A Model Predictive Control approach for semi-active suspension control problem of a full car

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M.Q. Nguyen[1*], M. Canale [2], O. Sename[1], L. Dugard [1]

***Abstract*— In this paper, a semi-active suspension Model Predictive Control (MPC) is designed for a full vehicle system equipped with 4 semi-active dampers. The main challenge in the semi-active suspension control problem is to tackle with the dissipativity constraints of the semi-active dampers. The constraints are here recasted as input and state constraints. The controller is designed in the MPC framework where the effects of the unknown road disturbances are taken into account. An observer approach allows to estimate the road disturbance information to be used by the controller during the prediction step. Then, the MPC suspension control law with road estimation (but without road preview) is computed by minimizing a quadratic cost function, giving a trade-off between the comfort and the handling, while guaranteeing phyiscal constraints of the semi-active dampers. Simulation results performed on a nonlinear full car model are presented in order to show the effectiveness of the proposed approach.**

Keywords: Semi-active suspension, road disturbance estimation, MPC control, Input Constraints.

I. Introduction

The suspension system plays a key role in enhancing the vehicle performance with regard to ride comfort and road handling. Semi-active suspensions are today more and more used in automotive industry because of their efficiency, while being less expensive and consumming less energy than pure active suspensions. However, the main challenge of semi-active suspension control problems is to handle the dissipativity constraints of the dampers. In this context, several control design problems have then been tackled with many different approaches. [1], [2] give extensive surveys on semi-active suspension control. In more detail, [3] proposed a classical control method based on the Skyhook control to improve the ride comfort. Then, several extensions of skyhook control have been presented in the past decades as in [4], or [5] where Mixed Skyhook-ADD (Acceleration Driven Damping) is proposed. More recently, some modern control approaches have been suggested. In [6] an LQ-based clipped optimal control is proposed. Some LPV techniques for semi-active suspension control problems have been presented as in [7], [8] and in [9] where an LPV approach with sector condition has been introduced to deal with the dissipativity constraints of the dampers.

As to actuator saturation control problems, Model Predictive Control (MPC), see e.g. [10], allows to explicitly take into account the effect of input and state constraints in the control design step. Several works have employed the MPC approach for semi-active suspension systems. Nevertheless, most of the studies considered a quarter-car model only. Let us mention here [11] where the constrained quarter-car semi-active suspension is modelled as a switching affine system and the MPC controller is computed thanks to mixed-integer quadratic programming techniques. In [12], a fast MPC is designed for a half car model, but the MPC controller is still designed based on a quarter car suspension model. However, in [11] and [12], the effects of road disturbances are not taken into account in the prediction horizon. On the other hand, [13] proposes a methodology for optimal semi-active suspension system based on MPC control for a quarter car model while assuming that the road disturbance is measured in advance and taken into account within the prediction horizon. It is worth noting that the quarter car model equipped with one semi-active damper is not able to describe the full dynamic of the vehicle with four semi-active dampers. In order to deal with the full car model case, one possibility is to design four separate controllers at the four corners. However, by this way, the effects of the coupling and the load transfer distribution between the corners during various driving situations (cornering, steering, accelerating, and braking...) may not be considered, which could lead to lower performance.

Concerning the full car dynamics, up to the authors knowledge, very few studies have been proposed to develop MIMO MPC semi-active control techniques. In particular, [14] employs a nonlinear programming approach (not suitable for implementation). To overcome such a problem, [14] introduces an approximate descripition of the constraints as well as the clipping of the control action. On the other hand, taking into account the disturbance effects for the computation of the MPC control action leads to better performance results. In this regard, [15] employs the road profile preview by means of expensive and not standard sensors (e.g. camera) for the case of active suspension. Disturbance estimation is not used, except in a case, to verify the preview obtained by the camera and still in the context of active suspension.

In this work, a semi-active suspension MPC controller is designed for a full vehicle model equipped with 4 semi-active dampers. The proposed solution integrates a state feedback control with an observer of the vehicle state variables and of the road disturbance. The paper contributions are twofolds:

- An observer approach is proposed to estimate both the

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system state (needed anyway by all MPC approaches) and road disturbances. It is worth mentioning that while the estimation of the road inputs allows to improve the efficiency of the predictive controller, it is here obtained using standard sensors and thus differs from the preview approach.

- A MPC suspension control with road disturbance estimation (but without road preview) is obtained by optimimizing a quadratic cost function. This cost describes the ride comfort and road holding performances, while ensuring the dissipativity constraints of the semi-active dampers. The controller is derived through the solution to a mixed integer quadratic programming (MIQP) which allows its implementation. The results are compared to those obtained by MPC with disturbance preview and by MPC without taking into account the disturbance during the prediction; they show the usefulness of the proposed approach.

The structure of the paper is given as follows. Section II describes a full vertical vehicle model and the problem formulation. Section III introduces the semi-active suspension control design using MPC. Section IV presents the observer design for state and road disturbance estimation. Simulation results are given in section V. Finally, some conclusions are drawn in section VI.

II. A FULL CAR MODEL EQUIPPED WITH 4 SEMI-ACTIVE SUSPENSIONS

A. Full car model

A full car vertical model is used for the analysis and control of the vehicle dynamic behavior. This is a classical 7 degrees of freedom (DOF) suspension model, obtained from a nonlinear full vehicle model (referred in [16]). This model involves the chassis dynamics (vertical (z_s), roll (θ) and pitch(ϕ)), and the vertical displacements of the wheels $z_{us_{ij}}$ at the front/rear ($i=(f,r)$)-left/right corner ($j=(l,r)$). The vertical 7 DOF full-car model is governed by the following dynamic equations:

$$\begin{cases} m_s\ddot{z}_s &= -F_{sfl}-F_{sfr}-F_{srl}-F_{srr} \\ I_x\ddot{\theta} &= (-F_{sfr}+F_{sfl})t_f+(-F_{srr}+F_{srl})t_r \\ I_y\ddot{\phi} &= (F_{srr}+F_{srl})l_r-(F_{sfr}+F_{sfl})l_f \\ m_{us}\ddot{z}_{us_{ij}} &= F_{s_{ij}}-F_{tz_{ij}} \end{cases} \quad (1)$$

where I_x, I_y are the moments of inertia of the sprung mass around the longitudinal and lateral axis respectively, h is the height of center of gravity (COG). l_f, l_r, t_f, t_r are COG-front, rear, left, right distances respectively.

$F_{tz_{ij}}$ are the vertical tire forces, given as:

$$F_{tz_{ij}} = k_{t_{ij}}(z_{us_{ij}}-z_{r_{ij}}) \quad (2)$$

where $k_{t_{ij}}$ are the stiffness coefficients of the tires, and $z_{r_{ij}}$ the road profiles.

The vertical suspension forces $F_{s_{ij}}$ at the 4 corners of the vehicle are modeled by a spring and a damper with non linear characteristics for simulation, and linear ones for control design. The equation (3) allows to model the suspension force used in the control design step:

$$F_{s_{ij}} = k_{ij}(z_{s_{ij}}-z_{us_{ij}})+F_{d_{ij}} \quad (3)$$

where k_{ij} is the nominal spring stiffness coefficient, $z_{s_{ij}}$ is the chassis position at each corner and $F_{d_{ij}}$ is the semi-active controlled damper force given by:

$$F_{d_{ij}} = c_{ij}(.)\dot{z}_{def_{ij}} = c_{ij}(.)(\dot{z}_{s_{ij}}-\dot{z}_{us_{ij}}) \quad (4)$$

where $\dot{z}_{def_{ij}}$ is the deflection speed and the damping coefficient $c_{ij}(.)$ is assumed to be varying for control purpose. To ensure the dissipativity constraint of each semi-active damper, the following constraint must be considered:

$$0 \leqslant c_{min_{ij}} \leqslant c_{ij}(.) \leqslant c_{max_{ij}} \quad (5)$$

Now, let us rewrite the damper force (4) as follows:

$$F_{d_{ij}} = c_{nom_{ij}}\dot{z}_{def_{ij}}+u_{ij} \quad (6)$$

where $c_{nom_{ij}} = (c_{max_{ij}}+c_{min_{ij}})/2$ is the nominal damping coefficient, u_{ij} is the incremental force and is considered as the control input. Then, the equation (3) becomes:

$$F_{s_{ij}} = k_{ij}(z_{s_{ij}}-z_{us_{ij}})+c_{nom_{ij}}(\dot{z}_{s_{ij}}-\dot{z}_{us_{ij}})+u_{ij} \quad (7)$$

where $z_{s_{ij}}$ are the sprung mass positions at each corner of the vehicle. By substituting the tire force equations (2), the suspension force equations (7) into the vehicle equations (1), the following LTI state-space representation with 14 state variables is given by:

$$\begin{cases} \dot{x}(t) = Ax(t)+B_1w(t)+B_2u(t) \\ y(t) = Cx(t)+D_1w(t)+D_2u(t)+\nu(t) \end{cases} \quad (8)$$

where

$$\begin{cases} x = [z_s,\theta,\phi,z_{usfl},z_{usfr},z_{usrl},z_{usrr},\dot{z}_s,\dot{\theta},\dot{\phi}, \\ \quad \dot{z}_{usfl},\dot{z}_{usfr},\dot{z}_{usrl},\dot{z}_{usrr}] \\ u = [u_{fl},u_{fr},u_{rl},u_{rr}] \\ w = [z_{rfl},z_{rfr},z_{rrl},z_{rrr}] \\ y = [\ddot{z}_{sfl},\ddot{z}_{sfr},\ddot{z}_{srl},\ddot{z}_{srr},z_{usfl},z_{usfr},z_{usrl},z_{usrr}] \end{cases}$$

are the state, the control input vector, the disturbance inputs, and output measurements vectors respectively. $\nu(t)$ is the measurement noise. A,B_1,B_2,C,D_1,D_2 are matrices of the state space representation.

Since in the MPC approach the optimization problem must be performed in discrete time domain, then the continuous time model (8) has been discretized with a sampling time T_s using the zero order hold method. The obtained discrete time model is denoted as:

$$\begin{cases} x_{k+1} = A_dx_k+B_{1d}w_k+B_{2d}u_k \\ y_k = C_dx_k+D_{1d}w_k+D_{2d}u_k+\nu_k \end{cases} \quad (9)$$

where $A_d,B_{1d},B_{2d},C_d,D_{1d},D_{2d}$ are matrices of the state space representation.

B. Input constraints

The dissipativity conditions of the semi-active damper given in (5) is here transformed into input constraints. Note that from (5-6), it follows that:

$$\begin{aligned} c_{min_{ij}}\dot{z}_{def_{ij}} \leq F_{d_{ij}} \leq c_{max_{ij}}\dot{z}_{def_{ij}} \quad &\text{if} \quad \dot{z}_{def_{ij}} \geq 0 \\ c_{max_{ij}}\dot{z}_{def_{ij}} \leq F_{d_{ij}} \leq c_{min_{ij}}\dot{z}_{def_{ij}} \quad &\text{if} \quad \dot{z}_{def_{ij}} < 0 \end{aligned} \quad (10)$$

The dissipativity constraint is now recast into:

$$\begin{aligned} c_{min_{ij}}\dot{z}_{def_{ij}} \leq c_{nom_{ij}}\dot{z}_{def_{ij}}+u_{ij} \leq c_{max_{ij}}\dot{z}_{def_{ij}} \quad &\text{if} \quad \dot{z}_{def_{ij}} \geq 0 \\ c_{max_{ij}}\dot{z}_{def_{ij}} \leq c_{nom_{ij}}\dot{z}_{def_{ij}}+u_{ij} \leq c_{min_{ij}}\dot{z}_{def_{ij}} \quad &\text{if} \quad \dot{z}_{def_{ij}} < 0 \end{aligned}$$

Since $c_{nom_{ij}} = \dfrac{(c_{max_{ij}} + c_{min_{ij}})}{2}$, it results:

$$|u_{ij}| \leq \frac{(c_{max_{ij}} - c_{min_{ij}})}{2} |\dot{z}_{def_{ij}}| \tag{11}$$

for i corresponding to front/rear and j to left/right

Actually, $\dot{z}_{def_{ij}} = \dot{z}_{s_{ij}} - \dot{z}_{us_{ij}}$ is a linear combination of the system state x, i.e. $\dot{z}_{def_{ij}} = \dot{z}_{s_{ij}} - \dot{z}_{us_{ij}} = C_{in}x$, where C_{in} is an appropriate matrix. Thus, in the discrete time domain, the control input u_k must satisfy the following constraints:

$$\begin{cases} H_1 u_k \leq \Gamma_1 x_k & \text{if} \quad C_{in} x_k \geq 0, \\ H_2 u_k \leq \Gamma_2 x_k & \text{if} \quad C_{in} x_k < 0 \end{cases} \tag{12}$$

where $H_1, H_2, \Gamma_1, \Gamma_2$ are appropriate matrices.

III. Semi-active suspension control using MPC

A. Performance index

The main objective of the suspension in the vehicle system is to isolate the body (comfort performance) from the road disturbances, without deteriorating the road holding (handling performance). Comfort and handling performance can be described through vehicle center of gravity heave acceleration $\ddot{z}_s$ and roll angle θ respectively. In particular, the following performance indices can be considered:

$$J_{comfort} = \int_0^T \ddot{z}_s^2(t)dt \tag{13}$$

$$J_{handling} = \int_0^T \theta^2(t)dt \tag{14}$$

However, it is a well known fact that (13) and (14) are conflicting objectives. For this reason, a control law has to be designed in order to optimize the overall performance through a suitable trade-off between (13) and (14) and taking into account the dissipativity constraint (**??**). Thus, the semi-active suspension control problem can be formulated as a constrained optimization problem that can be casted in the well known framework of MPC.

In this regard, by defining N_p as the prediction horizon, the following cost function is chosen as the performance index to be minimized:

$$J(U, N_p, x_{k|k}) = \sum_{i=0}^{N_p - 1} (1-\rho)(\ddot{z}^s_{k+i|k})^2 + \rho(\theta_{k+i|k})^2 \tag{15}$$

where $\ddot{z}^s_{k+i|k}, \theta_{k+i|k}$ denote the chassis acceleration and roll angle predicted by using the model (9), given the initial state $x_{k|k}$, and $U = [u_{k|k}, u_{k+1|k}, ..., u_{k+N_p-1|k}]$ is the vector of the control moves to be optimized. $\rho \in [0\ 1]$ is a weighting coefficient which can be tuned to achieve a suitable trade-off between comfort and handling performance.

It is worth mentioning that the tuning of the design parameter ρ can be usually obtained through a sequence of trial and error steps. However, in this work, an alternative approach is proposed which consists in considering ρ as the Load Transfer Ratio (LTR) of the vehicle. Actually, LTR can be computed by evaluating the roll load transfer while the vehicle is running. As soon as there exists a load transfer from the left to the right or vice-versa, it means that the vehicle is faced to roll motion. By defining the vertical forces acting on the left and right sides by F_{zl} and F_{zr} respectively, we have:

$$\begin{cases} F_{z_l} = m_s \frac{g}{2} + m_s h \frac{a_y}{l_f} \\ F_{z_r} = m_s \frac{g}{2} - m_s h \frac{a_y}{l_r} \end{cases} \tag{16}$$

that allows us to introduce the LTR as:

$$\rho := \left| \frac{F_{z_l} - F_{z_r}}{F_{z_l} + F_{z_r}} \right| \tag{17}$$

where a_y is the lateral acceleration of the vehicle at COG. Note that, according to (16), the LTR ratio can be evaluated online through the measurement of the lateral acceleration a_y. Since $\rho \in [0,\ 1]$, when $\rho \to 0$, there is neither lateral load transfer nor roll motion, i.e the cost function (15) minimizes the chassis acceleration, aiming at improving the comfort performance. On the other hand, when $\rho \to 1$, the vehicle is within a critical situation caused by the roll motion. In this case, the roll motion in (15) needs to be weighted in order to improve the road holding performance.

B. Optimization problem setup

Following the definitions given in the previous section, the optimization problem of the MPC design can be defined as:

$$\begin{aligned} &\min_U \quad J(U, N_p, x_k) \\ &\text{subject to} \begin{cases} x_{k+1} = A_d x_k + B_{1d} w_k + B_{2d} u_k \\ (12) \end{cases} \end{aligned} \tag{18}$$

In order to compute the control action in the MPC framework, the cost function J in (15) has to be evaluated along the state trajectory, within the prediction horizon, using the state equation in (9). In this regard, given the available measurements described in section II at each sampling time, the system state has to be estimated using a suitable observer. Regarding the disturbance contribution, differently from [14], it is not assumed that road profile preview measurements can be obtained using a camera. Thus, to be able to account for the disturbance effect during the prediction, an extended state observer is designed, considering exisiting standard sensors, allowing to estimate the road input and the state variables simultaneously. In this way, the overall state equation is employed for both prediction and state estimation. To this aim, a disturbance model is needed. One of the most common assumptions in MPC design is that the disturbance is considered to be constant within the prediction horizon, i.e $w_{k+i} = w_k,\ i = 0, ..., N_p - 1$. In this way, the following augmented state space model can be considered in the optimization problem (18):

$$\begin{bmatrix} x_{k+1} \\ w_{k+1} \end{bmatrix} = \begin{bmatrix} A_d & B_{1d} \\ 0 & I \end{bmatrix} \begin{bmatrix} x_k \\ w_k \end{bmatrix} + \begin{bmatrix} B_{2d} \\ 0 \end{bmatrix} u_k \tag{19}$$

The optimzation problem (18) now becomes:

$$\begin{aligned} &\min_U \quad J(U, N_p, x_k) \\ &\text{subject to} \quad (19) \quad \text{and } (12) \end{aligned} \tag{20}$$

The MPC control law is then computed by applying the receding horizon strategy, where only the first element of the computed optimal sequence U is applied as the actual control action: $u_k = u_{k|k}$. On the other hand, the dissipativity constraint (12) depends on the sign of the suspension deflection speed $C_{in}x_k$. Therefore, the switching between the constraints according to the sign of $C_{in}x_k$ must be satisfied. To this aim, the optimization problem (20) can be formulated as a quadratic problem involving logic constraints. In this regard, the optimization procedure is a mixed integer quadratic programming (MIQP) problem ([17]). Thanks to YALMIP [18] and using GUROBI optimization solver [19], the optimal control law can be computed.

IV. State and road disturbance estimation

This section presents the observer design methodology. Firstly, the output equation introduced in (9) can be augmented as follows:

$$y_k = \begin{bmatrix} C_d & D_{1d} \end{bmatrix} \begin{bmatrix} x_k \\ w_k \end{bmatrix} + \begin{bmatrix} D_{2d} \\ 0 \end{bmatrix} u_k + \nu_k \tag{21}$$

Within the available measurements, described in section II, the observability condition of the augmented system (19,21) is satisfied. Then, both the system state and road disturbances can be estimated by using an extended observer with the following structure:

$$\begin{cases} \begin{bmatrix} \hat{x}_{k+1} \\ \hat{w}_{k+1} \end{bmatrix} = A_o \begin{bmatrix} \hat{x}_k \\ \hat{w}_k \end{bmatrix} + B_{2o}u_k + L(y_k - \hat{y}_k) \\ \hat{y}_k = C_o \begin{bmatrix} \hat{x}_k \\ \hat{w}_k \end{bmatrix} + \begin{bmatrix} D_{2d} \\ 0 \end{bmatrix} u_k \end{cases} \tag{22}$$

where L is the observer gain to be designed and $A_o = \begin{bmatrix} A_d & B_{1d} \\ 0 & I \end{bmatrix}$; $B_{2o} = \begin{bmatrix} B_{2d} \\ 0 \end{bmatrix}$; $C_o = \begin{bmatrix} C_d & D_{1d} \end{bmatrix}$.

Let us define the estimation error of the augmented system: $e_k = \begin{bmatrix} x_k \\ w_k \end{bmatrix} - \begin{bmatrix} \hat{x}_k \\ \hat{w}_k \end{bmatrix}$

Then, the estimation error can be inferred from (19) and (22) as:

$$e_{k+1} = A_o \left(\begin{bmatrix} x_k \\ w_k \end{bmatrix} - \begin{bmatrix} \hat{x}_k \\ \hat{w}_k \end{bmatrix} \right) - LC_o \left(\begin{bmatrix} x_k \\ w_k \end{bmatrix} - \begin{bmatrix} \hat{x}_k \\ \hat{w}_k \end{bmatrix} \right) - L\nu_k \tag{23}$$

Finally, one has:

$$e_{k+1} = (A_o - LC_o)e_k - L\nu_k \tag{24}$$

It is well known that the state estimation of dynamic systems in the presence of measurement noise is one of the important problems in control engineering. One of effective solutions dealing with this problem is to use H_2 filtering approach. Therefore, the observer design consists in calculating observer gain L so that the transfer function $T_{\nu e}$ from the measurement noise ν_k to the estimation error e_k meets the H_2-norm upper bound constraint. Moreover, in order to improve the performance of the observer, the poles of the observer are placed in the circle $\mathbf{C}(\sigma, r)$(centered at σ and with the radius r) which is smaller than the unit circle (see Fig. 1). The following theorem allows to solve this problem:

Theorem 1: Consider the observer (22) and a given positive scalar $\gamma > 0$. The poles of the observer are inside the circle $\mathbf{C}(\sigma, r)$ and $\| T_{\nu e} \|_2 < \gamma$, if there exist symmetric positive definite matrices P and R, and matrix Y such that the following inequalities hold:

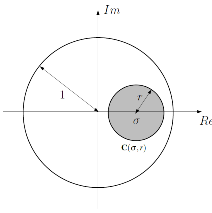

Fig. 1. Sub-region for pole location

$$\begin{bmatrix} P & P(\frac{A_o - \sigma I}{r}) - Y\frac{C_o}{r} & -Y \\ (\frac{A_o - \sigma I}{r})'P - Y'\frac{C_o}{r}' & P & 0 \\ -Y' & 0 & I \end{bmatrix} > 0, \tag{25}$$

$$\begin{bmatrix} R & I & 0 \\ I & P & 0 \\ 0 & 0 & I \end{bmatrix} > 0, \tag{26}$$

$$Trace(R) < \gamma \tag{27}$$

then the estimation error in (24) is asymptotically stable and the observer gain is computed by $L = P^{-1}Y$.

Proof: Consider the estimation error system (24), and by applying the H_2 performance conditions for discrete-time system (see [20]), one has $\| T_{\nu e} \|_2 < \gamma$ if:

$$\begin{bmatrix} P & P(A_o - LC_o) & -PL \\ (A_o - LC_o)'P & P & 0 \\ -L'P' & 0 & I \end{bmatrix} > 0 \tag{28}$$

$$\begin{bmatrix} R & I & 0 \\ I & P & 0 \\ 0 & 0 & I \end{bmatrix} > 0 \tag{29}$$

$$Trace(R) < \gamma \tag{30}$$

Now, to place the poles of the observer into the circle $\mathbf{C}(\sigma, r)$, a simple change in the LMI (28) is used: A_o is replaced by $\frac{A_o - \sigma I}{r}$ and C_o is replaced by $\frac{C_o}{r}$ (see [21]). Then, by denoting $Y = PL$, this ends the proof. □

By choosing $\sigma = 0.3, r = 0.5$ and minimizing γ subject to LMIs (25-27) in Theorem 1, one obtains $\gamma = 1.16$. The performance analysis of the observer will be shown in the next section.

Remark 1: Note that, due to the presence of the state observer (22) a suitable robust MPC design method that explicitly takes into account the state estimation error should be adopted. In this regard, a possible solution is given by the method introduced in [22], where a constraint tightening approach is adopted to deal with the state uncertainty induced by the observer. However, such an approach leads to a more conservative design procedure that may carry to unfeasibility issues of the optimization problem as well as a slight performance degradation. For this reason, similarly as done in [12] and [23], the MPC design will be performed without taking explicitly into account the effects of estimation errors, thus exploiting its inherent robustness properties. As it will be seen in section V, the proposed approach is able to provide quite good overall performance while satisfying the dissipativity constraint.

V. SIMULATION RESULTS

To assess the proposed observer-controller strategy, simulations are performed on a full non linear vehicle model [16] with non linear suspension forces, validated on a Renault Mégane Coupé. The simulations are performed with a sampling time $T_s = 0.005$ s, and a prediction horizon $N_p = 10$. The following scenario is used to test the effectiveness of the proposed MPC controller:

- The vehicle runs at 120 km/h in a straight line on dry road ($\mu = 1$, μ the adherence to the road).
- A 5cm bump occurs simultaneously on the left and right wheels (from $t = 0.5$ s to $t = 1$ s) to excite the bounce motion and chassis vibration.
- A 5 cm bump on the left wheels (from $t = 2$ s to $t = 2.5$ s) causes the roll motion.

Firstly, Fig. 2 shows that the road profiles are well estimated using the proposed observer approach. Such estimated road profile has been computed time by time, used for MPC design with road disturbance estimation and not used for road preview.

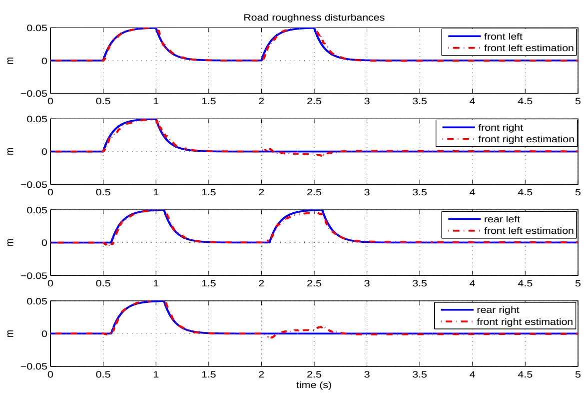

Fig. 2. Road profiles and their estimations

To demonstrate the efficiency of the proposed approach, we show comparison results of the following control strategies:

- MPC with road disturbance estimation, called *Proposed MPC*
- MPC without taking into account the road disturbance during the prediction horizon, called *MPC without w*
- MPC with road disturbance preview, called *MPC preview w*
- Uncontrolled passive suspensions (i.e $u_{ij} = 0$), called *Nominal damper*

Note that for *MPC preview w*, it has been assumed that the road disturbances are known in advance during the prediction horizon N_p.

Fig. 3 shows the time behavior of the chassis acceleration $\ddot{z}_s$. It can be seen that, within *Proposed MPC* the ride comfort of the vehicle behaves much better compared to Uncontrolled passive suspension case (*Nominal damper*). Moreover, *Proposed MPC* and *MPC preview w* have almost the same performance and have some improvements with respect to *MPC without w*. Fig. 4 shows the chassis position z_s response. *Proposed MPC* provides a better performance than *MPC without w, Nominal damper* and a similar behavior with respect to *MPC preview w*.

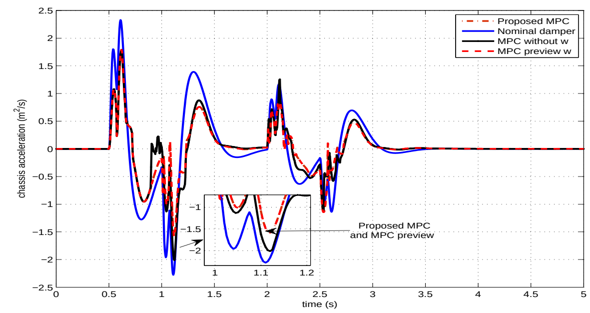

Fig. 3. Chassis acceleration

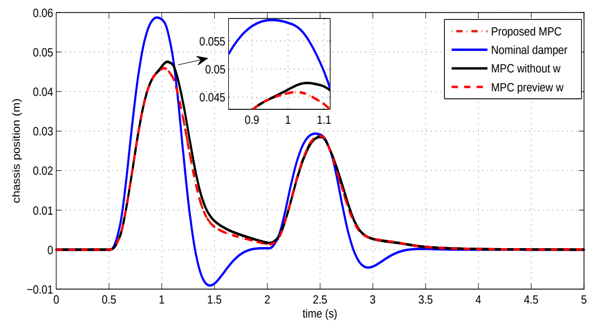

Fig. 4. Chassis position

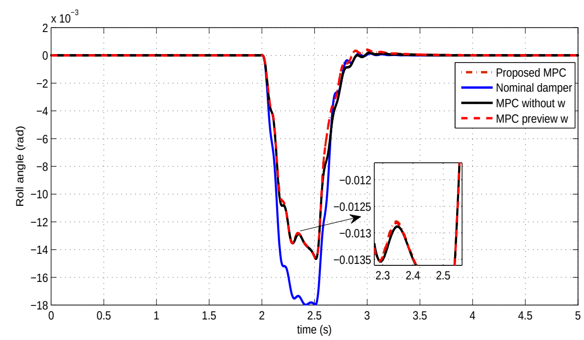

Fig. 5. Roll angle

Finally, Fig. 5 shows the obtained results for the roll motion dynamics described by θ. In particular, it can be noted that all the considered MPC strategies improve the road holding performance compared to *Nominal damper*. Moreover, we observe that the same behavior occurs for *Proposed MPC* and *MPC preview w*, while very slight improvement is obtained with respect to *MPC without w*.

Now, a deeper analysis is provided. Simulations are carried out using benchmark road profiles employed in standard industrial tests. In particular, the following road profiles (see Fig. 6) are taken into account:

- ISO road A (smooth runway), vehicle runs at 130 km/h.
- ISO road D (rough runway), maximum amplitude of 0.015 m and run at 90 km/h.

- A random road profile for comfort test, run at 60 km/h.

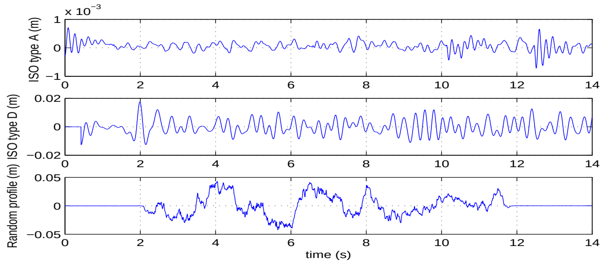

Fig. 6. Different benchmark road profiles

All the simulations last 14 s. To evaluate the effectiveness of each approach, the RMS (root mean square) of the chassis acceleration ($\ddot{z}_s$) is computed and results are presented in table I. Note that the RMS evaluation of the roll angle is not shown here since the roll motion is not excited due to the fact that the same road profile is applied at the four wheels within a delay between the front and rear axles.

TABLE I

RMS OF CHASSIS ACCELERATION FOR DIFFERENT ROAD PROFILES

	Proposed MPC	MPC preview w	MPC without w
ISO road A	0.0091	0.0091	0.0102
ISO road D	0.8140	0.7678	0.9129
Random road	0.7582	0.7542	0.8454

As shown in the presented simulation results, in the context of semi-active suspensions it seems that *MPC preview w* does not introduce significant improvements with respect to *Proposed MPC*. Moreover, the feedforward action obtained by *Proposed MPC* introduces improvements over the case of *MPC without w*. This demonstrates once again the usefulness of the proposed approach.

VI. Conclusion

In this work, a single MIMO state feedback control was designed for the semi-active suspension system of a full vertical vehicle using a MPC strategy. An observer was designed to estimate the system states and the road disturbances. The effects of the disturbances were taken into account in the control design step. The simulation results demonstrate the effectiveness of the presented approach. Thanks to predictive control techniques, multi-objective problems were considered where the control laws were computed to improve the passenger comfort and the road handling, while ensuring dissipativity constraints. For the future works, the implementation of this strategy on a testbed, available at Gipsa-lab Grenoble, will be made. It consists of a vehicle equipped with four controllable Electro-Rheological dampers, and of 4 DC motors generating separately different road profiles on each wheel.

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