New Valid Inequalities for the Two-Echelon Capacitated Vehicle Routing Problem

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Abstract

We introduce new valid inequalities for the two-echelon variant of the Capacitated Vehicle Routing Problem (CVRP)In particular, a first group of inequalities is obtained by extending to 2E-CVRP some of the most effective among the existing CVRP valid inequalities. A second group of inequalities is explicitly derived for the 2E-CVRP and concerns the flow feasibility at customer nodes and the satellite-customer route connectivity. The inequalities are then introduced in a Branch & Cut algorithm. Computational results show that the proposed algorithm is able both to solve to optimality many open literature instances and significantly reduce the optimality gap for the remaining instances.

Keywords: 2E-CVRP, valid inequalities.

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1 Introduction

In this paper we address the Two-Echelon Capacitated Vehicle Routing Problem (2E-CVRP), which is characterized by a single depot, a set of customers, and a set of intermediate depots, called satellites. The freight is not directly shipped to customers, but it is first consolidated at satellites. The first level routing problem addresses depot-to-satellite delivery, while the satellite-to-customer delivery is considered at the second level. The goal is to minimize the total transportation cost, and to satisfy demand and capacity constraints. This problem is faced in practice, both in long-term planning and real-time optimization. [?] introduced the first formal definition of the problem as a two-level, synchronized, multi-depot CVRP with time windows. The first flow model for the static, single-depot 2E-CVRP was introduced in [?], where two math-heuristics were also presented. From the point of view of exact methods, and cuts in particular, the main reference is still limited to [?]. In this paper some cuts derived from the MIP model defined in [?] are introduced. The results are promising, but, to our knowledge, no further work introducing additional families or adapting the standard CVRP cuts are present in the literature. The goal of this paper is to introduce valid inequalities to strengthen the 2E-CVRP continuous relaxation, which is widely used by implicit enumeration methods. The paper is organized as follows. After recalling the MIP model in Section 2, the valid inequalities are given in Section 3, while their computational performance are presented in Section 4. The variable definition, as well as the model definition are taken from [?].

2 Mathematical model of the 2E-CVRP

Let us define three set: $V_0 = \{v_0\}$ the depot, V_s the satellite set (its cardinality is n_s) and V_c the customer set (its cardinality is n_c). Further, let us consider the following variables:

- m_1 and m_2 the number of 1st-level and 2nd-level vehicles

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- m_{s_k} the maximum number of 2nd-level routes starting from satellite k
- K^1 the capacity of 1st-level vehicles and K^2 the capacity of 2nd-level vehicles
- d_i: demand of customer i, c_{ij}: unit cost of arc (i,j) and F_k: unit cost for freight operations at satellite k
- Q^1_{ij}: flow passing through the 1st-level arc (i,j) and Q^2_{ijk}: flow passing through the 2st-level arc (i,j) and coming from satellite k
- x_{ij}: number of 1st-level vehicles using the 1st-level arc (i,j)
- y^k_{ij}: boolean variable equal to 1 iff the 2st-level arc (i,j) is used by a 2nd-level route starting from satellite k
- z_{kj}: boolean variable equal to 1 iff customer j is served through satellite k
- $D_k = \sum_{j\in V_c} d_j z_{kj}$: total freight passing through satellite k.

The 2E-CVRP is as follows (see [**?**]):

$$\min \sum_{i,j\in V_0\cup V_s, i\neq j} c_{ij}x_{ij} + \sum_{k\in V_s}\sum_{i,j\in V_s\cup V_c, i\neq j} c_{ij}y^k_{ij} + \sum_{k\in V_s} F_k D_k \tag{1}$$

subject to:

$$\sum_{i\in V_s} x_{v_0 i} \leq m_1 \tag{2}$$

$$\sum_{k\in V_s}\sum_{j\in V_c} y^k_{kj} \leq m_2 \tag{3}$$

$$\sum_{j\in V_s\cup V_0, j\neq k} x_{jk} = \sum_{i\in V_s\cup V_0, i\neq k} x_{ki} \qquad k\in V_s\cup V_0 \tag{4}$$

$$\sum_{j\in V_c} y^k_{kj} \leq m_{s_k} \qquad k\in V_s \tag{5}$$

$$\sum_{j\in V_c} y^k_{kj} = \sum_{j\in V_c} y^k_{jk} \qquad k\in V_s \tag{6}$$

$$\sum_{i\in V_s\cup V_0, i\neq j} Q^1_{ij} - \sum_{i\in V_s\cup V_0, i\neq j} Q^1_{ji} = \begin{cases} \sum_{i\in V_c} -d_i & j \text{ is the depot} \\ D_j & \text{otherwise} \end{cases} \qquad j\in V_s\cup V_0 \tag{7}$$

$$\sum_{i\in V_c\cup\{k\}, i\neq j} Q^2_{ijk} - \sum_{i\in V_c\cup\{k\}, i\neq j} Q^2_{jik} = \begin{cases} -D_j & j \text{ is a satellite} \\ d_j z_{kj} & \text{otherwise} \end{cases} \qquad j\in V_c\cup V_s, k\in V_s \tag{8}$$

$$Q^1_{ij} \leq K^1 x_{ij} \qquad i,j\in V_s\cup V_0, i\neq j \tag{9}$$

$$Q^2_{ijk} \leq K^2 y^k_{ij} \qquad i,j\in V_s\cup V_c, i\neq j, k\in V_s \tag{10}$$

$$\sum_{i\in V_s} Q^1_{iv_0} = 0 \tag{11}$$

$$\sum_{j\in V_c} Q^2_{jkk} = 0 \qquad k\in V_s \tag{12}$$

$$y^k_{ij} \leq z_{kj} \qquad i\in V_s\cup V_c, j\in V_c, k\in V_s \tag{13}$$

$$y^k_{ji} \leq z_{kj} \qquad i\in V_s, j\in V_c, k\in V_s \tag{14}$$

$$\sum_{i\in V_s\cup V_c} y_{ij}^k = z_{kj} \quad k \in V_s, j \in V_c \qquad (15) \qquad \sum_{i\in V_s} y_{ji}^k = z_{kj} \quad k \in V_s, j \in V_c \qquad (16)$$

$$\sum_{k\in V_s} z_{kj} = 1 \quad j \in V_c \qquad (17) \qquad y_{kj}^k \leq \sum_{l\in V_s\cup V_0} x_{kl} \quad k \in V_s, j \in V_c \qquad (18)$$

$$y_{ij}^k \in \{0,1\} \quad k \in V_s \cup V_0, i,j \in V_c \qquad (19)$$

$$z_{kj} \in \{0,1\} \quad k \in V_s \cup V_0, j \in V_c \qquad (20) \qquad x_{kj} \in \mathbb{N} \quad k,j \in V_s \cup V_0 \qquad (21)$$

$$Q_{ijk}^1 \geq 0 \quad i,j \in V_s \cup V_0, k \in V_s \quad Q_{ijk}^2 \geq 0 \quad i,j \in V_s \cup V_c, k \in V_s \qquad (22)$$

The objective function (1) minimizes the sum of the transportation and operation costs. Constraints (2) and (3) impose that the number of routes at each level must not exceed the number of vehicles for that level. Constraints (4) require that each 1st-level route starts and ends at the depot if k is the depot v_0, while, if k is a satellite, the number of vehicles entering and leaving that satellite is the same. The respect of the satellite capacity is guaranteed by constraints (5). Constraints (6) force each 2nd-level route to start and end to a satellite and to be a flow. Constraints (7) and (8) indicate that the flow balance at each node is equal to the demand of that node, except for the depot, where the outgoing flow is equal to the total customer demand, and for each satellite, where the flow is equal to the demand assigned to that satellite. These constraints forbid subtours which do not contain the depot or a satellite, respectively. The arc capacity constraints are (9) for the 1st-level and (10) for the 2nd-level. Constraints (11) and (12) do not allow residual flows in the routes, making the returning flow of each route to the depot (1st-level) and to each satellite (2nd-level) equal to 0. Constraints (13) and (14) indicate that customer j is served by satellite k ($z_{kj} = 1$) only if it receives freight from that satellite (i.e. $y_{ij}^k = 1$ or $y_{ji}^k = 1$). Constraints (15) and (16) indicate that there is only one 2nd-level route passing through each customer, while constraints (17) assign each customer to an unique satellite. Constraints (18) allow a 2nd-level route to start from a satellite k only if a 1st-level route serves it.

3 Valid Inequalities for the 2E-CVRP

3.1 Extension of existing CVRP valid inequalities

Given any satellite k, let us consider the 2E-CVRP restricted to the 2nd-level network only, with root in k. Any solution of this problem can be seen as a solution of a corresponding CVRP, where satellite k is the depot and the customer set is not known, due to the presence in the CVRP solution of the unknown satellite-customer assignment variables $z_{kj}, \forall j$. In fact, these variables prevent the immediate extension of existing CVRP valid inequalities to the 2E-CVRP. By cope with this problem, let us consider a graph $\tilde{G}$ with node set $\tilde{V} = V_c \cup \{l\}$, where l is a macro-node obtained by collapsing the depot d_0 and all satellites k, and costs $\tilde{c}_{ij}$ are the original costs c_{ij} if nodes i and j are customers and 0 if one of them is l. Let w_{ij} be a boolean variable. It is equal to 1 iff arc (i, j) is used, and $Q^2_{ij} = \sum\limits_{k \in V_s} \mathrm{Q}^2_{ijk}$.

The 2E-CVRP restricted to the 2nd-level network with root in any given satellite is as follows

$$\min \sum_{i,j \in \tilde{V}, i \neq j} \tilde{c}_{ij} w_{ij} \tag{23}$$

$$w\left(\delta\left(\{i\}\right)\right) = 2 \quad i \in V_c \tag{24}$$

$$w\left(\delta\left(\{l\}\right)\right) \leq 2m_2 \tag{25}$$

$$\sum_{i \in \tilde{V}, i \neq j} \mathrm{Q}^2_{ij} - \sum_{i \in \tilde{V}, i \neq j} \mathrm{Q}^2_{ji} = \begin{cases} \sum_{i \in \tilde{V}} -d_i & \text{if } j = l \\ d_j & \text{otherwise} \end{cases} \quad j \in \tilde{V} \tag{26}$$

$$\mathrm{Q}^2_{ij} \leq K^2 w_{ij} \quad i, j \in \tilde{V}, i \neq j \tag{27}$$

$$w_{ij} \in \{0, 1\}, \quad \mathrm{Q}^2_{ij} \geq 0, \quad i, j \in \tilde{V}, i \neq j \tag{28}$$

where $w\left(\delta\left(V'\right)\right)$, with $V' = \{i\}$, is the sum of all variables w_{ij} such that the associated arcs (i, j) have a node incident in V' and the other in $\tilde{V} \backslash V'$. It is easy to see that (23)-(28) represents a CVRP defined on $\tilde{G}$, where the macro-node l acts as the depot.

Theorem 3.1 *Any feasible solution of the C-2ECVRP is a feasible solution of the continuous relaxation of* (23)-(28).

Proof. Let us consider a feasible solution $\bar{s} = (\bar{x}, \bar{y}, \bar{z})$ of the C-2ECVRP. Let us define $\zeta_{ij} = \sum\limits_{k \in V_s} \bar{y}^k_{ij}$, $i, j \in V_c$, $\zeta_{li} = \sum\limits_{k \in V_s} \bar{y}^k_{ki}$, $i \in V_c$, and $\zeta_{jl} = \sum\limits_{k \in V_s} \bar{y}^k_{jk}$, $j \in V_c$. Thus, by (15), (16) and (17) and being $0 \leq \bar{y}^k_{ij} \leq 1$, also $0 \leq \zeta_{ij} \leq 1$. The values $w_{ij} = \zeta_{ij}$ constitute a feasible solution of (23)-(28). In fact, (15), (16) and (17) impose $\sum\limits_{i \in \tilde{V}} \zeta_{ij} = \sum\limits_{k \in V_s} \sum\limits_{i \in V_c} \bar{y}^k_{ij} + \sum\limits_{k \in V_s} \bar{y}^k_{kj} = \sum\limits_{k \in V_s} \sum\limits_{i \in V_c} \bar{y}^k_{ji} + \sum\limits_{k \in V_s} \bar{y}^k_{jk} = \sum\limits_{i \in \tilde{V}} \zeta_{ji} = 1$, $j \in V_c$. Similarly, constraints (17) guarantee the equivalence between (8) and (26), and between (10) and (27). Moreover, (8), (10), (15), and (16) imply (24) and (3), (8), and (10) imply (25). Then, a feasible solution of the C-2ECVRP is also a feasible solution of the continuous relaxation of (23)-(28). □

By Proposition 3.1 we can state that the feasible set of the C-2ECVRP is a subset of the feasible set of (23)-(28). Then, any valid inequality for (23)-(28) is also valid for the C-2ECVRP. Moreover, the proof of Proposition 3.1 also gives us a procedure for building a feasible solution of (23)-(28) from any feasible solution of the C-2ECVRP. By means of the results in [?] we use the Capacity Inequalities and the Strengthened Comb Inequalities of the CVRP for our 2E-CVRP computational campaign.

3.2 Explicit valid inequalities for the 2E-CVRP

Explicit valid inequalities for the 2E-CVRP can be derived by considering the flow variables Q^2_{ijk} in the model (1)-(22).

3.2.1 Customer Node Feasibility Inequalities

The flow balance at any customer node j is guaranteed by constraints (8). For any integer feasible solution of the 2E-CVRP, we can state that in any triplet (i, j, k) $i,\ j\ \in V_c\ \cup V_s,\ k\ \in\ V_s$ only one variable between y^k_{ij} and y^k_{ji} is different from zero, if the route is a 2nd-level route which serves more than one customer.

Theorem 3.2 *[?] The following inequalities are valid for the 2E-CVRP*

$$Q^2_{ijk} - \sum_{m \in V_c \cup V_s, m \neq i} Q^2_{jmk} \leq d_j y^k_{ij} \quad i \in V_c \cup V_s, j \in V_c, k \in V_s \tag{29}$$

$$\sum_{i \in V_c \cup V_s, i \neq m} Q^2_{ijk} - Q^2_{jmk} \geq d_j y^k_{jm} \quad j \in V_c, m \in V_c, k \in V_s. \tag{30}$$

3.2.2 Satellite-Customer Route Connectivity Inequalities

In this section we introduce the new valid inequalities that constitutes the novelty of this paper.

Theorem 3.3 *Given any customer subset $V' \subset V_c$, $1 \leq |V'| \leq m_2$, the inequality*

$$\sum_{k \in V_s} \sum_{j \in V'} (y^k_{jk} + y^k_{kj}) \leq |V'| + m_2 - \left\lceil \frac{d\,(V_c \backslash V')}{K^2} \right\rceil \tag{31}$$

where $d\,(V_c \backslash V')$ is the total demand of customers which belong to $V_c \backslash V'$, is valid for the 2E-CVRP.

Proof. Given a customer subset $V' \subset V_c$, let us consider the maximum number of non-zero variables y_{ij}^k connecting V' to V_s. This can be obtained by computing the minimum number of arcs requested to serve $V_c \backslash V'$, which is $\left\lceil \frac{d(V_c \backslash V')}{K^2} \right\rceil$. □

Theorem 3.4 *The exact separation of inequality (31) is $NP-Hard$.*

Proof. To separate all valid inequalities (31) when V' changes is at least as complex as to find the inequality with the smallest violation. This corresponds to solve the following Knapsack Problem:

$$\min \sum_{k\in V_s}\sum_{j\in V'}\left(y_{jk}^k + y_{kj}^k - 1 + \frac{d_j}{K^2}\right)\zeta_j \;\; s.t. \sum_{k\in V_s}\sum_{j\in V'}\left(y_{jk}^k + y_{kj}^k - 1 + \frac{d_j}{K^2}\right)\zeta_j \geq m_2 + 1 \tag{32}$$

where $\zeta_j \in \{0,1\}$ $j \in V'$ is a boolean variable equal to 1 iff customer $j \in V'$. Being the Knapsack Problem $NP-Hard$, so it is the exact separation. □

Theorem 3.5 *Given a customer subset $V' \subset V_c$, such that $m_2 - \left\lceil \frac{d(V')}{K^2} \right\rceil = 0$, the inequalities*

$$\sum_{j\in V'}(y_{jk}^k + y_{kj}^k) + \sum_{i\in V'}\sum_{j\in V_c\backslash V'} y_{ij}^k + \sum_{i\in V_c\backslash V'}\sum_{j\in V'} y_{ij}^k \leq \sum_{j\in V'} z_{kj} \quad k \in V_s \tag{33}$$

are valid for the 2E-CVRP.

Proof. By constraints (15), (16), and (17), inequalities (33) become valid for the 2E-CVRP. □

Theorem 3.6 *The exact separation of inequalities (33) is $NP-Hard$.*

Proof. Let us define ζ_j as a boolean variable which is equal to 1 iff customer $j \in V'$. To separate all valid inequalities is at least as complex as to find the inequality with the smallest violation. This corresponds to solve, for every satellite k, the following problem

$$\min \sum_{j\in V'}(y_{jk}^k\zeta_j + y_{kj}^k\zeta_j) + \sum_{i,j\in V_c} m_{ij} \tag{34}$$

$$\sum_{j\in V'}(y_{jk}^k\zeta_j + y_{kj}^k\zeta_j) + \sum_{i,j\in V_c} m_{ij} \geq |V' + 1| \tag{35}$$

$$m_{ij} - \zeta_i + \zeta_j \geq 0 \;\; i,j \in V_c, (i,j) \tag{36}$$

$$m_{ij} \geq 0 \;\; i,j \in V_c, (i,j) \tag{37}$$

$$\zeta_j \in \{0,1\}. \tag{38}$$

By relaxing constraints (35), the remaining problem is a Knapsack Problem, which is $NP-Hard$, so it is the exact separation. □

Both valid inequalities (31) and (33) can be also heuristically separated (see [?] for a proposed heuristic).

4 Computational Results

4.1 General results and comparison with the literature

The valid inequalities presented in this paper have been implemented by the CPLEX Branch-and-Cut framework (BC). In this section, we present the results of BC compared with the State-of-the-Art (SOA) results. In particular, we consider the instances given in [?] and [?]: we consider Set 2 from [?], Set 3 from [?] and Set 4 from [?]. Set 1 from [?] is disregarded because their limited size. All instances are available in the OR-Library [?]. The results of Sets 2 and 3 are summarized in Tables 1, respectively. Columns 1 and 2 report the instance name and the number of satellites in each instance The SOA columns report the best results available in the literature, in terms of Final Solution, Best Bound, and Gap between the available optimum and the best bound, while the BC columns show the results of the Branch-and-Cut based on the proposed valid inequalities. The results of Set 4 are summarized in Table 2. Column 1 reports the instance name *Instance-x-sk-n*The remaining columns have the same meaning of Tables 1. From Table 1, we can notice that BC overcomes SOA. In fact, it is able to solve to optimality for the first time all five instances with 32 customers and three of the instances with 50 customers. For the remaining instances the mean gap is 0.40%. In particular, let us notice the significant gap reduction for the instances with 4 satellites, which are considered among the most difficult in the literature. The same behaviour is confirmed by Set 3 (see Table 1), where three instances are solved to optimality for the first time and the mean gap reduces from 4.17% to 1.54%. Finally, Set 4 (see Table 2) presents a mean gap reduction of more than 4 percentage points. The fact that this instance set shows the worst behaviour among the different instance sets is related to the vehicle capacity values. In fact, these values, when compared to the customer demands, generate a low value of the vehicle loading factor, which affects the effectiveness of our valid inequalities.

		SOA			BC		
Instance	Satellites	Final Solution	Best Bound	Gap	Final Solution	Best Bound	Gap
E-n22-k4-s6-17	2	417.07	417.07	0.00%	417.07	417.07	0.00%
E-n22-k4-s8-14	2	384.96	384.95	0.00%	384.96	384.96	0.00%
E-n22-k4-s9-19	2	470.6	470.6	0.00%	470.6	470.6	0.00%
E-n22-k4-s10-14	2	371.5	371.5	0.00%	371.5	371.5	0.00%
E-n22-k4-s11-12	2	427.22	427.22	0.00%	427.22	427.22	0.00%
E-n22-k4-s12-16	2	392.78	392.78	0.00%	392.78	392.78	0.00%
E-n33-k4-s1-9	2	730.16	730.16	0.00%	730.16	730.16	0.00%
E-n33-k4-s2-13	2	714.64	703.87	1.53%	714.63	714.63	0.00%
E-n33-k4-s3-17	2	707.49	695.77	1.69%	707.41	707.41	0.00%
E-n33-k4-s4-5	2	785.33	767.43	2.33%	778.73	778.73	0.00%
E-n33-k4-s7-25	2	756.85	744.4	1.67%	756.84	756.84	0.00%
E-n33-k4-s14-22	2	779.05	766.77	1.60%	779.05	779.05	0.00%
E-n51-k5-s2-17	2	597.49	582.21	2.63%	597.49	591.35	1.04%
E-n51-k5-s4-46	2	530.76	520.96	1.88%	530.76	530.76	0.00%
E-n51-k5-s6-12	2	554.81	531.83	4.32%	554.81	540.24	2.70%
E-n51-k5-s11-19	2	581.64	559.85	3.89%	581.64	573.75	1.37%
E-n51-k5-s27-47	2	538.2	527.32	2.06%	538.22	538.22	0.00%
E-n51-k5-s32-37	2	552.28	548.31	0.72%	552.28	552.28	0.00%
E-n51-k5-s2-4-17-46	4	541.07	515.67	4.93%	530.76	522.72	1.54%
E-n51-k5-s6-12-32-37	4	538.82	512.81	5.07%	531.92	529.16	0.52%
E-n51-k5-s11-19-27-47	4	531.12	519.59	2.22%	531.12	524.55	1.25%
Average				2.03%			0.40%

		SOA			BC		
Instance	Satellites	Final Solution	Best Bound	Gap	Final Solution	Best Bound	Gap
E-n22-k4-s13-14	2	526.1	526.1	0.00%	526.15	526.15	0.00%
E-n22-k4-s13-16	2	521.04	521.04	0.00%	521.09	521.04	0.01%
E-n22-k4-s13-17	2	496.34	496.34	0.00%	496.38	496.38	0.00%
E-n22-k4-s14-19	2	498.81	498.81	0.00%	498.8	498.8	0.00%
E-n22-k4-s17-19	2	512.8	512.8	0.00%	512.8	512.8	0.00%
E-n22-k4-s19-21	2	520.41	520.41	0.00%	520.42	520.42	0.00%
E-n33-k4-s16-22	2	672.17	634.09	6.01%	672.17	667.15	0.75%
E-n33-k4-s16-24	2	668.81	625.73	6.88%	666.02	658.04	1.21%
E-n33-k4-s19-26	2	680.89	648.2	5.04%	680.36	680.36	0.00%
E-n33-k4-s22-26	2	680.89	652.12	4.41%	680.89	671.07	1.46%
E-n33-k4-s24-28	2	672.6	633.03	6.25%	670.86	670.86	0.00%
E-n33-k4-s25-28	2	653.67	615.87	6.14%	650.95	650.95	0.00%
E-n51-k5-s12-18	2	692.56	662.16	4.59%	692.37	679.72	1.86%
E-n51-k5-s12-41	2	716.58	646.5	10.84%	691.37	661.35	4.54%
E-n51-k5-s12-43	2	712.48	690.28	3.22%	712.48	701.89	1.51%
E-n51-k5-s39-41	2	729.94	682.98	6.88%	729.94	695.95	4.88%
E-n51-k5-s40-41	2	732.42	678.19	8.00%	729.94	689.65	5.84%
E-n51-k5-s40-43	2	757.3	709.46	6.74%	761.54	716.64	6.27%
Average				4.17%			1.57%

Table 1
Comparison with State-of-the-Art on Set 2(on the left) and on Set 3 (on the right)

		SOA			BC		
Instance	Satellites	Final Solution	Best Bound	Gap	Final Solution	Best Bound	Gap
Instance50-s5-37.dat	5	1587.95	1405.64	12.97%	1528.73	1444.27	5.85%
Instance50-s5-38.dat	5	1185.58	1076.48	10.14%	1185.58	1081.53	9.62%
Instance50-s5-39.dat	5	1525.24	1421.58	7.29%	1525.24	1431.29	6.56%
Instance50-s5-40.dat	5	1199.42	1068.57	12.25%	1179.64	1082.17	9.01%
Instance50-s5-41.dat	5	1703.03	1541.88	10.45%	1681.04	1589.06	5.79%
Instance50-s5-42.dat	5	1223.09	1097.89	11.40%	1223.09	1126.94	8.53%
Instance50-s5-43.dat	5	1453.11	1283.21	13.24%	1422.29	1348.73	5.45%
Instance50-s5-44.dat	5	1039.39	935.42	11.12%	1039.39	970.71	7.08%
Instance50-s5-45.dat	5	1484.64	1299.06	14.29%	1444.82	1338.82	7.92%
Instance50-s5-46.dat	5	1095.69	930.53	17.75%	1068.5	992.83	7.62%
Instance50-s5-47.dat	5	1598.88	1444.15	10.71%	1581.57	1497.43	5.62%
Instance50-s5-48.dat	5	1096.96	998.69	9.84%	1092.32	1048.1	4.22%
Instance50-s5-49.dat	5	1479.16	1339.67	10.41%	1441.64	1373.65	4.95%
Instance50-s5-50.dat	5	1090.6	968.47	12.61%	1089.67	996.09	9.39%
Instance50-s5-51.dat	5	1436.3	1310.9	9.57%	1436.3	1318.45	8.94%
Instance50-s5-52.dat	5	1128.33	1003.03	12.49%	1109.52	1005.6	10.33%
Instance50-s5-53.dat	5	1552.75	1450.87	7.02%	1552.75	1486.25	4.47%
Instance50-s5-54.dat	5	1135.39	1034.88	9.71%	1135.39	1059.27	7.19%
Average				4.17%			1.57%

Table 2
Comparison with State-of-the-Art on Set 4

4.2 Analysis of the valid inequalities stratification

In the following, we analyse the relative performance of the two types of valid inequalities we have introduced. Let us call CVRP the extension of existing CVRP valid inequalities to the 2E-CVRP (see subsection 3.1) and NEW the explicit valid inequalities for the 2E-CVRP (see subsection 3.2). The results are presented in Table 3, where the mean values over the 50 customers

Set	Sat	NEW			NEW+CVRP		
		Time	Cuts	Impr. (%)	Time	Cuts	Impr. (%)
2	2	30	2995	1.07	128	5188	2.41
2	4	166	7496	1.22	533	14902	2.36
3	2	36	4006	1.91	129	6024	2.61
4	5	298	440	12.00	796	15174	3.39

Table 3
Analysis of valid inequalities stratification on Sets 2, 3, and 4

instances are reported and the columns have the following meaning: Columns 1-2: instance set and satellite number; Columns 3-5: mean computational time, number of cuts generated by the NEW valid inequalities and percentage improvement at the root node, calculated as (optimum of the C-2ECVRP with cuts - optimum of the C-2ECVRP without cuts)/ optimum of the C-2ECVRP without cuts *100; Columns 6-8: they have the same meaning of columns 3-5 for the combination NEW+CVRP valid inequalities.

As one can see from the results, the NEW valid inequalities explicitly derived for the 2E-CVRP better perform when the number of satellites increases. This is justified by the fact that, when the number of satellites increases, the importance of the first level routes increases too, so making the valid inequalities based on these routes more effective.