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Faster prediction of wildfire behaviour by physical models through application of proper orthogonal decomposition

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Abstract

Physical models of wildfires are of particular interest both in fire behavior research and application, but present an important drawback related to the large computational resources that are often necessary. The objective of this paper is to present the application of Proper Orthogonal Decomposition (POD) as a reduction technique for wildfire physical models, with the goal of reducing the computational time but keeping the main features of the original model. POD is applied to a simple one-dimensional full physical model in order to test its performances in terms of accuracy and computational efforts. First, the full physical model is tested with experimental data to check its ability to simulate forest fire behavior and then it is reduced using the POD technique. It is shown that the reduced model is able to reproduce the fire behavior with small deviations with respect to the physical model (about 6.4% deviation in the rate of spread) with a drastic reduction of the computational cost (about 85%).Comparisons between results obtained using POD model and full model demonstrate the interest to use reduction technique as a tool for new generations of operational systems, based on reduced physical approaches.

Additional keywords: wildfire, reduced model, Proper Orthogonal Decomposition, fast physical model

Introduction

Prediction of wildfire behavior (e.g., rate of spread, fire intensity, fire growth) is a crucial aspect in fire risk analysis, evacuation plans and optimization for fire extinction resources. The difficulty in building forest fire models is due to the complexity of the phenomena involved, which involve multiple length and time scales and present various sources of uncertainty. Different models have been proposed to predict fire behavior. They can be grouped in three classes (Pastor et al. 2003): empirical models, semi-empirical models and physics-based models.

Empirical models are obtained as correlations between data collected from experimental fires in laboratory, burning beds or from field fires. Semi-empirical models are based on some form of physical framework but they do not distinguish the different modes of heat transfer (Mell et al. 2007) and are built using correlations between experimental data (e.g., McArthur 1965, Rothermel 1972). They are able to provide an approximation of the rate of spread of a field fires if the fuel load, fuel characteristics, wind intensity and terrain slope are known and represent a valuable operational tool (Mandela et al. 2012) because they are based on algebraic equation and therefore they are fast and easy to manage.

Physics-based models are based on physical equations that describe mass momentum and energy conservation, radiative heat transfer, reaction and diffusion and essentially differs on the choice of the governing equation and on their implementation (Sullivan 2009*a*). Some of the most famous software based on physical models (Morvan 2011) are FIRESTAR (Morvan and Dupuy 2004), FIRETEC (Linn and Harlow 1997, FIRELES, WFDS (Mell et al. 2007). Physical models constitute an interesting option because empirical and semi empirical models when does not always provide satisfactory results, especially when there is an interaction between wind and slope (Morvan and Dupuy 2004). Indeed in these models wind-wildfire interaction is often predetermined and does not account of buoyancy and turbulence fluctuations. Furthermore semi-empirical models are usually able to evaluate the ROS only and no other local quantities such as temperature or heat exchanged with the environment in all the directions. Besides, a lot of wildfire problems are not included in the scope of empirical and semi-empirical models, as WUI fire, fuel treatments for wildfire intensity reduction and to study blow-up fires and fire spread through heterogeneous fuels (Mell et al. 2007). A comparative analysis between empirical and physical models showed that the development of wildfire management tools based on physical approaches is an important research field (Morvan et al. 2009). However, the physical mechanisms that occur in wildfires are very complex and they involve large domains. Therefore, even when some simplification are carried out, such as two dimensional models (Morvan and Dupuy 2004), complex models require very high computational resources. This currently precludes their use as operationally-oriented tool(Sullivan 2009c).

In this framework our paper proposes the application of a model reduction technique to physical fire behaviour model. This technique, named Proper Orthogonal Decomposition (POD)(Pearson 1901), reduces the computational cost of physical models without losing important information.POD is able to capture the main features of a complex problem using smaller degree of information (eigenfunctions) than the full model. This method has received much attention for the reduction of complex physical systems and it has been used in different field of science and engineering, such as the analysis of turbulent fluid flows (Lumley et al. 1967, Holmes P. et al 1993), multiphase flow (Brenner et al. 2012), unsteady thermal systems (Buljak et al. 2011), structural dynamics (Krysl et al. 2001), images processing (Rosenfeld and Kak2014), systems involving chemical reaction (Shvartsman et al. 2000) and many other fields.

No application of POD to wildfire science has been encountered. This paper aims at proposing its application in this field, by testing it on a simple case: a one-dimensional model based on conservation equations applied to a scenario with no-slope but for various wind velocities. Model effectiveness in fire behaviour prediction

has been checked using a set of data gathered during some field fire experiments, different than that used for parameter estimation. Therefore, experimental data here are used with the only aim of checking that the full model is able to predict fire behaviour evolution correctly, while the goal of this paper is to demonstrate that the POD reduced model allows one to obtain results close to that of the full model but using a much smaller computational time.

POD technique

POD method allows one to convert a physical model applied to N nodes into a reduced model of order K<<N that approximates the original one in a effective way i.e. without losing important information.

POD technique (Sirovich 1987)is based on the main idea that a physical field can be expressed as a linear combination of the mode. eigenfunctions, which describe the spatial aspects, and amplitude coefficients account for time dependence. Therefore it is possible to split the contribution of spatial coordinate and time contribution as reported in (1).

$$\mathbf{T}(x,y,z,t) \cong \mathbf{B}(x,y,z)\boldsymbol{\alpha}(t) \qquad (1)$$

The representation reported in (1) is not unique, in fact many set of functions and coefficients satisfy the relation. The set of eigenfunctions selected are orthomormal in order to exploit the advantages derived to their properties. The base is calculated in order to obtain, for any number K, the best approximation of T which minimize least square error. In other words, POD techniques allows to find the sequence of orthonormal functions such that the first two functions give the best possible two-term approximation, the first three function the best possible three-term approximation and so on. (Buljak et al. 2011). Equation (1) is then called Proper Orthogonal Decomposition. The best set of eigenfunctions is built using a collection of sampled value of the considered field, called snapshots. In the case described in (1), for instance, the set of snapshots is a set of solution S of the problem, T (x,y,z) at different time frames. The snapshots at M different time frames are collected in the so-called *snapshot matrix* S, which is a N x M matrix; snapshots can be obtained using experiments or simulations. If $B \in \mathbb{R}^{NxM}$ is a matrix that contains the basis vector of the model, matrix S can be expressed in the basis as:

$$\mathbf{S} = \mathbf{B}\boldsymbol{\alpha} \qquad (2)$$

where $\alpha \in \mathbb{R}^{MxM}$ are the coefficients of the expansion.

The procedure consists in considering only a set of eigenfunctions containing the largest amount of information of the system behavior. Therefore, a truncated basis matrix $\tilde{B} \in \mathbb{R}^{NxK}$ is considered in order to express the snapshots matrix using a coefficient matrix $\tilde{\alpha} \in \mathbb{R}^{KxM}$ characterized by smaller size than α.

$$\mathbf{S} \cong \widetilde{\mathbf{B}}\widetilde{\boldsymbol{\alpha}} \qquad (3)$$

$\tilde{B}$ contains relevant information since eigenfunctions are chosen in order to provide the best approximation of the physical field. The K best modes for system description are evaluated solving the eigenvalue problem;

$$R\vec{\mathbf{v}} = \boldsymbol{\lambda}\vec{\mathbf{v}} \tag{4}$$

where R is the correlation matrix and $\vec{\mathbf{v}}$ the eigenvector;

$$R = \mathbf{S}^{\mathrm{T}}\mathbf{S} \tag{5}$$

the base must in fact describe each snapshot with the minimum error.

The larger the eigenvalue, the larger the information of the system behavior provided by the corresponding eigenfunction. The K largest eigenvalues and the corresponding eigenvectors(casted in matrix $\tilde{B}$)are selected in order to satisfy the constraint on minimal energy collected, ε. This quantity represents the amount of information of the system behavior that is provided by the eigenfunctions selected. In order to obtain an energy value of 1 the complete information must be considered i.e. the full model. For details the interested reader can refer to Bialeki et al. (2005).

$$\frac{\sum_{i=1}^{K}\lambda_i}{\sum_{i=1}^{N}\lambda_i} > \varepsilon \tag{6}$$

Clearly the larger the number of eigenfunctions that are considered, the more detailed is the result obtained, but a high computational time is necessary to obtain the result. This effect is shown, for the considered model in the results section.

Model Description

Full model Selection and Description

This paper represents the first step on the evaluation of the POD capability to describe wildfire behavior, therefore a simple physical model has been selected. This is a one-dimensional model which considers mass (7) and energy conservation equation (8) for the fuel array. Similar model has been adopted in various works available in the literature (Balbi et al.1999, Simeoni et al2001).The choice of this model is due to the necessity of proving POD applicability to wildfire science. Once proved it, more detailed model can be considered, as two dimensional models, or models including momentum equation, radiative transfer equation and a more complex combustion model. In fact previous works dealing with POD (Holmes P. et al 1993,Shvartsman et al. 2000) show that turbulence and chemical reaction can be both modeled using this model reduction approach.

$$\rho c\frac{\partial T}{\partial t} + k_v v\frac{\partial T}{\partial x} = k\frac{\partial^2 T}{\partial x^2} - h(T - T_e) - \frac{H}{s}\frac{dM}{dt} + \Phi_{RAD} \tag{8}$$

$$\frac{dM}{dt} = -aM \qquad (9)$$

The energy equation include the storage term, convective term, diffusive term and a series of additional terms which take into account the contribution of other phenomena such as heat losses with the environment, energy delivered by the mass reaction and heat exchanged due to radiation. The presence of the radiative term (9) make the energy equation non linear.

$$\Phi_{RAD} = rF_{ij}\varepsilon\sigma T^4 \qquad (10)$$

The mass rate variation is characterized by a constant rate of change, according to Balbi et al. (1999).

In order to solve the problem, a numerical approach has been used. Energy equations has been discretized with a finite difference central scheme (Ferziger, Peric 2002) and a Newton-Raphson algorithm has been used in order to solve it. The matrix form of the energy equation of the full model is:

$$\boldsymbol{C\dot{T}} = \boldsymbol{AT} + \boldsymbol{DT}^4 + \boldsymbol{f} \qquad (10)$$

Dirichlet boundary conditions have been imposed at both the boundaries. A sinusoidal temperature distribution with a maximum of 700 K has been imposed as initial condition in the first 0.4 m of the domain in order to represent the fuel ignition while the environmental temperature is set at t=0 in all the other regions. The initial mass condition per unit area is 0.39 kg/m^2 everywhere.

The model has been applied to the system shown in Figure 1. The domain is 50 m long. This system is the one dimensional description of the domain used for the experimental data collection necessary to parameterize the physical model. A partition of the domain is also reported in the figure in order to better display results and compare results with field experiments.

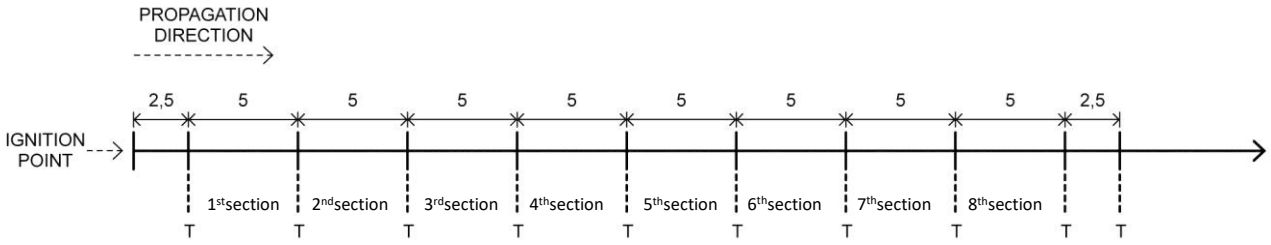


Figure 1 Experimental and model domain

Full model parameterization

The various parameters necessary for the full model have been obtained using data and information on the fuel, weather and fire behavior collected during field fire experiments on grassland fuels, dominated by *Molinia*

arundinacea Shrank (Figure 2) carried out under controlled conditions in winter 2009 in North-West Italy (Ascoli et al. 2009).

The values of H, ρ and s, have been computed by assessing the grass fuel structure and load in the field soon before burning. At each experimental site, fuels were harvested in six 1-m^2quadrats and oven dried in the laboratory at 90°C to determine the load (t ha^{-1}), while the fuel depth was assessed every 0.5 m along six linear transects (length =10 m) at each fire site. Fuel load and bulk density ranged between 4.29to 5.50 t ha^{-1}, and 2.2 to 7.1 kg/m^3, respectively. Flammability parameters (surface area to volume ratio, moisture of extinction, heat content) have been derived from published values for similar grass fuels (FCCS Inferred variables) available in (USDA, 2015). Density and specific heat have been evaluated considering the fuel array as a porous medium. Here a mean value was considered for each of them, in order to obtain results that describe the overall fire behavior.

Figure 2 Field fire experiment

Four fire experiments were conducted under moderate to dry weather on the same day in the winter dry season, when grass fuel was fully cured. Fuels were thus entirely constituted by dead grasses with a diameter< 6 mm (i.e., dead fine fuels) with a cover of 100% at each experimental site. Fuel moisture was assessed soon before each fire experiment by collecting five samples of dead *Molinia* leaves. Fresh samples were weighed in the field using a portable scale, and then oven dried in the laboratory at 90°C to constant weight. Fuel moisture was computed on a dry weight basis and ranged between 11% and 19%.To let the fire front spread freely through grasses, each fire experiment was ignited byline ignitions of 25 m in length, and the fire was allowed to spread for 50 m before being suppressed along a fuel break. At each fire experiment, the fire spread was assessed by measuring the arrival time of the fire front by using K-thermocouples (0.4 mm in diameter) positioned at fixed points (asterisk symbol in Figure 1) along a 50 m transect parallel to the spreading fire front. K-thermocouples were placed within the fuel bed (5-10 cm from the soil surface) and the environmental(air-gas-fuel) temperature was measured at intervals of 1 second. Consequently, at each sampling point the maximum temperature and the temperature-time profile were obtained (Figure 3).During each fire experiments, air temperature and moisture, and wind speed and direction were assessed every 10

seconds by two weather stations positioned at a height of 2 m upwind the experimental plot. Time since last rain was 19 days, air temperature and moisture, and wind speed ranged between20 to 27%, 19 to 25°C, and 2.8 to 7.1 km h^{-1}, respectively. By coupling weather data to fire behavior data allowed to separate thermocouple data recorded during no-wind conditions, backfire and headfire phases (i.e., against and with the wind respectively). Both no-wind conditions and headfire condition was considered for the model parameters. In total, 32 rate of spread observations and 32 time-temperature profiles were collected, each associated with fuel characteristics (load; bulk density; moisture), environmental conditions (slope; wind speed) fire behavior (rate of spread; no-wind, back or head fire phase) and effects (fuel consumption). Rate of spread ranged between 0.8 to 14.2 m min^{-1}. Fuel consumption was assessed soon after the fire by collecting remaining charred fuels in six 1 m^2 quadrats and ranged between 75 and 90% of the pre-fire mean fuel load. Maximum flame temperature ranged between 244°C and 733°C. Average residence time above 60°C and 300°C were 183 and 21 seconds, respectively. A total of 3 observations were retained for parameter evaluation.

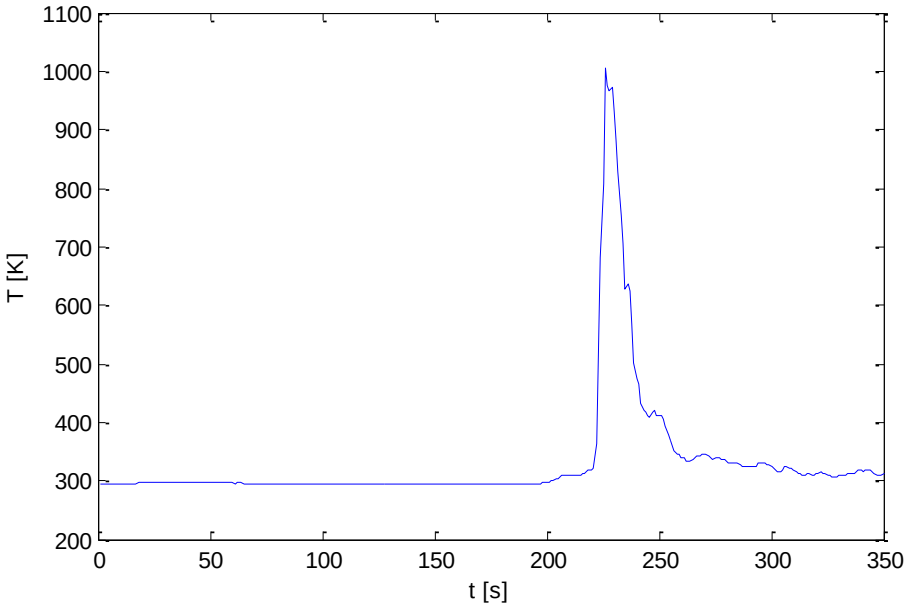


Figure 3 Temperature evolution collected to a thermocouple

The values of the diffusive coefficient k, the convective losses coefficient h, the linearized reaction rate α, the radiative coefficient r, and the convective coefficient k_v have been determined using a comparison between simulation and experimental data. A genetic algorithm has been used in order to obtain the best set of coefficients to reproduce the fire front spread. Indeed, genetic algorithms have been already successfully used to obtain the coefficients for wildfire prediction models (Wendt et al. 2013, Ascoli et al. 2015). The differences in terms of burning period, arrival time, maximum temperature and integral of the temperature in time have been evaluated. The sum of these deviations has been considered as the indicator of the error performed to the model respect to experimental data in fire behavior prediction. The genetic algorithm has been used to find the set of coefficients that minimizes the error. An initial population of 60 random cases has been chosen. The optimization has been conducted considering 40 generations.

In order to determine k_v the optimization has been carried out considering three different wind velocities. The same set of coefficients has been used to simulate fire propagation with wind velocities of 0 m/s, 1.4 m/s and 2.0 m/s. Results have been compared with experimental results collecting during the same wind velocities. The average value of the three error indicators has been computed and the set of coefficient that minimize the error has been evaluated; in this way the physical model has been set.

POD model Description

Using POD procedure a reduction of the model has been performed. The system is solved for the temporal amplitude α.

$$T = \sum_{i=1}^{K} \alpha_i B_i \tag{11}$$

Therefore rewriting (10) using (11) and multiplying times $\tilde{B}^T$ the energy equation becomes:

$$\tilde{\mathbf{C}}\boldsymbol{\alpha}' = \tilde{\mathbf{A}}\boldsymbol{\alpha} + \tilde{\mathbf{B}}^{\mathrm{T}}\mathbf{D}(\tilde{\mathbf{B}}\boldsymbol{\alpha})^{4} + \tilde{\mathbf{B}}^{\mathrm{T}}\mathbf{f} \tag{12}$$

where

$$\tilde{\mathbf{C}} = \tilde{\mathbf{B}}^{\mathrm{T}}\mathbf{C}\tilde{\mathbf{B}} \text{ and } \tilde{\mathbf{A}} = \tilde{\mathbf{B}}^{\mathrm{T}}\mathbf{A}\tilde{\mathbf{B}} \tag{13}$$

A smaller set of equation is then solved with a consequent decrease of computational cost.

Results

Full Model results analysis

In this first part of the section, the outcomes of the coefficients setting are analyzed. At first, a discretization analysis has been performed in order to select the time step and the grid size that provide an acceptable error. Results are plotted in figure 4.The choice of the time step and spatial discretization have been selected in order to keep both errors due to time and space below 10%.

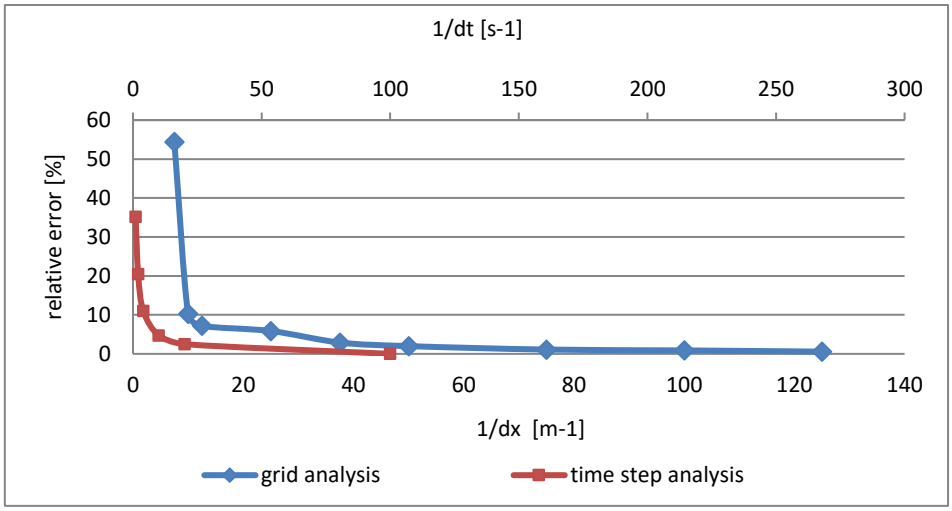


Figure 4 Time step and Grid analysis results

Once the space and time discretization has been defined, the model coefficients have been set through genetic algorithm optimization. A comparison of the ROS obtained with the physical model and the experimental ROS is shown in figure 5 for three different wind velocities which shows that the physical model is able to capture the behaviour of the experimental fire front evolution A small error is committed for each wind speed, particularly for the use of 1,4 m/s. Deviations are due to the inaccuracy of the model but also the uncertainties in the parameters and in the measurements, such as the non-homogeneous fuel characteristics, humidity, wind velocity, etc.

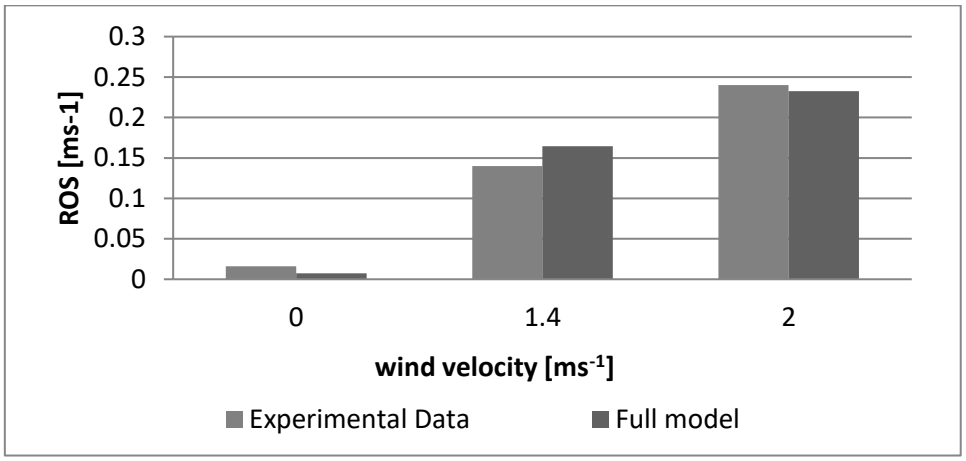


Figure 5 ROS obtained with the set full model and the ROS obtained during the experiments

In order to show this concept a collection of the relevant rate of spread data gathered during the experiments are reported in Figure 6 on the basis of wind velocity. A general trend of the fire evolution as the function of the wind speed can be notices, but an average deviation of 15% is registered with respect to a possible linear trend (Sullivan, 2009b), with a maximum difference in the ROS of about 0.075 m/s.

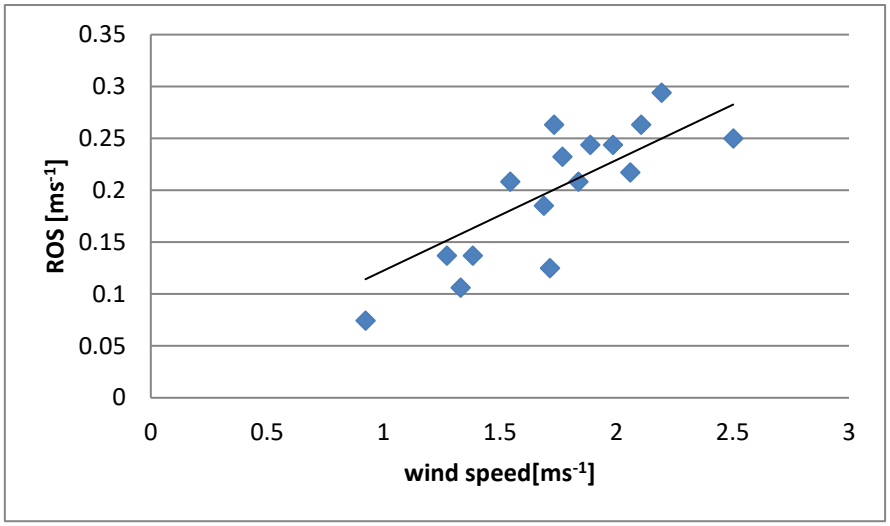


Figure 6 Collection of ROS obtained during all the experiments in different wind conditions

POD model results analysis

In this section the results obtained using the POD model are reported and analyzed with the aim of providing information on its effectiveness in predicting the wildfire field. A comparison with both full model results and experimental data is performed. Temperature distributions at different time frames obtained with both the full and the POD model considering 2 ms^{-1} wind speed are reported in Figure 7.

Figure 7 shows that the evolution predicted with the full model rapidly reaches a shape that tends to not change with time; this condition will be later on called the pseudo steady state. The temperature profile in the combustion region varies only in the first part of the simulation then the rate of spread is constant. Figure 7 indicates that a good agreement between full model and POD model is achieved, although the POD model slightly over-predicts the fire front velocity. The differences that can be observed in Fig. 7 are due to the fact that only a part of the total amount of eigenfunctions is used in order to reduce the computational cost.

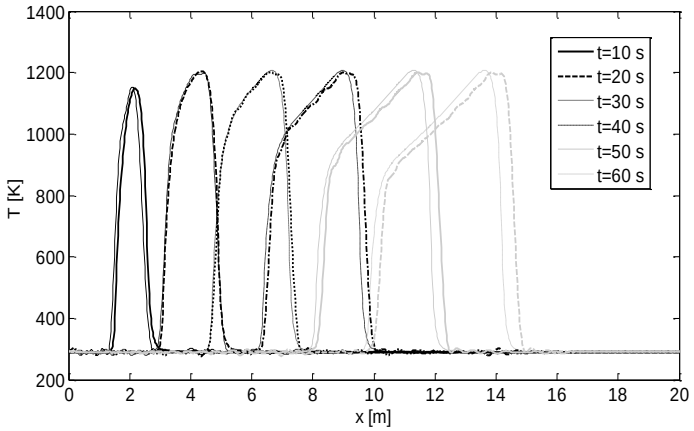
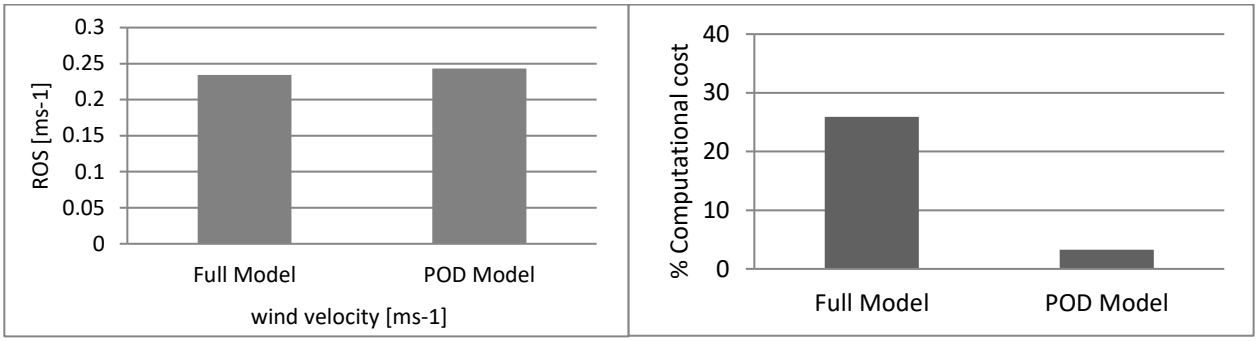


Figure 7 Temperature distribution at different times Full model (plain line)and POD model (bold lines)

In order to better compare the results obtained with the full and the POD model the rate of spread comparison is reported in Figure 8 a. It indicates that a good agreement between experimental ROS and the rate of spread calculated with the proposed model is achieved.



(a) (b)

Figure 8.Comparison between Full and POD model in terms of relative error and computational cost

In figure 8b a comparison of computational time requested to solve the problem using the two models in the same domain is shown. The computational cost is expressed as the ratio between the computational time and the fire propagation time in the domain. A reduction of 85% in the computational time is obtained using the POD model. This result suggests that model reduction can be a useful approach to be used in order to make physical models more suitable for fast (or faster) simulation.

The error, as previously indicated in Figure 8a, depends on the number of eigenfunctions used to evaluate the results. The higher is the number of eigenfunctions the higher is the results accuracy but also the computational cost. Therefore is necessary to find a compromise between accuracy and computational cost. In order to chose the number of eigenfunctions, an error analysis has been performed which results are reported in Figure 9. The square in the figures represents the number of eigenfunctions that is interesting to select due to the low relative error performed and the high computational cost reduction.

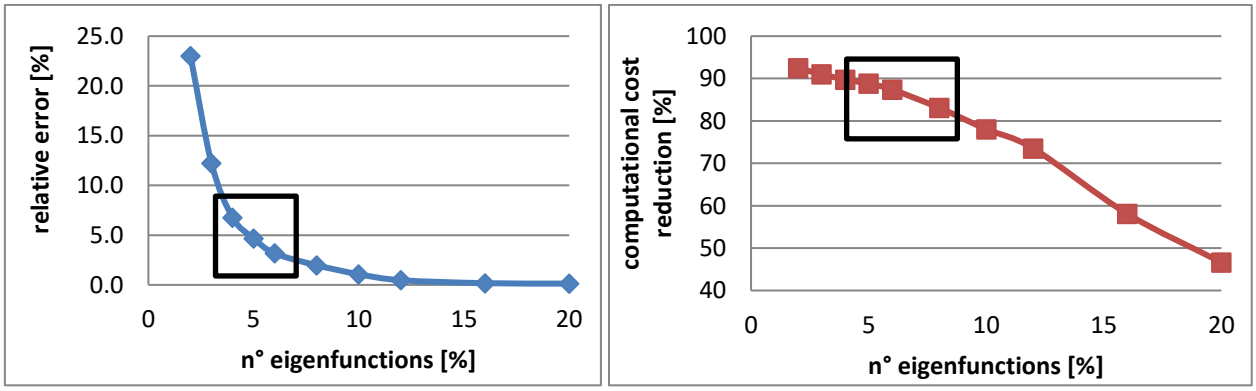


Figure 9 Relative error and computational cost reduction performed to the POD model with different eigenfunctions numbers

POD model application

The POD model has been used in order to predict fire evolution of two particular experiments carried out on the entire domain shown in Figure 1. During the experiments, wind speed had changed varying from 0,9 m/s to 2,5 m/s. It is worth remarking that the POD model has been created considering the snapshots corresponding to a single wind velocity, 2.0 m/s. Therefore only a single full model simulation is required in order to obtain a reduced model that is used in a wide range of wind speeds. The values of ROS obtained with the POD model in the different domain sections of the domain are compared with the full model results and the experimental values. Each domain section is characterized by a particular wind velocity, due to the variations that were registered during the experiments. Results are reported in Figure 10.

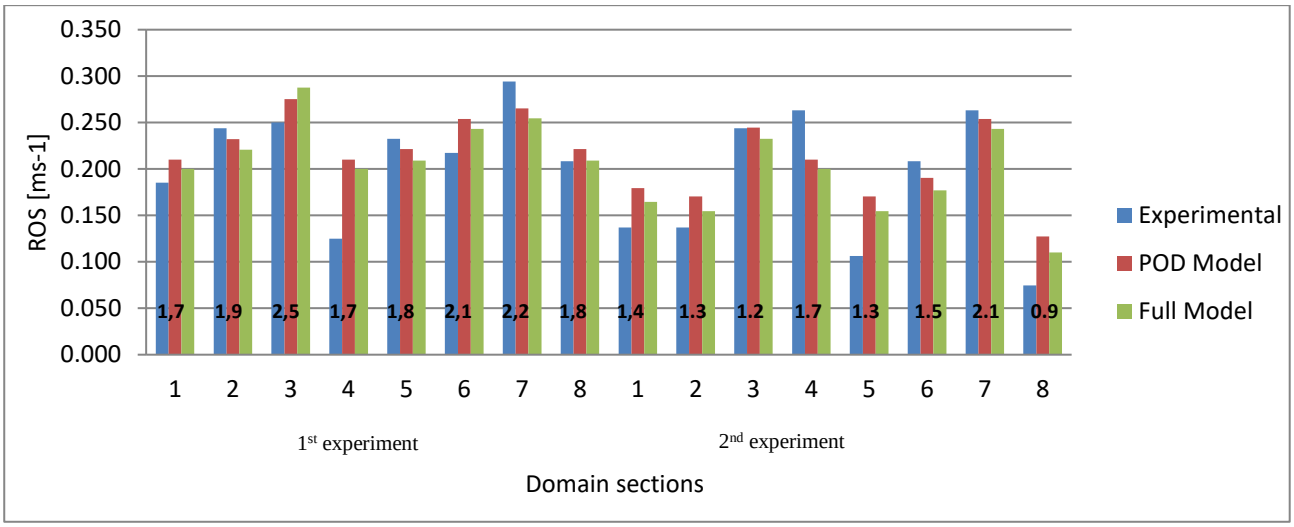


Figure 10 ROS in simulations vs experiments(in bold the respective wind speeds)

In order to test the POD model performance the most interesting comparison is that with the full model results. In this regard, it is possible to notice that the average deviation of the two models is about 6.4%, with an average difference in the ROS of about 0.013 m/s, while the maximum difference is about 0.017 m/s. Deviations are slightly larger in the case of the smallest velocities (0.9-1.3 m/s), since the snapshots that have been used in model reduction corresponds to a larger velocity (2.0 m/s). In general, these results prove that POD method can be used as a model reduction technique for wildfire evolution physical model.

Comparison between both models and experimental results is performed with the purpose of showing that the physical models (both the full and the reduced ones) are suitable for representing the events. The average deviations of the two models with respect to the experiments is about 20%, with an average difference in the ROS of about 0.03 m/s.

In some cases, the level of accuracy is smaller, with a maximum difference in the ROS of about 0.09 m/s. As already discussed, this is due to the non-homogeneities in fuel characteristics, humidity and wind (which value was read each 10 s), as shown by the not precise relation between wind speed and rate of spread(Figure 6). For instance with the same wind speed (1,7 m/s) two very different experimental rate of spread have been obtained.

Conclusions

The present paper reports for the first time the use of POD technique for the reduction of computational time in wildfire physical models. POD has been applied to a simple 1D problem based on unsteady energy conservation equation, where the terms accounting for diffusion, convection, radiation, and heat release associated with the mass rate and wind speed contribution have been considered. Coefficients are determined using a comparison between time evolutions in simulations and experiments. The full model shows that it is able to estimate with good accuracy the pseudo steady-state behavior of the experimental fire for different wind conditions.

POD model is able to provide results very similar to the full model with an average deviation of 15%. The size of the problem that is solved is significantly reduced, in fact only 6% of the available eigenfunctions have been used. This allows a reduction of the computational time to solve the energy equation to 15% of the full model.

Snapshots provided by the full model in a single wind condition (2.0 m/s) have been used to create the reduced model, which has been then used for the simulations in different wind conditions (0.9-2.5 m/s). Results show that POD model is able to estimate the ROS with good accuracy, even when the wind speed differs from that used to obtain the snapshots. The average deviation in the ROS prediction by the two models is of about 0.013 m/s, i.e. about 6.4% difference, while the maximum difference is about 0.017 m/s.

These are encouraging result which suggest the applicability of the POD to more complex and multi-dimensional models and its use for the parametric analysis of fire scenarios and risk assessment, definition of evacuation plans and resources.

Nomenclature

a	mass loss rate parameter	$[s^{-1}]$
c	specific heat of the fuel array	$[J\ kg^{-1}K^{-1}]$
B	basis matrix	[-]
$\widetilde{B}$	truncated basis matrix	[-]
F_{ij}	view factor	[-]
h	convective coefficient	$[W\ m^{-3}K^{-1}]$
H	heat content of fuel	$[J\ kg^{-1}]$
k	equivalent conductivity coefficient	$[Wm^{-1}K^{-1}]$
k_v	Convective term coefficient	$[J\ m^{-3}K^{-1}]$
M	mass of fuel per unit of surface	$[kg\ m^{-2}]$
s	fuel array depth	[m]
S	snapshots matrix	[-]

r	radiation coefficient	$[m^{-1}]$
R	correlation matrix	[-]
T	temperature of the fuel array	[K]
T_e	temperature of environment	[K]
ν	wind velocity	$[m\ s^{-2}]$
$\vec{v}$	eigenvector	[-]
α	POD coefficient matrix	[-]
$\tilde{\alpha}$	POD truncated coefficient matrix	[-]
ε	energy of the POD decomposition	[]
λ	eigenvalue	[]
ρ	density of the fuel array	$[kg\ m^{-3}]$
σ	Stefan-Boltzman constant	$[Wm^{-2}K^{-4}]$
Φ_{RAD}	radiative heat flux	$[W\ m^{-3}]$

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