# Determining the Quantum Expectation Value by Measuring a Single Photon

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(Dated:)

**Quantum mechanics, one of the keystones of modern physics, exhibits several peculiar properties, differentiating it from classical mechanics. One of the most intriguing is that variables might not have definite values. A complete quantum description provides only probabilities for obtaining various eigenvalues of a quantum variable. These and the corresponding probabilities specify the expectation value of a physical observable, which is known to be a statistical property of an ensemble of quantum systems. Here we demonstrate, in contrast to this paradigm, a unique method allowing to measure the expectation value of a physical variable on a single particle, namely, the polarisation of a single protected photon. This is the first realisation of quantum protective measurements, paving the way for applications both to quantum mechanics foundations and quantum-enhanced measurements.**

Despite its unprecedented success in accurately predicting experimental results, there is no consensus about quantum mechanics foundational concepts. The reality of the wavefunction is still under hot debate [1–4]. In stark contrast with classical physics, quantum observables lack definite values. A complete description of a quantum system only predicts the spectrum and probabilities for the measurement outcomes of a physical observable. Given the quantum state of the system $|\Psi\rangle$, which, according to standard quantum mechanics, comprises its complete description, to each observable $A$ we can associate a definite number: $\langle\Psi|A|\Psi\rangle = \sum p_i a_i$ ($p_i$ being the probability to obtain the (eigen)value $a_i$ as the result of the measurement of $A$). The meaning of this number is statistical: for finding the expectation value of $A$ one needs to measure an ensemble of identically-prepared systems.

Single measurement yielding the expectation value of a physical variable seems to be against the spirit of quantum mechanics. However, it has been suggested that, in certain special situations, one can find the expectation value of an observable performing only a single measurement. This is the method of *protective measurement* (PM), originally proposed as an argument supporting the reality of the quantum wavefunction [5].

This is, however, a highly controversial issue [6–12], namely, does the procedure allow observing the state or only the protection mechanism? Nevertheless, the PM idea triggered and helped studying various foundational topics beyond exploring the meaning of the wavefunction, such as Bohmian trajectories [13], stationary basis determination [12] and analysis of measurement optimisation for minimising the state disturbance [14]. The concept of protection was also extended to measurement of a two-state vector [15].

Protection can be realised [5] both actively or passively: here we employ a variant of an active protection technique based on the Zeno effect [16]. Since in our Zeno protection method we strongly project on a particular state, our experiment corresponds to a protocol where Bob wants to measure the expectation value of an observable on a quantum state unknown to him, in presence of a protection mechanism designed for such state. The state, together with its protection mechanism, is provided by Alice, who needs to know the state to set up the protection. Thus, Bob actually measures the photon in conjunction with the "protection apparatus".

In spite of the rich and diverse analysis of the theory behind PM, to this date PMs have not been realised experimentally. Indeed, although weak measurements (WMs) [17–20] and Zeno effect [21–26] have been largely considered in experiments for several physical systems, up to now no experiment joining them in a PM has been realised yet.

Our main result is the extraction of the expectation value of the photon polarisation by means of a measurement performed on a single protected photon (see Fig. 1), that survived the Zeno-type protection scheme. The polarisation operator is defined by

$$P = |H\rangle\langle H| - |V\rangle\langle V|, \tag{1}$$

where $H$ ($V$) is the horizontal (vertical) polarisation. Because of the presence of the active protection in our experiment, the single click of a multi-pixel camera tells us that the expectation value of the polarisation operator of the single protected photon is $\langle P\rangle = -0.3 \pm 0.3$ (see Fig. 1), in agreement with the theoretical predictions ($\langle P\rangle = -0.208$).

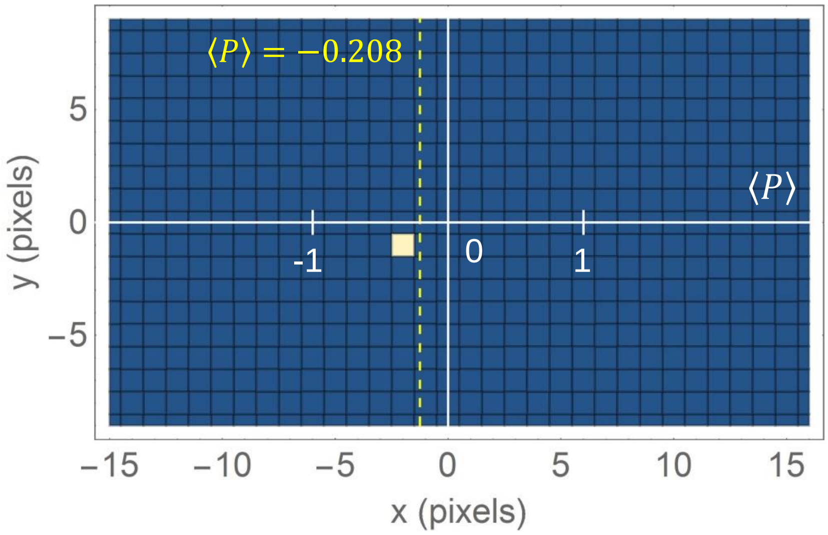


FIG. 1: **Estimation of the polarisation expectation value $\langle P \rangle$ by means of a single protective measurement.** The $x$ coordinate of the pixel which detected the single photon tells us - without the need of any statistics - the expectation value of the polarisation operator, $\langle P \rangle = -0.3(3)$, where the uncertainty is estimated from the width of the photon counts distribution presented in the paper, the theoretical value being (for $\theta = \frac{17\pi}{60}$) $\langle P \rangle = -0.208$.

In our experiment (see Fig. 2), heralded single photons [27], prepared in the polarisation state $|\psi_\theta\rangle = \cos\theta|H\rangle + \sin\theta|V\rangle$, pass through a birefringent material shifting them in the transverse direction $x$ (according to their polarisation). The spatial mode is close to a Gaussian one with a 4.1 pixels $\sigma$ (being $\sigma$ the source of uncertainty associated to the estimation of $\langle P \rangle$ presented in Fig. 1). The WM interaction is obtained exploiting $K = 7$ birefringent units, while the state protection is implemented via the quantum Zeno scheme, i.e. by inserting a thin-film polariser after each birefringent unit, realising a state filtering equivalent to the one made at the preparation stage. Finally, the photons are detected by a spatial-resolving single-photon detector prototype [28]. Without protection, the photons end up in one of the two regions corresponding to the vertical and horizontal polarisations, centered around $x = \pm a$ (see Fig. 3a). Then, the expectation value can be statistically

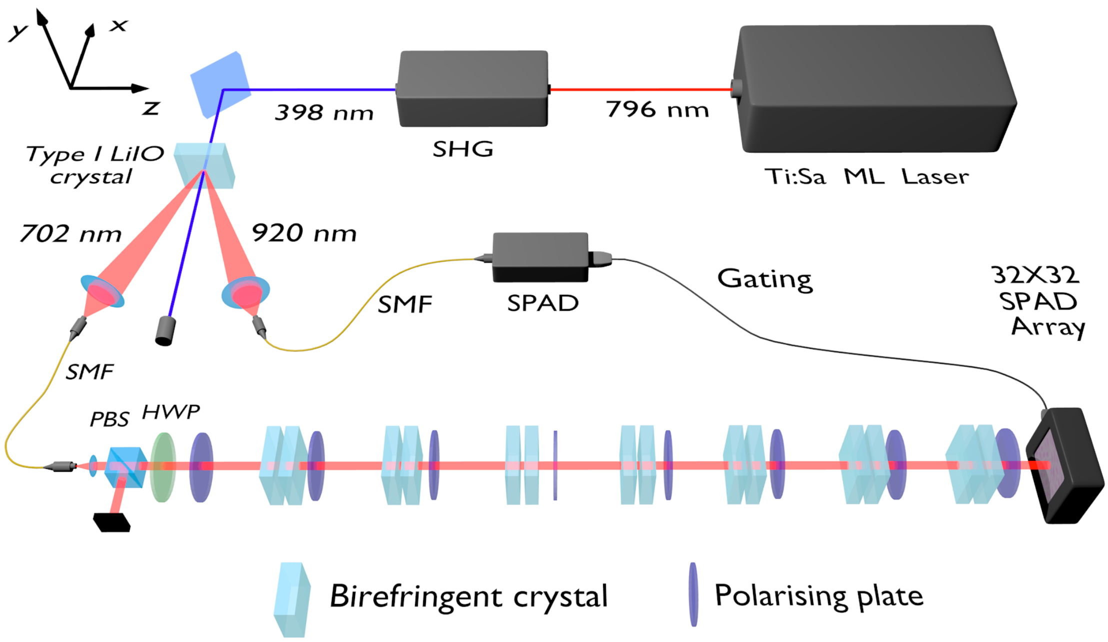


FIG. 2: **Experimental setup.** Heralded single photons are produced by type-I Parametric Down-Conversion in a $LiIO_3$ crystal, then properly filtered, fiber coupled and addressed to the open-air path where the experiment takes place. After being prepared in the polarisation state $|\psi_\theta\rangle = \cos\theta|H\rangle + \sin\theta|V\rangle$, they pass through a birefringent material shifting them in the transverse direction $x$ (according to their polarisation). The weak interaction is obtained by means of $K = 7$ birefringent units, each unit composed of a first crystal separating the beam by 1.66 pixels (less than the beam width) and a second one used to compensate the phase and time decoherence induced by the first crystal: only the action of all units together allows separating orthogonal polarisations. The protection of the quantum state, implementing the quantum Zeno scheme, is realised by inserting a thin-film polariser after each birefringent unit, projecting the photons onto the same polarisation as the initial state $|\psi(\theta)\rangle$. At the end of the optical path, the photons are detected by a spatial-resolving single-photon detector prototype, i.e., a two-dimensional array of $32 \times 32$ "smart pixels".

found by the counts ratio:

$$\langle P\rangle = \frac{N_H - N_V}{N}. \tag{2}$$

In contrast, with PM the photons end up in a region centered at $x = a\langle P\rangle$ (see Fig. 3b). A

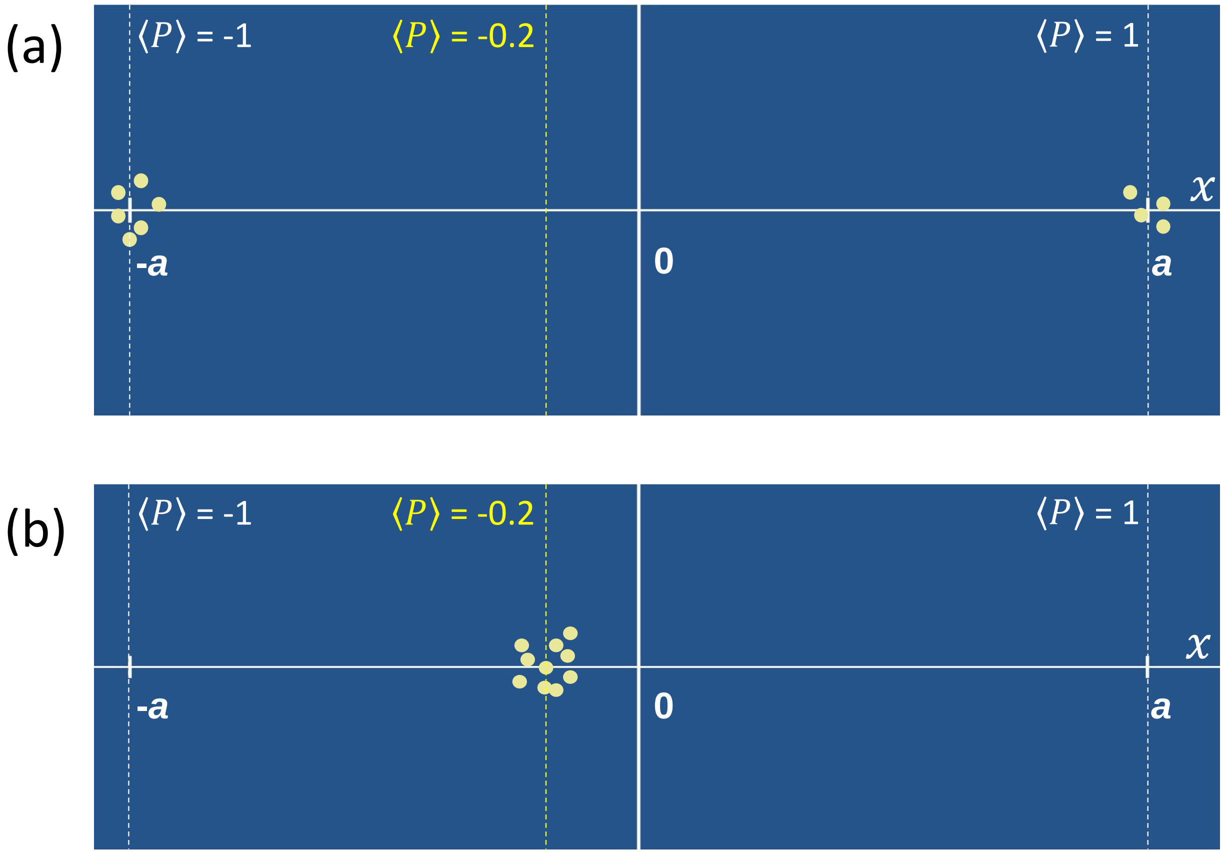


FIG. 3: **Illustrative drawing showing the measurement of unprotected (panel (a)) and protected (panel (b)) photons.** In the first case, 6 photons fall close to $x = -a$ (corresponding to $P = -1$), while 4 photons fall near $x = a$ (corresponding to $P = 1$), giving the expectation value $\langle P \rangle = -0.2$. In the second case, instead, all the photons accumulate close to $x = -0.2a$, the $\langle P \rangle = -0.2$ position; this indicates that, with PM, we can estimate the expected value of our observable even with a single photon.

large ensemble of measurements allows finding the center with arbitrarily good precision, but even a single photon detection provides information about $\langle P \rangle$, albeit with a finite precision defined by the distribution width.

In Fig. 4(a-d) we show the results obtained collecting heralded single photons for a measurement time of 1200 s. Panels (a) and (c) show, respectively, a histogram and a contour plot of the photon counts distribution observed in the unprotected case for the input state $|\psi_{\frac{17\pi}{60}}\rangle = 0.629|H\rangle + 0.777|V\rangle$. As in a standard Stern-Gerlach experiment, we observed photons only in two regions corresponding to the eigenvalues of $P$. The polarisation expectation value $\langle P\rangle$ evaluated using (2) from this distribution (dark counts subtracted) is $\langle P_{\frac{17\pi}{60}}\rangle = -0.21(4)$, in agreement with theoretical expectations, $\langle P_{\frac{17\pi}{60}}\rangle = -0.208$. Panels (b) and (d) show histogram and contour plot of the photon counts distribution obtained in the protected case for the same polarisation state. Instead of two distributions around $x = \pm a$, here we find a single distribution of photon detections centered very close to $x = \langle P\rangle a$. The measured expectation value is $\langle P_{\frac{17\pi}{60}}\rangle = -0.19(2)$ (dark counts subtracted). This result demonstrates that we have been able to realise and exploit the PM concept, providing the estimation of the polarisation operator, $\langle P\rangle$, by detecting a single photon.

This is further confirmed in Fig. 4 (e) and (f), presenting typical photon detection maps for the input state $|\psi_{\frac{17\pi}{60}}\rangle$ obtained from a small number of detected photons. Specifically, Fig. 4(e) and (f) correspond respectively to the unprotected ($N = 14$ detection events) and protected ($N = 17$ detection events) case; the circles drawn in the two figures represent the width of the distributions reported in Fig. 4(a-d). As expected, counts are clearly concentrated inside the circles -despite the non-negligible dark counts level of our non-ideal SPAD array, likely responsible for the detection events outside the circles-, demonstrating the validity of PM even when just few detections are considered. The first detected photons in the runs are signified with white pixels. We see that, while the white pixel of Fig. 4(f) provides a good estimate of $\langle P\rangle$, we cannot learn much from the white pixel of Fig. 4(e).

Indeed, using a single photon is what makes PMs special. However, Zeno protection ensures survival of the photon only for the ideal case of noiseless devices in the infinite protection operations limit $K \to \infty$. For our, non-ideal case, one could argue that our experiment concerns a single *post-selected* photon (i.e. that survived all protection stages) and, allowing post-selection, one can perform a measurement yielding the expectation value in the case of both weak and strong interaction.

To discuss quantitatively the performance of PM we compare it with the straightforward alternative, a projective measurement exploiting, e.g., a polarising beam-splitter (note that,

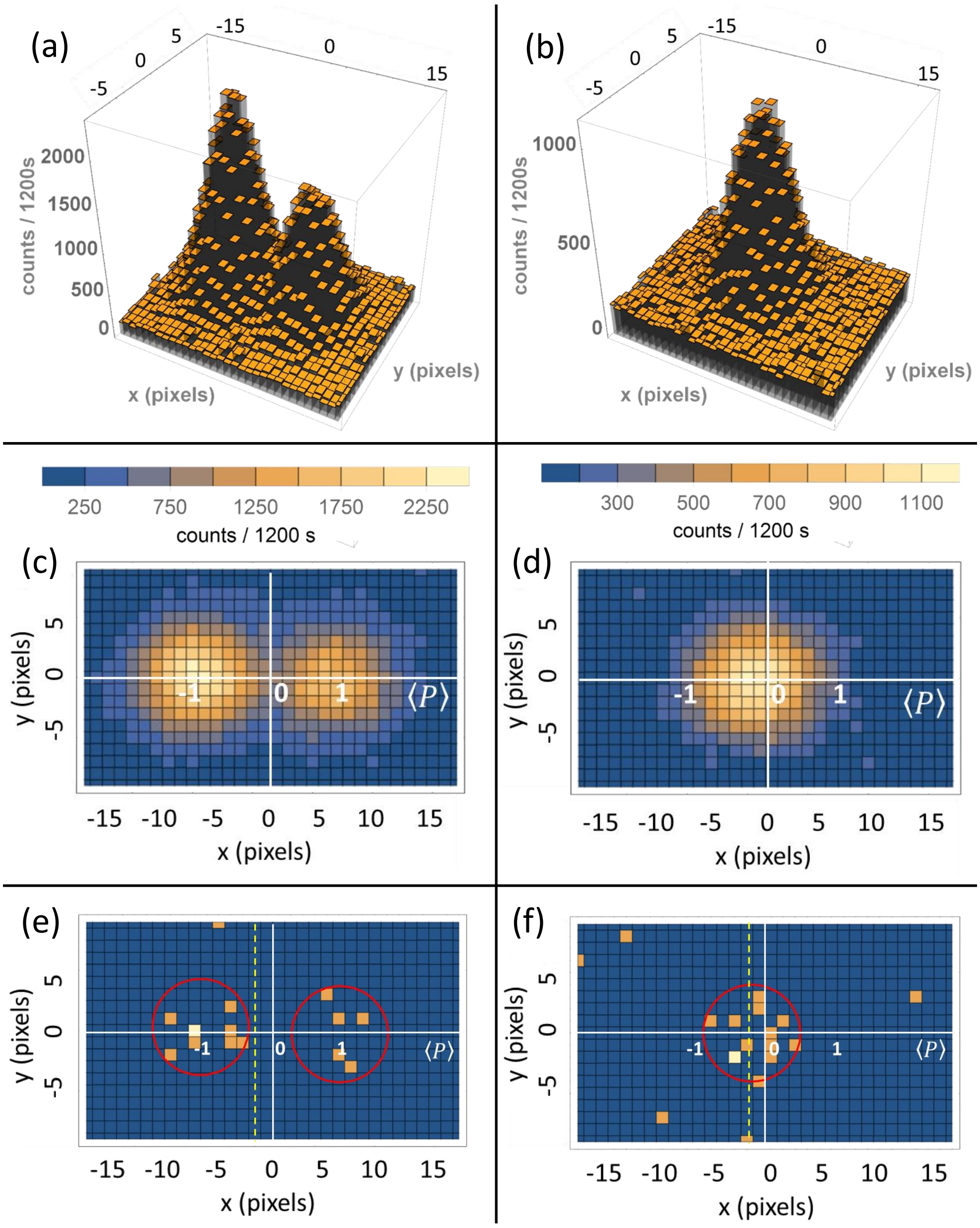


FIG. 4: **Results obtained for the input state** $|\psi_{\frac{17\pi}{60}}\rangle = \mathbf{0.629}|\mathbf{H}\rangle + \mathbf{0.777}|\mathbf{V}\rangle$**.** Panels (a) and (c): histogram and contour plot of the photon counts distribution obtained for the unprotected state. Panels (b) and (d): histogram and contour plot of the photon counts distribution obtained for the protected state. Panel (e): experiment with 14 single events (the first one in white), without protection. The yellow dashed line indicates the $x = a\langle P_{\frac{17\pi}{60}}\rangle$ value. Panel (f): experiment with 17 single events (the first one in white), with protection: as expected, all the photons accumulate around the $x = a\langle P_{\frac{17\pi}{60}}\rangle$ position (yellow dashed line).

when estimating $P$, the simple projective measurement saturates the Quantum Cramér-Rao bound [29]). For this purpose we plot in Fig. 5 the ratio $R = \frac{u_{\mathrm{PBS}}(P)}{u(P)}$ between the uncertainties on $\langle P \rangle$ in the two cases. We consider in both cases the same initial number of photons, taking into account the photons lost in PM (see Methods). We consider two different scenarios: $K = 7$ (yellow surface) and $K = 100$ (blue surface) interaction-protection stages. In both cases, PM is almost always advantageous ($R > 1$) with respect to the projective measurement, going below the $R = 1$ plane (in magenta) only for extremely weak interactions. In our experiment, with $\xi \sim 0.4$ and just $K = 7$, a 10% advantage is already present for most of the possible states, even if the maximum for $R$ corresponds to $\xi \sim 1$. For $K = 100$, instead, the reasonably weak interaction $\xi \sim 0.4$ grants the maximum of the advantage ($R > 8.5$ almost everywhere), while for stronger interaction the advantage is reduced to $R < 4$. The advantage of PM stems from the very high survival probability of the protected photons (see Methods). We also point out that our experiment is the first realisation of a "robust" WM [30] at single photon level.

This is the first experimental realisation of PMs [5]. Our results demonstrate that a single-event detection can provide reliable information regarding a certain property of a quantum system -the expectation value of the polarisation operator- supposed to be only statistical, belonging to an ensemble of identically-prepared quantum systems. In doing this, PMs require that prior information on the preparation stage is exploited in realising the protection. Although our results may not resolve the controversy regarding the meaning of PMs, they are of interest for all approaches. Proponents of the quantum state ontic interpretation should be excited to see this first single-particle measurement (they will argue that the necessity of protection is not surprising: every measurement obeys the Heisenberg uncertainty principle). At the same time, proponents of the minimalist approach, where only measurement outcomes exist, should also be interested to see a property of various preparation/protection methods of the quantum state directly inferred from a single photon detection as the pointer shift.

Furthermore, we demonstrate that PMs outperform the (traditional) optimal quantum-

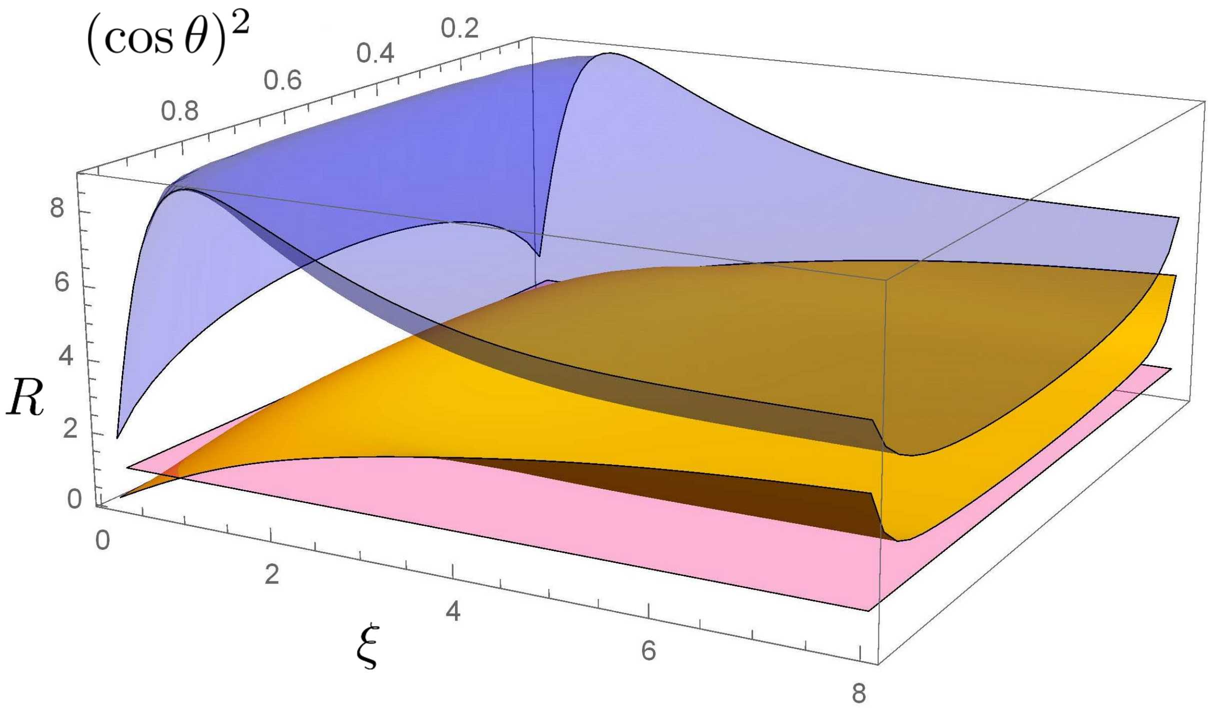


FIG. 5: **Comparison between the uncertainty on P with the PM approach (u(P)) and the one given by projective measurement ($\mathbf{u}_{\mathrm{PBS}}$(P)).** Yellow surface: ratio $R = \frac{u_{\mathrm{PBS}}(P)}{u(P)}$ for a PM scheme with $K = 7$ interaction-protection stages (as in our experiment), plotted versus the interaction strength $\xi$ and the $H$-polarisation component $(\cos\theta)^2$ of the single-photon state $|\psi\rangle$. Blue surface: ratio $R$ for a PM with $K = 100$ stages. Magenta surface: $R = 1$ bound, discriminating the part where PM approach is advantageous (above) and disadvantageous (below) with respect to the projective measurement.

measurements and could find several significant applications, e.g. when testing an unknown state preparation-protection procedure, given that both the state preparation and protection exploit the same projective measurement system (or equivalently a set of identical projective measurements, as in our case).

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**Author Contributions**

IPD, MGram, MGen (responsible of the laboratories) and LV (responsible for the theoretical framework) planned the experiment, with the support of FP, AA and EC. The experimental realization was achieved (supervised by IPD, GB, MGram and MGen) by FP (leading role), AA and ER. The SPAD camera was developed and optimized for this experiment by RL, FV and AT. The manuscript was prepared with inputs by all the authors. They also had a fruitful systematic discussion on the progress of the work.

**Competing financial interests**

The authors declare no competing financial interests.

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