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A MULTILAYERED PLATE ELEMENTS ACCOUNTING NODE-DEPENDENT KINEMATICS FOR STATIC ANALYSIS OF PIEZOELECTRIC STRUCTURES

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Abstract. The present work deals with the static analysis of multilayered plates embedding piezoelectric layers as actuators and sensors. Finite Elements (FE) with equivalent-single-layer and layer-wise capabilities are employed to ensure an accurate description of the mechanical and electric fields in the layers. A new class of finite elements for the analysis of composite and piezoelectric multilayered plates is proposed. By making use of node-dependent plate theory assumptions, the new finite element allows for the simultaneous analysis of different subregions of the problem domain with different kinematics and accuracy, in a global/local sense. The structural theory of the plate element is a property of the FE node in this present approach, and the continuity between two adjacent elements is ensured by adopting the same kinematics at the interface nodes. The main advantage of the present variable-kinematics element and related global/local approach is that no ad-hoc techniques and mathematical artifices are required to mix the fields coming from two different and kinematically incompatible adjacent elements, because the plate structural theory varies within the finite element itself. It is possible to reduce the computational costs by assuming refined theories only in those zones/nodes of the structural domain where the resulting strain and stress states present a complex distribution. At the same time, computationally cheaper, low-order models can be used in the remaining parts of the plate where a localized detailed analysis is not necessary. With node-dependent kinematics it is possible to keep the order of the expansion of the state variables and models along the main reference plane of the plate structure as a parameter of the model. The electrical potential assumption for the layered actuators and sensors has been extended, from a layer-wise (LW) modeling, to an equivalent-single-layer (ESL) description, in the same way the displacements assumptions on the composite layers are described by a ESL and LW models. Some results from the static analysis of plates under electro-mechanical loads will be provided, in order to show the efficiency of models presented.

1 INTRODUCTION

Plate structures have a predominant role in a variety of engineering applications. Nevertheless, the use of new materials, such as composites layered materials and/or piezoelectric layers, leads to increasingly complex structural designs that require careful and detailed analysis. The analysis of layered composite structures is complicated in practice. In some cases, structures may contain regions where three-dimensional (3D) stress fields occur. To accurately capture these localized 3D stress states, solid models or higher-order theories are necessary. However, the high computational costs represent the drawback of refined plate theories or three-dimensional analyses. Analytical solution for general smart structural problems is a very tough task, and they exist, only, for very few specialized and idealized cases. Meanwhile, the finite element method has become the most widely used technique to model various physical processes, including piezoelectricity. The introduction of piezoelectric material into a passive structure naturally leads to a multilayered component, and it has been recognized that classical models are not suitable for an accurate design of such structures, see for example the review article of Noor and Burton [1]. The fundamentals of the modeling of piezoelectric materials have been given in many contributions, in particular in the pioneering works of Mindlin [2], EerNisse [3], Tiersten and Mindlin [4], and in the monograph of Tiersten [5]. The embedding of piezoelectric layers into plates and shells sharpens the requirements of an accurate modeling of the resulting adaptive structure due to the localized electro-mechanical coupling, see e.g. the review of Saravanos and Heyliger [6]. Some of the latest contributions to the Finite Elements (FEs) analysis of piezoelectric shells that are based on exact geometry solid-shell element was developed by Kulikov et al. [7, 8], and a piezoelectric solid-shell element with a mixed variational formulation and a geometrically nonlinear theory was developed by Klinkel et al. [9]. Although the enormous improvements and formulations of higher-order plate structural theories, considerable work has been recently directed towards the implementation of innovative solutions for improving the analysis efficiency for complex geometries and assemblies, possibly in a global/local scenario. In this manner, the limited computational resources can be distributed in an optimal manner to study in detail only those parts of the structure that require an accurate analysis. In general, two main approaches are available to deal with a global/local analysis: (1) refining the mesh or the FE shape functions in correspondence with the critical domain; (2) formulating multi-model methods, in which different subregions of the structure are analysed with different mathematical models. The first mentioned approaches can be addressed as single-theory or single-model methods. Differently, in the case of multi-theory methods, in which different subregions of the structure are analysed with different structural theories with kinematically incompatible elements, the compatibility of displacements and equilibrium of stresses at the interface between dissimilar elements have to be achieved. A wide variety of multiple model methods have been reported in the literature. In general, multi-theory methods can be divided into sequential or multistep methods, and simultaneous methods. In a sequential multi-model, the global region is analysed with an adequate model with a cheap computational cost to determine the displacement or force boundary conditions for a subsequent analysis at the local level. The local region can be modeled with a more refined theory, or it can be modeled with 3-D finite elements, see [10, 11]. The simultaneous multi-model methods are characterized by the analysis of the entire structural domain, where different subregions are modeled with different mathematical models and/or distinctly different

levels of domain discretization, in a unique step. A well-known method to couple incompatible kinematics in multi-model methods, is the use of Lagrange multipliers, which serve as additional equations to enforce compatibility between adjacent subregions. In the three-field formulation by Brezzi and Marini [12], an additional grid at the interface is introduced. The unknowns are represented independently in each sub-domain and at the interface, where the matching is provided by suitable Lagrange multipliers. Ben Dhia *et al.* [13, 14] proposed the Arlequin method to couple different numerical models by means of a minimization procedure. This method was adopted by Biscani *et al.* [15] for the mechanical analysis of plates and by Biscani *et al.* [16] for the electro-mechanical analysis of plates.
In the present work, the hierarchical characteristics of Unified Formulation (UF) by Carrera are used to develop a new simultaneous multiple-model method for 2D elements with node-dependent kinematics. This node-variable capability given by UF enables one to vary the kinematic assumptions within the same finite plate element. The expansion order of the plate element is, in fact, a property of the FE node in the present approach. Therefore, between finite elements, the continuity is ensured by adopting the same expansion order in the nodes at the element interface. This method has shown its potentiality in the mechanical analysis of plate structure, see [17]. Here the node-dependent variable kinematics is extended to the electro-mechanical analysis of multilayered plate structure. In this manner, global/local models can be formulated without the use of any mathematical artifice. As a consequence, computational costs can be reduced assuming refined models only in those zones with a quasi-three-dimensional stress field, whereas computationally cheap, low-order kinematic assumptions are used in the remaining parts of the plate structure.

2 REFINED AND HIERARCHICAL THEORIES FOR PLATES

This work proposes a class of new finite elements which allows employing different kinematic assumptions in different subregions of the problem domain. To highlight the capabilities of the novel formulation, a four-node plate elements with node-dependent kinematics is shown in Figure 1. The element proposed in this example makes use of a second-order ESL refined theory at node 1 and 8. On the other hand, a fourth-order ESL refined theory are employed at node 6 and 7, respectively. Therefore, a first-order layer-wise theory is used at node 3 and 4. Finally, a second-order layer-wise plate theory is assumed at node 2 and 5. As it will be clear later in this paper, thanks to the hierarchical capabilities of UF by Carrera, the choice of the nodal plate theory is arbitrary and variable-kinematics plate elements will be used to implement multi-model methods for global-local analysis. Before discussing the present formulation, a brief overview of classical and higher-order plate theories is given below or the sake of completeness. Plates are bi-dimensional structures in which one dimension (in general the thickness in the z direction) is negligible with respect to the other two dimensions.

Higher Order Theories written in Unified Formulation framework. According to Unified Formulation by Carrera [18], refined models can be formulated in a straightforward manner by assuming an expansion of each of the primary variables by arbitrary functions in the thickness direction. Thus, each variable can be treated independently from the others, according to the required accuracy. This procedure becomes extremely useful when multifield problems are in-

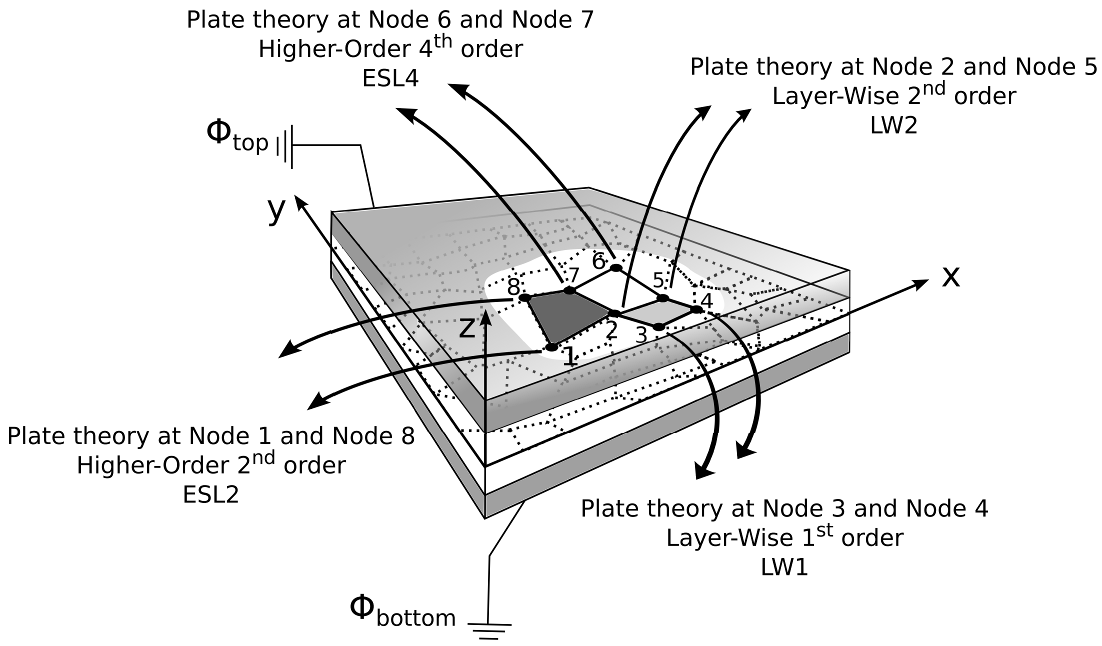

Figure 1: Node-dependent kinematic finite element example for a piezoelectric plate.

vestigated such as thermoelastic and piezoelectric applications [19, 20]. In a displacement-based formulation, UF states, in fact, that the three-dimensional displacement field and the electric potential are the combination of through-the-thickness functions weighted by the generalized unknown variables:

$$\begin{aligned} u(x,y,z) &= F_0(z)\,u_0(x,y) + F_1(z)\,u_1(x,y) + ... + F_N(z)\,u_N(x,y) \\ v(x,y,z) &= F_0(z)\,v_0(x,y) + F_1(z)\,v_1(x,y) + ... + F_N(z)\,v_N(x,y) \\ w(x,y,z) &= F_0(z)\,w_0(x,y) + F_1(z)\,w_1(x,y) + ... + F_N(z)\,w_N(x,y) \\ \Phi(x,y,z) &= F_0(z)\,\Phi_0(x,y) + F_1(z)\,\Phi_1(x,y) + ... + F_N(z)\,\Phi_N(x,y) \end{aligned} \tag{1}$$

Similarly, in a compact form one has:

$$\begin{aligned} \boldsymbol{u}(x,y,z) &= F_s(z)\boldsymbol{u}_s(x,y) \qquad s = 0,1,...,N \\ \Phi(x,y,z) &= F_s(z)\Phi_s(x,y) \qquad s = 0,1,...,N \end{aligned} \tag{2}$$

where $\boldsymbol{u}(x,y,z)$ is the three-dimensional displacement vector, and $\Phi(x,y,z)$ is the electric potential; F_s are the thickness functions depending only on z; $\boldsymbol{u}_s$ is the generalized displacement vector of the variables; s is a sum index; and N is the number of terms of the theory expansion. Depending on the choice of the thickness functions, F_s, and the number of terms in the plate kinematics, N, various theories can be implemented.

For example, higher-order theories can be expressed by making use of Taylor-like expansions of the generalized unknowns along the thickness. In the case of generic expansions of N terms, HOT displacement field and the electric potential can be expressed as in equation 3.

$$
\begin{aligned}
u(x,y,z) &= u_0(x,y) + z\,u_1(x,y) + ... + z^N\,u_N(x,y)\\
v(x,y,z) &= v_0(x,y) + z\,v_1(x,y) + ... + z^N\,v_N(x,y)\\
w(x,y,z) &= w_0(x,y) + z\,w_1(x,y) + ... + z^N\,w_N(x,y)\\
\Phi(x,y,z) &= \Phi_0(x,y) + z\,\Phi_1(x,y) + ... + z^N\,\Phi_N(x,y)
\end{aligned}
\tag{3}
$$

The classical models, CLT and FSDT kinematics, are particular cases of the full linear expansion, obtained from equation 3 imposing $N = 1$. For more details see [18].
The limitations, due to expressing the unknown variables in function of the midplane position of the plate, can be overcome in several ways. A possible solution can be found employing the Legendre polynomials. They permit to express the unknown variables in function of the top and bottom position of a part of the plate thickness. In the case of Legendre-like polynomial expansion models, the displacements and the electric potential are defined as follows:

$$
\begin{aligned}
\boldsymbol{u} &= F_0\,\boldsymbol{u}_0 + F_1\,\boldsymbol{u}_1 + F_r\,\boldsymbol{u}_r = F_s\,\boldsymbol{u}_s, \qquad s = 0,1,r\ , \quad r = 2,...,N\\
\Phi &= F_0\,\Phi_0 + F_1\,\Phi_1 + F_r\,\Phi_r = F_s\,\Phi_s, \qquad s = 0,1,r\ , \quad r = 2,...,N
\end{aligned}
\tag{4}
$$

$$
F_0 = \frac{P_0 + P_1}{2}, \quad F_1 = \frac{P_0 - P_1}{2}, \quad F_r = P_r - P_{r-2}. \tag{5}
$$

in which $P_j = P_j(\zeta)$ is the Legendre polynomial of j-order defined in the ζ-domain: $-1 \leq \zeta \leq 1$. $P_0 = 1$, $P_1 = \zeta$, $P_2 = (3\zeta^2 - 1)/2$, $P_3 = (5\zeta^3 - 3\zeta)/2$, $P_4 = (35\zeta^4 - 30\zeta^2 + 3)/8$.
For the Layer-Wise (LW) models, the Legendre polynomials and the relative top and bottom position are defined for each layer k. For the Equivalent Single Layer model, the Legendre polynomials and the relative top and bottom position are defined for the whole multilayer.

3 FINITE ELEMENTS WITH NODE-DEPENDENT KINEMATICS

Thanks to UF by Carrera, FEM arrays of classical to higher-order plate theories can be formulated in a straightforward and unified manner by employing a recursive index notation. By utilizing an FEM approximation, the generalized displacements of equation2 can be expressed as a linear combination of the shape functions to have

$$
\begin{aligned}
\boldsymbol{u}_s(x,y) &= N_j(x,y)\boldsymbol{u}_{s_j} \qquad j = 1,...,\text{(nodes per element)}\\
\boldsymbol{\Phi}_s(x,y) &= N_j(x,y)\boldsymbol{\Phi}_{s_j} \qquad j = 1,...,\text{(nodes per element)}
\end{aligned}
\tag{6}
$$

where $\boldsymbol{u}_{s_j}$ is the vector of the generalized displacements nodal unknowns, $\boldsymbol{\Phi}_{s_j}$ is the vector of the generalized electric potential nodal unknowns and N_j can be the usual Lagrange shape functions. j denotes a summation on the element nodes. Since the principle of virtual displacements in used in this paper to obtain the elemental FE matrices, it is useful to introduce the finite element approximation of the virtual variation of the generalized displacement vector $\delta\boldsymbol{u}_\tau$, and the virtual variation of the generalized electric potential $\delta\boldsymbol{\Phi}_\tau$,

$$
\begin{aligned}
\delta\boldsymbol{u}_\tau(x,y) &= N_i(x,y)\delta\boldsymbol{u}_{\tau_i} \qquad i = 1,...,\text{(nodes per element)}\\
\delta\boldsymbol{\Phi}_\tau(x,y) &= N_i(x,y)\delta\boldsymbol{\Phi}_{\tau_i} \qquad i = 1,...,\text{(nodes per element)}
\end{aligned}
\tag{7}
$$

In equation 7, δ denotes the virtual variation, whereas indexes τ and i are used instead of s and j, respectively, for the sake of convenience.
In this work, and according to equations 2, 6 and 7, the thickness functions F_s and F_τ, which determine the plate theory order, are independent variables and may change for each node within the plate element. Namely, the three-dimensional displacement field, the electric potential and their related virtual variations can be expressed to address FE node-dependent plate kinematics as follows:

$$
\begin{aligned}
\boldsymbol{u}(x,y,z) &= F_s^j(z)N_j(x,y)\boldsymbol{u}_{s_j} \qquad s=0,1,...,N^j \quad j=1,...,(\text{nodes per element})\\
\Phi(x,y,z) &= F_s^j(z)N_j(x,y)\boldsymbol{\Phi}_{s_j} \qquad s=0,1,...,N^j \quad j=1,...,(\text{nodes per element})\\
\delta\boldsymbol{u}(x,y,z) &= F_\tau^i(z)N_i(x,y)\delta\boldsymbol{u}_{\tau_i} \qquad \tau=0,1,...,N^i \quad i=1,...,(\text{nodes per element})\\
\delta\Phi(x,y,z) &= F_\tau^i(z)N_i(x,y)\delta\boldsymbol{\Phi}_{\tau_i} \qquad \tau=0,1,...,N^i \quad i=1,...,(\text{nodes per element})
\end{aligned}
\tag{8}
$$

where the subscripts τ, s, i, and j denote summation. Superscripts i and j denote node dependency, such that for example F_τ^i is the thickness expanding function and N^i is the number of expansion terms at node i, respectively.

For the sake of clarity, the displacement field of a variable kinematic plate element as discussed in Figure 1 is described in detail hereafter. For example, taking into account the description of the displacement field of the darkest element, the global displacement field of the element is approximated as follows:

- Node 1 Plate Theory = HOT with $N^1 = 2$ Eq. (3)
- Node 2 Plate Theory = LW with $N^2 = 2$ Eq. (4)
- Node 7 Plate Theory = HOT with $N^7 = 4$ Eq. (3)
- Node 8 Plate Theory = HOT with $N^8 = 2$ Eq. (3)

In a CUF-based FE framework and according to equation 8, it is easy to verify that the displacements at a generic point belonging to the plate element can be expressed as given in equation 9. In this equation, only the displacement component along x-axis is given for simplicity reasons:

$$
\begin{aligned}
u(x,y,z) = &\left(u_{0_1} + z\,u_{1_1} + z^2\,u_{2_1}\right)\,N_1(x,y) + \left[\left(\frac{1+\zeta_k}{2}\right)u_{0_2} + \left(\frac{1-\zeta_k}{2}\right)u_{1_2} + \right.\\
&\left. + \left(\frac{3\zeta_k^2-1}{2}-1\right)u_{2_2}\right] N_2(x,y) + \left(u_{0_7} + z\,u_{1_7} + z^2\,u_{2_7} + z^3\,u_{3_7} + z^4\,u_{4_7}\right)\,N_7(x,y)+\\
&+ \left(u_{0_8} + z\,u_{1_8} + z^2\,u_{2_8}\right)\,N_8(x,y)
\end{aligned}
\tag{9}
$$

It is intended that, due to node-variable expansion theory order, the assembling procedure of each finite element increases in complexity with respect to classical mono-theory finite elements. In the present FE approach, in fact, it is clear that both rectangular and square arrays are handled and opportunely assembled for obtaining the final elemental matrices.
Therefore, to overcome the numerical problems related to the shear locking, it is possible to use many computational procedures, such as reduced integration, selective integration [21], and the mixed interpolation of tensorial components (MITC) [22]. In this paper, a MITC technique is used to overcome the shear locking phenomenon, for more details see [20].

3.1 FUNDAMENTAL NUCLEUS OF THE STIFFNESS MATRIX

Given UF and FE approximation, the governing equations for the static response analysis of the multi-layer plate structure can be obtained by using the principle of virtual displacements, which states:

$$\int_{\Omega_k}\int_{A_k} \left\{\delta\boldsymbol{\epsilon}_k^T\boldsymbol{\sigma}_k - \delta\boldsymbol{\mathcal{E}}_k^T\boldsymbol{\mathcal{D}}_k\right\} d\Omega_k dz_k = \delta L_e \tag{10}$$

where the term on the left-hand side represents the virtual variation of the strain energy; Ω and A are the integration domains in the plane and the thickness direction, respectively; $\boldsymbol{\epsilon}$ and $\boldsymbol{\sigma}$ are the vector of the strain and stress components; $\boldsymbol{\mathcal{E}}$ and $\boldsymbol{\mathcal{D}}$ are the vector of the electric field and electric displacements components; and δL_e is the virtual variation of the external loadings; k represents the n^{th} layer, and T means the vector transposition. The constitutive equations are the following:

$$\begin{aligned}\boldsymbol{\sigma}_k &= \boldsymbol{C}_k\boldsymbol{\epsilon}_k - \boldsymbol{e}_k\boldsymbol{\mathcal{E}}_k\\ \boldsymbol{\mathcal{D}}_k &= \boldsymbol{e}_k\boldsymbol{\epsilon}_k + \boldsymbol{\varepsilon}_k\boldsymbol{\mathcal{E}}_k\end{aligned} \tag{11}$$

where $\boldsymbol{C}$ is the vector of the material stiffness coefficients, $\boldsymbol{e}$ is the vector of the piezoelectric stiffness coefficients, and $\boldsymbol{\varepsilon}$ is the vector of the permittivity coefficients. The mechanical strains $\boldsymbol{\epsilon}$ and the electric field $\boldsymbol{\mathcal{E}}$ are related to the mechanical displacements $\boldsymbol{u}$ and the electric potential Φ via the geometrical relations as follows:

$$\begin{aligned}\boldsymbol{\epsilon} &= \boldsymbol{D}_g\boldsymbol{u}\\ \boldsymbol{\mathcal{E}} &= -\boldsymbol{D}_{eg}\Phi\end{aligned} \tag{12}$$

where $\boldsymbol{D}_g$ and $\boldsymbol{D}_{eg}$ are the vectors containing the differential operators defined as follows:

$$\boldsymbol{D}_g = \begin{bmatrix} \frac{\partial}{\partial_x} & 0 & 0 \\ 0 & \frac{\partial}{\partial_y} & 0 \\ \frac{\partial}{\partial_y} & \frac{\partial}{\partial_x} & 0 \\ \frac{\partial}{\partial_z} & 0 & \frac{\partial}{\partial_x} \\ 0 & \frac{\partial}{\partial_z} & \frac{\partial}{\partial_y} \\ 0 & 0 & \frac{\partial}{\partial_z} \end{bmatrix} \qquad \boldsymbol{D}_e g = \begin{bmatrix} \frac{\partial}{\partial_x} \\ \frac{\partial}{\partial_y} \\ \frac{\partial}{\partial_z} \end{bmatrix} \tag{13}$$

By substituting the constitutive equations 11 for composite elastic materials, the linear geometrical relations 12 as well as equation 8 into equation 10, the linear algebraic system in the form of governing equations is obtained in the following matrix expression:

$$\begin{aligned}\delta\boldsymbol{u}_{\tau i}^k &: \boldsymbol{K}_{uu}^{k\tau sij}\boldsymbol{u}_{sj}^k + \boldsymbol{K}_{u\Phi}^{k\tau sij}\Phi_{sj}^k = \boldsymbol{P}_{u_{\tau i}}^k\\ \delta\boldsymbol{\Phi}_{\tau i}^k &: \boldsymbol{K}_{\Phi u}^{k\tau sij}\boldsymbol{u}_{sj}^k + \boldsymbol{K}_{\Phi\Phi}^{k\tau sij}\Phi_{sj}^k = \boldsymbol{P}_{\Phi_{\tau i}}^k\end{aligned} \tag{14}$$

In compact form:

$$\delta \boldsymbol{u}^k_{\tau i} \, : \, \boldsymbol{K}^{k\tau sij} \, \boldsymbol{u}^k_{sj} \, = \, \boldsymbol{P}^k_{\tau i} \tag{15}$$

where

$$\boldsymbol{K}^{k\tau sij} = \begin{bmatrix} \boldsymbol{K}_{uu} & \boldsymbol{K}_{u\Phi} \\ \boldsymbol{K}_{\Phi u} & \boldsymbol{K}_{\Phi\Phi} \end{bmatrix}^{k\tau sij} \tag{16}$$

where $\boldsymbol{K}^{k\tau sij}$ is a 4×4 matrix, called fundamental nucleus of the electro-mechanical stiffness matrix, and its explicit expression is given in [23]. The mechanical part $\boldsymbol{K}^{k\tau sij}_{uu}$ is a 3×3 matrix, the coupling matrices $\boldsymbol{K}^{k\tau sij}_{u\Phi}$, $\boldsymbol{K}^{k\tau sij}_{\Phi u}$ have dimension 3×1 and 1×3 respectively, and the electrical part $\boldsymbol{K}^{k\tau sij}_{\Phi\Phi}$ is a 1×1 matrix. The nucleus is the basic element from which the stiffness matrix of the whole structure is computed. The fundamental nucleus is expanded on the indexes τ and s to obtain the stiffness matrix of each layer k. Then, the matrixes of each layer are assembled at the multi-layer level depending on the approach considered. $\boldsymbol{P}^k_{\tau i}$ is a 3×1 matrix, called fundamental nucleus of the external load.

4 NUMERICAL RESULTS

To assess this new finite element, a four-layer cross-ply square plate with a cross-ply Gr/Ep composite core [0°/90°] and PZT-4 piezoelectric external skins, simply-supported boundary condition is considered. The static analysis of the plate structure is evaluated in sensor configuration. For the sensor case, a bi-sinusoidal transverse normal pressure is applied to the top surface of the plate:

$$p\,(x, y, z_{top}) = p^o_z\, sin(m\pi x/a) sin(n\pi y/b) \tag{17}$$

with amplitude $p^o_z = 1$ and wave numbers $m = 1$, $n = 1$. The potential at top and bottom position is imposed $\Phi_t = \Phi_b = 0$. The square plate has the following geometrical data: $a = b = 4.0$, and $h_{tot} = 1.0$. In respect to the total thickness, a single piezoelectric skin is thick $h_p = 0.1 h_{tot}$, while the single core layer is thick $h_c = 0.4 h_{tot}$. The material properties of the plate are given in Table 1. The results are evaluated in the plate center.

In order to compare the results with other solutions present in literature [16], the mid-plane domain of the plane structure was subdivided into two zones along the axes x and y, as shown in figure 2, and multi-theory models $Case\,A$ and $Case\,B$ are depicted on the FE discretization of a quarter of the plate. Some results of the transverse mechanical displacement w, transverse normal stress σ_{zz}, electric potential Φ, and tranvserse electric displacement $\mathcal{D}_z$ evaluated along the plate thickness in the center plate, are given in figure 3, Results are compared with the exact $3D$ solutions provided by Heyliger [24]. Figures 4(a) and 4(b) show the three-dimensional distributions of the transverse shear stress σ_{xz} of the variable kinematic multi-model $Case\,A$ and $Case\,B$ configuration. The results show the enhanced global/local capabilities of the $Case\,A$ model, which is able to predict correctly the stress state in the center zone where the loading is bigger.

5 CONCLUSIONS

A new methodology for global/local analysis of composite plate structure under electro-mechanical loadings has been introduced in this work. This approach makes use of advanced finite plate elements with node-dependent kinematics. The finite element arrays of the generic

Table 1: Material data for multilayered plate.

	Mechanical Properties								
	$E_{11}\,[GPa]$	$E_{22}\,[GPa]$	$E_{33}\,[GPa]$	$\nu_{12}\,[-]$	$\nu_{13}\,[-]$	$\nu_{23}\,[-]$	$G_{12}\,[GPa]$	$G_{13}\,[GPa]$	$G_{23}\,[GPa]$
Gr/EP	132.38	10.756	10.756	0.24	0.24	0.49	5.6537	5.6537	3.606
$PZT-4$	81.3	81.3	64.5	0.329	0.432	0.432	30.6	25.6	25.6
	Electrical Properties								
	$e_{15}\,[C/m^2]$	$e_{24}\,[C/m^2]$	$e_{31}\,[C/m^2]$	$e_{32}\,[C/m^2]$	$e_{33}\,[C/m^2]$	$\tilde{\varepsilon}_{11}/\varepsilon_0\,[-]$	$\tilde{\varepsilon}_{22}/\varepsilon_0\,[-]$	$\tilde{\varepsilon}_{33}/\varepsilon_0\,[-]$	$\varepsilon_0\,[C/Vm]$
Gr/EP	0	0	0	0	0	3.5	3.0	3.0	$8.85*10^{-12}$
$PZT-4$	12.72	12.72	-5.20	-5.20	15.08	1475	1475	1300	$8.85*10^{-12}$

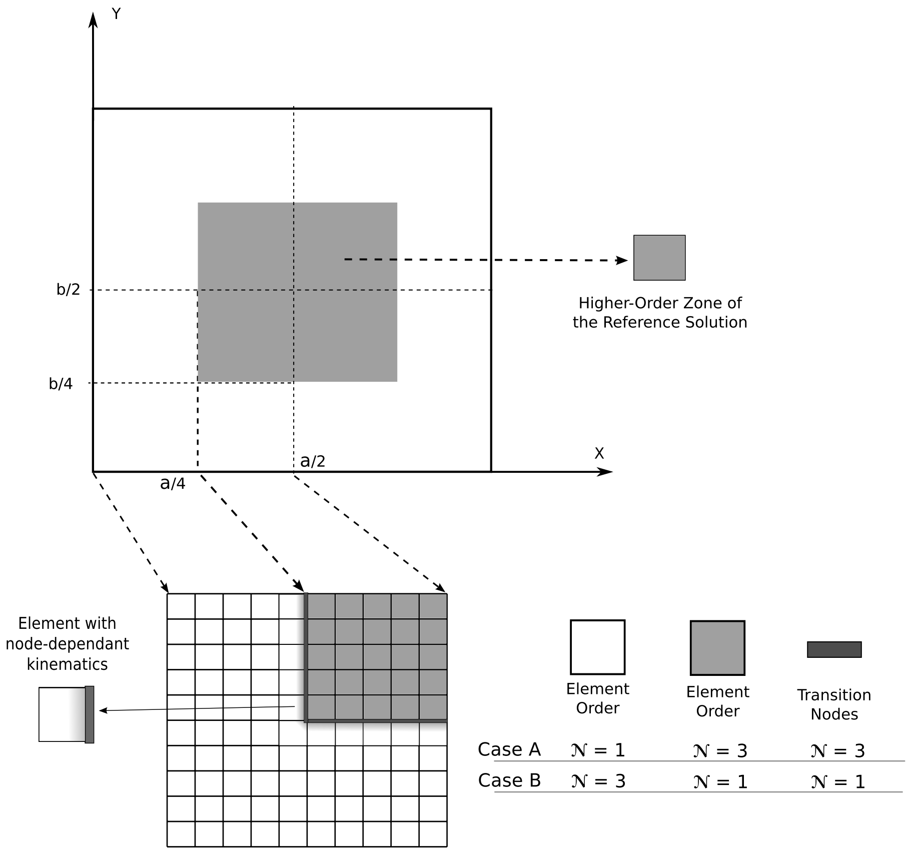

Figure 2: Mesh zones of the composite plate with piezoelectric skins and graphical representation of the multi-theory models, based on layer-wise models.

plate element are formulated in terms of fundamental nuclei, which are invariants of the theory approximation order. In this manner, the plate theory can vary within the same finite elements

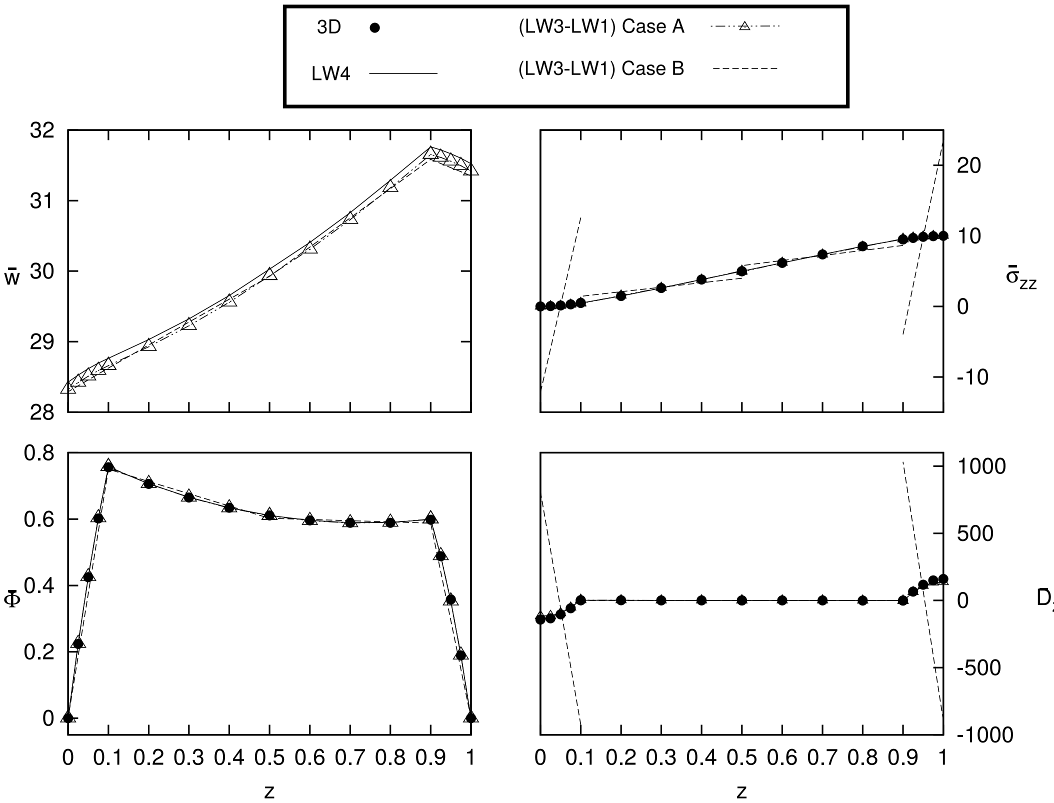

Figure 3: Transverse mechanical displacement w, transverse normal stress σ_{zz}, electric potential Φ, and tranvserse electric displacement $\mathcal{D}_z$ evaluated in the center plate.

with no difficulties. Thus, given a finite element model, the theory approximation accuracy can be enriched locally in a very straightforward manner by enforcing the same kinematics at the interface nodes between kinematically incompatible plate elements. The resulting global/local approach is very efficient because it does not employ any mathematical artifice to enforce the displacement/stress continuity, such as those methods based on Lagrange multipliers or overlapping regions. The proposed methodology has shown good potentiality in the analysis of composite plate embedding piezoelectric skins.

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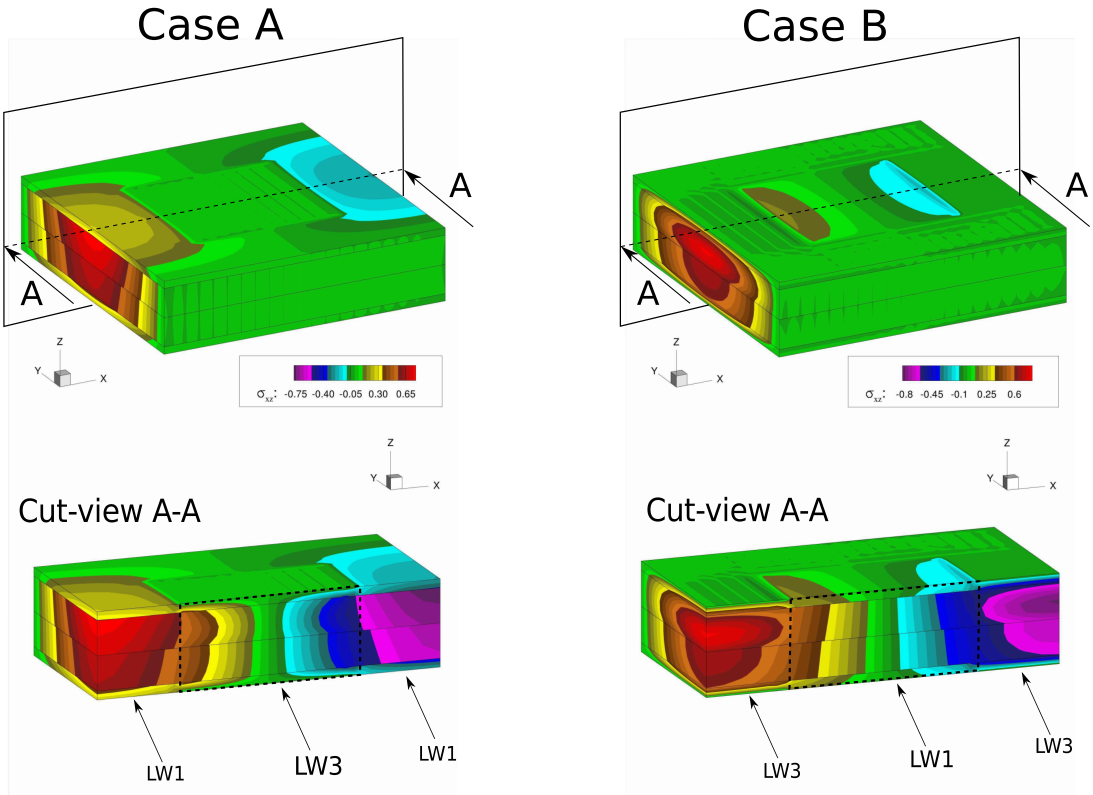

(a) *Case A* multi-theory model

(b) *Case B* multi-theory model

Figure 4: Composite four-layered plate. Transverse shear stress σ_{xz}. Multi-model theories.

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