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Loewner-based macromodeling with exact interpolation constraints

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Abstract—The so-called Loewner framework has been recently revitalized, providing an alternative approach to the more standard Vector Fitting scheme for building compact macromodels of interconnect networks. The matrix approximation process that is embedded in the Loewner method produces however models whose accuracy cannot be locally controlled at a desired frequency point. This paper proposes a simple solution for constraining the model response at any selected frequency value, with emphasis on the DC point. Two application examples illustrate the effectiveness of this approach.

I. INTRODUCTION AND MOTIVATION

The physical processes causing signal and power degradation due to interaction between signals and fields with complicated interconnect geometries and non-ideal material characteristics can be effectively captured by low-order dynamical models, expressed as ordinary differential equations or equivalent circuits. Such compact models are usually identified from tabulated frequency responses available from full-wave field solvers, using one of the many available macromodeling methods that have been introduced and refined over the last two decades. Once a macromodel is available, system-level simulation becomes tractable using off-the-shelf circuit solvers of the SPICE class. Hence, macromodeling schemes form a basic functional block in most advanced software packages for Signal and Power Integrity (SPI) applications.

Vector Fitting (VF) [1] and its variants [2], [3] has become the method of choice for macromodeling, due to its exceptional robustness and versatility. Recently, the Loewner framework [4], [5] can be used to solve this problem. We partition the set of frequencies \(\{f_1, \ldots, f_N\}\) into right frequencies \(\lambda_k, k = 1, \ldots, N/2\) and left frequencies \(\mu_h, h = 1, \ldots, N/2\). It is advised [6, Ch. 2.1] to use odd frequencies and their complex conjugates as right data and even frequencies with their complex conjugates.
Choosing point model of size \(s\) satisfying the right and left interpolation conditions [4], we select right tangential directions as \(\ell_k\) and left data \(\ell_k^*H_k = v_h\). These quantities are collected into the following matrices

\[
A = \text{diag} \left[ \lambda_1 \ldots \lambda_s \right], \quad M = \text{diag} \left[ \mu_1 \ldots \mu_s \right], \quad (3)
\]

\[
R = \begin{bmatrix} r_1 \ldots r_s \end{bmatrix}, \quad W = \begin{bmatrix} w_1 \ldots w_s \end{bmatrix}, \quad (4)
\]

\[
L = \begin{bmatrix} \ell_1 \ldots \ell_s \end{bmatrix}, \quad V = \begin{bmatrix} v_1 \ldots v_s \end{bmatrix}, \quad (5)
\]

Next, the Loewner matrix is defined entry-wise as

\[
\mathbb{L}_{\ell_k} = \frac{v_k r_k - \ell_k w_k}{\mu_h - \lambda_k}, \quad (6)
\]

and shifted Loewner matrix is defined as

\[
\mathbb{L}_{\ell_k} = \frac{\mu_h v_k r_k - \ell_k w_k}{\mu_h - \lambda_k}. \quad (7)
\]

We can immediately (with no required computation) write a (non-minimal) descriptor realization

\[
H(s) = W (L_s - sL)\omega V \quad (8)
\]

satisfying the right and left interpolation conditions [4]

\[
H(\lambda_k) = w_k r_k \quad (10)
\]

and \(H(\mu_h) = v_h\). To obtain a minimal realization, we perform a singular value decomposition

\[
[Y, \Sigma, X] = \text{svd}(\mathbb{L}_x - x\mathbb{I}_s), \quad x \in \{ f_i \}. \quad (9)
\]

Choosing \(n\) as the singular value where to truncate the SVD (\(n\) is application-dependent), we define (in Matlab notation) \(X_n = X(:, 1:n)\) and \(Y_n = Y(:, 1:n)^*\). The model of size \(n\) in descriptor form is

\[
E = -Y_n^*X_n = -\mathbb{I}_n, \quad (10)
\]

\[
A = -Y_n^*L_n X_n = -\mathbb{I}_{sn}, \quad (11)
\]

\[
B = Y_n V = V_n, \quad C = WX_n = W_n, \quad D = 0. \quad (12)
\]

III. Exact Interpolation at a Finite Point

A D term can be introduced with the parametrization

\[
E_d = -L_n, \quad (13)
\]

\[
A_d = -\mathbb{L}_sn + (Y_n L_n) D (R_n X_n) = -\mathbb{L}_sn + L_n DR_n, \quad (14)
\]

\[
B_d = V_n - (Y_n L_n) D = V_n - L_n D, \quad (15)
\]

\[
C_d = W_n - D (R_n X_n) = W_n - DR_n, \quad (16)
\]

\[
D_d = D. \quad (17)
\]

where \(D \in \mathbb{C}^{p \times p}\) is a free parameter matrix [4, Th. 5.2]. We wish to fix \(D\) to impose exact interpolation at a finite point \(s_0\) (at DC or any arbitrary finite frequency):

\[
C_d(s_0E_d - A_d)^{-1}B_d + D_d = H_0. \quad (18)
\]

Using the quantities described in (13)-(17), we obtain:

\[
(W_n - DR_n) (\mathbb{L}_{sn} - s_0 \mathbb{L}_n - L_n DR_n)^{-1} (V_n - L_n D) + D = H_0.
\]

The Sherman Morrison Woodbury formula can be used to compute the inverse of \(\mathbb{L}_{sn} - s_0 \mathbb{L}_n - L_n DR_n\) as a rank 1 correction of \(\mathbb{L}_{sn} - s_0 \mathbb{L}_n\)

\[
(\Phi - L_n DR_n)^{-1} = \Phi^{-1} + \Phi^{-1} L_n D (I - R_n \Phi^{-1} L_n D)^{-1} R_n \Phi^{-1},
\]

where \(\Phi = \mathbb{L}_{sn} - s_0 \mathbb{L}_n\). We define \(\Phi_{WL} = W_n \Phi^{-1} L_n\), \(\Phi_{RV} = R_n \Phi^{-1} V_n\) and \(\Phi_{RL} = R_n \Phi^{-1} L_n\) and \(\Phi_{WV} = W_n \Phi^{-1} V_n\). After some matrix manipulations, we find

\[
D_d = \left[ (\Phi_{WL} - I) + (S_0 - \Phi_{WV}) (\Phi_{RV} - I)^{-1} \Phi_{RL} \right]^{-1}.
\]

The final realization is computed from (13)-(17). The interpolation condition (18) is verified by direct substitution.

IV. Numerical Results

The performance of the proposed algorithm is illustrated on two different interconnect examples. The first testcase is a via field underneath an LGA connector, known through 301 linearly spaced scattering frequency samples (from DC to 30 GHz). We disregard the DC point and use the remaining samples, together with their complex conjugates, to build \(\Lambda, M, R, L, W, V, L\) and \(\mathbb{L}_s\) in the real approach [5, App. B].

The normalized singular values of the Loewner, the shifted Loewner matrix and the linear combination \(\mathbb{L}_s - f_{301} \mathbb{L}\) are shown in Fig. 1. We truncate the SVD at \(n = 94\), yielding a model as in (10)-(12), which, when plotted against the raw data (Fig. 2a), yields an error below 20dB. After enforcing the exact value at DC as

![Normalized Singular Values](image-url)
described in Sect. III, a model of the form (13)-(17) is obtained (Fig. 2b) with an accuracy of −18dB.

The errors of the two models are plotted in Fig. 3a, showing the singular values of the matrices obtained by subtracting the model evaluated at each frequency from the corresponding measurement. The error at DC for the D ≠ 0 model is below −300dB, as expected. Fig. 3b shows the poles of the two systems, both being stable.

The second example is a 4-port package interconnect, characterized through 467 samples of its S-parameters, ranging from 0 to 30GHz, and computed via a field solver. The samples are processed as for the LGA via field. The normalized singular values of the Loewner, the shifted Loewner matrix and the linear combination LL s − f 467 LL s are shown in Fig. 4. We truncate the SVD to n = 39, yielding a model as in (10)-(12), which, when plotted against the measurements (Fig. 4a), yields an error below −47dB. After enforcing the exact DC value following Sect. III, a model as in (13)-(17) is obtained (Fig. 5b), with an accuracy below −43dB.

The errors of the two models are shown in Fig. 6a. The error at DC for the D ≠ 0 model is below −300dB, as expected. Fig. 6b shows the poles of the two systems. Interestingly, for this example, the original system is unstable, while after enforcing the DC condition, all poles become stable.

V. CONCLUSION

We demonstrated how the state-space matrices of a Loewner-based macromodel can be redefined to enforce exact interpolation conditions at DC (more generally, at any arbitrary frequency point). This condition is crucial whenever a very aggressive accuracy is desired, e.g., when the model is terminated with nonlinear device models, whose bias level must be carefully controlled.

REFERENCES