Applying The Analytic Hierarchy Process to Rank Natural Threats to Power System Security

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APPLYING THE ANALYTIC HIERARCHY PROCESS TO RANK NATURAL THREATS TO POWER SYSTEM SECURITY

Simona - Louise VORONCA[1], Mihai - Marius VORONCA[2], Tao HUANG[3], Anca Alexandra PURCĂREA[4]

Power systems are contemporarily exposing to multiple threats and schematic changes per se. Always modest rather than adequate financial resources are allocated to justified and easy-to-adopt effective actions "against the most serious threat". Selecting a limited number of natural threats to work with, authors described in this paper the principal steps to achieve a related threats' ranking by using the analytical hierarchy process method. The key outcome is the knowledge provided to decision-makers to consolidate their actions of allotting the available financial resources to restrict possible effects of the most vigorous menace impact.

Keywords: power system, natural threats, analytic hierarchy process

1. Introduction

Severe failures in critical energy infrastructures operation dramatically impact the human kind and generate large-range undesired impacts. After power system blackout occurrence [1, 2], appreciable economic losses are directly and indirectly striking the society. Collateral damaging consequences in other infrastructures interfering with the power system are also significant (i.e. sanitary services, communications, transports etc.).

Different threats are the contemporary critical energy infrastructures exposing to. Several have a common denominator: the human factor. Accidental failures of aging equipment due to the lack of investment might appear most commonly as a result of inappropriate action. But last decade, a growth of malicious actions as result of the human factor aggressive and voluntary intervention was detected too. Other threats are the natural ones, which are closely associated with geologic phenomena [3]. Disasters of this kind are random, and

[1] Eng., National Power Grid Co. Transelectrica SA, Bucharest, Romania, e-mail: simona.voronca@transelectrica.ro

[2] Eng., Romanian Energy Efficiency Fund, Bucharest, Romania, e-mail: mihai.voronca@free.org.ro

[3] Senior Assistant Researcher, Department of Energy, Polytechnic of Turin, Italy, e-mail: tao.huang@polito.it

[4] Prof., Dean - Faculty of Entrepreneurship, Business Engineering and Management, University POLITEHNICA of Bucharest, Romania, e-mail: apurcarea@gmail.com

approximate information about their appearance frequencies coming from the statistical interpretation of related historical records seems to be reasonable [3].

To prepare the critical infrastructures against this last category of possible threats, namely the natural ones, the identification of the most important ones becomes the first and foremost step to launch effective and efficient actions with limited resources that can be used to decrease the exposure to those threats [4].

Given the characteristics already mentioned with regard to the natural threats and for elaboration purposes of this paper, they will be employed as examples to illustrate the threat ranking method proposed by the authors.

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2. The analytic hierarchy process: brief overview

The analytical hierarchy process is an application of the theory of decision-making using multi-criteria analysis [5, 6].

The method relies on the transformation of a multiple conflicting objectives decision-making problem into element (i.e. criteria, sub-criteria or alternatives) scores and weights, of assessment results extracted from subjective qualitative judgments on relative element importance [7].

To order items along a dimension such as preference or importance using an interval-type scale (Table 1), the pairwise comparison needs to be deployed.

Table 1

The fundamental scale of absolute numbers depicting the intensity of importance

Intensity	Definition	Comments
1	Element of equal importance with other	In the observer's judgement, both elements are practically identical
3	Element slightly more important than other	The judgement is in the favour of the element that the observer considers to be moderately more important
5	Element more important than other	The observer considers that the element is without any doubt of greater importance
7	Element strongly more important than other	In the observer's view, the element generously exceeds in importance the other
9	Element extremely more important than other	The observer considers the element as being of the greatest importance
2; 4; 6; 8;	Intermediate values	For refined judgements regarding the importance comparison, the observer can use intermediate values
1/2; 1/3; 1/4; 1/5 etc.	Reciprocal fractions	When an element is compared with another, the importance of this another element is a fraction having as denominator the importance of the initial element relative to another

Thurstone [8] was the first introducing a scientific approach of using pairwise comparison for measurement in connection with the psychophysical theory developed by Weber [9] and Fechner [10].

Through the analytic hierarchy process, a decision-making problem is decomposed into components and an ascending multi-level hierarchic component order is settled. At each hierarchic level, the related components are compared pairwise [5]. At any given level, components are related to an adjacent upper level, integration across the hierarchy levels thereby being generated. A set of priorities of relative importance, or a scaling method between various actions or alternatives is obtained [5].

For critical energy infrastructures possibly exposed to natural threats, assigning each of them with relative priority weights reveals the most damaging one. Finally, the financial resources allocation could target the most urgent needs.

2.1. Approximative eigenvectors and eigenvalues

Table 1 indicates the fundamental scale of absolute numbers used to perform the pairwise comparison. Let's consider n elements E_i $\{i = 1,2,\dots, n\}$.

Based on the observer's individual preferences, qualitative attributes of each considered pair of elements from the same level (E_i, E_j) $\{i, j = 1,2,\dots, n\}$ are converted into quantitative attributes stored in a square comparison matrix $\boldsymbol{E}$:

$$\boldsymbol{E} = \left(e_{ij}\right),\ e_{ij} = 1/e_{ji},\ i \neq j;\ e_{ii} = 1;\ (\forall)\, i = 1,2,\dots,n;\ j = 1,2,\dots,n. \tag{1}$$

By dividing each element by the sum of elements in the corresponding column in the comparison matrix $\boldsymbol{E}$, its normalised form $\boldsymbol{E}^{norm}$ is obtained (2). $\boldsymbol{E}^{norm}$ is a left stochastic matrix [11], and each matrix column is then a stochastic vector.

$$\frac{\boldsymbol{E} = \begin{pmatrix} 1 & e_{12} & \cdots & e_{1n} \\ e_{21} & 1 & \cdots & e_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ e_{n1} & e_{n2} & \cdots & 1 \end{pmatrix}}{\sum\limits_{i=1}^{n} e_{i1} \quad \sum\limits_{i=1}^{n} e_{i2} \quad \cdots \quad \sum\limits_{i=1}^{n} e_{in}} \Rightarrow \boldsymbol{E}^{norm} = \begin{pmatrix} \dfrac{1}{\sum\limits_{i=1}^{n} e_{i1}} & \dfrac{e_{12}}{\sum\limits_{i=1}^{n} e_{i2}} & \cdots & \dfrac{e_{1n}}{\sum\limits_{i=1}^{n} e_{in}} \\ \dfrac{e_{21}}{\sum\limits_{i=1}^{n} e_{i1}} & \dfrac{1}{\sum\limits_{i=1}^{n} e_{i2}} & \cdots & \dfrac{e_{2n}}{\sum\limits_{i=1}^{n} e_{in}} \\ \cdots & \cdots & \cdots & \cdots \\ \dfrac{e_{n1}}{\sum\limits_{i=1}^{n} e_{i1}} & \dfrac{e_{n2}}{\sum\limits_{i=1}^{n} e_{i2}} & \cdots & \dfrac{1}{\sum\limits_{i=1}^{n} e_{in}} \end{pmatrix}. \tag{2}$$

If $\boldsymbol{E}$ is fully consistent [12], its corresponding eigenvalue $\lambda^{\boldsymbol{E}}_{max}$ [13, 14] is identical with the matrix order n. Under such hypothesis, by calculating the arithmetic mean of elements from each row of $\boldsymbol{E}^{norm}$, an approximation $\boldsymbol{F}^{\approx}$ of the left stochastic eigenvector $\boldsymbol{F}$ associated with $\boldsymbol{E}$ is generated.

The operation is equivalent to (3) and represents the first iteration within the power method used [15]. $\boldsymbol{F}^{\approx}$ is then a left stochastic vector, too.

$$\boldsymbol{F}^{\approx} = \frac{1}{n}\left(\boldsymbol{E} \times \boldsymbol{F}_0^{\approx}\right); \boldsymbol{F}_0^{\approx} = \begin{pmatrix} 1 \\ 1 \\ \cdots \\ 1 \end{pmatrix}; \tag{3}$$

$$\boldsymbol{F}^{\approx} = \begin{pmatrix} F_1^{\approx} \\ F_2^{\approx} \\ \cdots \\ F_n^{\approx} \end{pmatrix}; F_j^{\approx} = \frac{1}{n}\left(\frac{e_{j1}}{\sum_{i=1}^{n} e_{i1}} + \frac{e_{j2}}{\sum_{i=1}^{n} e_{i2}} + \cdots + \frac{e_{jn}}{\sum_{i=1}^{n} e_{in}}\right), e_{jj} = 1, (\forall)\ j = \overline{1,2,...,n}. \tag{4}$$

When $\boldsymbol{E}$ is not fully consistent, a Perron - Frobenius theorem based methodology [5, 6], for checking matrix consistency is needed [13, 14]. The theorem stipulates that a square real matrix with nonnegative elements admits a dominant eigenvalue $\lambda^{\boldsymbol{E}}_{max}$, the dominant eigenvector having also nonnegative elements; moreover $\lambda^{\boldsymbol{E}}_{max}$ is a root [12] of the matrix characteristic equation (5).

$$\boldsymbol{E}\ \boldsymbol{F} = \lambda^{E}_{max}\ \boldsymbol{F}. \tag{5}$$

Using $\boldsymbol{F}^{\approx}$ in (5), a column vector $\boldsymbol{\lambda}^{\boldsymbol{E}\approx}_{\boldsymbol{max}}$ can be obtained as:

$$\boldsymbol{\lambda}^{\boldsymbol{E}\approx}_{\boldsymbol{max}} = \frac{\boldsymbol{E}\ \boldsymbol{F}^{\approx}}{\boldsymbol{F}^{\approx}} = \begin{pmatrix} \lambda^{E}_{max1} \\ \lambda^{E}_{max2} \\ \cdots \\ \lambda^{E}_{maxn} \end{pmatrix}, \tag{6}$$

the approximation of the dominant eigenvalue $\lambda^{\boldsymbol{E}\approx}_{max}$ of $\boldsymbol{E}$, resulting from $\boldsymbol{\lambda}^{\boldsymbol{E}\approx}_{\boldsymbol{max}}$ in (6), as arithmetic mean of elements. Based on approximate eigenvalue $\lambda^{\boldsymbol{E}\approx}_{max}$, the consistency index $CI^{\boldsymbol{E}}$ and the consistency ratio $CR^{\boldsymbol{E}}$ are determined in (7):

$$CI^{\boldsymbol{E}} = \frac{\lambda^{\boldsymbol{E}\approx}_{max} - n}{n-1}; CR^{\boldsymbol{E}} = \frac{CI^{\boldsymbol{E}}}{RI^{\boldsymbol{E}}} < 10\%, \tag{7}$$

where RI^{E} is the appropriate value of the random index [12]. The matrix consistency is usually acceptable if the consistency ratio CR^{E} is smaller than 10% [5, 6].

2.2. Hierarchic levels: goal, criteria and alternatives

To resolve a multi-criteria decision-making problem by using the analytic hierarchy process means to identify its components and to establish an ascending multi-level hierarchic component order.

In Fig. 1, one goal, n criteria and m alternatives on a three - level hierarchic order were considered.

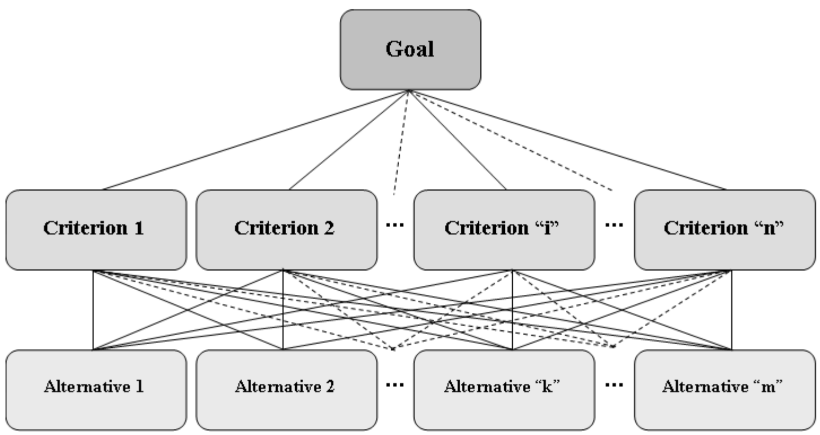

Fig. 1. The analytic hierarchy process: network model for "*m*" alternatives prioritization.

Calling the pairwise comparison for criteria in relation with the goal, a square criteria comparison matrix $\boldsymbol{C}$ is obtained:

$$\boldsymbol{C} = \left(c_{ij}\right),\ c_{ij} = 1/c_{ji},\ i \neq j;\ c_{ii} = 1;\ (\forall)\ i = 1,2,...,n;\ j = 1,2,...,n. \tag{8}$$

Based on (4) in section 2.1., in (9), the criteria priority vector $\boldsymbol{P^{C}}$ which is an approximation of eigenvector of matrix $\boldsymbol{C}$ from (8) is determined.

$$\boldsymbol{P^{C}} = \begin{pmatrix} P_1^C \\ P_2^C \\ \cdots \\ P_n^C \end{pmatrix};\ P_j^C = \frac{1}{n}\left(\frac{c_{j1}}{\sum\limits_{i=1}^{n} c_{i1}} + \frac{c_{j2}}{\sum\limits_{i=1}^{n} c_{i2}} + \cdots + \frac{c_{jn}}{\sum\limits_{i=1}^{n} c_{in}} \right),\ c_{jj} = 1,\ (\forall)\ j = 1,2,...,n. \tag{9}$$

Using (5-7, 9), the approximation of matrix $\boldsymbol{C}$ dominant eigenvalue $\lambda^{\boldsymbol{C}\approx}{}_{max}$ is calculated and the consistency can be verified. Calling the pairwise comparison for m alternatives in relation with each of the n criteria, n alternatives comparison square matrices $\boldsymbol{A}^{\boldsymbol{C}}{}_{\boldsymbol{i}}$ (i=1, 2…n), are obtained:

$$\boldsymbol{A}^{\boldsymbol{C}_i} = \left(a^{\boldsymbol{C}_i}{}_{kl}\right),\, a^{\boldsymbol{C}_i}{}_{kl} = 1/a^{\boldsymbol{C}_i}{}_{lk},\, k \neq l;\, a^{\boldsymbol{C}_i}{}_{kk} = 1;\, (\forall)\, k, l = 1,2,...,m;\, i = 1,2,...n. \quad (10)$$

For all matrices $\boldsymbol{A}^{\boldsymbol{C}}{}_{\boldsymbol{i}}$ from (10), and similarly to (4), n alternatives priorities vectors $\boldsymbol{P}^{\boldsymbol{AC}}{}_{\boldsymbol{i}}$ (i=1, 2…n) are determined in (11).

$$\boldsymbol{P}^{\boldsymbol{AC}_i} = \begin{pmatrix} P_1^{AC_i} \\ P_2^{AC_i} \\ \cdots \\ P_m^{AC_i} \end{pmatrix};$$

$$P_k^{AC_i} = \frac{1}{m}\left(\frac{a_{k1}^{C_i}}{\sum\limits_{k=1}^{m} a_{k1}^{C_i}} + \frac{a_{k2}^{C_i}}{\sum\limits_{k=1}^{m} a_{k2}^{C_i}} + \cdots + \frac{a_{km}^{C_i}}{\sum\limits_{k=1}^{m} a_{km}^{C_i}}\right),\ a_{kk}^{C_i} = 1, \quad (11)$$

$$(\forall)\ k = 1,2,...,m,\ i = 1,2,...,n.$$

Using (5-7, 11), n approximations of matrices $\boldsymbol{A}^{\boldsymbol{C}}{}_{\boldsymbol{i}}$ dominant eigenvalue $\lambda^{\boldsymbol{AC}\approx}{}_{\boldsymbol{i}\,max}$ (i=1, 2…n) are calculated and matrices $\boldsymbol{A}^{\boldsymbol{C}}{}_{\boldsymbol{i}}$ consistencies are verified. Aggregating all n alternatives' priority vectors $\boldsymbol{P}^{\boldsymbol{AC}}{}_{\boldsymbol{i}}$ (i=1, 2…n) from (11) into a *(n × m)* order matrix and weighting it with the criteria priority vector $\boldsymbol{P}^{\boldsymbol{C}}$, left stochastic vector $\boldsymbol{P}^{\boldsymbol{A}}$ of the alternatives' priorities relative to the goal is in (12).

$$\boldsymbol{P}^{\boldsymbol{A}} = \begin{pmatrix} P_1^{AC_1} & P_1^{AC_2} & \cdots & P_1^{AC_n} \\ P_2^{AC_1} & P_2^{AC_2} & \cdots & P_2^{AC_n} \\ \cdots & \cdots & \cdots & \cdots \\ P_m^{AC_1} & P_m^{AC_2} & \cdots & P_m^{AC_n} \end{pmatrix}\begin{pmatrix} P_1^C \\ P_2^C \\ \cdots \\ P_n^C \end{pmatrix} = \begin{pmatrix} \sum\limits_{i=1}^{n} P_1^{AC_i} P_i^C \\ \sum\limits_{i=1}^{n} P_2^{AC_i} P_i^C \\ \cdots \\ \sum\limits_{i=1}^{n} P_m^{AC_i} P_i^C \end{pmatrix} = \begin{pmatrix} P_1^A \\ P_2^A \\ \cdots \\ P_m^A \end{pmatrix}. \quad (12)$$

Based on the priorities of relative importance from (12), the observer could easily identify the greatest priority alternative and make decisions accordingly.

3. Case study: power sector threats prioritization

The proposed framework and methodology allow taking key decisions, by scaling between various alternatives and setting priorities or relative importance on a given selection of natural threats with extensive potential impact on the power system. A case study using the analytic hierarchy process method was developed. The most frequent causes of system disturbances appear to be natural phenomena, than communication/control failure, design and application error, operator error and primary equipment failure [16]. To mitigate risks due to natural phenomena, there are possible countermeasures enhancing preparedness, to reduce consequences or gravities, such as increasing the resilience of transmission equipment to remain reliable under a wider range of ambient conditions and/or operating the power system in a more secure mode than normal when such severe natural phenomena are forecast. For a power system geographically located in a temperate continental climate and seismically active area, five imminent natural threats were considered to exemplify the threat ranking method proposed: earthquakes (T_1), floods (T_2), blizzards (T_3), wild fires (T_4) and heat waves (T_5). With regard to criteria, the selection from [3] is used: likelihood (C_1), gravity (C_2) and preparedness (C_3).

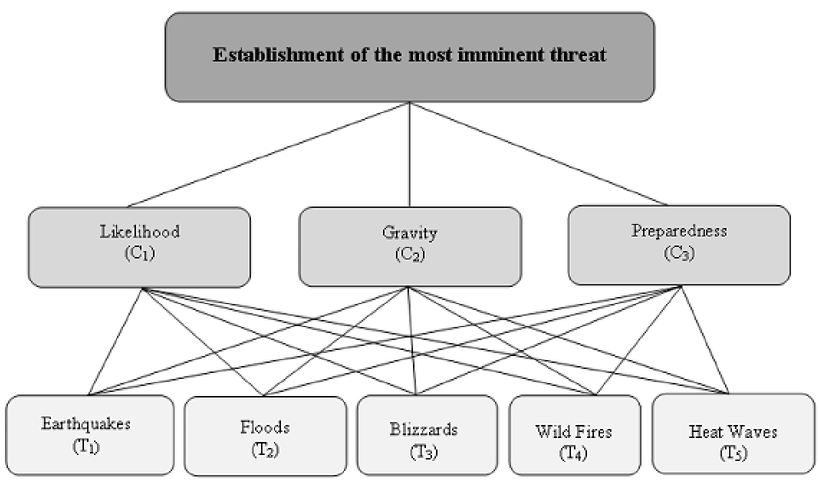

Fig. 2. The analytic hierarchy process: network model for threats prioritization.

In Fig. 2, the resulting multi-level hierarchic component order is presented. As indicated in Chapter 2, based on observers' judgements (Table 1), the criteria pairwise comparison with respect to the case study and their corresponding prioritization are to be performed. The threats pairwise comparison with respect to each of the criteria and their related relative prioritization are to be obtained, too.

3.1. Criteria prioritization

Assuming there is no dependence among the criteria [3], "Gravity" was considered more important than "Likelihood"; thus the number 7 was designated when comparing these two criteria. Likewise, "Preparedness" was considered more important than "Likelihood"; therefore the number 5 was used to mark their relative importance.

Based on (8-9), the criteria comparison matrix $\boldsymbol{C}$ of order n=3 and the criteria priority vector $\boldsymbol{P}^{\boldsymbol{C}}$ are found: The comparing results are reported in (13).

$$\boldsymbol{C} = \begin{pmatrix} 1 & 1/7 & 1/5 \\ 7 & 1 & 2 \\ 5 & 1/2 & 1 \end{pmatrix}; \boldsymbol{P}^{C} = \begin{pmatrix} 0.0755 \\ 0.5907 \\ 0.3338 \end{pmatrix}. \tag{13}$$

The most influential criterion is in line with each threat' consequences, while the likelihood is the least concern when considering the threats ranking [3].

Using (5-7, 9, 13), the approximate dominant eigenvalue $\lambda^{\boldsymbol{C}\approx}{}_{max}$ of matrix $\boldsymbol{C}$ gets the value 3.014177. The random index $RI^{\boldsymbol{C}}$ is equal to 0.5245 [12], and the resulting consistency ratio $CR^{\boldsymbol{C}}$ from (7) is 1.35%. The matrix $\boldsymbol{C}$ is consistent [5, 6].

3.2. Natural threats prioritization

With respect to criterion "Likelihood", within the considered geographical area, "Floods" and "Blizzards" were considered twice important than "Earthquakes"; similarly "Wild Fires" was awarded with the mark 2 and "Heat waves" received the mark 5.

Using (10-11), the resulting threats comparison matrix $\boldsymbol{A}^{\boldsymbol{C}}{}_{\boldsymbol{1}}$ and the threats priority vector $\boldsymbol{P}^{\boldsymbol{AC}}{}_{\boldsymbol{1}}$ are indicated in (14).

$$\boldsymbol{A}^{C_1} = \begin{pmatrix} 1 & 1/2 & 1/2 & 2 & 5 \\ 2 & 1 & 1 & 3 & 8 \\ 2 & 1 & 1 & 4 & 9 \\ 1/2 & 1/3 & 1/4 & 1 & 2 \\ 1/5 & 1/8 & 1/9 & 1/2 & 1 \end{pmatrix}; \boldsymbol{P}^{AC_1} = \begin{pmatrix} 0.1819 \\ 0.3288 \\ 0.3559 \\ 0.0926 \\ 0.0408 \end{pmatrix}. \tag{14}$$

Based on (5-7, 11, 14), the approximate dominant eigenvalue $\lambda^{AC}{}_{1}{}^{\approx}{}_{max}$ of matrix $\boldsymbol{A}^{\boldsymbol{C}}{}_{\boldsymbol{1}}$ is 5.015516. As the matrix $\boldsymbol{A}^{\boldsymbol{C}}{}_{\boldsymbol{1}}$ order is m=5, for the related random index value $RI^{AC}{}_{1}$ equal to 1.1086 [12]. With (7), the resulting consistency ratio $CR^{AC}{}_{1}$ is 0.35%, smaller than 10% [5, 6]. The matrix $\boldsymbol{A}^{\boldsymbol{C}}{}_{\boldsymbol{1}}$ consistency is verified.

With respect to criterion “Gravity”, “Earthquakes”, “Blizzards” and “Heat waves ” were considered of equal importance against “Floods” and “Wild Fires” which were judged twice and three times less important. With (10-11), the threats comparison matrix $\boldsymbol{A}^{\boldsymbol{C}}{}_{\boldsymbol{2}}$ and the threats priority vector $\boldsymbol{P}^{\boldsymbol{AC}}{}_{\boldsymbol{2}}$ are showed in (15).

$$\boldsymbol{A}^{C_2} = \begin{pmatrix} 1 & 2 & 1 & 3 & 1 \\ 1/2 & 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 3 & 1 \\ 1/3 & 1/2 & 1/3 & 1 & 1/3 \\ 1 & 1 & 1 & 3 & 1 \end{pmatrix}; \boldsymbol{P}^{AC_2} = \begin{pmatrix} 0.2672 \\ 0.1881 \\ 0.2308 \\ 0.0831 \\ 0.2308 \end{pmatrix}. \tag{15}$$

With relations (5-7, 11, 15) is obtained the approximation of matrix $\boldsymbol{A}^{\boldsymbol{C}}{}_{\boldsymbol{2}}$ dominant eigenvalue $\lambda^{AC}{}_{2}{}^{\approx}{}_{max}$ which is 5.055456. For the matrix $\boldsymbol{A}^{\boldsymbol{C}}{}_{\boldsymbol{2}}$ order m=5, the related random index value $RI^{AC}{}_{2}$ is equal to 1.1086 [12], and from (7), the resulting consistency ratio $CR^{AC}{}_{2}$ is 1.25%. As $CR^{AC}{}_{2}$ is smaller than 10% [5, 6], the matrix $\boldsymbol{A}^{\boldsymbol{C}}{}_{\boldsymbol{2}}$ shows consistency.

With respect to criterion “Preparedness”, “Earthquakes” and “Blizzards” are of identical importance, “Blizzards” and “Floods” were awarded with the mark 2 and the mark 5 for “Heat waves”. Using (10-11), the threats comparison matrix $\boldsymbol{A}^{\boldsymbol{C}}{}_{\boldsymbol{3}}$ and the threats priority vector $\boldsymbol{P}^{\boldsymbol{AC}}{}_{\boldsymbol{3}}$ are obtained.

$$\boldsymbol{A}^{C_3} = \begin{pmatrix} 1 & 1 & 2 & 2 & 5 \\ 1 & 1 & 2 & 2 & 5 \\ 1/2 & 1/2 & 1 & 1 & 3 \\ 1/2 & 1/2 & 1 & 1 & 2 \\ 1/5 & 1/5 & 1/3 & 1/2 & 1 \end{pmatrix}; \boldsymbol{P}^{AC_3} = \begin{pmatrix} 0.3122 \\ 0.3122 \\ 0.1624 \\ 0.1498 \\ 0.0634 \end{pmatrix}. \tag{16}$$

Using relations (5-7, 11, 16), the approximate dominant eigenvalue $\lambda^{AC}{}_{3}{}^{\approx}{}_{max}$ of matrix $\boldsymbol{A^{C}}_{3}$ gets the value 5.013279. For the order m=5 of the matrix $\boldsymbol{A^{C}}_{3}$, the related value [12] of random index value, $RI^{AC}{}_{3}$ equal to 1.1086 and with (7), the resulting consistency ratio $CR^{AC}{}_{3}$ is 0.30% and the matrix $\boldsymbol{A^{C}}_{3}$ consistency results valid [5, 6]. With (12), the threats priorities vector $\boldsymbol{P^{A}}$ is determined (17).

$$\boldsymbol{P}^{A} = \begin{pmatrix} 0.1819 & 0.2672 & 0.3122 \\ 0.3288 & 0.1881 & 0.3122 \\ 0.3559 & 0.2308 & 0.1624 \\ 0.0926 & 0.0831 & 0.1498 \\ 0.0408 & 0.2308 & 0.0634 \end{pmatrix} \begin{pmatrix} 0.0755 \\ 0.5907 \\ 0.3338 \end{pmatrix} = \begin{pmatrix} 0.2758 \\ 0.2401 \\ 0.2174 \\ 0.1061 \\ 0.1606 \end{pmatrix}. \tag{17}$$

Finally, the natural threats prioritisation is presented in Table 2.

Table 2

The ranking of natural threats

Ranking	Natural Threat	Natural Threat Priority Value
1	Earthquake	0.2758
2	Floods	0.2401
3	Blizzards	0.2174
4	Heat waves	0.1606
5	Wild Fires	0.1061

4. Discussions

Based on (5), all characteristic equations associated to matrices $\boldsymbol{C}$, $\boldsymbol{A^{C}}_{1}$, $\boldsymbol{A^{C}}_{2}$ and $\boldsymbol{A^{C}}_{3}$ have been resolved and solutions for each dominant eigenvalue $\lambda^{C\approx}{}_{max}$, $\lambda^{AC}{}_{1}{}^{\approx}{}_{max}$, $\lambda^{AC}{}_{2}{}^{\approx}{}_{max}$, $\lambda^{AC}{}_{3}{}^{\approx}{}_{max}$ have been obtained. Compared with the values associated to each dominant eigenvalue from section 3, relative errors vary within 0.0002 % - 0.3216 %. As each dominant eigenvalue is the arithmetic mean of related approximate eigenvector elements, results (17) accuracy is satisfactory.

5. Conclusions

A multi-criteria decision making problem has been resolved using the analytic hierarchic process. As a study case, five natural threats under three independent criteria - likelihood, gravity and preparedness were selected as representative natural threats menacing a power system located in a certain geographical area. Based on expert judgements, their ranking has been settled. The most imminent one out of the five natural threat evaluated to which the power system security is exposed is “Earthquakes”, followed by “Floods”, “Blizzards”, “Heat waves ” and “Wild Fires”. Since the power systems from all over the world are exposed to more than 40 natural, accidental, malicious threats [3], this replicable approach, rather than prescribing a “correct” decision, helps to set the priorities that best suits the goals and allows extended scientific investigations in order to tackle more complicated multi-criteria decision making challenges.

The proposed framework and threats ranking model showed a great potential in finding the most imminent threats. Also, the multi-criteria decision could provide the ranking on preventive countermeasures in terms of allotting financial resources to reduce both likelihood and impacts before the materialization of the most damaging threat occurs.

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