AIMETA 2017
XXIII Conference
The Italian Association of Theoretical and Applied Mechanics
Luigi Ascione, Valentino Berardi, Luciano Feo, Fernando Fraternali and Antonio Michele Tralli (eds.)
Salerno, Italy, 4–7 September 2017

THE ROLE OF COMPLEXITY IN RANDOM ELEMENT REMOVAL IN STATICALLY INDETERMINATE STRUCTURES

V. De Biagi

Dept. of Structural, Geotechnical and Building Engineering
Politecnico di Torino, Corso Duca degli Abruzzi 24, 10129 Torino
e-mail: valerio.debiagi@polito.it

Keywords: robustness, structural complexity, statically indeterminate structures.

Abstract. *Modern requirements on constructions impose that proper design strategies must be adopted in order to obtain a robust structure: in this sense, consequence-based design focuses the attention on the structural response to damage. The behavior of statically indeterminate structural systems under damage is non-linear because the load paths intertwine each other, even if each component behaves linearly. As much as the structural complexity increases, the presence of effective ways of carrying the load becomes crucial for the robustness of the structural system under a damage process acting at random on the structure. In addition, the size of the system plays an important role: although tendentially more fragile, large systems are able to redistribute and absorb the effects of damage even with low complexity.*

The theoretical considerations are applied to a real case: a statically indeterminate truss structure is presented and randomly damaged. The results are discussed and the possibility of using the normalized structural complexity index as a potential metrics for testing the efficacy of alternate load path strategy in structural design is critically debated.

1 INTRODUCTION

The current design practice requires that the structures has to be robust. This design philosophy is described and imposed by various national and international standards. For example, ISO 2394 considers the possibility of a structure not to be damaged to an extent disproportionate to the original cause [1], the Eurocode 1 proposes a similar idea, considering the ability of the structure to withstand events, instead of being damaged [2].

During its life, a construction can be subjected to various loading scenarios of extremely different magnitude, e.g. [3, 4, 5, 6]. In this sense, the concept of “black swan” introduced by Taleb [7] perfectly fits some of the present and future engineering problems and issues. In a society in which anthropogenic hazards are possible, in recent times, specific attention has been paid to the response of structures to unexpected events, e.g., terroristic attacks. These scenarios are unforecastable and the basic hypotheses of reliability-based design are false since the probability of occurrence of the cause of damage is not known a priori. A possible strategy for dealing with these situations considers the use of consequence-based design, which is a threat independent design procedure [8]. A damage is imposed to the structure and the structural response is evaluated. The structure is iteratively redesigned in order to be collapse safe. In this sense, damage tolerance is a concept very close to structural robustness.

In some common beliefs, the idea of robustness is often associated to the degree of static indeterminacy. Biondini and colleagues confirmed the fact that only a certain degree of static indeterminacy provides a significant contribution to structural robustness [9, 10]; at the same time, it has been proved that the a large degree of static indeterminacy (only) it not a sufficient condition for ensuring structural robustness [8], thus it is not a good indicator for robustness. Bertero and Bertero remarked that redundancy alone cannot confer robustness without providing ductility and over-strength [11].

In a recent paper, the author highlighted the fact that the behavior of statically indeterminate structural systems under damage is non-linear because the load paths intertwine each other, even if each component behaves linearly [12]. He studied the effects of a random damage on a truss structure through two metrics, one devoted to quantify the degree of differentiation of the load paths, the other dedicated to measure the average impact of damage.

In order to complete the previous studies on statically indeterminate frame structures, the aim of the present paper is to investigate the relationship between the amount on interaction between the load paths in the structure (i.e., the complexity of the structure) and the effects of random element removal for statically indeterminate structures. In the present paper the results of new tests performed on a frame structure are presented and commented in the light of the previous tests on a truss structure [12], which are summarized and detailed.

2 THEORETICAL BACKGROUND

2.1 Structural complexity

A complex structure is defined as a system made up of a large number of parts that interact in a non-simple way under an arbitrary loading scheme [13]. De Biagi and Chiaia identified the internal elastic energy, or the deformation work, as the parameter that better describes the behavior of a structure subjected to loads. In fact, it is a single-value parameter that accounts both for the configuration of the single elements (structural topology), their stiffness and the way the structure is loaded. The interaction between the parts is measured in terms of ratio between the deformation work in a given structure (i.e., the whole structure) and the deformation work in each of the substructures that are “extracted” from the given structural scheme, namely the performance ratio ψ. The substructures, called "fundamental structures" are statically determinate structures that span all the nodes of the statically indeterminate scheme. The performance ratio ranges from 0 to 1, since the denominator is always larger than the numerator: ψ tends to one in representative substructures; viceversa, ψ tends to zero in the cases in which the substructure does not adequately replicate the original scheme. The performance ratio of the n fundamental structures acts as an information functional. Following the milestone theory by Shannon [14] on information entropy and one of its developements proposed by Dehmer [15] on graph information content, the measure of the amount of information required to describe the structural behavior, is named Structural Complexity Index SCI, and is computed as

$$\mathrm{SCI} = -\sum_{i=1}^{n} \left(\frac{\psi_i}{\sum_{j=1}^{n} \psi_j} \log \frac{\psi_i}{\sum_{j=1}^{n} \psi_j} \right) \tag{1}$$

where ψ_i is the performance ratio of the i-th fundamental structure, as defined previously. In the theory, the amount of information turns into the amount of interaction between the various load paths [13].

The maximum complexity is attained when all the possibile fundamental structures have the same effectiveness, i.e., the same performance factor, leading to $\mathrm{SCI} = \log n$. Thus, the Normalized Structural Complexity Index, NSCI, is expressed as

$$\mathrm{NSCI} = \frac{\mathrm{SCI}}{\log n}. \tag{2}$$

The NSCI ranges between 0 and 1. As much as the parameter approaches to 0^+, the structural system is simple. On the opposite side, values of NSCI tending to 1^- refer to complex structures. A complete treatment on structural complexity metrics can be found in [13].

Structural complexity indices have geometric scaling invariance properties, as described in [16]; with force-only external loads, this allows to set cross-section size and material properties of a reference element, as described in the methodological section of the paper.

2.2 Average impact of element removal

Various approaches for quantifying structural robustness are available in the literature. For example, Starossek and Haberland [17] proposed an energy-based metrics that accounts for the energy required for damage propagation. Biondini and Restelli [10] compared the effectiveness of various structural performance indicators: it results that the ratio between stored energies in the undamaged and damaged configurations is a suitable parameter for damage-tolerance analysis.

In analyzing damage tolerance in parallel systems, De Biagi and Chiaia [18] evaluated the effects of element removal on a system of rods through the average increment of internal elastic energy in the system. In a structure composed by r elements (say rods or beams), the average structural response in terms of deformation work, M is given by

$$M = \frac{1}{r} \sum_{i=1}^{r} \frac{W_i - W_0}{W_0} \tag{3}$$

where r is the total number of beams, W_0 is the elastic energy in the undamaged (original) structure, W_i is the energy in the structure after the removal of the i-th element. The value of M tends to zero in those systems into which random element removal has no impact, while it tends to infinite in systems in which element removal has dramatic consequences (e.g., collapse).

3 METHODS

The normalized structural complexity, which indicates the amount of interaction between the load paths, and the M-value, which quantifies the effects of random element removal, are computed for different structures with similar topology and external loads, in order to highlight trends in the structural behavior. Two different structural configurations are considered: a planar frame structure and a truss. The detailed calculations related to the second structural configuration are reported in [12].

3.1 Frame structure

The frame, depicted in Figure 1, is a two-stories plane structure made of 13 elements (6 beams and 7 columns). The frame is clamped in three foundation nodes and it is loaded by a set of vertical and horizontal loads (blue arrows), as sketched. The interstorey height is variable, while the intercolumn width is kept constant to 5 m. The cross-section area of each element is equal to 0.5 m^2, i.e., the axial stiffness is kept constant for all the elements. The flexural stiffness varies across the various elements since the shape of the cross section is defined through the size ratio $\rho = \frac{h}{b}$. The width and the height of the cross section are $b = \sqrt{0.5/\rho}$ [m] and $h = \sqrt{0.5\rho}$ [m], respectively. The first-inertia moment, which determines the flexural stiffness, is $J = \frac{0.5^2}{12}\rho$ [m^4]. For $\rho \to \infty$, the height h of the cross section increases (thus, the width b reduces), with consequent larger flexural stiffness. On the contrary, for $\rho \to 0$, the decrease of cross section height h presupposes no flexural stiffness on the element. A script was predisposed

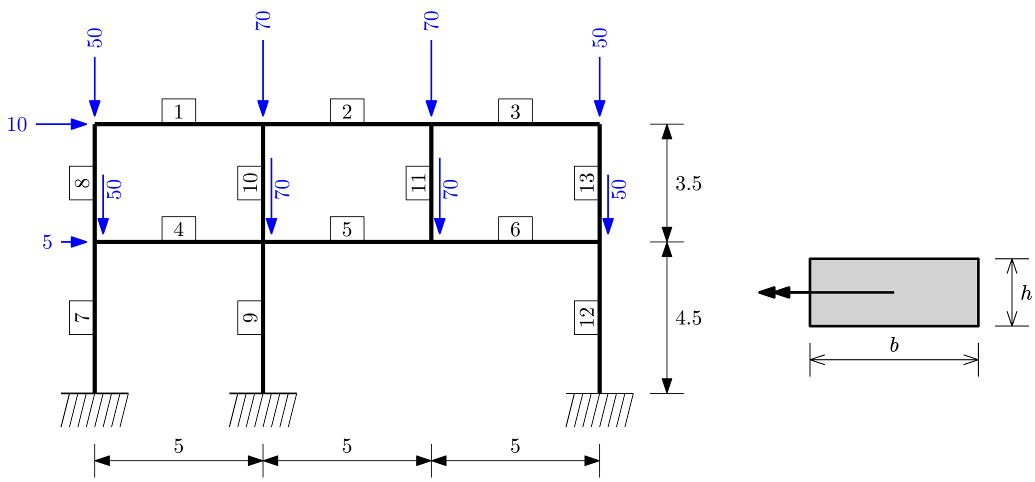

Figure 1: Planar frame structure: the sizes are in meters and the forces in kN. On the right, a cross section is illustrated. The out-of-plane flexural axis is marked with the doublearrow.

on Matlab to perform the following steps:

1. generate and store a set of 12 random numbers in the range $(0.00001; 1)$;
2. assign to element no.1 a unit size ratio and to elements from 2 to 13 the random numbers previously generated. The assumption of unit size ratio of element no.1 derives from scaling invariance properties of the complexity metrics. The sizes of the elements, i.e., the width and the height of each cross section, were computed and attributed to the elements. Young's Modulus of frame elements was set equal to 30 GPa.

3. compute the normalized structural complexity index and the M-value.

This algorithm was repeated 3×10^5 times.

3.2 Truss structure

The truss, depicted in Figure 2, is composed of 6 nodes and 15 rods. The truss is statically indeterminate and it is supported by two pinned support. The system is loaded by a vertical force (50 kN) acting at the free end of the structure. The elements are subjected to axial force, only. For the i-th rod, an area ratio $\phi = A_i/A_1$ determines the cross section area of the element A_i with respect to the reference area attributed to rod no.1, i.e. A_1. For computing purposes, $A_1 = 0.001$ m^2 and Young's modulus is 210 GPa. A set of 2×10^5 different trusses was generated and their performance to random element removal was evaluated using the following procedure:

1. a random area ratio in the range $(0; 20)$ was assigned to each rod from 2 to 15. The area of each rod was then computed.

2. the normalized structural complexity index and the M-value are computed.

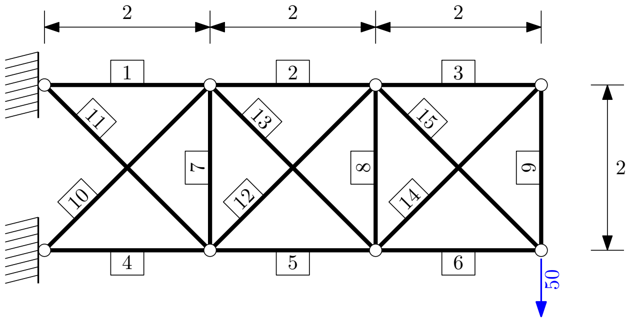

Figure 2: Reference structures: the sizes are in meters and the forces in kN.

4 RESULTS

The parametric structural generation gave a sample of 300k frame structures and 200k truss structures: the values of the normalized structural complexity and M were stored for each scheme. In addition, the cross-section ratios, i.e., the size ratios ρ of each beam of the frame structure or the area ratio ψ of each rod of the truss were saved. The results are presented in a graphical way. For each structure, the first figure plots the values of M against the corresponding NSCI. That is, the plot related to frame structure has 300k different points, i.e., the NSCI-M couples. Similarly, the plot referring to the truss structure has 200k points. Since M-value ranges from 10^{-1} to 10^5, the ordinate axes of both plots are in logarithmic scale.

A second set of plots relates to the geometric properties of a subsets of random generated structures. For each structural type (i.e., frame or truss), three different cases are analyzed. In particular, the cases refer to those schemes having the NSCI and the M within precise bounds. Structures with either low complexity and M-value belong to Case A group; structures with low complexity but high M-value are classed in Case B; structures with high complexity are classed in Case C, independently from the value of M. The results are presented in boxplots: in descriptive statistics, this graphics serves for indicating quartiles of datasets [19]. For each element (beam or rod), the lower and the upper bounds of each box mark the first q_1 and the third quartile q_3 of size or area ratios, respectively. The median value, corresponding to the second quartile q_2, is indicated with a red line. The whiskers of each box mark either the minimum and maximum values of each dataset or the bounds $q_1 - 1.5\,(q_3 - q_1)$ and $q_3 + 1.5\,(q_3 - q_1)$ (in case of normal distributed values, approximatively 99% of the values would be within such bounds). The outliers, marked with red crosses, are all the entries that are smaller, or larger, than the previously mentioned bounds.

4.1 Frame structure

Figure 3 plots the M-values against the computed NSCI for each of the 3×10^5 randomly generated frame structures. It is observed that the normalized structural complexity ranges from 0.4663 and 0.9557 and the M-value ranges from 1.12 and 5.52×10^3. Observing the plot and, in particular the density of the points, it emerges that the majority of the structures has normalized complexity larger than 0.7 and M lower than 10. Meanwhile, it can be noted that for lower complexities two trends are highlighted: on one side, there are structure with both low complexity and low M, on the other structures with low complexity and large M.

Such situations are analyzed in detail, in particular, three subsamples are considered. Case A considers those frame structures with $\mathrm{NSCI} < 0.52$ and $M \leq 10$ (162 of the 300k generated structures), Case B considers structures having $\mathrm{NSCI} < 0.52$ and $M > 10$ (61 structures), Case C considers structures with NSCI $\geq$0.945 (55 structures). The boxplots of the three mentioned cases are illustrated in Figure 4.

4.2 Truss structure

Figure 5 plots the M-values against the computed NSCI for each of the 2×10^5 randomly generated truss structures [12]. It is observed that the normalized structural complexity ranges from 0.7891 and 0.9993 and the M-value ranges from 0.18 and 6.71×10^3. Observing the plot, it emerges that the majority of the structures has normalized complexity larger than 0.9 and M lower than 10. Meanwhile, it can be noted that for lower complexities two trends are highlighted: on one side, there are structure with both low complexity and low M, on the other structured with low complexity and large M.

Such situations are analyzed in detail, in particular, three subsamples are considered. Case A considers those frame structures with $\mathrm{NSCI} < 0.872$ and $M \leq 2$ (89 of the 200k generated structures), Case B considers structures having $\mathrm{NSCI} < 0.82$ and $M > 10$ (52 structures), Case C considers structures with $\mathrm{NSCI} \geq 0.99$ (164 structures). The boxplots of the three mentioned cases are illustrated in Figure 6.

5 DISCUSSION

As mentioned previously, the aim of the numerical tests was to assess the relationship between the amount of structural complexity in a statically indeterminate structure and the effects of random removal. The structural complexity was measured as information entropy through the NSCI index, while the consequences of random damage were quantified in terms of average increment of internal energy through parameter M. Two different statically indeterminate structures were considered. In general, it appears that as much as the structural complexity increases,

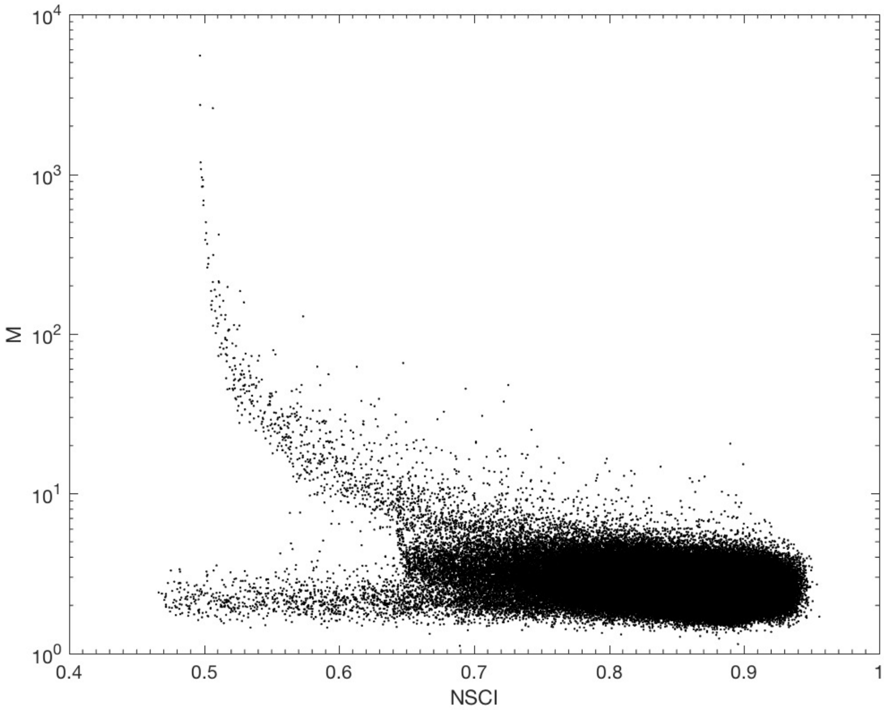

Figure 3: Planar frame structure: plot of the values of M with respect to the corresponding NSCI.

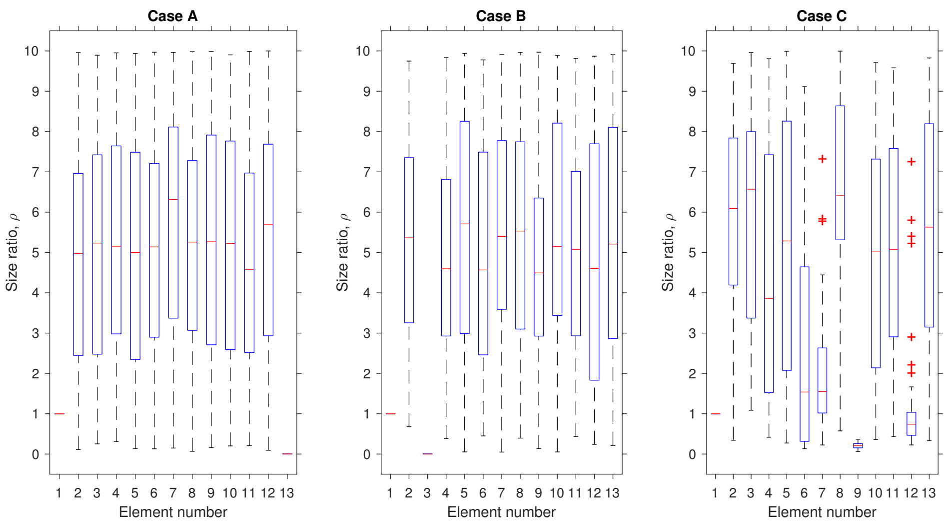

Figure 4: Planar frame structure: boxplots of the size ratios related to three relevant cases identified in Figure 3. Element no.1 has square cross section (i.e., $\rho = 1$).

the effect of element removal tends to weaken. This appears more clear in the considered truss structure (Figure 5) rather than in the frame structure (Figure 3) since, in the frame, a constant trend of M appears for normalized complexities larger than 0.7. On the contrary, similar trends

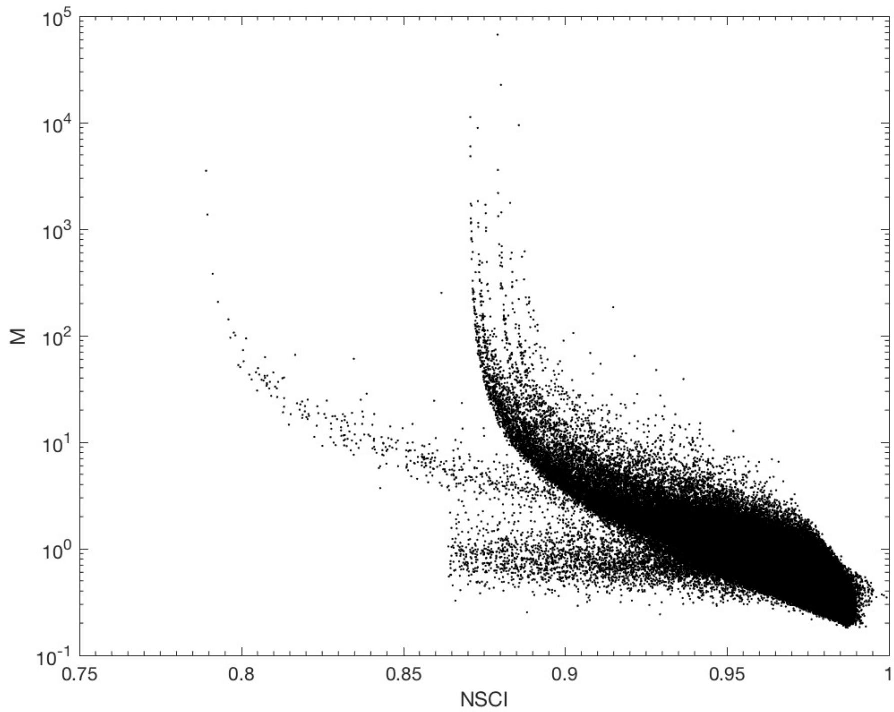

Figure 5: Truss structure: plot of the values of M with respect to the corresponding NSCI.

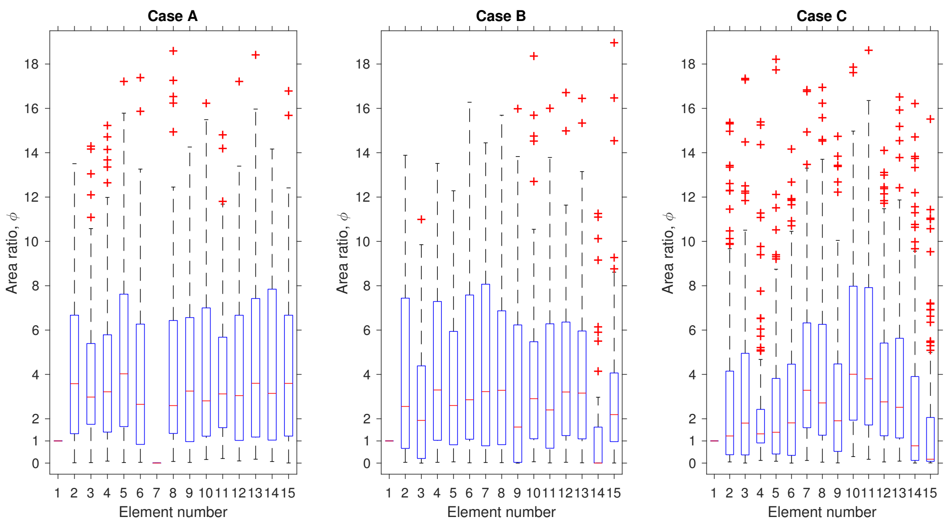

Figure 6: Truss structure: boxplots of the size ratios related to three relevant cases identified in Figure 5. Element no.1 has the reference area, thus its area ratio is $\phi = 1$.

were observed in both structures for low complexities. As emerged in a previous detailed analysis by the author [12], for lower complexities, two opposite behaviors are present: there are structures that have low complexity, meaning less efficiency in the load paths, but are still able to redistribute the effects of random damage (Cases A); at the same time, there are structures that present low complexity but are not able to tolerate random element removal (Cases B). Such cases are analyzed in detail in the following paragraph.

The frame structures identified in the subsample A, i.e., the structures having NSCI smaller than 0.52 and M-value smaller than 10 have a common mark: the size ratio of element no.13, which is the rightmost top column, is quasi-null, independently from the size ratio of the remaining elements. This can be seen in the left-hand side boxplot of Figure 4: the box of element no.13 collapses into a single line. That is, the flexural stiffness of element 13 is null (but the axial stiffness is not). The low M-value observed for this configuration can be justified considering that, if element no.3 is removed, the 50 kN vertical external force is carried by element no.13, while if element no.13 is removed, the high flexural stiffness of element no.3 is able to carry the external vertical load. On the contrary, the frame structures identified in the subsample B, i.e., the structures having NSCI smaller than 0.52 and M-value larger than 10 have size ratio of element no.3, which is the rightmost third level beam, quasi-null, independently from the size ratio of the remaining elements. As before, the central boxplot of Figure 4 depicts this trend: the box of element no.3 collapses into a single line at zero. This fact can be straightforwardly explained: the removal of element no.3 does not cause large increments in the internal energy because element no.13 is able to carry vertical loads only; on the contrary, the removal of element no.13 causes a large increment of work of deformation due to the fact that the 50 kN vertical force has to be supported by element no.3, which has no flexural stiffness (since its size ratio is close to zero).

The truss structures identified in the subsample A, i.e., the structures having NSCI smaller than 0.872 and M-value smaller than 2 have a common mark: the area ratio of rod no.7, which is a vertical rod, is quasi-null, independently from the area ratio of the remaining elements, as can be seen in the left-hand side boxplot of Figure 6. This implies low complexity values as well as low M-values. In order to comment this behavior is it necessary to note that rods no. 2, 8, 5, 12 and 13 create a "rigid" cluster and that rods no. 1, 4, 10, 11 are connected to the external pin supports. Studying the behavior of the damaged structures, if rod no.7 has no axial stiffness, the removal of one of the left rods (i.e., rod no. 1, 4, 10 or 11) would not cause a mechanism since the ends of rod no.7 are kept fixed by the previously identified "rigid" cluster, and because of the external pin supports. The truss structures identified in the subsample B, i.e., the structures having NSCI smaller than 0.82 and M-value larger than 10 are characterized by having area ratio of rod no.14 close to zero (Figure 6). In detail, the median of the 52 considered trusses is zero, while higher values were recorded. The impact of element removal is large because the diagonal rods are essential for ensuring the stability of the unsupported end of the truss cantilever. Similar trend is observed for the configurations in which the area ratio of rod no.15 is close to zero, at $\mathrm{NSCI} = 0.8857$.

Referring to higher complexities, the boxplot related to the frame structures identified in the subsample C (see Figure 4), i.e., structures having NSCI larger than 0.945, shows that axial stiffness outnumbers flexural stiffness of element no.9 and, to a lesser extent, of element no.12. Element no.9 is a central first level column, element no.12 is a lateral first level column. High flexural stiffness is observed in the top beam (elements no. 1, 2 and 3) as to element no.8, which is the leftmost top column that would carry the lateral load in case of removal of element no.1, high flexural stiffness is observed. The truss structures belonging to the subsample C, i.e., the structures having NSCI larger than 0.99, in general, do not have a common mark. It can be noted that the median value of the 164 considered trusses has area ratio of rod no.15 close to zero, meaning the need, but not necessarily the large size, of diagonal elements.

6 CONCLUSIONS

In summary, it has been shown that structures in which there is a strong differentiation of the load paths, i.e., in structures with high complexity, the average impact of element removal is reduced. For each structural type, the random generated structures have the same degree of static indeterminacy, but the response to damage is different. This means that high degree of static indeterminacy is a needed but not sufficient condition for robustness. Additional parameters, such the distribution of stiffnesses (both axial and flexural) has to be considered in the design. The implementation of the results presented in the present paper in construction engineering works would permit to help practitioners in preliminary design. As mentioned in [12], the ability of the structural engineer lies in its capacity to combine materials, cross-section sizes, and structural details in order to respect the distribution of stiffnesses found in the complexity analysis. Despite apparently simple, linear analysis was proved to be sufficient to assess the possibility that a sudden column removal engenders a disproportionate collapse [20].

REFERENCES

[1] International Standard Organisation. ISO 2394: General Principles on Reliability for Structures, 1998.

[2] Comitée Européen de Normalisation. Eurocode 1, Part 1-7: General actions - accidental actions, 2006.

[3] T. Krauthammer, *Modern protective structures*, CRC Press, 530 pp., 2008.

[4] A. Ventura, V. De Biagi, B. Chiaia, Effects of rockfall on an elastic-plastic member: A novel compliance contact model and dynamic response. *Eng. Structures*, **148**, 126–144.

[5] V. De Biagi, B. Chiaia, B. Frigo, Impact of snow avalanche on buildings: Forces estimation from structural back-analyses. *Eng. Structures*, **92**, 15–28.

[6] V. De Biagi, M.L. Napoli, M. Barbero, A quantitative approach for the evaluation of rockfall risk on buildings, *Natural Hazards*, 1–28, DOI 10.1007/s11069-017-2906-3.

[7] N.N. Taleb, *The Black Swan: The Impact of the Highly Improbable*, Random House Trade Paperbacks, 2010.

[8] D.M. Frangopol, J.P. Curley, Effects of damage and redundancy on structural reliability. *J. Struct. Eng.*, **113**, 1533–1549, 1987.

[9] F. Biondini, D.M. Frangopol, S. Restelli, On structural robustness, redundancy and static indeterminacy. *Crossing Borders*, 237, 2008.

[10] F. Biondini, S. Restelli, Damage propagation and structural robustness. In: F. Biondini, D.M. Frangopol eds. *Life-Cycle Civil Engineering: Proceedings of the International Symposium on Life-Cycle Civil Engineering, IALCCE'08*, Varenna, Italy, June 11-14, p.131, 2008.

[11] R.D. Bertero, V.V. Bertero, V.V., Redundancy in earthquake-resistant design. *J. Struct. Eng.*, **125**(1), 81–88, 1999.

[12] V. De Biagi, Structural behavior of a metallic truss under progressive damage. *International Journal of Solids and Structures*, **82**, 56–64, 2016.

[13] V. De Biagi, B. Chiaia, Complexity and robustness of frame structures. *Int. J. Solids Struct.*, **50**, 3723–3741, 2013.

[14] C.E. Shannon, A mathematical theory of communication. *Bell Syst. Tech. J.*, **27**, 379–423, 1948.

[15] M. Dehmer, Information-theoretic concepts for the analysis of complex networks. *Applied Artificial Intelligence*, **22**, 684–706, 2008.

[16] V. De Biagi, B. Chiaia, Scaling in structural complexity. *Complexity* **20**, 57–63, 2014.

[17] U. Starossek, M. Haberland, Approaches to measures of structural robustness. *Struct. Infrastruct. Eng.*, **7**, 625–631, 2011.

[18] V. De Biagi, B. Chiaia, Damage tolerance in parallel systems. *Int. J. Dam. Mech.*, **25**, 1040–1059, 2016.

[19] D. Zwillinger, S. Kokoska, *CRC Standard Probability and Statistics Tables and Formulae*, CRC Press, 568 pp., 1999.

[20] G. Powell, Disproportionate collapse: The futility of using nonlinear analysis. In: *Proceedings of the Structures Congress 2009, Don't Mess with Structural Engineers: Expanding Our Role*, ASCE, 1–10, 2009.