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Dispersion of solid particles in agitated vessels

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In this paper, after a review of the work done on the dispersion of solids in the last years, we present a model able to interpret the quality of the suspension, and show some experimental results obtained in a 0.050 m$^3$ vessel with a dished bottom. We have determined radial and axial profiles of solid concentration by sampling. Different particle sizes, densities, shapes and concentration were tested and four different types of stirrers were employed.

The radial concentration profiles were flat enough to justify the adoption of a monodimensional model, based on turbulent disturbances; for a given geometrical situation, the main velocity field may be neglected.

The shape of the axial concentration profiles (see fig. 1 as an example) showed that the particle diffusivity cannot be considered as a constant; this changes in the vessel because the turbulence intensity also changes. Besides, large scale eddies also play a role in solid dispersion.

Starting from the continuity equation for the solid phase, by means of a dimensional analysis we obtained that the solid distribution in the vessel may be considered a function of a $K$ parameter so defined:

$$K = \frac{\psi^{1/3}N_D}{u_c}$$

Fig. 2 shows an example of axial concentration profiles with $K$ as parameter.

The $K$ number was employed successfully to correlate experimental value of suspension quality $\sigma$; in spite of the remarkable variation in particle size, density and shape (see fig. 3). The $B^{-0.13}$ factor was introduced in order to take into account the effect of the mean concentration. In fact it has been confirmed that in dilute suspensions axial concentration profiles are independent of total solid hold up, if referred to $N/N_{hs}$.

Data from other authors were also satisfactorily correlated.

Exploratory tests on bimodal suspensions showed that fine and coarse particles interact.

Notation

- $D$: stirrer diameter, m
- $N$: stirrer velocity, rev/s
- $u_c$: terminal settling velocity, m/s
- $\psi$: power number
Introduction
Suspension of solid plays a decisive part in mixing technology. Kipke /1/ reported, as trend assessment, that most of the stirring tests carried out in industrial pilot plants deal with this problem. It is convenient to point out a differentiation between "off bottom suspension" and "dispersion" of the solid throughout the vessel.
The former regards lifting of the particles from the bottom, so that no solid remains at rest on it for more than a few seconds. The conditions for complete suspension have been widely investigated and many works were published in the last years. It has been shown that this is the most profitable operating situation whenever mass-transfer is the controlling process.
A few data, instead, are available on the dispersion of the solid particles and there is a lack of knowledge about the solid distribution in the vessel.
A slurry liquid interface can form, with a pseudo particle-free region irrespective of whether all particles are whirled up or not. On increasing the power input, the particle bed expands to fill the vessel. The layer thickness criterion was employed by some authors to evaluate the homogeneity of the suspension /1, 2/. Weisman and Efferding /2/ regarded the slurry concentration below the interface as uniform and found initial solid suspension and bed expansion to be controlled by similar dimensionless groupings.
But the two phenomena are governed by different mechanisms: eddies of scale comparable with particle size (i.e. eddy of inertial subrange for small particles) are responsible for suspension, while in solid dispersion large scale eddies are of importance /3/.
Einiken and Mersmann /4/, also, adopted the layer thickness criterion in their dimensional analysis; they investigated the influence of solid hold up and found that in high concentration suspensions the terminal settling velocity of the particle in a swarm is an influencing parameter.
This technique gives no quantitative data on the solid distribution in the vessel; even if there is not a solid-free zone, suspension is far from homogeneity and the concentration profiles are different for the various size fraction /5/.
Homogeneous suspensions would be useful in many operations, such as continuous crystallization. Aeschbach and Bourne /6/ and Bourne and Sharma /7/ investigated the conditions for the homogeneity and developed an apparatus to approximate the ideal conditions.
Bohnet and Niesmak /8/ pointed out that it is not possible to obtain perfect homogeneity. They used the variance of the axial concentration profile, $\sigma$, as a measure of the quality of distribution, as suggested by Einiken /9/, and increasing the stirrer speed they observed for all solids tested a minimum of $\sigma$, depending on the particle physical properties.
Suspensions near homogeneity, anyway, are very difficult to obtain in an industrial plant, and they would be too expensive, because power input for stirring increases with the third power of the impeller speed.
In many processes as for example crystallization and polymerization, however, it is necessary to control the suspension quality, because this influences the characteristics of the product. If a slurry is withdrawal continuously from the vessel, the solid concentration profiles must be known to place rightly the openings in order to assure stable operating conditions and the projected solid hold up. Baldi and coworkers /10/ noted that complete suspension is associated with a pseudohomogeneity zone, where concentration is more or less constant and close to the averaged concentration in the vessel. The height of this zone increases with the stirrer speed and seems not to be influenced by solid hold up /3/.
Solid distribution is a function of stirrer speed and physical properties of the system, but geometrical characteristics of the apparatus have a strong influence. Machoń and coworkers /11, 12/ measured the solid particle concentration in a stirred crystallizer, studying the effect of the draft tube and comparing flat and conical bottom.
Several authors /8, 13, 14, 15/ tested the influence of bottom shape, impeller type, stirrer diameter and clearance on the suspension quality, looking for the most profitable conditions.
The best technical solution with respect to energy consumption seems to be $T/D=3$ and $h/D=0.5$. The bottom shape also is very important: the required power decreases employing a flat bottom, a dished bottom, a profiled bottom or a cone and fillet tank respectively.

It has been shown /8, 11/ that, at constant stirrer speed, an increase in solid hold up causes an increase in the concentration gradients and makes worse the suspension quality.

But, at least in dilute suspensions, the axial concentration profiles and the mean residence time of the solid at different height in a CSTR, seem to be independent of total solid hold up, if referred to $N/N_{js}$ /16,10/.

Several attempts have been made to calculate the solid distribution inside the vessel.

Kudrna et al. /17/ proposed a monodimensional stochastic model, to point out the statistical nature of the problem. They obtained a diffusion equation and solved it under different boundary conditions.

A deterministic approach was tried by Peñáz et al. /18/ and by Šysová et al. /19/.

Starting from the knowledge of the velocity field in the vessel, they used the continuity equation to obtain a bidimensional model. The turbulent flux of particles has been expressed by introducing turbulent diffusivity.

Correction coefficients for the particle settling velocity and for the coefficient of turbulent diffusivity were introduced as a model parameter and estimated from experimental data; some results seem not to agree very well with theoretical previsions.

The necessity to know the local fluid velocities make these models very cumbersome to use.

Several authors used a monodimensional sedimentation dispersion model to describe the solid concentration profiles; they all assumed the coefficient of turbulent diffusion as a constant along the vessel, differently from Šysová et al. /19/.

Tojo and Mlyanami /20/ and Shamslou and Koutsakos /21/ investigated a vessel stirred by axial flow impellers; Fajner et al. /22/ studied a staged batch column with multiple radial impeller.

Tojo and Mlyanami gave also a correlation for the model parameters. An improved correlation is due to Magelli et al. /23/; they tested the scale effects and measured the coefficient of turbulent diffusivity of the particles independently from the solid terminal velocity.

The quality of the solid distribution can be described by the value of a function, for example the variance of the concentration profile. Then it would be usefull to express the suspension quality as a function of solid physical properties and operating conditions.

Bohnet and Niesmak /8/ assumed that the suspension quality is affected by the ratio between the power consumption for stirring and that for suspending and distributing the solid particles, determined by taking into account the terminal settling velocity of the solid; but they did not get satisfactory results.

Another attempt was made by Niesmak /24/, who starting from the experimental result $N_k D_p^{2/3}$ (at constant $c$), proposed to correlate the suspension quality with the fluid (Re•Fr) number and the particle (Re$_p$•Fr$_p$•c/δp) number.

All the previous paper refer to monomodal solid suspensions; very little literature data are available about the effect of mixtures of particle sizes /25/.

It would be of great industrial importance the knowledge of a scale up role for transferring the experimental data to a large scale set up. But the rules proposed by different authors scatter between $P/V_e D^{0.5}$ (corresponding to a constant value of Proude number) and $P/V_e D^{-1}$ (corresponding to a constant stirrer tip velocity).

Einenkel /26/ recommended $P/V_e D^{-0.33}$. Similarly Kipke /1/ proposed $-0.3$ as diameter exponent, but observed that it is incorrect to extrapolate this result to very large tanks; he also pointed out that the problem is more complicated in the case of multiple impeller arrangements because hysteresis effects may be present.

Niesmak /24/ and Buurman et al. /3/ suggested respectively $N_1 D$=cost and $N^2 D^{1.55}$=cost as a scale up rule.

Gerstemberg et al. /27/ suggested that the discrepancies arise from the use of different criteria for the quality of the suspension.
Voit and Mersmann /28/ pointed out that it is not correct to extrapolate the results by using the methods of the theory of similarity, but it is necessary to model the stirred vessel taking into consideration the different regions by which it is composed with its different physical laws. In their opinion there are two basic case of suspensions, depending on the target efficiency of the solid particle, and therefore two limiting laws: if target efficiency is zero, constant Froude number criterion is valid, if target efficiency is one, the specific power consumption increases with the square root of the tank diameter. Most authors found different laws because carried out their experiments in the transition region. The aim of this paper is to present a model able to interpret the quality of the suspension.

Measurement of concentration
The simplest method to measure the solid concentration is taking samples /3, 6, 7, 10, 16, 25, 29, 30/; but the measure is reliable only if the sampling is isokinetic. The shape of the sampling tubes and the withdrawal velocity are of strong importance /31/. Other methods are based on a change in the physical properties of the suspension due to the solid particles. Several authors /11, 12, 18, 19/ employed a conductivity probe, observing that nonconductive particles suspended in a conducting liquid reduce the conductance of the system. Tojo and Miyamani /20/ inserted into the vessel a probe measuring the extinction of a light beam in the passage through the suspension. Basing on the intensity of light attenuation /8, 21, 22/ and on the γ-ray adsorption /14, 24/ also non intrusive techniques have been developed; as the ray crosses all the vessel, these methods can give only integral informations. All these physical methods are advantageous because make more quick the measure, but are usefull only if a monomodal suspension is employed. Brown and Felton /32/ developed a laser diffraction technique by which both the volume concentration and the particle size distribution may be measured.

Experimental set-up
A 0.390 m diameter vessel with torispherical bottom and four baffles at 90° was employed; the maximum height of liquid H was 0.465 m. A first series of tests was carried out on monomodal water suspensions of glass beads stirred by four different types of impeller: an A 310 Lightnin axial propeller (ψ=0.33); a 45° pitched four blade turbine (ψ=1.2) and a disk turbine with six flat blades inclined at 45° (ψ=2.1), mixed flow impellers; a Rushton type turbine (ψ=4.8). Water suspensions of crushed silica and barytes were tested to verify the influence of the solid density and of the particle shape, in order to obtain a confirmation of the model reliability; the Lightnin propeller and the four blade turbine were employed. The stirrer diameter was T/3 and the clearance H/3; for the axial impeller the pumping was directed toward the bottom. The particle classes were obtained by sieving; their settling velocity was determined experimentally in a little semibatch elutriator /33/. The solid physical properties are listed in table 1. Only dilute suspensions (B=0.50; 1.51; 5.10) were tested. Local solid concentration was determined by sampling; the experimental technique has been described in detail elsewhere /34, 35/. Tests on the effects of the withdrawal velocity and of the shape of the sampling tube were carried out to verify the reliability of the measures. The axial concentration profiles near the wall at various stirrer velocities multiple of Ns were obtained. Several radial profiles were also determined. The degree of radial homogeneity was generally good; only for disk turbines, in the zone below the stirrer, there were significant radial gradients of concentration. The concentration profiles depend on the particle physical properties, concentration of the solid, type and velocity of the stirrer. It was confirmed that, in dilute suspensions, solid distribution is independent of total solid hold up if referred to N/Ns.
Fig. 1 shows an example of the axial concentration profiles at \( N_{fs} \) for different types of impeller.

**Model and discussion**

As the radial concentration profiles are rather flat, the adoption of a monodimensional model seems justified, at least for axial stirrers and, for disk turbines, in the zone above the impeller.

The complete theoretical development has been reported elsewhere /35, 36/.

From the continuity equation it derives:

\[
- \frac{d\bar{u}_\ell \bar{C}}{dz} + \bar{u}_{pz} \frac{d\bar{C}}{dz} = 0
\]  

(1)

It was observed that the terminal velocity of a particle in a turbulent field is smaller than that in a still fluid; but with our current knowledge it is not possible to evaluate the former. Therefore, in the following discussion the experimental \( u_\ell \) value in still fluid will be used.

With reference to eq. (1) the convective term due to turbulent accelerations is usually written using a gradient type law, by introducing a coefficient of turbulent diffusion; in the monodimensional case:

\[
\bar{u}_{pz} \frac{d\bar{C}}{dz} = -\mathcal{D}_p \frac{d\bar{C}}{dz}
\]  

(2)

This model is generally satisfactory for small scale turbulent perturbations, that are isotropic. But there is a probable convective contribution of large scale anisotropic motion also, so that \( \bar{u}_{pz} \frac{d\bar{C}}{dz} \) may be more correctly expressed as follow:

\[
\bar{u}_{pz} \frac{d\bar{C}}{dz} = -\mathcal{D}_p \frac{d\bar{C}}{dz} + \mathcal{V}_{pz} \frac{d\bar{C}}{dz}
\]  

(2bis)

where \( \mathcal{V}_{pz} \) is the contribution of large scale turbulent motions. Nevertheless, let us assume, as a first approximation, the validity of eq. (2).

Substitution of eq. (2) into (1) gives:

\[
\bar{u}_\ell \bar{C} = \mathcal{D}_p \frac{d\bar{C}}{dz}
\]  

(3)

Therefore the local concentration depends on \( \mathcal{D}_p / u_\ell \), or, introducing into the equation dimensionless variables, on the Péclet number defined as

\[
\text{Pe}^* = \frac{u_\ell L^*}{\mathcal{D}_p}
\]  

(4)

with \( L^* \) a characteristic linear dimension of the system.

Since inside the vessel the dissipated power and the turbulence intensity change with the spatial coordinates, very likely \( \mathcal{D}_p \) and \( \text{Pe}^* \) are not constant.

The experimental axial concentration profiles confirm this (see fig. 1); they are very complicated and can not be described by the simple exponential function that would derive from the solution of eq. (3) assuming \( (\mathcal{D}_p / u_\ell) \) as a constant along the vessel.

Experimental concentration profiles very similar to that of the authors were described well by Sysova et al. /19/ assuming \( \mathcal{D}_p \) as a function of the axial coordinate. Of course \( \mathcal{D}_p \) and \( \text{Pe}^* \) are also a function of the system geometry.

In a dilute suspension, i.e. when the fluid motion is unaffected by the presence of solid, the particle diffusivity arises only from the particle-fluid interaction. \( \mathcal{D}_p \) is proportional to \( \mathcal{D}_f \), even if the proportionality coefficient may be a function of turbulence microscale and solid characteristics.

Applying the Taylor theory for the fluid diffusion, the coefficient of fluid turbulent diffusion has the following expression /37/:

\[
\mathcal{D}_f = \nu \frac{f_\ell}{f_\zeta} \frac{L^*}{u_\ell}
\]  

(5)

This relationship is valid for homogeneous turbulence and long diffusion times.

From the homogeneity hypothesis it also follows that \( v_\ell f_\zeta = u_\ell f_\zeta \); moreover, assuming isotropy and taking the power locally dissipated by eddies as proportional to the total power dissipated by the impeller, it can be written /37/:

\[
u_\ell = (\nu \zeta D)^{1/3} N
\]  

(6)

The substitution of the previous equation in (4) gives:
\[
\text{Pe}^* = \frac{u_t L^*}{\psi^{1/3}N_0^{2/3} \epsilon^{1/3} A_T}
\] (7)

Eq. (7) would be strictly valid in an homogeneous isotropic turbulent field, where \text{Pe}^* would not change with the spatial coordinates. In an actual stirred vessel, \text{Pe}^* would change, as said before, according to the distribution of the turbulence intensity. However, an average value of this parameter may be useful to describe the average quality of the suspension.

In eq. (7), the two turbulent scales are of the same order and related /37/; further, the mean value of \( A_T \) was found to be proportional to \( D /38/ \). Hence assuming \( D \) as a characteristic length of the system, eq. (7) becomes:

\[
\text{Pe}^* = \frac{u_t}{\psi^{1/3}N_D}
\] (10)

Moreover, it can also be observed that the fluctuating velocity of the large scale eddies may be assumed as proportional to \( ND \), for a given geometrical system, as can be observed from the results of Schwartzberg and Treybal /39/. So according to this discussion, the solid distribution in the vessel may be considered a function of a \( K \) parameter so defined:

\[
K = \frac{\psi^{1/3}N_D}{u_t}
\] (11)

It is more convenient, in our opinion, not to call this number a Péclét number, in order to stress that the dispersing phenomenon is not solely due to the turbulent diffusion, but also to the anisotropic turbulent motion.

Fig. 2 shows how the axial concentration profiles depend on the \( K \) number; increasing this number, the quality of the suspension will increase.

Figs. 3-6 show the distribution quality, calculated from experimental concentration profiles using

\[
\sigma = \sqrt{\frac{1}{\sum_i (C_i - \bar{C})^2}}
\]

versus the dimensionless number \( K \); the \( B^{-0.13} \) factor has been introduced in order to take into account the effect of the mean concentration.

The experimental points are well correlated, in spite of the remarkable variation in particle size, density and shape; therefore it is confirmed that the particle physical properties can be taken into account by means of \( u_t \).

It is also confirmed that in a dilute suspension it is sufficient to correct the stirrer velocity by the factor \( B^{-0.13} \) to describe the effect of the mean concentration. It is interesting to note that the kind of relationship between \( \sigma \) and \( K \) does not change when, in more concentrated suspensions, the effect of the concentration has a more complex law.

The curves in the figures are not very close to each other because the \( K \) parameter takes into account only the total dissipated power, while the stirrers employed promote a non similar fluiddynamic in the vessel causing a different average turbulent intensity and a different distribution of the local dissipated power. In fig. 7 the \( \sigma \) data of Bohnet and Niesmak /8/ are drawn as a function of \( K/B^{0.13} \).

The data refer to bronze, glass and styropor® beads. A marine propeller was employed. Also in this case, the data are well correlated by the proposed group.

Figs. 3-6 allow us to make a comparison between the efficiency of the different types of impellers, that is to compare the power consumption required to obtain a given distribution quality. For given \( D \) and \( u_t \), in fact, \( K \) is proportional to the cubic root of the dissipated power.

About the effect of the mean concentration, it is difficult to explain the results and give a justification of the reference to \( N_{js} \). The solid distribution in the vessel depends on the turbulent intensity, but also on the boundary conditions, i.e. the concentration on the bottom, that are affected by the suspension mechanism. The experimental data show that at a constant value of \( N/N_{js} \) the dimensionless concentration near the bottom increases when the suspension becomes more concentrated; therefore the boundary conditions on the bottom also change with
Bimodal suspensions

Some exploratory tests on bimodal glass beads suspensions were carried out in a 5 litres flat bottom vessel; a Rushton turbine was employed. They seem to indicate that small and large particle interact strongly. At constant \( N/N_{js} \), the solid concentration on the bottom decreases. In the rest of the vessel the suspension quality is worse than that of the corresponding monomodal suspension. Particle concentration decreases in the upper part and increases in the lower part. In the lower part of the vessel the increase in the solid concentration is stronger for the coarse particles.

Conclusions

The axial profiles of solid concentration cannot be described by a simple dispersion model, with a constant turbulent diffusion coefficient. In the examined geometrical systems, the coefficient of turbulent diffusion of the particles changes remarkably along the vessel. Furthermore the dispersion of the solid is probably strongly affected by the large scale eddies or macroscopic turbulences which is not easy to describe with a model.

From theoretical considerations, we proposed a model based on the effects of the turbulent disturbances, that allowed the correlation of the whole suspension quality as a function of the stirrer speed and the settling velocity of the particles; the latter, in fact, can describe satisfactorily all the physical characteristics of the solid liquid system.

Though the settling velocity in a still fluid is probably different from that in a turbulent fluid, nevertheless the results of our tests showed that the former can be employed satisfactorily to correlate the experimental data. The problem of complete description of the solid concentration is still far from being solved satisfactorily, but it seems that the suggested theoretical treatment can be a sufficient basis for an evaluation of the suspension quality.

Notation

- \( B \): weight of the solid in suspension, per weight of liquid, kg/kg
- \( c \): turbulent concentration fluctuation, \( m^3/m^3 \)
- \( C \): local volumetric concentration, \( m^3/m^3 \)
- \( C_{av} \): mean volumetric concentration, \( m^3/m^3 \)
- \( D \): stirrer diameter, m
- \( D' \): particle diameter, m
- \( D'' \): turbulent diffusion coefficient, \( m^2/s \)
- \( H \): height of liquid in the vessel, m
- \( K \): parameter defined in (11)
- \( L^* \): characteristic linear dimension of the system, m
- \( l_e \): average size of the energy-containing eddies, m
- \( N \): stirrer velocity, rev/s
- \( N_{js} \): minimum suspension stirrer velocity, rev/s
- \( T \): tank diameter, m
- \( u \): turbulent component of Eulerian velocity, m/s
- \( u' \): root-mean square Eulerian turbulent velocity component, m/s
- \( u_t \): terminal settling velocity, m/s
- \( v' \): root mean square Lagrangian turbulent velocity component, m/s
- \( z \): Cartesian coordinate in axial direction, m

Greek symbols

- \( \lambda_L \): Lagrangian integral length scale, m
- \( \rho \): density, kg/m^3
- \( \sigma \): suspension quality
- \( \psi \): power number

Subscripts

- \( f \): fluid
- \( p \): particle
- \( z \): z-axis direction

Superscripts

- \( \text{mean} \): time and section mean value

References

2. J. Weisman, L. E. Efferding, 1960, A.I.Ch.E. J. 6, 419-426
Table 1 - Solid physical properties.

<table>
<thead>
<tr>
<th>Material</th>
<th>Density kg/m³</th>
<th>D₃₀₁₈₀ m</th>
<th>D₂₁₂½₂₅₀ m</th>
<th>D₄₂₀½₅₀₀ m</th>
<th>B kg/kg</th>
<th>uₑ m/s</th>
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</thead>
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<tr>
<td>Crushed silica</td>
<td>2720</td>
<td>125/180</td>
<td>0.50</td>
<td>1.29.10⁻²</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>212/250</td>
<td>0.50</td>
<td>2.77.10⁻²</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>420/500</td>
<td>0.50</td>
<td>6.11.10⁻²</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Crushed barytes</td>
<td>4280</td>
<td>125/180</td>
<td>0.50</td>
<td>2.97.10⁻²</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>212/250</td>
<td>1.51</td>
<td>4.76.10⁻²</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Glass beads</td>
<td>2670</td>
<td>100/177</td>
<td>0.50</td>
<td>1.38.10⁻²</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2600</td>
<td>208/250</td>
<td>0.50</td>
<td>2.40.10⁻²</td>
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</tr>
<tr>
<td></td>
<td>2600</td>
<td>417/500</td>
<td>0.50</td>
<td>6.03.10⁻²</td>
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</tr>
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</table>
Fig. 4 - Distribution quality versus K number.

Fig. 5 - Distribution quality versus K number.

Fig. 6 - Distribution quality versus K number.

Fig. 7 - Distribution quality versus K number. Zohmet and Siemen's data [16].