Loewner-based macromodeling with exact interpolation constraints

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***Abstract*—The so-called Loewner framework has been recently revitalized, providing an alternative approch to the more standard Vector Fitting scheme for building compact macromodels of interconnect networks. The matrix approximation process that is embedded in the Loewner method produces however models whose accuracy cannot be locally controlled at a desired frequency point. This paper proposes a simple solution for constraining the model response at any selected frequency value, with emphasis on the DC point. Two application examples illustrate the effectiveness of this approach.**

I. Introduction and Motivation

The physical processes causing signal and power degradation due to interaction between signals and fields with complicated interconnect geometries and non-ideal material characteristics can be effectively captured by low-order dynamical models, expressed as ordinary differential equations or equivalent circuits. Such compact models are usually identified from tabulated frequency responses available from full-wave field solvers, using one of the many available macromodeling methods that have been introduced and refined over the last two decades. Once a macromodel is available, system-level simulation becomes tractable using off-the-shelf circuit solvers of the SPICE class. Hence, macromodeling schemes form a basic functional block in most advanced software packages for Signal and Power Integrity (SPI) applications.

Vector Fitting (VF) [1] and its variants [2], [3] has become the method of choice for macromodeling, due to its exceptional robustness and versatility. Recently, the Loewner framework [4], [5] has been revitalized as a possible alternative to VF. Based on a solid theoretical basis, this approach produces a macromodel in descriptor form, whose transfer function interpolates exactly the given frequency samples. To reduce the model order, a truncated singular value decomposition is applied, yielding an approximation error which cannot be controlled directly at all frequencies (unless when processing noise-free samples of purely rational transfer functions). This is undesirable, since SPI simulations require a very aggressive accuracy at specific frequencies, in particular at the DC point.

This paper proposes a simple yet effective process that, starting from a macromodel known via its state-space (descriptor) realization, as usually obtained by the Loewner method, produces a new macromodel characterized by different state-space matrices, and whose frequency response matches exactly a prescribed value at a desired finite frequency point. The final result is a macromodel that admits an approximation error at any frequency, as implied by the order reduction process that is applied in the model construction, but which is exact at least at one frequency point. This model will then behave more robustly in system-level simulations, especially in the presence of nonlinear termination networks.

II. Review of the Loewner framework

Given tabulated data (S-, Y- or Z- parameters)

$$(f_i, \mathbf{H}_i), \quad i = 1, \ldots, N, \tag{1}$$

where $\mathbf{H}_i \in \mathbb{C}^{p\times p}$ is the multiport parameter measured at f_i, macromodeling seeks a rational model whose transfer function evaluated at f_i approximates $\mathbf{H}_i$:

$$\mathbf{H}_i \approx \mathbf{C}\left(j2\pi f_i \mathbf{E} - \mathbf{A}\right)^{-1}\mathbf{B} + \mathbf{D}, \quad \forall i = 1, \ldots, N.$$

The Loewner framework [4], [5] can be used to solve this problem. We partition the set of frequencies

$$\{f_1, \ldots, f_N\} = \{\lambda_1, \ldots, \lambda_{N/2}\} \cup \{\mu_1, \ldots, \mu_{N/2}\} \tag{2}$$

into right frequencies λ_k, $k = 1, \ldots, \frac{N}{2}$ and left frequencies μ_h, $h = 1, \ldots, \frac{N}{2}$. It is advised [6, Ch. 2.1] to use odd frequencies and their complex conjugates as right data and even frequencies with their complex conjugates as left data. We select right tangential directions as column vectors $\mathbf{r}_k$ and left directions as row vectors ℓ_h. They can be chosen, for simplicity, as vectors of the identity matrix [5]. Matrix data (1) is converted to right vector data $\mathbf{H}_k\mathbf{r}_k = \mathbf{w}_k$ and left vector data $\ell_h\mathbf{H}_h = \mathbf{v}_h$. These quantities are collected into the following matrices

$$\Lambda = \text{diag}\begin{bmatrix}\lambda_1 & \dots & \lambda_{\frac{N}{2}}\end{bmatrix}, M = \text{diag}\begin{bmatrix}\mu_1 & \dots & \mu_{\frac{N}{2}}\end{bmatrix}, \quad (3)$$

$$\mathbf{R} = \begin{bmatrix}\mathbf{r}_1 & \dots & \mathbf{r}_{\frac{N}{2}}\end{bmatrix}, \mathbf{W} = \begin{bmatrix}\mathbf{w}_1 & \dots & \mathbf{w}_{\frac{N}{2}}\end{bmatrix}, \quad (4)$$

$$\mathbf{L} = \begin{bmatrix}\ell_1 \\ \vdots \\ \ell_{\frac{N}{2}}\end{bmatrix}, \mathbf{V} = \begin{bmatrix}\mathbf{v}_1 \\ \vdots \\ \mathbf{v}_{\frac{N}{2}}\end{bmatrix}. \quad (5)$$

Next, the Loewner matrix is defined entry-wise as

$$\mathbb{L}_{hk} = \frac{\mathbf{v}_h\mathbf{r}_k - \ell_h\mathbf{w}_k}{\mu_h - \lambda_k} \quad (6)$$

and shifted Loewner matrix is defined as

$$\mathbb{L}_{s_{hk}} = \frac{\mu_h\mathbf{v}_h\mathbf{r}_k - \lambda_k\ell_h\mathbf{w}_k}{\mu_h - \lambda_k}. \quad (7)$$

We can immediately (with no required computation) write a (non-minimal) descriptor realization

$$\mathbf{H}(s) = \mathbf{W}\left(\mathbb{L}_s - s\mathbb{L}\right)^{-1}\mathbf{V} \quad (8)$$

satisfying the right and left interpolation conditions [4] $\mathbf{H}(\lambda_k)\mathbf{r}_k = \mathbf{w}_k$ and $\ell_h\mathbf{H}(\mu_h) = \mathbf{v}_h$. To obtain a minimal realization, we perform a singular value decomposition

$$[\mathbf{Y}, \Sigma, \mathbf{X}] = \text{svd}(\mathbb{L}_s - x\mathbb{L}), \quad x \in \{f_i\}. \quad (9)$$

Choosing n as the singular value where to truncate the SVD (n is application-dependent), we define (in Matlab notation) $\mathbf{X}_n = \mathbf{X}(:, 1:n)$ and $\mathbf{Y}_n = \mathbf{Y}(:, 1:n)^*$. The model of size n in descriptor form is

$$\mathbf{E} = -\mathbf{Y}_n\mathbb{L}\mathbf{X}_n = -\mathbb{L}_n, \quad (10)$$

$$\mathbf{A} = -\mathbf{Y}_n\mathbb{L}_s\mathbf{X}_n = -\mathbb{L}_{sn}, \quad (11)$$

$$\mathbf{B} = \mathbf{Y}_n\mathbf{V} = \mathbf{V}_n, \mathbf{C} = \mathbf{W}\mathbf{X}_n = \mathbf{W}_n, \mathbf{D} = \mathbf{0}. \quad (12)$$

III. Exact interpolation at a finite point

A $\mathbf{D}$ term can introduced with the parametrization

$$\mathbf{E}_d = -\mathbb{L}_n, \quad (13)$$

$$\mathbf{A}_d = -\mathbb{L}_{sn} + (\mathbf{Y}_n\mathbf{L})\mathbf{D}(\mathbf{R}\mathbf{X}_n) = -\mathbb{L}_{sn} + \mathbf{L}_n\mathbf{D}\mathbf{R}_n, \quad (14)$$

$$\mathbf{B}_d = \mathbf{V}_n - (\mathbf{Y}_n\mathbf{L})\mathbf{D} = \mathbf{V}_n - \mathbf{L}_n\mathbf{D} \quad (15)$$

$$\mathbf{C}_d = \mathbf{W}_n - \mathbf{D}(\mathbf{R}\mathbf{X}_n) = \mathbf{W}_n - \mathbf{D}\mathbf{R}_n \quad (16)$$

$$\mathbf{D}_d = \mathbf{D}, \quad (17)$$

where $\mathbf{D} \in \mathbb{C}^{p\times p}$ is a free parameter matrix [4, Th. 5.2]. We wish to fix $\mathbf{D}$ to impose exact interpolation at a finite point s_0 (at DC or any arbitrary finite frequency):

$$\mathbf{C}_d(s_0\mathbf{E}_d - \mathbf{A}_d)^{-1}\mathbf{B}_d + \mathbf{D}_d = \mathbf{H}_0. \quad (18)$$

Using the quantities described in (13)-(17), we obtain:

$$(\mathbf{W}_n - \mathbf{D}\mathbf{R}_n)\left(\mathbb{L}_{sn} - s_0\mathbb{L}_n - \mathbf{L}_n\mathbf{D}\mathbf{R}_n\right)^{-1}(\mathbf{V}_n - \mathbf{L}_n\mathbf{D}) + \mathbf{D} = \mathbf{H}_0.$$

The Sherman Morrison Woodbury formula can be used to compute the inverse of $\mathbb{L}_{sn} - s_0\mathbb{L}_n - \mathbf{L}_n\mathbf{D}\mathbf{R}_n$ as a rank p correction of $\mathbb{L}_{sn} - s_0\mathbb{L}_n$

$$(\mathbf{\Phi} - \mathbf{L}_n\mathbf{D}\mathbf{R}_n)^{-1} = \mathbf{\Phi}^{-1} + \mathbf{\Phi}^{-1}\mathbf{L}_n\mathbf{D}\left(\mathbf{I} - \mathbf{R}_n\mathbf{\Phi}^{-1}\mathbf{L}_n\mathbf{D}\right)^{-1}\mathbf{R}_n\mathbf{\Phi}^{-1},$$

where $\mathbf{\Phi} = \mathbb{L}_{sn} - s_0\mathbb{L}_n$. We define $\mathbf{\Phi}_{WL} = \mathbf{W}_n\Phi^{-1}\mathbf{L}_n$, $\mathbf{\Phi}_{RV} = \mathbf{R}_n\Phi^{-1}\mathbf{V}_n$, $\mathbf{\Phi}_{RL} = \mathbf{R}_n\Phi^{-1}\mathbf{L}_n$ and $\mathbf{\Phi}_{WV} = \mathbf{W}_n\Phi^{-1}\mathbf{V}_n$. After some matrix manipulations, we find

$$\mathbf{D}_d = \left[(\mathbf{\Phi}_{WL} - \mathbf{I}) + (\mathbf{S}_0 - \mathbf{\Phi}_{WV})(\mathbf{\Phi}_{RV} - \mathbf{I})^{-1}\mathbf{\Phi}_{RL}\right]^{-1} (\mathbf{S}_0 - \mathbf{\Phi}_{WV})(\mathbf{\Phi}_{RV} - \mathbf{I})^{-1}. \quad (19)$$

The final realization is computed from (13)-(17). The interpolation condition (18) is verified by direct substitution.

IV. Numerical results

The performance of the proposed algorithm is illustrated on two different interconnect examples. The first testcase is a via field underneath an LGA connector, known through 301 linearly spaced scattering frequency samples (from DC to 30 GHz). We disregard the DC point and use the remaining samples, together with their complex conjugates, to build Λ, M, $\mathbf{R}$, $\mathbf{L}$, $\mathbf{W}$, $\mathbf{V}$, $\mathbb{L}$ and $\mathbb{L}_s$ in the real approach [5, App. B].

The normalized singular values of the Loewner, the shifted Loewner matrix and the linear combination $\mathbb{L}_s - f_{301}\mathbb{L}$ are shown in Fig. 1. We truncate the SVD at $n = 94$, yielding a model as in (10)-(12), which, when plotted against the raw data (Fig. 2a), yields an error below -20dB. After enforcing the exact value at DC as

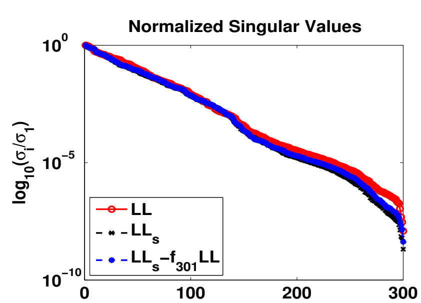
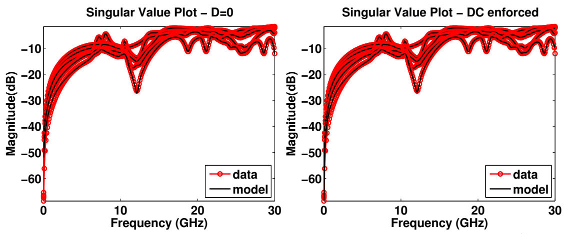


Figure 1: Drop of the normalized singular values

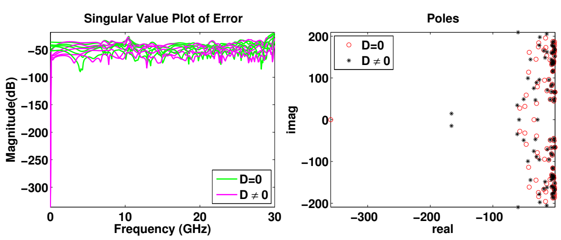


(a) Model with $\mathbf{D} = \mathbf{0}$ (b) Model with $\mathbf{D} \neq \mathbf{0}$

Figure 2: Responses of LGA via models of order 94

described in Sect. III, a model of the form (13)-(17) is obtained (Fig. 2b) with an accuracy of -18dB.

The errors of the two models are plotted in Fig. 3a, showing the singular values of the matrices obtained by subtracting the model evaluated at each frequency from the corresponding measurement. The error at DC for the $\mathbf{D} \neq \mathbf{0}$ model is below -300dB, as expected. Fig. 3b shows the poles of the two systems, both being stable.



(a) Error plots of the models (b) Poles of the models

Figure 3: LGA via model with DC constraint.

The second example is a 4-port package interconnect, characterized through 467 samples of its S-parameters, ranging from 0 to 30GHz, and computed via a field solver. The samples are processed as for the LGA via field. The normalized singular values of the Loewner, the shifted Loewner matrix and the linear combination $\mathbb{L}_s - f_{467}\mathbb{L}$ are shown in Fig. 4. We truncate the SVD to $n = 39$, yielding a model as in (10)-(12), which, when plotted against the measurements (Fig. 5a), yields an error below -47dB. After enforcing the exact DC value following Sect. III, a model as in (13)-(17) is obtained (Fig. 5b), with an accuracy below -43dB.

The errors of the two models are shown in Fig. 6a. The error at DC for the $\mathbf{D} \neq \mathbf{0}$ model is below -300dB, as expected. Fig. 6b shows the poles of the two systems. Interestingly, for this example, the original system is unstable, while after enforcing the DC condition, all poles become stable.

V. Conclusion

We demonstrated how the state-space matrices of a Loewner-based macromodel can be redefined to enforce

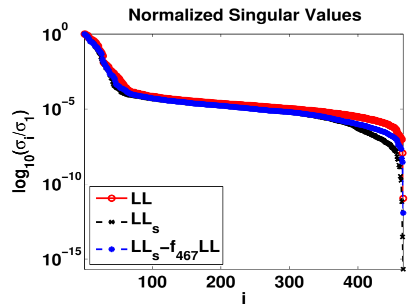
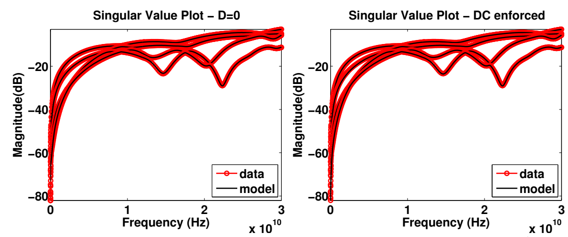
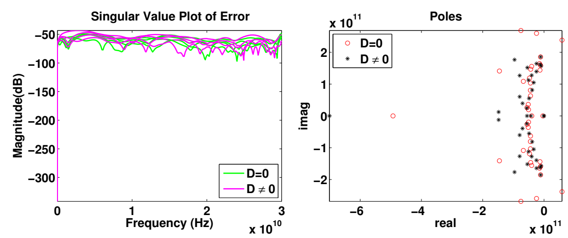


Figure 4: Drop of the normalized singular values



(a) Model with $\mathbf{D} = \mathbf{0}$ (b) Model with $\mathbf{D} \neq \mathbf{0}$

Figure 5: Responses of interconnect models of order 39



(a) Error plots of the models (b) Poles of the models

Figure 6: Package model with DC constraint.

exact interpolation conditions at DC (more generally, at any arbitrary frequency point). This condition is crucial whenever a very aggressive accuracy is desired, e.g., when the model is terminated with nonlinear device models, whose bias level must be carefully controlled.

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