonding between the FRP sheet and the masonry substrate were observed during the test.

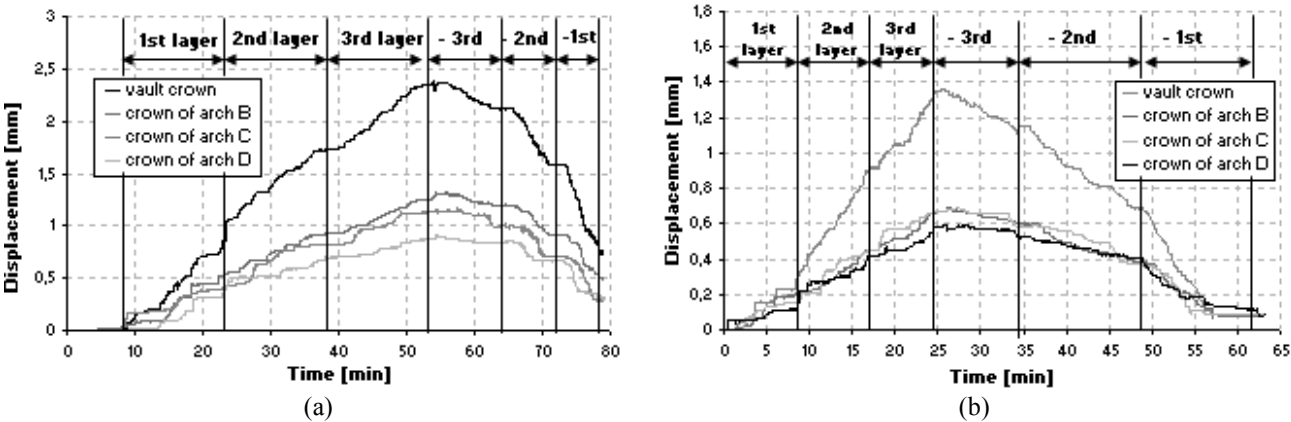


Figure 5 – Displacements of the arch crowns and of the vault crown vs. time for the unstrengthened (a) and strengthened (b) vaults.

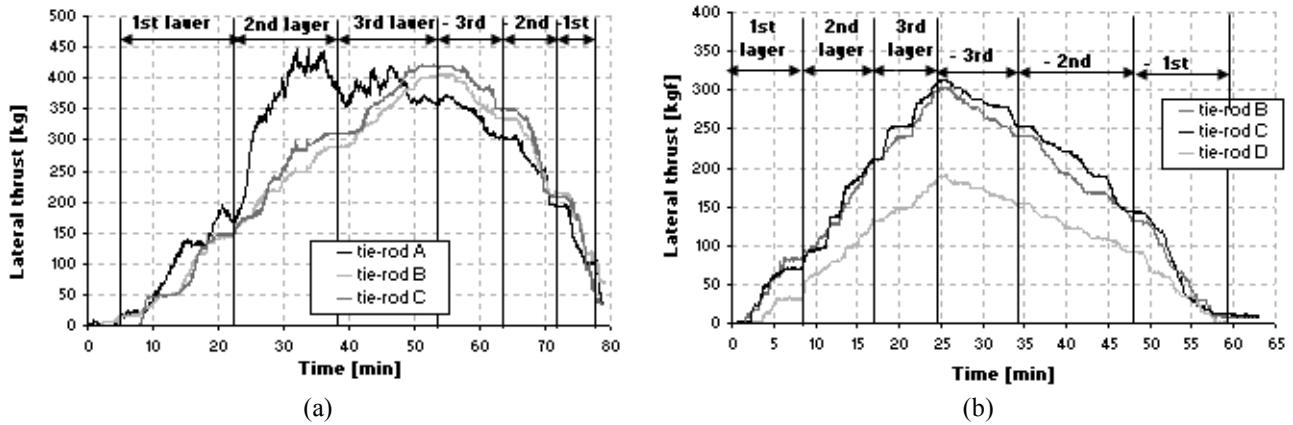


Figure 6 – Lateral thrusts on the tie-rods vs. time for the unstrengthened (a) and strengthened (b) vaults.

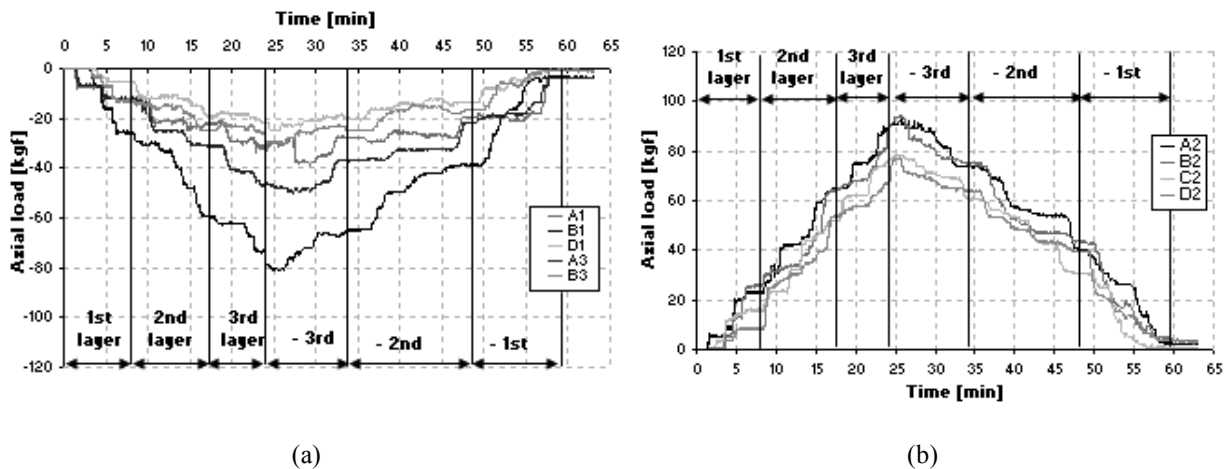


Figure 7 – Axial load in the FRP at the haunches (a) and at the crown (b) of the lateral arches, vs. time.

The theoretical values of the minimum thrust under the three steps of loading were computed as previously illustrated. In the tested vault, the measured unit weight of the tuff was  $1428 \text{ kgf/m}^3$ , and the same value was assumed for the unit weight of the fill which was made of the same material. The compressive strength of masonry was taken equal to that of the tuff material, considering that in the lateral arches the presence of the mortar joints is believed to exert no appreciable influence on the masonry strength.

Table 2 shows the comparison between the experimental thrust (obtained as the average of the measurements on the four arches) and the theoretical minimum thrust. It can be noted that the measured values of the thrust are always (except in one case) slightly larger than the theoretical values. This was expected because the theoretical analysis delivers the minimum possible thrust. However, the difference between experimental and theoretical values is very limited, demonstrating that the simple illustrated procedure yields accurate predictions of the actual thrusts for both unstrengthened and strengthened vaults. In particular, the effect of the FRP strengthening on the reduction of the thrust is fully captured.

Table 2 – Experimental and theoretical values of the lateral thrust.

Loading step	Experimental lateral thrust [kgf]			Theoretical minimum lateral thrust [kgf]		
	Unstrengthened vault*	Strengthened vault*	Percent reduction	Unstrengthened vault	Strengthened vault	Percent reduction
First step ( $200 \text{ kg/m}^2$ )	162	61	62%	176	64	63%
Second step ( $400 \text{ kg/m}^2$ )	330	186	44%	277	159	43%
Third step ( $562.5 \text{ kg/m}^2$ )	398	269	32%	359	244	32%

\*average of three functioning strain gages

## 6. CONCLUSIONS

Based on the theoretical and experimental study summarized above, the following conclusions can be drawn:

- Strengthening the four lateral arches of an edge vault with FRP sheets at the intrados produces a significant reduction of the thrust transmitted to the piers;
- The use of FRP anchor spikes is effective in preventing debonding of an FRP sheet applied at the intrados of a masonry arch. The application of FRP at the intrados can then be regarded as an effective solution. In many cases, strengthening of a vault at the extrados is unfeasible or significantly onerous, as it implies removal of floor finishes and spandrel fill;
- The simple analytical model adopted in the research can be used to evaluate with satisfactory accuracy the reduction of thrust obtained by the use of the FRP, and can then be considered a useful design tool.

## ACKNOWLEDGMENTS

The authors wish to acknowledge Mr. Giampiero and Mr. Piero Montalbano of Real Property (Manduria, Ta, Italy), for providing the site instrumentation and making the experiments possible, Mr. Emilio Pisconti and Mr. Giuseppe Ruggieri for constructing the scaled prototypes of edge vault, and MAC Italia S.p.A. for providing the FRP system free of charge.

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