Performance Assessment of Short-Reach Multi-Level PAM Transmission Exploiting LMS Equalization

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Abstract: We study the performance of pulse amplitude modulation (PAM), with 2, 4 and 8 levels per symbol, for short reach applications with propagation over up to 2 km of standard single mode fiber. We consider the three modulation formats operated both at the same bit-rate of 25 Gbps and at same symbol rate of 25 Gbaud, supposing 3.125 % FEC overhead and bit error rate threshold of $10^{-4}$. We analyze transmitters (Tx) and receivers (Rx) based on digital signal processing, including a training-driven finite impulse response (FIR) filter equalizer set through the least mean square (LMS) algorithm. We analyze the sensitivity impairments induced by a limited Tx/Rx electrical bandwidth and evaluate the minimum number of taps – and consequent latency – needed by the FIR equalizer in order to obtain sensitivity benefits.

1. Introduction

The growth in the Internet (IP) traffic is driving an increasing need for additional capacity in all communications systems. Regarding data-center, IP traffic is envisioned to growing at a compound annual growth rate (CAGR) of 27 percent until 2020 [6]. This increased IP traffic demand also for very short-reach links is addressing data rate standardization at 100 Gbps and 400 Gbps, as investigated by the IEEE 802.3bs 400 GbE task force [1]. While the only option as transport medium is the optical fiber possibly exploiting multiple wavelength ($\lambda$) transmission, the choice for line rate and modulation format is still open. Proposed line rates are $4\lambda \times 25$ Gbps/$\lambda$ for 100 Gbps and $4\lambda \times 100$ Gbps/$\lambda$ for 400 Gbps [7]. The presented analysis is mainly targeted to intra-data-center networking, so components’ cost is indeed a constraint. Consequently, modulation formats must rely only on direct-detection, keeping apart – at least for present and mid-term future – the option of coherent receivers. More specifically, potential transmission techniques for the considered applications are Pulse Amplitude Modulation (PAM) [11–14, 21], Discrete Multi-Tone (DMT) [14, 15, 23, 24] and Carrier-less Amplitude and Phase Modulation (CAP) [15, 16]. Nyquist-shaped PAM – the spectral efficient variant of PAM – will also be an option to improve spectral efficiency [17, 20]. Performances of different modulation formats have been extensively investigated and compared, as, for instance, presented in [7, 8, 10, 15, 16].

In this work, we focus our attention on the implementation of multilevel PAM using digital signal processing (DSP) at transmitter and receiver, including a training-driven finite impulse response (FIR) equalizer whose filter taps are set through the application of the least-mean-square (LMS) algorithm on a training sequence. We suppose to rely on silicon-photonics modulators characterized by limited extinction ratio (ER), and investigate on the effect of electrical bandwidth limitations of
transmitter (Tx) and receiver (Rx) for the 2-, 4- and 8-level PAM, given the bit-rate ($R_b = 25 \text{ Gbps}$) or given the symbol-rate ($R_s = 25 \text{ Gbaud}$). For all considered options, we also analyze benefits of the number of taps ($N_{taps}$) of the FIR equalizer in order to establish the minimum required $N_{taps}$ – and the consequent latency – still granting a sensitivity advantage. Such an analysis is a specific need for the analyzed scenario, as low-latency is a primary requirement for distributed and high performance computing in order to enable distributed big-data processing [25].

The article is structured as follows. In Sec. 2 we describe the considered setup and present the simulative analyses we have carried out. We also show the bit error rate (BER) vs. optical signal-to-noise ratio (OSNR) curves for the considered PAM configurations obtained with large Tx/Rx bandwidth and large $N_{taps}$. In Sec. 3, we show the effects of electrical bandwidth reduction in terms of sensitivity penalty, while contents of Sec. 4 display performances vs. the number of FIR equalizer taps.

2. Analyzed setup and simulative analyses

Fig. 1 shows the block diagram for the PAM-n setup we have considered for simulative analyses whose results are presented in this paper. The transmitter is made of a DSP unit followed by a digital-to-analog converter (DAC) that drives a Mach-Zehnder modulator (MZM) shaping the optical signal generated by a continuous wave (CW) laser. The DSP unit takes at the input logical data and properly shapes the modulated signal delivered to the DAC. As logical data, we used $k$ streams of pseudo random sequences (PRBS) with degree of 17 corresponding to $2^{17} - 1 = 131071$ bits where $k = 1, 2, 3$ for PAM-2, PAM-4 and PAM-8 respectively. We evaluate BER through error counting on 400000 symbols, getting reliable results down to the target BER of $10^{-4}$. The DSP unit is supposed to operate at 2 sample-per-symbol (SpS) and includes a pre-emphasis filtering counteracting the DAC unwanted effects such as resolution, conversion speed, difference between the desired and actual output, linearity between input and output signal etc and the $\text{ArcSin}$ operation compensating for the sinusoidal electro-optic (EO) transfer function of the MZM.

For the DAC we suppose a number of quantizing bits sufficiently large i.e. 10, consequently reducing the impairments, i.e. quantizing noise, to focus on bandwidth limitations’ effects. Since commercially available DACs usually generate output swing of few hundred mV, a RF driver/amplifier is needed to match the switching voltage of a typical MZM. In order to consider electrical bandwidth limitations at transmitter and receiver, we insert a 5th order Bessel filter, whose 3 dB bandwidth $BW$ defines the Tx/Rx bandwidth limit for the system. At the Tx, such bandwidth limited electrical signal will drive the modulator. For the MZM we suppose a value for the extinction ratio (ER) that is typical for silicon-photonics devices, so we set ER = 6 dB [18]. After the electro-optical conversion operated by the MZM, optical PAM modulated signals are transmitted through 2 km of standard single mode fiber. Fiber propagation effects are always negligible for the considered scenarios, except for the possible lumped reflections [19] that we did not include in the presented analyses since penalties are generally low and smaller than 1 dBo for PAM-2 and PAM-4 [3]. At the same time we neglected the effect of relative intensity noise, assuming the receiver to be noise-limited. The maximum allowable RIN to respect such assumption is around -134 dB/Hz for PAM-4 and -142 dB/Hz for PAM-8 [4].

The receiver front-end is in charge of the opto-electrical (OE) conversion that is obtained by a PIN photodetector followed by a trans-impedance amplifier (TIA). The photodetector is supposed to have a responsivity $R = 0.7 \text{ A/W}$, while the TIA is characterized by a trans-impedance $R_t = 285$
and by an input-referred noise factor of 32 pA/√Hz. The OE conversion is the main source of noise impairing the system, as it introduces shot-noise through the photodetection and thermal noise generated by the amplification. The bandwidth limitation for the OE conversion is equal to the symbol rate, eliminating the need for the use of a cascaded anti-aliasing electric filter.

The filter after OE conversion is used to bandwidth-limiting the electrical signal that is then digitally converted and received. The 3 dB bandwidth of the filter at the receiver is the same as the 3 dB bandwidth $BW$ of the bandwidth-limiting Bessel filter at Tx. After filtering, the received electrical signal is converted to a digital format, for DSP and Rx data-decision, by an analog-to-digital converter (ADC) operating at 2 SpS. As for the Tx DAC, the Rx ADC is supposed to work with a sufficiently large number of bits i.e. 10, allowing to neglect the quantizing noise. The digitalized signal is processed by a DSP unit including the training-driven FIR equalizer, the algorithm defining decision thresholds and the hard-decision unit returning the detected data sequence. The pre-FEC BER is calculated comparing the received data-sequence to the transmitted one and performing error-counting. As we simulated 400000 symbols, the BER estimation was sufficiently accurate down to the target BER = $10^{-4}$ corresponding to observe 35, 71 and 106 errors for PAM-2, PAM-4 and PAM-8 respectively.

At the receiver FIR based equalizer is used to remove the noise and intersymbol interference. It is a fractionally spaced Feed Forward Equalizer (FFE). Depending on the considered scenario i.e. bit-rate or the symbol rate case, the spacing between the taps is one DSP sample period i.e.: $T_b/2$ or $T_s/2$ respectively. Since FFE requires less power and cost as compared to a Decision Feedback Equalizer (DFE), FFE is used in the analysis.

The filtering coefficients of the FIR filter are defined during a training sequence by the use of the least-mean square (LMS) algorithm [9]. The training sequence is set to 45000 symbols, that proved to be a proper duration to ensure convergence of tap weights to optimal values using an LMS converge factor of $10^{-3}$. Once convergence is achieved, updates of the equalizer taps though LMS is performed by switching from the training sequence to the decision-directed mode. If $N_{taps}$ is the number of equalizer taps, this algorithm introduces a latency of $T_L = 1/2 \cdot (N_{taps} - 1) \cdot T_s$, as the DSP works at 2 SpS, where $T_s$ is the symbol duration.

The DSP unit includes also an algorithm calculating the decision thresholds. As well as the LMS equalizer, it exploits the training sequence and calculates the thresholds starting from the most-significant bit (MSB) to the least-significant bit (LSB) decision threshold. It is an adaptive algorithm that took a sample of the received data, defines $N$ number of threshold levels between the extreme values. For each threshold value, the BER is calculated using error counting. The threshold value providing the minimum BER is selected as the threshold value to be used for the whole set of data. Finally, the Rx DSP operates the decision on the received bits by comparing the equalized signal to the related threshold. We have performed the simulative analyses presented in

![Fig. 1. Block diagram of the setup used for the presented simulative analyses.](image-url)
this article on the setup described in Fig. 1. We have considered PAM modulation formats with increasing number of bit-per-symbol (BpS), examining PAM-2 (BpS = 1), PAM-4 (BpS = 2) and PAM-8 (BpS = 3). We have also considered different data-rate scenarios, setting in one case (i) the bit-rate R_b = 25 Gbps and in the other (ii) the symbol rate R_s = 25 Gbaud. In (i) the symbol rate – and the required DSP speed R_{DSP} = 2 \cdot R_s – scales with BpS as R_s = R_b / BpS, while in the other case the DSP speed is set and is used to scale the bit rate as R_b = R_s \cdot BpS. For all the analyzed rates, we suppose the adoption of a forward error-correction (FEC) code characterized by an overhead OH = 3.125 % [2] and target pre-FEC BER = 10^{-4}. The simulated line-rates (R_{s,l}) are enlarged accordingly, i.e., \( R_{s,l} = R_s \cdot (1 + OH/100) \).

We have investigated performance impairments supposing Tx and Rx DSP bandwidths to be the same and equal to BW, that in results are shown as percentage of the symbol rate, i.e., as normalized bandwidth NBW = BW / R_s \cdot 100 that has been swept in the range [50%; 100%]. In order to emulate electrical bandwidth limitations at Tx/Rx, 5^{th} order Bessel filters are used. BW represents the 3 dB bandwidth of such filters. The bandwidth is varied by varying the bandwidth of the Bessel filter whose bandwidth is normalized (NBW) to the symbol rate. We consider NBW = 100% as reference scenario.

Then, we have assessed the equalizer benefits vs. the number of FIR filter taps \( N_{taps} \), with the specific target to evaluate the minimum \( N_{taps} \) still granting performance advantages. All results are summarized using the sensitivity defined as the received power level required to obtain the target BER = 10^{-4}.

In order to establish a reference sensitivity \( P_{ref} \) for each scenario, we first performed simulations minimizing the impairments and maximizing the equalizer strength by setting \( NBW = 100\% \) and \( N_{taps} = 39 \). Results of this reference analyses displayed as BER vs. \( P_{Rx} \) are shown for all the three modulation formats in Fig. 2(a) and Fig. 2(b) for the case of same bit-rate of 25 Gbps and of same symbol-rate of 25 Gbaud, respectively. For \( R_b = 25 \) Gbps, \( P_{ref} \) is -6.5 dBm, -4.5 dBm and -1.75 dBm for PAM-2, PAM-4 and PAM-8, respectively. While, for \( R_s = 25 \) Gbaud, \( P_{ref} \) is -6.5 dBm, -1.8 dBm and 3.2 dBm for PAM-2, PAM-4 and PAM-8, respectively. Note that for PAM-2 the two scenarios coincide being the symbol rate equal to the bit rate when transmitting 1 bit per symbol.

![Fig. 2. BER vs. the received power for the same bit-rate of 25 Gbps (a) and same symbol-rate of 25 Gbaud (b) analyses.](image-url)
3. Results on performance impairments of Tx/Rx electrical bandwidth

After establishing the reference sensitivity for each PAM-\(n\) scenario, we carried out the analysis aimed at defining the sensitivity penalty due to a reduced electrical bandwidth of the Tx and Rx DSP units. We reduced the percentage bandwidth \(NBW\) from the reference value of 100\% down to 50\%. As the purpose of this analysis was to show penalties induced by the lack of bandwidth only, we set the number of equalizer taps to the reference value of \(N_{\text{taps}} = 39\). Figs. 3 show the evaluated BER vs. received power for the three considered PAM-\(n\) and for the case of given bit-rate \(R_b = 25\) Gbps and of given symbol rate \(R_s = 25\) Gbaud. Different curves refer to value of \(NBW\) from 100 \% down to 50 \%. For PAM-2, the two scenarios overlap, so related results are displayed in Fig. 3(a) only.

For all the considered scenarios, the same general behavior can be noticed: with the reduction of \(NBW\) we observe a detaching of BER curves from the optimal ones displayed in Fig. 2 showing a progressive power penalty. It happens also with the use of the equalizer, as it recovers for the lack of bandwidth, but it enhances higher frequency noise components. Such a penalty is almost negligible for PAM-2 (Fig. 3(a)) while it increases with the number of PAM levels because of the larger sensitivity of multilevel modulation formats to the noise.

The described behavior characterizes both the given-\(R_b\) and the given-\(R_s\) cases, but in the first scenario (Figs. 3(a), 3(b), 3(d)) it is much more limited as the occupied bandwidth determined by \(R_s = R_b/BpS\) scales down with the decrease of the number \(n = 2^{BpS}\) of PAM levels. Impairments are much more relevant for the given-\(R_s\) scenario for which a flooring effect that prevents to reaching the target BER can be observed for PAM-8 and \(NBW < 70\%\) (see Fig. 3(e)).

Figs. 4 summarize the analysis with respect to the reduction of Tx/Rx electric bandwidth showing the power penalty at the target BER = \(10^{-4}\) with respect to the ideal scenarios whose performances are displayed in Fig. 2. For each PAM-\(n\) and for the two scenarios, we considered as \(P_{\text{ref}}\) the sensitivity values for ideal cases listed in Sec. 2, and consequently calculate the power penalty as \(P_{\text{pen}} = P_{\text{sens}} - P_{\text{ref}}\) dB, where \(P_{\text{sens}}\) is the needed received power that varies with \(NBW\) derived from results of Figs. 3. Fig. 4(a) refer to the same-\(R_b\) case and shows how PAM-2 thanks to the equalizer experiences a penalty limited to 0.2 dB down to \(NBW = 50\%\). For PAM-4, \(P_{\text{pen}}\) grows quite regularly with the \(NBW\) decrease, reaching a maximum of 0.9 dB for \(NBW = 50\%\). PAM-8 shows a behavior similar to the PAM-4’s down to \(NBW = 60\%\) while \(P_{\text{pen}}\) grows more rapidly for narrower bandwidths reaching 1.8 dB for \(NBW = 50\%\). Power penalties for the given-\(R_s\) case are plotted in Fig. 4(b). Except for the PAM-2, for which given-\(R_s\) coincides with given-\(R_b\), the power penalty estimated for given-\(R_s\) is larger than the one observed for same-\(R_b\). Such a worsening is limited for PAM-4 that thanks to the equalizer operates also at \(NBW = 50\%\) with a penalty limited to 1.4 dB. While PAM-8, as already commented, experiences a BER flooring limiting the operation to a minimum tolerable \(NBW = 70\%\) to which corresponds a penalty of 1.8 dB.

4. Results on performance effects of LMS taps

In the analyses presented so far we assumed the use of a FIR filter equalizer based on a large number of taps, more precisely, we used \(N_{\text{taps}} = 31\). Such an assumption was aimed at exploring potentialities of the equalizer without the drawbacks of a limited length FIR filter. On the other hand, the larger \(N_{\text{taps}}\) the worse for the latency and – most important – an increased DSP complexity. Regarding the latency, it is presently not a major issue using state-of-the-art FEC codes [5].
Fig. 3. BER vs. the received power for different value of normalized Tx/Rx electric bandwidth, for PAM-2 (a), PAM-4 (b, c) and PAM-8 (d, e). And for the case of given-$R_b$ $R_b = 25$ Gbps (a, b, d) and of given-$R_s$ $R_s = 25$ Gbps (a, c, e)
that are largely dominant in terms DSP latency. While DSP complexity – and related power consumption – is indeed an issue for the considered system scenario where low-cost and low-power equipment are firm requirements.

Because of these, as last analysis, we studied the power penalty vs. the number of equalizer taps in order to establish the minimum $N_{\text{taps}}$ still granting benefits shown in previous sections. For the evaluation of penalties, we refer to the sensitivity values for ideal cases listed in Sec. 2 as we have done in Sec. 3 for the bandwidth-induced penalties. Results are plotted in Figs. 5 as $P_{\text{pen}}$ vs. $N_{\text{taps}}$.

Fig. 5(a) displays results for PAM-2 for which given-$R_b$ and given-$R_s$ cases coincide. It can be observed that for PAM-2 the restrained penalty of 0.2 dB is granted for all the bandwidths down to $NBW = 50\%$ using only 7 taps for the FIR filter equalizers. As we suppose the DSP operates a 2 sample-per-symbol, $N_{\text{taps}} = 7$ introduces 3 bits of DSP excess latency and it also requires a limited complexity and power consumption. Fig. 5(b) and 5(c) present results for PAM-4 for the same-$R_b$ and same-$R_s$ cases, respectively. In terms of number FIR filter taps needed to grant the equalizer benefits, both cases behave similarly indeed requiring only 7 taps to grant the minimum penalty for each bandwidth limitation. For the PAM-4, 7 taps corresponds to 6 bits of DSP latency. Finally, results for PAM-8 are displayed in Fig. 5(d) and 5(e) for the same-$R_b$ and same-$R_s$ cases, respectively. As already noted, the increase of number of levels implies a lower resilience to bandwidth limitations, that is more evident in the same-$R_s$ case for which we observed also a BER flooring. In terms of requested $N_{\text{taps}}$ granting the minimum penalty, PAM-8 with given-$R_b$ needs a minimum of 9 taps corresponding to 12 bits of extra latency, while PAM-8 with given-$R_s$ needs at least 19 taps implying 27 bits of extra latency.

Observing Figs. 5 it can be noted that above the minimum required number of taps, hierarchy in families of curves referring to different bandwidth limitation follows the intuitive behavior: the smaller $NBW$, the larger the penalty. While analyzing penalties for small number of taps in some cases intuitive behaviors are subverted. More specifically, cases for larger $NBW$ display behaviors worse than smaller bandwidths’. Even if these are scenarios without specific practical interests, it
Fig. 5. Power Penalty vs. the number of equalizer taps for different value of normalized Tx/Rx electric bandwidth, for PAM-2 (a), PAM-4 (b, c) and PAM-8 (d, e). And for the case of given-Rb $R_b = 25$ Gbps (a, b, c) and of given-Rs $R_s = 25$ Gbps (a, d, e)
is worth to comment these apparently counter-intuitive behaviors.

The equalizer, thanks to the LMS algorithm, aims at implementing an electric filter that is matched to the reference pulse, so the larger $NBW$, the larger the bandwidth of the required matched filter. Hence, the implementation of a filter matched to a wide-bandwidth pulse is not adequate for a small number of available taps implying uncontrolled results. As an example of such a behavior, we compare the equalizer filters for PAM-2 in case of using $N_{taps} = 3$ to the ones generated for $N_{taps} = 15$. Results are displayed in Fig. 6(a) and Fig. 6(b), respectively. In case of $NBW$ from 60% to 80%, the equalization requires a relevant emphasis for high frequency components as it can be observed for the proper-behavior case of Fig. 6(b) derived for $N_{taps} = 15$. So, if the number of taps is too small, such a requested filter behavior cannot be satisfied and the LMS algorithm drives the filter to an improper behavior characterized by a strong filtering effects on low frequencies, consequently implying a large penalty. Such a behavior is clearly displayed in Fig. 6(b) and clearly explains results of Fig. 5(a).

5. Comments and conclusions

We studied performances of multilevel PAM-$n$ with $n = 2, 4, 8$ for the use in short-reach scenarios, specifically targeting intra data-center transmission. We supposed to implement a simple DSP-based transmitter using a low extinction-ratio (ER= 6 dB) Mach-Zehnder modulator for the electro-optical conversion, and delivering PAM signaling. While at the receiver we suppose to take advantage of a DSP unit implementing a FIR filter equalizer set by a training-driven LMS algorithm. With a simulative campaign, we analyzed the cases of given-$R_b$ of 25 Gbps and of given-$R_s$ of 25 Gbaud, and studied the system resilience to Tx/Rx electrical bandwidth limitations and to the variations of equalizer benefits with the number of FIR filter taps.

We observed that PAM-2 can operate with only 0.2 dB of power penalty with a normalized bandwidth down to 50%, requiring an equalizer of only 7 taps. For the PAM-4, the requirement on the equalizer is still only 7 taps, but the power penalty for $NBW = 50\%$ increases up to 0.9 dB for the given-$R_b$ case and up to 1.4 dB for the given-$R_s$ scenario. As expected, PAM-8 resulted
to be the most sensitive to lack of bandwidth and to minimize penalties it requires an equalizer with at least a 9-tap FIR filter for the given-\( R_0 \) case and 19 taps the given-\( R_s \) scenario. But for the given-\( R_b \), the equalizer enables to work down to \( NBW = 50\% \) with 1.8 dB penalty, while for the given-\( R_s \) a BER flooring effect occurs, limiting the minimum \( NBW \) to 70\% corresponding with 1.8 dB penalty.

6. References


