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Crack deflection in brittle materials by Finite Fracture Mechanics

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Abstract

When dealing with mixed-mode brittle fracture of cracked elements, T-stress affects both the stress field and the energy balance. This problem is investigated here through the coupled Finite Fracture Mechanics (FFM) criterion by varying mode mixity of the main crack. Results are presented in terms of the critical stress intensity factors (SIF) and the critical kinking angle. As concerns pure mode I loading conditions, if $T > 0$ is large enough, the crack ceases to propagate collinearly and the critical SIF deviates from the fracture toughness of the material. On the other hand, for mode II loading conditions, if $T < 0$ is sufficiently low, the critical SIF ceases to increase and the critical kinking angle jumps to an infinitesimal value.

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Keywords: Crack kinking, brittle fracture, FFM;

1. Introduction

T-stress effects on crack kinking in brittle fracture mechanics have been investigated since seventies (Williams and Ewing, 1972; Carpinteri et al., 1979; Cotterell and Rice, 1980; Kariahaloo, 1981; Yukio et al., 1983; Sumi et al., 1985; He et al., 1991; Becker et al., 2001; Christopher et al., 2007; Lazzarin et al., 2009), but it was only since the middle of nineties, that failure criteria based on a linear-elastic analysis combined with an internal material length have been successfully proposed (Kosai et al., 1993; Seweryn, 1998; Smith et al., 2001).

More recently, also coupled stress and energy approaches of FFM were formalized in this framework. Leguillon and Murer (2008) modified the criterion proposed in Leguillon (2002) to include T-stress effects: the analysis was carried out numerically, by a two-scale asymptotic matching procedure (Leguillon, 1993). On the other hand, in the present work, the problem is faced by the approach put forward in Cornetti et al. (2006): the criterion is similar to that presented in Leguillon and Murer (2008), but the stress condition is averaged and not of punctual type (Cornetti et al., 2014; Sapora and Mantic, 2016). It is important to remark that according to FFM, the crack advance becomes a structural parameter, allowing to remove some inconsistencies related to the criteria previously introduced.

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The FFM analysis is carried out by exploiting asymptotic expressions for the asymptotic stress field and the crack driving force available in the Literature (Amestoy and Leblond, 1992; Seweryn, 1998). The coupled equations providing the critical load and kinking angle are derived analytically and then solved numerically. It is found that positive T-stresses decrease both the critical failure load and the critical kinking angle, whereas an opposite trend is observed for negative T-values. Furthermore, in pure mode I loading conditions, there exists a critical threshold $T_+ > 0$ above which the crack ceases to propagate collinearly and the critical mode I SIF K_{If} deviates from the fracture toughness K_{Ic} of the material (Cotterell and Rice, 1980; Smith et al., 2001; Leguillon and Murer, 2008; Cornetti et al., 2014). On the contrary, under mode II loading conditions (indeed, note that $K_I = 0$ does not represent, strictly speaking, a pure mode II condition since $T = 0$ corresponds to a symmetrical load), theoretical predictions show an infinitesimal critical kinking angle and a unit limit value for the ratio between the critical mode II SIF K_{IIf} and K_{Ic}, below a critical value $T_- < 0$ (Sapora and Mantic, 2016).

2. FFM criterion

The coupled FFM criterion by Cornetti et al. (2006); Carpinteri et al. (2008) is based on the assumption of a finite crack extension Δ and on the contemporaneous fulfilment of two conditions. The former is a stress requirement: the average circumferential stress $\sigma_{\theta\theta}(r,\theta)$ on Δ, prior to the crack extension, must be greater than the material tensile strength σ_u. By referring to a cracked element with a polar reference system placed at the notch root (Fig.1), we have in formulae:

$$\int_0^{\Delta} \sigma_{\theta\theta}(r,\theta)\mathrm{d}r \geq \sigma_u \Delta. \tag{1}$$

The latter is the energy balance: the integral of the crack-driving force on Δ, representing the energy available for a crack increment, must be higher than the fracture energy (G_c) times the crack increment Δ. By means of Irwin's relationships, the condition can be expressed in terms of the SIFs related to the kinked crack, k_I and k_{II} for mode I and mode II, respectively, and of the fracture toughness K_{Ic}, namely:

$$\int_0^{\Delta} [k_I(c,\theta)^2 + k_{II}(c,\theta)^2]\mathrm{d}c \geq K_{Ic}^2 \Delta. \tag{2}$$

The FFM criterion is thus described by the coupled inequalities (1) and (2), and in order to be implemented the functions $\sigma_{\theta\theta}$, k_I and k_{II} are required.

2.1. Stress field and SIFs functions

By taking the T-stress effects into account, the circumferential stress field $\sigma_{\theta\theta}(r,\theta)$ at the crack tip can be approximated as (see Fig.1 with $c = 0$):

$$\sigma_{\theta\theta}(r,\theta) = \frac{K_I}{\sqrt{2\pi r}} f_{\theta\theta}^{I}(\theta) + \frac{K_{II}}{\sqrt{2\pi r}} f_{\theta\theta}^{II}(\theta) + T\sin^2\theta, \tag{3}$$

where K_I,K_{II} are the SIFs related to the main crack and $f_{\theta\theta}^{I}$,$f_{\theta\theta}^{II}$ are two angular functions (see the Appendix, Eq. (A.1)). On the other hand, by dimensional analysis concepts and the principle of superposition, the SIFs related to a

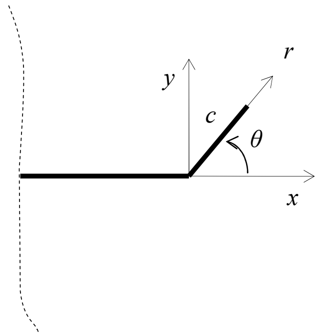

Fig. 1. Cracked element with polar coordinate system and kinked crack of length c.

kinked crack of length c can be expressed as (He et al., 1991; Amestoy and Leblond, 1992):

$$k_I(c,\theta) = \beta_{11}(\theta)K_I + \beta_{12}(\theta)K_{II} + \beta_1(\theta)T\sqrt{c}, \tag{4}$$

and

$$k_{II}(c,\theta) = \beta_{21}(\theta)K_I + \beta_{22}(\theta)K_{II} + \beta_2(\theta)T\sqrt{c}. \tag{5}$$

Approximating analytical expressions for the angular functions β presented by Amestoy and Leblond (1992) are reported in the Appendix (Eqs. (A.2)-(A.7)). Tabulated values can be also found in Tada et al. (1985); Melin (1994); Fett et al. (2004). Note that β_2, β_{12} and β_{21} are odd functions, whereas β_1, β_{11} and β_{22} result to be even.

Before proceeding, let us now introduce, for the sake of clarity:

- the functions $\overline{f}^i_{\theta\theta} = \sqrt{2/\pi} f^i_{\theta\theta}$ $(i = I, II)$;
- the mode-mixity related to the main crack, $\psi = \arctan(K_{II}/K_I)$.
- the characteristic length, $l_{ch} = (K_{Ic}/\sigma_u)^2$;
- the dimensionless crack advance, $\delta = \Delta/l_{ch}$;
- the dimensionless T-stress, $\tau = T\sqrt{l_{ch}}/\sqrt{K_I^2 + K_{II}^2}$;
- the combinations for the angular functions,

$$\overline{\beta}_1 = \beta_1\beta_{11} + \beta_2\beta_{21}, \quad \overline{\beta}_2 = \beta_1\beta_{12} + \beta_2\beta_{22}, \quad \overline{\beta}_{11} = \beta_{11}^2 + \beta_{21}^2, \quad \overline{\beta}_{22} = \beta_{12}^2 + \beta_{22}^2, \quad \overline{\beta}_{12} = 2(\beta_{11}\beta_{12} + \beta_{21}\beta_{22}).$$

2.2. Implementation and results

At incipient failure ($K_I = K_{If}$), the coupled conditions (1) and (2) become a system of two equations in two unknowns: the critical crack advancement δ_c and the failure load, implicitly embedded in the K_{If} function. The substitution of Eqs. (3), (4) and (5) into Eqs. (1) and (2) provides after some simple manipulations (Cornetti et al.,

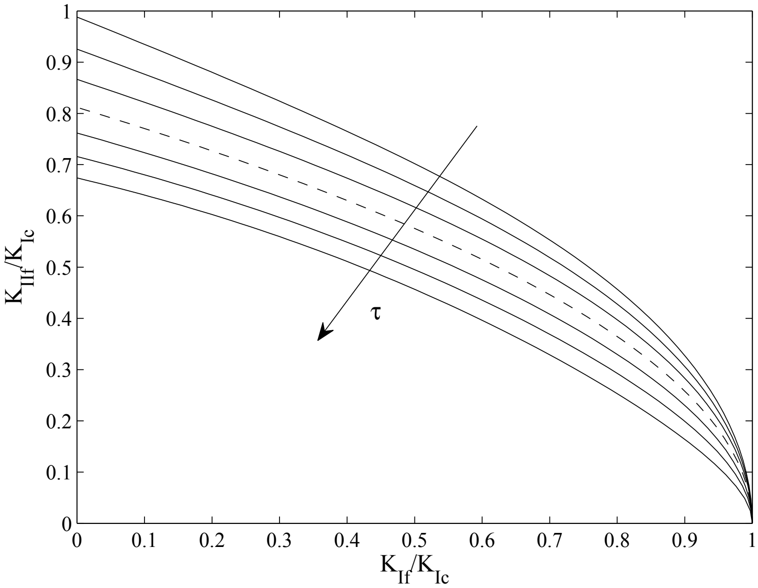

Fig. 2. T-stress effects on FFM fracture loci. From the top to the bottom, curves refer to $\tau = -0.3, -0.2, -0.1, 0$ (dashed line), 0.1, 0.2, 0.3.

2014):

$$\begin{cases} \dfrac{K_{If}}{K_{Ic}} = \dfrac{\sqrt{\delta}}{\overline{f}^{I}_{\theta\theta} + \tan\psi \overline{f}^{II}_{\theta\theta} + \bar{\tau}\sin^2\theta}, \\ \delta = \dfrac{(\overline{f}^{I}_{\theta\theta} + \tan\psi \overline{f}^{II}_{\theta\theta} + \bar{\tau}\sin^2\theta)^2}{(\overline{\beta}_{11} + \overline{\beta}_{12}\tan\psi + \overline{\beta}_{22}\tan^2\psi) + \dfrac{4\bar{\tau}}{3}(\overline{\beta}_1 + \overline{\beta}_2\tan\psi) + \dfrac{\bar{\tau}^2}{2}(\beta_1^2 + \beta_2^2)}, \end{cases} \tag{6}$$

where $\bar{\tau} = \tau\sqrt{\delta(1+\tan\psi)}$, for the sake of simplicity.

Observe that, for given loading and structural properties, ψ and τ are fixed. In order to implement FFM, the latter equation in (6) should be firstly solved: a different crack advance δ corresponds to a different kinking angle θ. Each couple (δ, θ) must be substituted into the former equation: the actual crack advance δ_c and critical kinking angle θ_c are those which minimize the K_{If} function. The relationship $K_{IIf} = \tan\psi K_{If}$ then provides the corresponding value for K_{IIf}.

FFM results are presented in Figs. 2 and 3, for the fracture loci and the critical kinking angle, respectively. By assuming $K_I, K_{II} > 0$, as T increases, the failure load decreases, as well as the critical kinking angle θ_c, which tends asymptotically towards $-90°$.

As concerns pure mode I loading conditions ($K_{II} = 0$), if $T > 0$ is sufficiently large,$\tau \geq \tau_+ = 0.42$,the crack does not propagate collinearly any more (θ_c different from 0°) and K_{If} deviates from K_{Ic}. This phenomenon has been already described in the Literature (Cotterell and Rice, 1980; Smith et al., 2001; Leguillon and Murer, 2008; Cornetti et al., 2014) on the basis of some experimental observations (Selvarathinam and Goree, 1998; Chao et al., 2001). FFM predictions are presented in Figs. 4 and 5. The present results, showing nearly continuous functions, are in qualitative agreement with those derived in Smith et al. (2001), but slightly differ from those proposed in Leguillon and Murer (2008) where the existence of a θ_c-jump from 0° to $-72°$ was detected at a threshold tensile $\tau_+ = 0.704$. . On the other hand, in the case of a compressive T-stress ($T < 0$), the straight crack path always reveals to be stable.

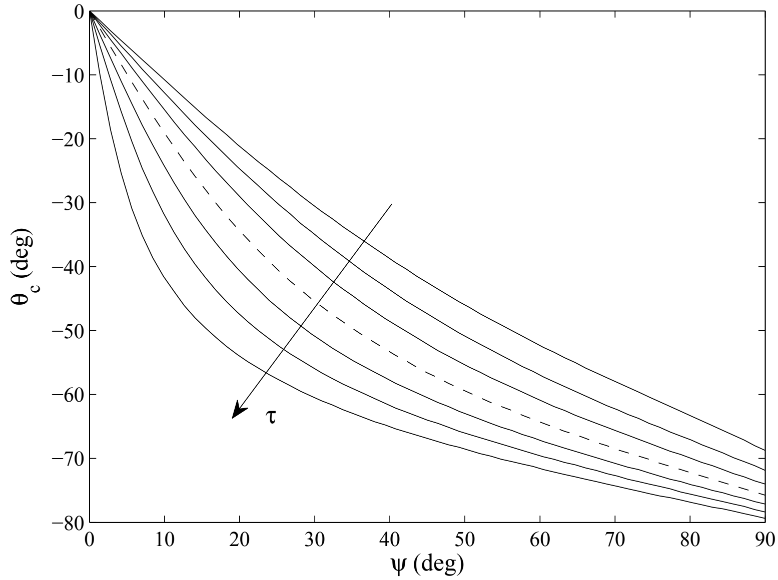

Fig. 3. T-stress effects on FFM critical kinking angle. From the top to the bottom, curves refer to $\tau = -0.3, -0.2, -0.1, 0$ (dashed line), 0.1, 0.2, 0.3.

As regards mode II loading conditions ($K_I = 0$), an increasing tensile T-stress provides decreasing kinking angles θ_c from $-75.5°$ ($T = 0$, Sapora et al. (2014)) to $-90°$ ($T \to \infty$). The trend is similar for compressive T-stress till the threshold $\tau = \tau_- \simeq -0.325$. Below τ_-, the kinking angle becomes infinitesimal and K_{IIf} keeps equals to K_{Ic} (Figs. 4 and 5). The reason of this behavior is imputable to the fact that the shear contribution to the strain energy release rate prevails and the maximum released energy corresponds to $\theta_c = 0°$. In order for the stress requirement in (1) to match this condition, the crack advance (which is not reported here) must become infinitesimal too, so that tensile stresses result to be high enough.

In order to overcome this drawback, as suggested by Sapora and Mantic (2016), let us observe that estimates of the toughening of elements under shear should consider possible local plastic and viscoelastic dissipation, crack face asperity shielding and frictional effects: the assumption of G_c to be constant is reasonable only if the G_I-contribution to the energy release rate (ERR) prevails, whereas a larger amount of dissipated energy should be associated to crack kinking dominated by G_{II} (Hutchinson and Suo, 1992; Liechti and Chai, 1992; Banks-Sills and Ashkenazi, 2000; Mantič et al., 2006). One of the most implemented fracture criterion writes (Hutchinson and Suo, 1992):

$$\frac{G_I}{G_{Ic}} + \frac{G_{II}}{G_{IIc}} = 1, \tag{7}$$

where $G_{IIc} = G_{Ic}/\gamma$ has the interpretation of pure mode II toughness and γ is a parameter weighting the mode II-contribution. It vanishes for $\gamma \to 0$, whereas $\gamma = 1$ corresponds to an ideally brittle material. Note that the condition $\gamma \to 0$ provides the basis for the well-known $k_{II} = 0$ criterion proposed on the basis of simple symmetry arguments by Goldstein and Salganik (1974), and that an analogous relationship to (7) was adopted by Seweryn (1998) and suggested by Leguillon and Murer (2008).

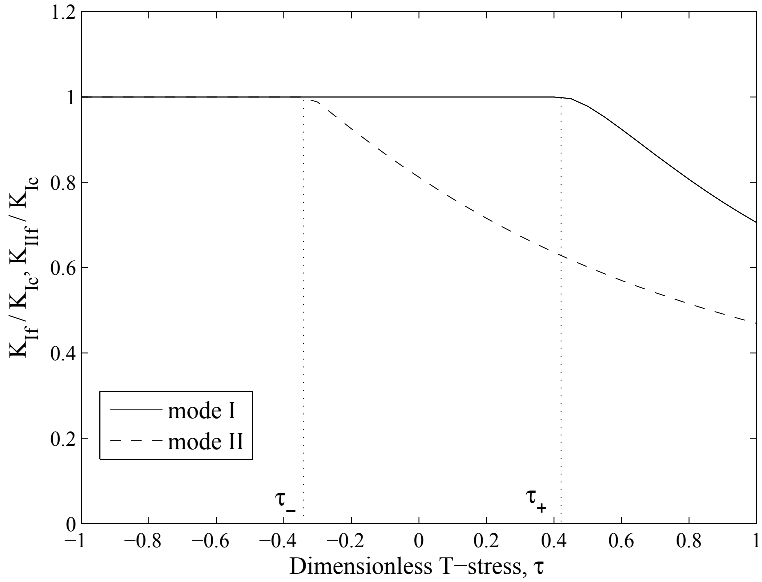

Fig. 4. T-stress effects on FFM failure loads for pure mode I and mode II loading conditions.

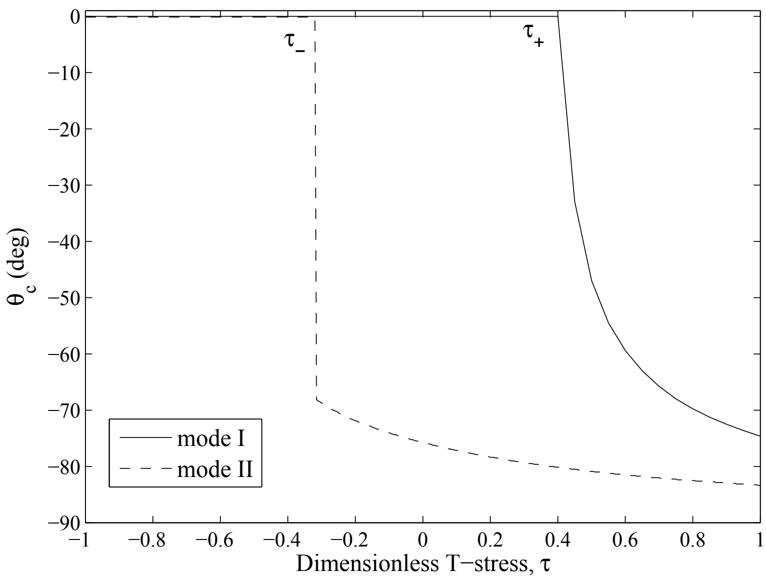

Fig. 5. T-stress effects on FFM critical kinking angle for pure mode I and mode II loading conditions.

In order to improve FFM predictions (Sapora and Mantic, 2016), from an equivalent point of view, one could consider the following modified ERR in the energy balance:

$$\overline{G} = G_I + \gamma G_{II}. \tag{8}$$

3. Conclusions

It was shown that T-stress effects reveal to be more significant for i) sufficiently high T-magnitudes; ii) prevailing mode II conditions; iii) less brittle materials (i.e., higher l_{ch}). Indeed, for a specific test, since all the parameters K_I, K_{II} and T generally vary as the mode mixity ψ varies, the real impact of T-stress on predictions should be discussed from case to case, after evaluating these three parameters. Whereas for mode I loading conditions the crack propagation reveals to be unstable for $\tau \geq 0.42$, for mode II loading conditions a singular behavior was observed for $\tau \leq -0.325$.

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Appendix A. Angular functions

The following equations hold for what concerns the angular functions related to the stress field:

$$f^{I}_{\theta\theta}(\theta) = \cos^3(\theta/2), \qquad f^{II}_{\theta\theta}(\theta) = -3\sin(\theta/2)\cos^2(\theta/2) \tag{A.1}$$

whereas the approximating expressions for the functions β in Eq. (4) and (5) can be found in Amestoy and Leblond (1992), with $m = \theta/\pi$:

$$\begin{aligned}\beta_{11} &= 4.1m^{20} + 1.63m^{18} - 4.059m^{16} + 2.996m^{14} - 0.0925m^{12} - 2.88312m^{10} + 5.0779m^8 + \\ &+ (\pi^2/9 - 11\pi^4/72 + 119\pi^6/15360)m^6 + (\pi^2 - 5\pi^4/128)m^4 - 3\pi^2 m^2/8 + 1,\end{aligned} \tag{A.2}$$

$$\begin{aligned}\beta_{12} &= 4.56m^{19} + 4.21m^{17} - 6.915m^{15} + 4.0216m^{13} + 1.5793m^{11} - 7.32433m^9 + 12.313906m^7 + \\ &+ (-2\pi - 133\pi^3/180 + 59\pi^5/1280)m^5 + (10\pi/3 + \pi^3/16)m^3 - 1.5\pi m,\end{aligned} \tag{A.3}$$

$$\begin{aligned}\beta_{21} &= -1.32m^{19} - 3.95m^{17} + 4.684m^{15} - 2.07m^{13} - 1.534m^{11} + 4.44112m^9 - 6.176023m^7 + \\ &+ (-2\pi/3 + 13\pi^3/30 - 59\pi^5/3840)m^5 - (4\pi/3 + \pi^3/48)m^3 + \pi/2m,\end{aligned} \tag{A.4}$$

$$\begin{aligned}\beta_{22} &= 12.5m^{20} + 0.25m^{18} - 7.591m^{16} + 7.28m^{14} - 1.8804m^{12} - 4.78511m^{10} + 10.58254m^8 + \\ &+ (-32/15 - 4\pi^2/9 - 1159\pi^4/7200 + 119\pi^6/15360)m^6 + (8/3 + 29\pi^2/18 - 5\pi^4/128)m^4 - (4 + 3\pi^2/8)m^2 + 1,\end{aligned} \tag{A.5}$$

$$\begin{aligned}\beta_{1} &= -13.7m^{20} - 2.85m^{18} + 10.947m^{16} - 7.314m^{14} - 6.0205m^{12} + 26.66807m^{10} - 50.70880m^8 + \\ &+ 63.665987m^6 - 47.93339m^4 + (2\pi)^{3/2}m^2,\end{aligned} \tag{A.6}$$

$$\beta_2 = 6.62m^{19} + 9.69m^{17} - 12.781m^{15} + 3.043m^{13} + 15.6222m^{11} - 39.90249m^9 + 61.174444m^7 + \\ -59.565733m^5 + 12\sqrt{2\pi}m^3 - 2\sqrt{2\pi}m. \tag{A.7}$$

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