

Modified compact Genetic Algorithm for Thinned Array Synthesis

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Abstract—In this paper, a new optimization algorithm, the Modified compact Genetic Algorithm (M-cGA) is introduced and applied to the synthesis of thinned arrays. The M-cGA has been derived from the compact Genetic Algorithm (cGA), properly modified and improved by implementing more than one probability vector (PV) and adding suitable learning scheme between these PVs. The so obtained algorithm has been applied to the optimized synthesis of different size linear and planar thinned arrays: in all the considered cases it outperforms not only the cGA, but also the other optimization schemes previously applied to this kind of problems, both in terms of goodness of the solution (minimization of the peak side-lobe level) and of computational cost.

Index Terms—Compact genetic algorithm, optimization algorithm, thinned array, antenna.

I. INTRODUCTION

IN recent years, thinned arrays have attracted significant attention from researchers because of their advantages such as the reduction of the array weight and of the complexity of the feeding network. However, array thinning has also some disadvantages, the main of which is the decreasing of the maximum gain value, that corresponds to an increase of the side-lobe level (SLL) with respect to a fully populated array with the same equivalent size [1].

To circumvent this drawback, several techniques have been proposed, aimed to find the best location of the active elements inside the array grid [2]–[12]. Deterministic approaches have been firstly adopted, but they do not show significant improvements with respect to the random element placement [2], [3]. Recently, dynamic program [4] and stochastic optimization techniques, including Genetic Algorithm (GA) [5], simulated annealing (SA) [6], [7], and Ant Colony Optimization (ACO) [8], [9], have been applied to the optimization of thinned array. The obtained results are remarkable, even if they could be further improved.

The combination of deterministic approaches and stochastic optimization have been proposed exploiting the available knowledge of Different Sets (DS) or Almost Different Sets using GA [10]–[12]. These combinations proved that this procedure most of the times is very effective. However,

the operations performed by the optimizer still presents the inherent disadvantage of stochastic based optimization, i.e. the process convergence can be really slow, with a resulting increase of its computational cost.

In [13] the previous hybrid approach has been extended to planar thinned arrays, while in [14] the synthesis of these last have been carried out by the combination of others optimization algorithm (PSO) and combinatorial method. Also [15] deals with the synthesis of planar thinned arrays, proposing two techniques that are the hybridization of a deterministic approach (the density tapering) in one case with the random location of the elements, in the other with the iterative Fourier Transform. This last is instead used alone for the design of large planar arrays in [16].

In this framework, the compact Genetic Algorithm (cGA) [17] seemed to be a good candidate for the optimized synthesis of thinned arrays. The authors have recently introduced an improved version, named Modified cGA (M-cGA), with the aim of overcoming the limitation of the former one [18]. Some preliminary results have been presented in [19], [20], showing that M-cGA provided good solutions with a reduced computational cost, i.e. it converged faster. In view of these encouraging results on its application to different test functions and simple electromagnetic problems, the use of M-cGA was further investigated: in this paper the results of its application to the optimization of several, different-size linear and planar thinned arrays are reported, and compared with the results obtained with other approaches. The paper is structured as follows. In Section II, the compact genetic algorithm (cGA) is firstly introduced, followed by the description of the M-cGA; in Section III the results of the optimization of planar and linear thinned arrays through the M-cGA are shown, while in Section IV some conclusions are drawn.

II. THE MODIFIED COMPACT GENETIC ALGORITHM

Despite of its name, the compact Genetic Algorithm, first presented in [17], belongs to the Estimation Distribution Algorithms, since, in order to get the distribution of good solutions, it uses a probability vector (PV) to represent a possible solution; this PV is managed in place of the population of entities typical of Evolutionary Algorithms. The length of the PV corresponds to the number N of variables of the problem, and the value of the PV elements represents the probability of a variable to get a particular value. A full treatment of the method can be found in [17], [21], but for the sake of clarity and uniformity of notation it is briefly summarized in the following subsection.

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Step 1:   Initial PV: PV=0.5*ones(1,N)
Step 2:   Generate two individuals: a & b
Step 3:   Compete between individuals
            winner, loser:= compete(a,b)
Step 4:   Update PV:
            for i = 1:N
              if winner(i)#loser(i)
                if winner(i)==1
                  PV(i)=PV(i)+1/n;
                else
                  PV(i)=PV(i)-1/n;
                end
              end
Step 5:   Check convergence
Parameters:  n: population size  N: PV length

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Fig. 1. Pseudo code of the cGA

A. Compact Genetic Algorithm

The pseudo code of the cGA is shown in Fig. 1. Initially, each element of the PV is set equal to 0.5, assuming a uniform distribution for each one. At the following step, two individuals are generated from each element of the current PV. They compete each other, and the winner is responsible for the updating the corresponding PV's element: its value is increased or decreased by a factor $1/n$ (where n is the population size) according to the value of the winner. The cGA will stop when all the PV's elements are equal to 0 or 1, i.e. the optimal solution is found.

A first variation of the standard cGA has already been introduced in [17], by increasing the number of generated offspring and applying tournament competition, i.e. simulating higher selection pressure. This modification however has a high computational cost since it needs to store and evaluate a considerable number of individuals.

In [21], Ahn proposed new versions of cGA introducing elitism. He created two different approaches, i.e. the persistent elitism cGA (pe-cGA) and the non-persistent elitism cGA (ne-cGA). The elitism-based cGAs outperform the original cGA in term of function evaluations but they do not perform better in term of solution quality.

B. Modified compact Genetic Algorithm

The idea behind the M-cGA is to enhance the exploration capability of the cGA, that tends to stagnate, by adding one of the operator typical of the stochastic algorithms. Therefore, starting from the ne-cGA, the Modified cGA was implemented by introducing more PVs and integrating a learning scheme in the update procedure. In Fig. 1, the new "step 4" of the M-cGA is reported, which describes the updating procedure, different from that of the standard cGA. In fact, in M-cGA each element of each PV is update according to the rule used

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Step 4:   Update PVs:
            % local update
            for i = 1:P & for j = 1:N
              if winner(i,j)#loser(i,j)
                if winner(i,j)==1
                  PV(i,j)=PV(i,j)+1/n;
                else
                  PV(i,j)=PV(i,j)-1/n;
                end
              end
            % global update
            pv_best = best(PVs)
            for i = 1:P
              PV(i) = PV(i)+c*(pv_best-PV(i))
            end
Parameters:  P: number of PVs  c: learning factor

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Fig. 2. New updating rules for the M-cGA

in the standard cGA, that represents its self-knowledge, but it is also influenced by the elements of the other PVs, i.e. by a global knowledge, and in particular by the best element among those of all the PVs. In this way it is possible to enhance the exploration properties of the algorithm and increase the ability to avoid local optimum, with a reduced increase of the computational cost: in fact, the number of operations performed by the M-cGA is equal to that carried on by the cGA, just multiplied by the number of PVs, that is generally very small (2-6).

III. THINNED ARRAY SYNTHESIS

In view of the preliminary results reported in [18], [19], [20], the application of the M-cGA to thinned array has been further investigated. Several configurations of both linear and planar thinned arrays have been considered. The performance of the M-cGA has been compared with results available in literature, obtained by other approaches on the same configurations. They have been compared both in terms of their capability to obtain a good solution, i.e. a configuration that minimize the Peak Sidelobe Level (PSL), and of their computational cost. In all the considered situations, the M-cGA uses 4 PVs and the reported results are the average values over 50 independent trials.

A. Synthesis of Linear Thinned Array

For what concerns linear arrays, five different configurations have been considered: arrays with 96, 198 and 502 element, the 50% of which is turned on, a 198 element array, with 79 elements switched off, and an array with 200 elements, 46 of which are off. These configurations were chosen due to the availability of previous results in literature, therefore it was possible to compare the performance of the M-cGA not only with the standard cGA but also with other established approaches [12].

TABLE I
PSL [dB] OBTAINED WITH DIFFERENT METHODS FOR ARRAYS WITH THE
50% OF THE ELEMENTS SWITCHED OFF

Array	GA[12]	ADS-GA [12]	cGA	M-cGA
98/49	-19.82	-20.4	-19.8	-20.45
198/99	-18.20	-19.24	-19.9	-21.9
502/251	-20.83	-21.31	-20.4	-23.53

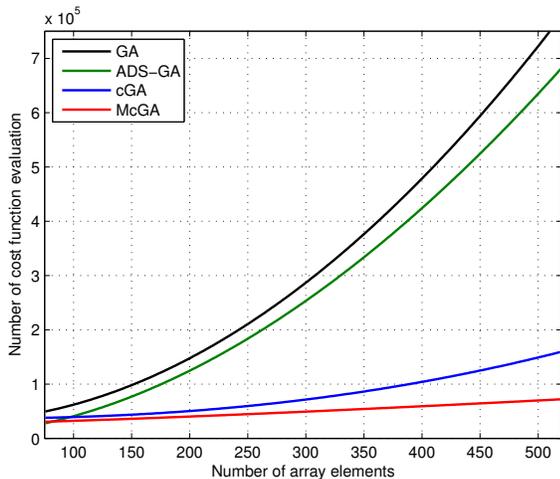


Fig. 3. Function Evaluation of different thinned arrays

Table I reports the PSL of the first three configurations, i.e. the arrays with the 50% of elements switched off, obtained with the M-cGA, the cGA, the GA and the hybrid ADS-GA [12] respectively. These results show that the cGA works almost always as the GA, while the M-cGA outperforms both the GA and the cGA in all cases, most significantly when the size of the array increases; its performance is comparable with those of the ADS-GA for the smallest array, but it becomes better than the latter when increasing the problem size.

Fig. 3 gives an information about the computational cost of the four considered methods applied to arrays with the 50% of elements switched off, since it shows the variation of the number of cost function evaluations vs. the total number of array elements. This plot highlights the advantage of using the probability vector instead of the population, since it allows a drastic reduction of the workload. Moreover, it proves that M-cGA outperforms cGA, since the use of more PVs speeds up the convergence.

Finally, Tab. II summarizes the results for the last two considered arrays, for what concerns both the minimum PSL and the number of cost function evaluations, relative to the M-cGA and compared with those for the ADS-GA [12]; in fact, from the above analysis, the latter appears to provide better results than the cGA and the GA. Also in these two cases, the PSL values obtained with the M-cGA are slightly better than those given by the ADS-GA, but the M-cGA outperforms the ADS-GA for what concerns the computational cost, that is reduced to one half in the first case and even to one third

TABLE II
COMPARISON OF THE M-CGA AND THE ADS-GA [12] IN TERMS OF
MINIMUM PSL AND COMPUTATIONAL COST

Array	PSL [dB]		No. of cost function eval.	
	ADS-GA [12]	M-cGA	ADS-GA [12]	M-cGA
198/79	-20.25	-21.10	126, 126	60, 000
200/46	-23.05	-23.75	305, 600	100, 000

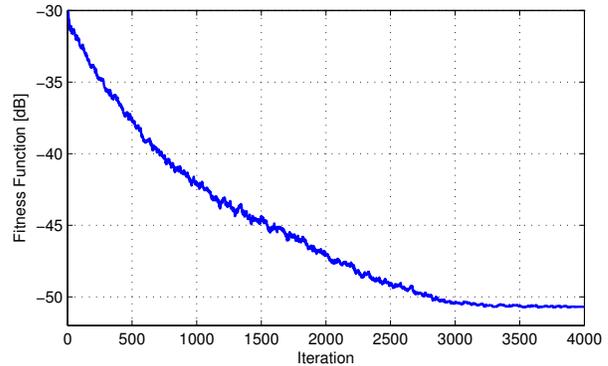


Fig. 4. Average curve of convergence of the M-cGA applied to the optimization of the 20×10 thinned array

for the second array.

B. Synthesis of Planar Thinned Array

In this section, results on the synthesis of planar, i.e. square and rectangular, thinned arrays, are shown. Similarly to the linear case, different configurations have been considered based on previous literature availability. In all the cases, the fitness function optimized by the M-cGA is the sum of PSLs in two main planes, i.e. $\phi = 0^\circ$, and $\phi = 90^\circ$ and the probability vectors are one-dimensional vectors as for the linear array.

The first configuration considered is a 20×10 element planar array, in which 108 elements are turned on. In Fig. 4 the M-cGA average curve of convergence is plotted: the value of the fitness function after 3000 iterations corresponds to an array configuration whose radiation pattern is shown in Fig. 5. The PSL is equal to -26.6 dB in the $\phi = 0^\circ$ plane, and to -23.5 dB in the $\phi = 90^\circ$ plane. These achieved values are lower than those obtained with the GA in [5], and with the modified real genetic algorithm (MGA) that optimized also the position of the elements switched on [22]. Moreover, the number of fitness function evaluations required to converge is around 12000 for the M-cGA, i.e. less than half of those needed by the MGA [22].

As a last example of application of the M-cGA to the optimized synthesis of planar thinned arrays, different square arrays have been considered, with different size and percentage of switched off elements. The obtained PSLs, which in these cases is equal in the two planes, are reported in the third column of Tab. III. In the columns 4–7 the results obtained with the cGA, the HPSO [14], the ACO [9] and the IFTDT [15] are also shown.

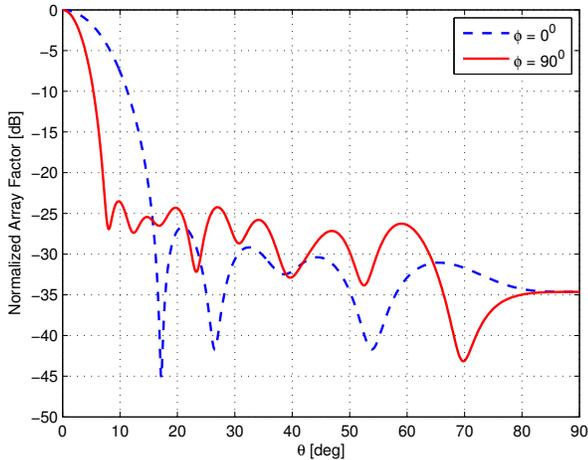


Fig. 5. Far-field patterns in the two main planes for the 20×10 optimized thinned array

TABLE III
MINIMUM PSL [DB] OBTAINED WITH DIFFERENT METHODS APPLIED TO
DIFFERENT SIZE PLANAR THINNED ARRAYS

array size	% of ON elements	M-cGA	cGA	HSPSO [14]	ACO [9]	IFTDT [15]
12×12	48	-19.4	-17.9	-16.7	-	-17.6
24×24	44	-23.3	-22.0	-19.0	-	-22.8
30×30	60	-24.6	-23.9	-	-23.5	-24.3

IV. CONCLUSION

In this paper, the M-cGA, an enhanced version of the cGA, recently introduced integrating learning mechanism in cGA, is applied to the synthesis of thinned arrays. The results here presented reveal that the M-cGA is able to well control the PSL of both linear and planar thinned array, with a reduced computational cost.

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