# Distorsion analysis of non-linearly loaded antennas



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DISTORSION ANALYSIS OF NON-LINEARLY LOADED ANTENNAS

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## INTRODUCTION

Non-linearly loaded antennas and scatterers are of considerable interest in a number of technical problems and applications. Non-linear effects must be taken into account in systems of antennas which include semiconductor components, integrated circuits and voltage limiters with very strong incident fields such as those produced by lightning or by NEMP (Nuclear ElectroMagnetic Pulse) [1] . Moreover, some field measurement techniques such as the EDM (Energy Density Meter) now being developed at the NBS [2] make use of short dipoles loaded by a diode. It must also be noted that some radar applications with cooperative targets are based on the use of non-linearly loaded scatterers [3] . Examples are provided by the transponders with frequency duplication, used in anti-collision systems [4] in order to distinguish between the required signal and the clutter coming from the environment. Further applications regard transistorized or active antennas which are highly qualified elements in directional antenna arrays [5].

A suitable numerical approach in the steady state condition can be found as a generalization of the piecewise harmonic balance method, which is successfully applicable to non-linearly loaded antennas and scatterers [6]. In addition the simultaneous presence of several signals can be examined when carrier frequencies are close enough. In fact the intermodulation products can be approximately evaluated by means of the monochromatic circuit responses.

## NUMERICAL ALGORITHMS FOR THE SYSTEM ANALYSIS

A new general formulation has been recently proposed [6] which allows strongly non-linear multiports (described through implicit equations) to be taken into account and leads to numerical algorithms quite fast even with hard non-linear constraints. According to this harmonic balance procedure the whole network is split into a subnetwork composed only of linear elements and subnetworks which include all the non-linearities. The electrical variables $\underline{v}(t)$ and $\underline{i}(t)$ at the M connecting ports can be assumed as unknowns. Let $\underline{x}(t)$ be the equivalent generators at the connecting ports.

Taking into account the constraints $\mathscr{L}\{\underline{v}(t),\underline{i}(t)\} = \underline{x}(t)$ imposed by the linear subnetwork and the hypotesis of a steady state response, the set of non-linear equations describing the non-linear subnetworks, becomes

$$\underline{G}\,[\,\underline{I}(\omega)\;,\;\underline{X}(\omega)\,] = \underline{0} \tag{1}$$

of M(N+1) non-linear time-independent equations, where $[I(\omega),X(\omega)] = \mathscr{F}[\underline{i}(t),\underline{x}(t)]$ and N is the maximum number of harmonics necessary to correctly represent time functions according to the Shannon's theorem.

To solve eq.(1), the use of the second-order convergence algorithm

$$(2) \quad \left(\frac{\partial \underline{G}}{\partial \underline{I}}\right)_{\underline{I}^k} \cdot (\underline{I}^{k+1} - \underline{I}^k) = -\underline{G}(\underline{I}^k) \text{ with : } \frac{\partial \underline{G}}{\partial \underline{I}} \rightarrow [g_{ij}] \ : \ g_{ij} = \frac{\partial G_i}{\partial I_j}$$

and $i,j = 1 \div M(N+1)$, reduces the problem to the search for the solution of a sequence of linear systems such that the matrix of the coefficients can be evaluated numerically or analytically, the unknowns being the differences between the variables at two successive iterations (k and k+1).

The algorithm (2) has a second order covergence rate provided that an accurate starting point for the iterative process is chosen. In the case of strong non-linearities a choice of $\underline{I}^o$ which is not close enough to the true value the iterative process may not converge or may stop at a local minimum which is not zero. To overcome these difficulties the problem of solving system (2) with the forcing term $\underline{X}(\omega)$ as parameter can be transformed into a sequence of problems with the same structure, but with different values of the signal amplitude, that is

$$(3) \qquad \underline{X}(\omega) = \underline{X}^I(\omega) + h\left[\underline{X}^F(\omega) - \underline{X}^I(\omega)\right] \qquad 0 \leqslant h \leqslant 1$$

where $\underline{X}^I(\omega)$ and $\underline{X}^F(\omega)$ are the initial and final values of the vector of the forcing terms and h is a number ranging from 0 to 1. Usually a solution of (2) is known for a particular value $\underline{X}(\omega)=\underline{X}^I(\omega)$, for example, for zero signal amplitude. By starting from this value and increasing gradually by step the value of h, the direct search for the solution of system (1) can be split into a sequence of solutions of well-conditioned systems. In fact, every solution obtained with a particular signal amplitude ($h=h_0$) is a good starting point for the system having a slightly higher signal ($h=h_0+\Delta h$). In this way convergence problems are eliminated and the whole process is speeded-up. The technique can be refined by improving the prediction of the correct starting point for the iterative process, using an extrapolation procedure on the solutions obtained for weaker signals. The procedure described provides as a further result, the responses of the system for different values of the input signal amplitude, that is a very important feature in order to evaluate intermodulation products.

## INTERMODULATION ANALYSIS FOR NARROW-BAND SIGNAL

A special feature of the above described harmonic analysis algorithm is that it provides the non-linear system response for a whole set of input signal amplitudes. Such a characteristic is useful not only by itself, but also because it allows a straightforward computation of the intermodulation products arising from a narrow-band input signal.

In order to show how the intermodulation analysis can be effected, let us suppose that a periodic steady state analysis has been carried out for a set of sinusoidal input signals

$$(3) \qquad x_0(t) = b \cos \omega_0 t$$

with different values of the amplitude b within a range $0 < b \leqslant B$. The corresponding system output can be expressed throught a Fourier series in the form

$$(4) \qquad y_0(t) = \sum_{n=0}^{N} \mathcal{R}\{\underline{H}_n(b, \omega_0)\, e^{jn\omega_0 t}\}$$

where the dependence of the output harmonic components on the actual input signal amplitude b and angular frequency $\omega_0$ has been pointed out throught the "harmonic transfer functions" $\underline{H}_n(b,\omega_0)$.

Once a sufficient number of samples of these harmonic transfer functions has been computed by a periodic steady state analysis, it is quite straightforward to compute the system response to any narrow-band input signal with spectrum centered at $\omega_0$ and expressed in the form

$$x(t) = \int_{\omega_0-\Delta\omega}^{\omega_0+\Delta\omega} \mathcal{R}\{\underline{X}(\omega)\, e^{j\omega t}\}\, d\omega = \int_{-\Delta\omega}^{\Delta\omega} \mathcal{R}\{\underline{X}(\Omega+\omega_0)\, e^{j(\Omega+\omega_0)t}\}\, d\Omega$$

with $\Delta\omega \ll \omega_0$. In fact, the narrow-band signal x(t) can be expressed as a modulated signal with carrier frequency $\omega_0$, amplitude a(t) and phase $\alpha(t)$ in the form :

$$x(t) = \mathcal{R}\{\underline{A}(t)\, e^{j\omega_0 t}\}$$

where the complex envelope , given by

$$\underline{A}(t) = \underline{a}(t)\, e^{j\alpha(t)} = \int_{-\Delta\omega}^{\Delta\omega} \underline{X}(\Omega+\omega_0)\, e^{j\Omega t}\, d\Omega$$

can be easily computed by using the fast Fourier transform numerical algorithm. It should be noted that, since x(t) is a narrow-band signal, $\underline{A}(t)$ is a time function whose variations are much "slower" than those of x(t); therefore, the non-linear system response can be computed as a sequence of quasi-steady-state periodic responses. More in detail the output signal can be obtained as the sum of "slowly" modulated harmonic components in the form:

$$(5) \qquad y(t) = \sum_{n=0}^{N} \mathcal{R}\{\underline{D}_n(t)\, e^{jn\omega_0 t}\}$$

where the time dependent vector $\underline{D}_n(t)$ representing the n-th harmonic component is given by

$$(6) \qquad \underline{D}_n(t) = \underline{H}_n\left[a(t), \frac{d\alpha}{dt} + \omega_0\right] e^{jn\alpha(t)} \simeq \underline{H}_n[a(t), \omega_0]\, e^{jn\alpha(t)}$$

The approximation in eq.(6) of the exact input signal angular frequency $\omega(t) \equiv (d\alpha/dt) + \omega_0$ with center band frequency $\omega_0$ is reasonable under the assumption of the narrow band.
According to eqns (5) and (6) the system response to a given narrow-band input signal x(t) can be easily obtained once the harmonic transfer function $\underline{H}_n$ has been computed by using the harmonic analysis algorithm previously described. If, as it is more useful, the frequency domain representation of the intermodulation products is required, this can be ob

tained by a numerical transformation of the time domain output signal computed through eqns (5) and (6).

*STRAGHT-WIRE ANTENNA WITH A NON-LINEAR LOAD*

As an example of application of the proposed method, a straght-wire antenna of length h and diameter 2a has been considered; the antenna is connected at center to a non-linear load (fig.1). In this simple case, the non-linear subnetwork is given by the antenna itself and the connecting section reduces to a single port, so that the network unknowns are directly given by $V(\omega)$ and $I(\omega)$, that is the voltage and the current at the antenna and load terminals. A monochromatic incident field at frequency 100 MHz is assumed. The wire-antenna analysis program is based on the solution of the integral equation for the wire current via the moment's method with triangular test and expansion functions. The load is a backward diode: the non-linear transition capacitance has been taken into account and a parallel inductance is included in the linear part to assure the short circuit condition for the DC current. Fig.2 presents the harmonic containt in terms of the back radiated power versus the incident available power for an antenna with h=1.5 m and a=0.0025 m.

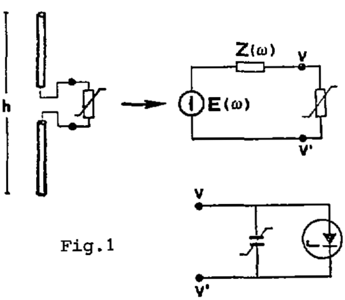


Fig.1

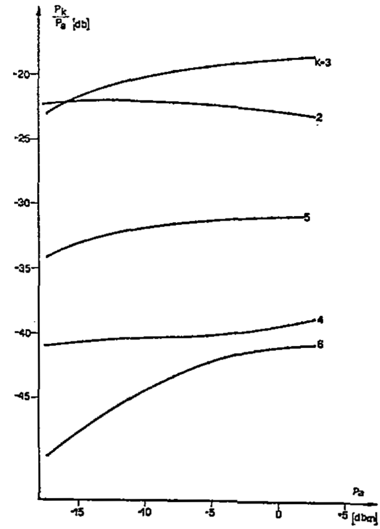


Fig.2

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