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PERFORATED SCREENS AS FREQUENCY SELECTIVE SURFACES
A SCATTERING MATRIX APPROACH

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Modern reflector antenna systems make use of Frequency Selective Surfaces (FSS) as free space diplexers. Two classes of FSS exist. One class has the form of a periodic distribution of conducting patches, sandwiched between dielectric support structures, and has a low pass frequency response. The other class consists of a periodically perforated metallic screen and has a dual frequency behaviour. The frequency responses of the two complementary structures are related by the Babinet Principle if the dielectric environment is symmetrical with respect to the grid. In general however a direct formulation is needed for the solution of the scattering problem. In principle either the the currents induced on the conductors or the aperture fields can be assumed as the unknowns of the problem. However it is obvious that FSS of the first class are more easily studied using the induced current formulation [1], while the converse is true for FSS of the second class. In this paper we apply the aperture formulation to the analysis of high pass structures consisting of periodically perforated metallic screens embedded in a stratified dielectric medium.

This problem has been addressed by several authors in the past [2,3]. Our effort has been to develop a more general formulation which leads to the definition of a generalized scattering matrix, a necessary concept for the analysis of multiple grid structures.

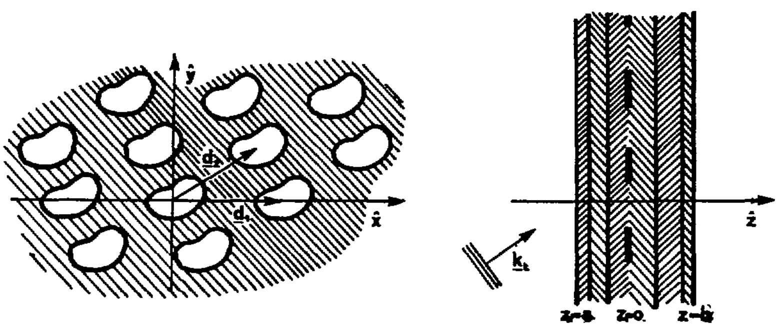

Fig.1 Geometry of the structure

Consider a periodically perforated metallic screen, defined by the basis vectors $\underline{d}_1$, $\underline{d}_2$, embedded in a layered dielectric medium, illuminated by an arbitrary plane wave with transversal wavevector $\underline{k}_t$ (see Fig. 1). In these conditions the fields in the regions z<0 and z>0 can be represented in terms of a discrete spectrum of plane waves (Floquet modes). In virtue of the Equivalence Theorem for the region z>0, the grid can be substituted by a solid metallic plane, on the left side of which an unknown distribution of magnetic currents ($\underline{M} = \hat{z} \times \underline{E}$) is present. By using a transmission line formulation the original scattering problem can be represented as indicated in Fig. 2, where the equivalent circuits refer to a generic Floquet mode with modal impedance Z_∞. From now on we use the symbols ^ and ~ to denote the quantities related to the regions z<0 and z>0 respectively. The voltage generators $\hat{V}^i$ and $\hat{V}$ are the projections on the relevant Floquet mode of the incident electric field at z=a and of the magnetic current distribution at z=0 respectively. The two circuits a) and b) are equivalent to the regions z<0 and z>0 respectively and moreover the modal voltage $\tilde{V}$ is equal to the impressed voltage $\hat{V}$ because of the continuity of the tangential electric field at z=0. Notice that in the definition of the scattering matrices of the dielectric layers on both sides of the grid it is convenient from a numerical point of view to use the free space modal impedances for normalization. This means that in the computations the grid can always be assumed to be free standing.
From circuit a) and b) we get the modal currents $\hat{I}$ and $\tilde{I}$ at z=0

$$\hat{I} = \hat{Y}_t \hat{V}^i - \hat{Y} \hat{V} \qquad\qquad \tilde{I} = \tilde{Y} \tilde{V} \qquad\qquad (1a,b)$$

Notice that $\hat{I} \neq \tilde{I}$ because of the magnetic field discontinuity at z=0. In order to proceed in the solution of our problem, we use the standard mode matching technique. The tangential aperture fields are expanded as follows

$$\underline{E}_t = \Sigma_k \mathring{V}_k \underline{\phi}_k \qquad\qquad \underline{H}_t = \Sigma_k \mathring{I}_k \underline{\psi}_k \qquad\qquad (2)$$

where $\{\underline{\phi}_k\}$ and $\{\underline{\psi}_k\}$ are convenient sets of aperture expansion functions, which depend on the shape of the aperture.

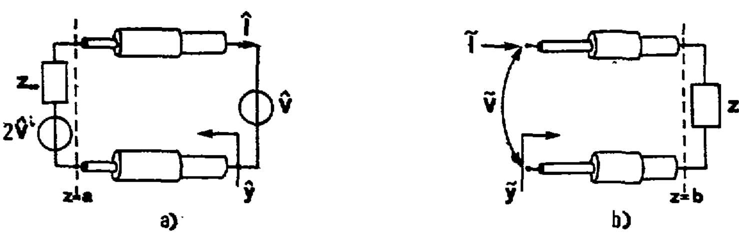

Fig.2 Modal equivalent circuit of structure of Fig.1

Enforcing the boundary conditions yields:

$$\Sigma_k \hat{V}_k \underline{e}_k = \Sigma_n \mathring{V}_n \underline{\phi}_n \quad (3a)$$

$$\Sigma_k \tilde{V}_k \underline{e}_k = \Sigma_n \mathring{V}_n \underline{\phi}_n \quad (3b)$$

on the elementary cell

$$\Sigma_k \hat{I}_k \hat{z} \times \underline{e}_k = \Sigma_n \mathring{I}_n \underline{\psi}_n \quad (3c)$$

$$\Sigma_k \tilde{I}_k \hat{z} \times \underline{e}_k = \Sigma_n \mathring{I}_n \underline{\psi}_n \quad (3d)$$

on the aperture (A)

where $\underline{e}_k$ are the electric field eigenfunctions of the k-th Floquet mode. Notice that k is here a multiple index and stands for (mpq) where m=1,2 denotes TE and TM modes and p,q= 0,±1,±2,... identify the order of the mode.

In order to obtain a matrix formulation, we project eqs.(3a,b) on the set $\{\underline{e}_k\}$ since these equations hold for every point of the cell, and eqs.(3c,d) on the set $\{\underline{\psi}_k\}$ since these equations hold on the aperture only. The result is

$$\hat{V} = Q\, \mathring{V} \qquad \tilde{V} = Q\, \mathring{V} \quad (4a,b)$$

$$P\, \hat{I} = \mathring{I} \qquad P\, \tilde{I} = \mathring{I} \quad 4c,d)$$

where Q and P are the projection matrices

$$Q_{ij} = \int_A \underline{\psi}_j \cdot \underline{e}_i^* \, dS \quad (5a)$$

$$P_{ij} = \int_A \hat{z} \times \underline{e}_j \cdot \underline{\phi}_i^* \, dS \quad (5b)$$

and the other quantities are column vectors, the elements of which were introduced before. From a numerical point of view it is convenient to assume $\underline{\psi}_k = \hat{z} \times \underline{\phi}_k$, which implies that $P = Q^+$, where the symbol + denotes Hermitian adjoint. Notice that Eq.(1a,b) can be read as matrix equations if $\hat{Y}$, $\tilde{Y}$ and $\hat{Y}_t$ are interpreted as diagonal matrices. Hence by substituting these equations into eqs.(4c,d) and eliminating $\mathring{I}$, we get an equation containing the unknown vectors $\hat{V}$ and $\tilde{V}$. Finally, by using eqs.(4a,b) we obtain a linear system of equations in the unknown coefficients $\mathring{V}_n$

$$B\, \mathring{V} = Q^+ \hat{Y}_t \hat{V}^i \quad (6)$$

where

$$B = Q^+ [\hat{Y} + \tilde{Y}]\, Q \quad (7)$$

By solving eq.(6) for $\mathring{V}$ and by substituting into eqs.(4a,b), we obtain the modal voltage vectors $\hat{V}$ and $\tilde{V}$ at z=0. Hence the scattered modal voltage vector at the sections z=a ($\hat{V}^s$) and z=b ($\tilde{V}^s$) can be computed by solving an elementary transmission line problem, as shown in Fig.2 :

$$\hat{V}^s = [\hat{\Gamma} + \hat{T}\, W\, \hat{Y}_t]\, \hat{V}^i \qquad \tilde{V}^s = [\tilde{T}\, W\, \hat{Y}_t]\, \hat{V}^i \quad (8a,b)$$

where

$$W = Q\, B^{-1}\, Q^+ \quad (9)$$

$\hat{\Gamma}$ is a diagonal matrix containing the reflection coefficients for each Floquet mode of the structure constituted by the dielectric layers of the region z<0 backed by a solid metallic plane. $\hat{T}$ and $\tilde{T}$ transmission diagonal matrices relating the scattered voltages $\hat{V}^s$ and $\tilde{V}^s$ at z=a and z=b respectively to the total voltages at z=0.
In order to compute the Generalized Scattering Matrix (GSM) of the structure we must consider also the case where the source is on the right side ($\tilde{V}^i$). It is easy to see that the solution in this case is obtained by exchanging the symbols ^ and ~ in eqs.(8a,b). Notice that the system matrix B is the same in both case, because it depends only on the structure and not on the excitation.
In conclusion, the Generalized Scattering Matrix can be written as the sum of two terms, $S = S_p + S_a$. The first one is the GSM of the structure where the perforated screen has been substituted with a solid one; hence S_p is a diagonal matrix. The second one (S_a) takes into account the effect of the apertures and is related to the radiation of the equivalent magnetic currents in presence of the solid metallic screen. Its expression is

$$S_a = \left[\begin{array}{c|c} \hat{T}\,W\,\hat{Y}_t & \hat{T}\,W\,\tilde{Y}_t \\ \hline \tilde{T}\,W\,\hat{Y}_t & \tilde{T}\,W\,\tilde{Y}_t \end{array}\right] \tag{10}$$

Obviously this decomposition is a consequence of our particular application of the Equivalence Theorem.

This approach has been applied to the case of round apertures arranged in a skewed lattice. We have chosen the mode functions of a circular waveguide as aperture functions. In this case the elements of the B matrix are given by double summations of terms which decrease for large k (Floquet mode transversal wavenumber) as fast as k^{-4}. Since this type of scattering problem can be given a spectral interpretation, the relative convergence problem is solved directly in the spectral domain taking into account the spectral bandwidth of the aperture functions involved. Results relative to this case will be presented.

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