

A GTD Analysis of discontinuities in rectangular waveguides

*Original*

A GTD Analysis of discontinuities in rectangular waveguides / Daniele, V., Montrosset, I., Zich, R.. - STAMPA. - (1980), pp. 419-423. (10th European Microwave Conference Varsavia (PI) 1980, 8-12 Sept.) [10.1109/EUMA.1980.332898].

*Availability:*

This version is available at: 11583/2666090 since: 2017-02-27T16:15:24Z

*Publisher:*

IEEE

*Published*

DOI:10.1109/EUMA.1980.332898

*Terms of use:*

This article is made available under terms and conditions as specified in the corresponding bibliographic description in the repository

*Publisher copyright*

IEEE postprint/Author's Accepted Manuscript

©1980 IEEE. Personal use of this material is permitted. Permission from IEEE must be obtained for all other uses, in any current or future media, including reprinting/republishing this material for advertising or promotional purposes, creating new collecting works, for resale or lists, or reuse of any copyrighted component of this work in other works.

(Article begins on next page)

# A GTD ANALYSIS OF DISCONTINUITIES IN RECTANGULAR WAVEGUIDES.

V. Daniele , I. Montrosset and R. Zich

**ABSTRACT :** The problem of the junction between two rectangular waveguides is examined by ray methods. Results are presented taking into account only a first order approximation (singly diffracted rays) .

## INTRODUCTION

Discontinuities in waveguides can be examined by ray methods ,as suggested by Yee,Felsen and Keller [1] . However this kind of analysis has been extensively adopted either for parallel plane waveguides or for circular ones [2,3,4] . In this work ,the case of rectangular waveguides is thoroughly examined with ray optics.

The reason why the method was previously confined to parallel plane or circular geometry follows from some troubles which occur in expressing the scattered field in any angular direction for rectangular discontinuity by following the procedure indicated by Yee,Felsen and Keller that is by considering the GTD field as radiated by equivalent nonisotropic sources on the edges. This difficulty can be overcome by phrasing the problem in a quite different way , that is by expressing initially the coupling coefficients in terms of a Double Fourier Transform of the field on the aperture and then by showing that this transform can be evaluated again in terms of the GTD contribution expressed as field radiated by the non isotropic line sources located only on a couple of opposite edges. Therefore , since the coupling coefficients depends on the field radiated by the discontinuity in the free space , the edge currents are evaluated by a GTD analysis and then the radiation pattern is computed by integrating the effect of these currents along a couple of edges. This allows to evaluate the field in the directions associated with the modal description of the guided field.

## EVALUATION OF THE MODAL EXCITATION COEFFICIENTS.

The evaluation of the modal field excited in a guide by an aperture is straightforward if the field on the aperture is known. In this case  $V_\alpha$  , the the modal amplitude of the  $\alpha^{\text{th}}$  mode can be written as follows

$$V_\alpha(z=0) = \int_{\rho} \underline{E}_t(\underline{\rho}) P_A(\underline{\rho}) \cdot \underline{e}_\alpha^*(\underline{\rho}) d\rho \quad (1)$$

where  $\underline{e}_\alpha(\underline{\rho})$  is the modal eigenfunction ,the integral is extended to the complete  $\underline{z}=0$  plane and  $P_A(\underline{\rho})$  is the pupil function of the region of the aperture A (fig.1).

The equation (1) can be rewritten by introducing a Fourier representation ; if  $(\underline{E}_t(\underline{\sigma}), \underline{e}_\alpha(\underline{\sigma})) = F(P_A(\underline{\rho})\underline{E}_t(\underline{\rho}), \underline{e}_\alpha(\underline{\rho}))$  it follows :

$$V_\alpha(z=0) = (2\pi)^{-2} \int_{\underline{\sigma}} \underline{E}_t(\underline{\sigma}) \cdot \underline{e}_\alpha^*(\underline{\sigma}) d\underline{\sigma} \quad (2)$$

It is worthwhile observing that in geometries characterized by rectangular cross sections,  $\underline{e}_\alpha(\underline{\sigma})$  reduces to the superposition of a few  $\delta$ -Dirac functions, in the form:

---

Istituto di Elettronica e Telecomunicazioni, Politecnico di Torino,  
C.so Duca degli Abruzzi 24 , 10129 TORINO, Italy .

$$\underline{e}_\alpha(\underline{\sigma}) = \sum_i \underline{A}_{\alpha i} \delta(\underline{\sigma} - \underline{\sigma}_i) \quad (3)$$

The eq.(3) allows eq.(2) to be written as:

$$V_\alpha(z=0) = (2\pi)^{-1} \sum_i \underline{E}_t(\underline{\sigma}_i) \underline{A}_{\alpha i} \quad (4)$$

The relationship (4) indicates that the modal amplitude  $V(z=0)$  is closely related to the Fourier Transform of the aperture field<sup>α</sup>, which, in turns, is strictly connected with the field radiated by the same aperture distribution in the free-space. As matter of fact, if we consider the aperture field  $P_A(\rho)\underline{E}_t(\rho)$  over an infinite perfect conducting screen in  $z=0$ , the field radiated is given by:

$$\underline{E}_t(\underline{\rho}, z) = (2\pi)^{-2} \int_{\underline{\sigma}} \underline{E}_t(\underline{\sigma}) \exp\{-j(\underline{\sigma} \underline{\rho} + (K_0^2 - \sigma^2)^{\frac{1}{2}} z)\} d\underline{\sigma} \quad (5)$$

This relation, evaluated through a saddle point technique, allows  $\underline{E}_t(\underline{\sigma}_i)$  to be evaluated according to:

$$\underline{E}_x(\underline{\sigma}_i) = (2\pi r / jK_0) \{ \cos\phi_i E_\theta(\theta_i, \phi_i) - \sin\phi_i E_\phi(\theta_i, \phi_i) / \cos\theta_i \} e^{jK_0 r} \quad (6a)$$

$$\underline{E}_y(\underline{\sigma}_i) = (2\pi r / jK_0) \{ \sin\phi_i E_\theta(\theta_i, \phi_i) - \cos\phi_i E_\phi(\theta_i, \phi_i) / \cos\theta_i \} e^{jK_0 r} \quad (6b)$$

where  $E_\theta$  and  $E_\phi$  are the far field components of the radiated field and  $\theta_i, \phi_i$  are defined by:

$$\underline{\sigma}_i \cdot \hat{x} = K_0 \sin\theta_i \cos\phi_i, \quad \underline{\sigma}_i \cdot \hat{y} = K_0 \sin\theta_i \sin\phi_i$$

It follows that the values of the Fourier Transform of the field required in eq.(4) are directly evaluated in terms of the far-field components.

In the case of the transition between two rectangular waveguides with same symmetry axis, the use of eq.s (6) and (4) leads to the following expression of the coupling coefficients  $V'_{mn}$  and  $V''_{mn}$  respectively for  $E_{mn}$ - and  $H_{mn}$ -modes:

$$V'_{mn} = (j/2 C_{mn}) \left\{ \frac{m}{a} [E_x(\xi_m, \eta_n) \exp(j\frac{m+n}{2}\pi) - E_x(\xi_m, -\eta_n) \exp(j\frac{m-n}{2}\pi) + E_x(-\xi_m, \eta_n) \exp(j\frac{n-m}{2}\pi) - E_x(-\xi_m, -\eta_n) \exp(j\frac{-m-n}{2}\pi)] + \frac{n}{a} [E_y(\xi_m, \eta_n) \exp(j\frac{m+n}{2}\pi) + E_y(\xi_m, -\eta_n) \exp(j\frac{m-n}{2}\pi) - E_y(-\xi_m, \eta_n) \exp(j\frac{n-m}{2}\pi) - E_y(-\xi_m, -\eta_n) \exp(-j\frac{m+n}{2}\pi)] \right\}$$

$$V''_{mn} \frac{(\epsilon_m \epsilon_n)^{\frac{1}{2}}}{4j C_{mn}} \left\{ \frac{n}{b} [E_x(\xi_m, \eta_n) \exp(j\frac{m+n}{2}\pi) - E_x(\xi_m, -\eta_n) \exp(j\frac{m-n}{2}\pi) + E_x(-\xi_m, \eta_n) \exp(j\frac{n-m}{2}\pi) - E_x(-\xi_m, -\eta_n) \exp(-j\frac{m+n}{2}\pi)] + \frac{m}{b} [E_y(\xi_m, \eta_n) \exp(j\frac{m+n}{2}\pi) + E_y(\xi_m, -\eta_n) \exp(j\frac{m-n}{2}\pi) - E_y(-\xi_m, \eta_n) \exp(j\frac{n-m}{2}\pi) - E_y(-\xi_m, -\eta_n) \exp(-j\frac{m+n}{2}\pi)] \right\}$$

with  $C_{mn} = (m^2 b/a + n^2 a/b)^{\frac{1}{2}}$ ,  $\epsilon_m = 1$  for  $m=0$  and  $=2$  for  $m \neq 0$

$$\xi_m = m \pi / a = K_0 \sin \theta_{mn} \cos \phi_{mn} \quad \text{and}$$

$$\eta_n = n \pi / b = K_0 \sin \theta_{mn} \cos \phi_{mn}$$

## EVALUATION OF THE RADIATION FIELD.

The approach presented in the previous section is useful if the field radiated by the aperture can be evaluated directly by overcoming the need of expressing the actual field on the aperture. It follows that a GTD approach presents many advantages. However, the case of a rectangular aperture has received little attention in the past and essentially only results on the two symmetry planes are available. The fact is that if a  $H_{10}$  incident mode is assumed, a GTD description of the field scattered by the aperture leads to ray congruences defined only on particular surfaces, that is either on the plane  $(x,z)$  and  $(y,z)$  or the planes  $(x,z)$  and two cones with  $x$ -axis if the incident field is viewed as superposition of two plane waves. Conversely, the evaluation of  $V_{mn}$ , as given by eq.(7), requires the field in quite arbitrary directions, not necessarily belonging to the congruences of rays connected with a pure GTD description.

These difficulties can be overcome by using the concept of the "equivalent edge currents". However, it is interesting to observe that these currents are not "physical" ones, so that the far field cannot be obtained by integration of them over the four edges of the aperture. Rather, since the radiated field is given by a double integral in the form

$$\underline{E}_t(\underline{r}) = \frac{\omega \mu_0 (\hat{\theta}\hat{\theta} + \hat{\phi}\hat{\phi})}{2j\pi r} e^{-jk_0 r} \int_{-a/2}^{a/2} \int_{-b/2}^{b/2} \underline{E}_t(x,y,0) \times \hat{z} \exp\{jk_0 \hat{r} \cdot (x\hat{x} + y\hat{y})\} dx dy \quad (8)$$

If in eq.(8) the integration over  $y$  is performed firstly, the result can be interpreted as superposition over  $x$  of contributions of the type

$$\left[ \int_{-b/2}^{b/2} \underline{E}_t(x,y,0) \times \hat{z} \exp(jk_0 \sin\theta \sin\phi y) dy \right] dx \quad (9)$$

This integration (9) does not formally differ from the one occurring in a bidimensional case, where GTD is absolutely valid and where it can be interpreted in terms of "edge point" contributions. It turns out that the field radiated everywhere can be obtained by integrating over  $x$  the elementary contributions (9) which can be interpreted in terms of radiation from the non-isotropic equivalent currents on the edges parallel to the  $x$ -axis. Obviously, the same procedure could be applied by starting with the integration over  $x$ : in that case the field is given as superposition of contributions along the vertical edges (parallel to  $y$ -axis) and only the equivalent edge currents along them have to be considered.

In order to check the procedure, the field radiated by an aperture of considerable extent has been considered. The results, presented in fig. 2 a and 2b, compare the field evaluated by using the two alternative formulations of the GTD approach and the physical optics formulations (electric field, magnetic field and both fields). The agreement is very good. Conversely if the aperture is very small the results are, as usual, not so good and especially the GTD with edge currents over the horizontal edges seems to be inadequate (fig. 3a and 3b).

## NUMERICAL RESULTS.

In fig. 4a and 4b are presented the results for the junction between two guides of rectangular cross section with the same  $a/b$  ratio and symmetry axis. The results obtained for the two formulations of the GTD (taking into account only the first order diffracted rays) and for the physical optics are presented.

## ACKNOWLEDGMENTS.

This research was supported by CSELT (Centro Studi Laboratori Comunicazioni). The authors thank Dr. P. Bielli and S. Depadova for the useful discussions.

REFERENCES

- [1] Yee H.Y., Felsen L.B. and Keller J.B. : "Ray theory of reflection from the open end of a waveguide.", SIAM J. Appl. Math., vol.16, pp.268-300, 1968.
- [2] Felsen L.B. and Yee H.Y.: "Ray method for sound wave reflection in an open ended circular pipe.", J.O.S.A., vol.40, pp.1028-1039, 1968.
- [3] Felsen L.B., Yee H.Y.: "Ray optics a novel approach to scattering by discontinuities in a waveguide.", IEEE Trans on M.T.T., vol.MTT-17, pp. 73-85, 1969.
- [4] Felsen L.B., Yee H.Y.: "Ray-optical analysis of electromagnetic scattering in waveguides.", IEEE Trans. on M.T.T., vol.MTT-17, pp.671-683, 1969.

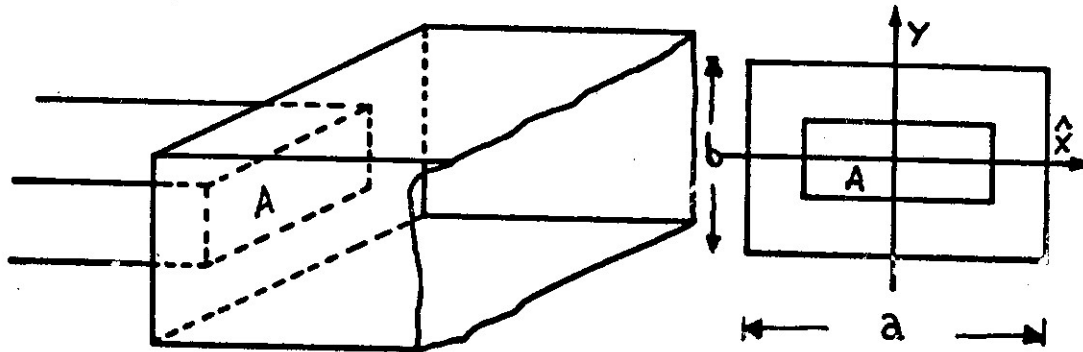


Fig.1 . Typical geometry of the problem.

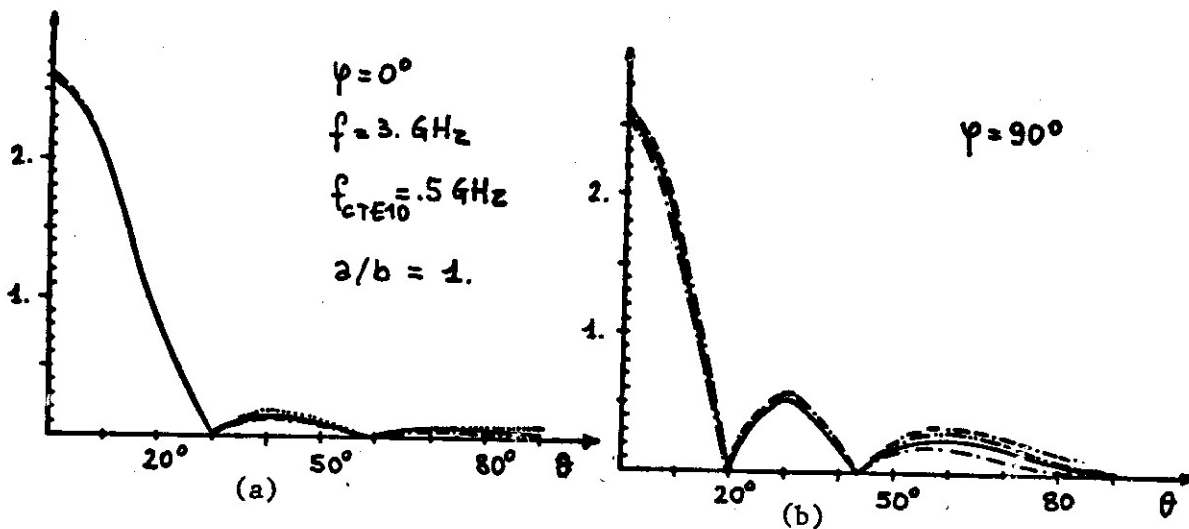


Fig.2 . Field radiated by flanged rectangular waveguide. Results obtained with : GTD horizontal edges (---) ,GTD vertical edges (---) physical optics(PO) electric field(---) PO magnetic field (.....) and PO electric and magnetic field(\_\_\_\_\_).

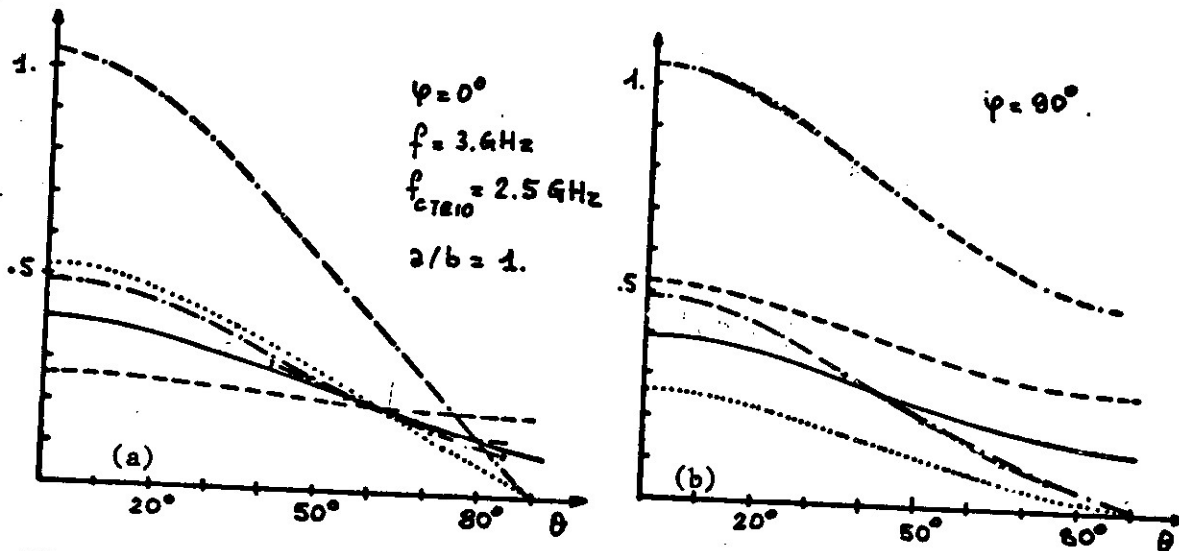


Fig.3 Field radiated from monomodal rectangular waveguide. Same notations of fig.2.

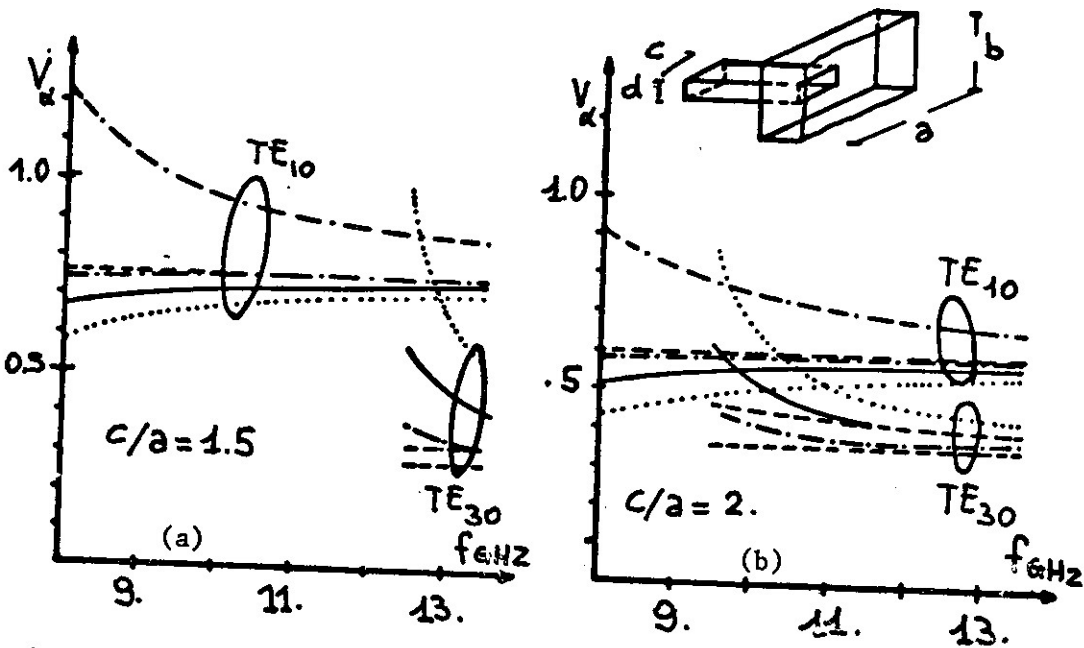


Fig.4 Excitation coefficients for the  $H_{10}$  mode incident ( $c=2.5 \text{ cm}$ ). Same notation of fig.2.