System-Level Vulnerability Assessment for EME: From Fault Tree Analysis to Bayesian Networks—Part I: Methodology Framework

System-Level Vulnerability Assessment for EME: From Fault Tree Analysis to Bayesian Networks—Part I: Methodology Framework

Congguang Mao, and Flavio Canavero

Abstract—**The intense electromagnetic environments (EMEs), such as the intentional electromagnetic interference and electromagnetic pulse, pose severe threats to the normal functions of electric and electronic systems. A system is usually composed of numbers of interdependently linked subsystems or equipments. The interactions of the system and the high-power EME involve large quantities of parameters and scenarios, so the complete tests or computations are usually difficult to fulfill, which leads to a hard mission to assess the system-level electromagnetic vulnerability. This paper provides the thought of divide-and-rule to cope with this problem. First, it divides the system into relatively independent and manageable subsystems, and after respective tests and computations, the subsets of data are fused to characterize the whole system. The key point for this assessment methodology is to set up one model or framework to unify all the activities, which is completed here by the causal Bayesian networks (BNs). The system-level effects and the environment threats are characterized with the probability theory. The modeling and parameter determining techniques are presented. Since fault tree analysis (FTA) is also utilized in the electromagnetic risk assessment, the assessment procedures based on relatively BN and FTA are compared. The final results indicate that BN is capable of extending the modeling and analysis power of FTA.**

Index Terms—**Assessment, bayesian networks (BNs), E3, FTAEMI, IEMI, vulnerability.**

I. Introduction

THE vulnerability of electric and electronic systems subjected to the intense electromagnetic environments (EMEs), such as the intentional electromagnetic interference (IEMI) and electromagnetic pulse (EMP), may cause the serious disorder or disaster to the society, such as the power grid, communication, and traffic command systems [1]–[4]. It is regarded as one hard and complex task to assess such system-level EME effects (E3) because the task has the following features.

First, the so-called system level means it is composed of numbers of interdependent subsystems. During the interactions with the EMEs, considerable quantities of couplings, propagations, and effects will occur, and massive data need to be processed. The scale of the complete computation and test is so huge that sometimes they cannot be carried out, such as the distributional or geographic systems.

Second, the parameters can vary in large ranges. For example, incident wave can impact the system from any possible direction, and similarly, the failure threshold of one device is never a fixed value, but distributes in a certain interval. Perhaps just for there is uncertainty or inaccuracy in the intermediate processes and the results, the quantification of the system-level effects should be called the assessments, compared with the precise measurement and calculation.

Aiming at the hardest part of the high-power electromagnetic problem, the tests are recommended instead of the pure analysis [1]. However, in order to organize a large-scale test, first of all a considerate plan should be drawn up, which means one rational and feasible analysis and assessment method to unify all the test activities is badly necessary. This paper tries to find some tools to help to answer this question.

Through deep observation and investigation of interactions of the system with EMEs, it can be found that the process involves two major societies: one is the reliability engineering and the other is the electromagnetic compatibility (EMC). Each of them has its own study interests and subjects. The former puts the emphasis on the logical model and analysis of the system structure and safety, including the reliability physics, mathematics, test and design, etc. The latter mainly considers the control of the electromagnetic interferences (EMIs), devoting itself to the interference reproducing, coupling computation, protection design and tests, etc. To build up the assessment framework, both of them have to be integrated.

As one of the classical tools, fault tree analysis (FTA) has been extensively applied in the reliability engineering and the risk assessment since 1960s [5]. Recently, this method is introduced to assess the system-level risk for IEMI [6]. On the other hand, the Bayesian networks (BNs) theory is proposed in 1980s in artificial intelligence (AI) fields [7], [8], and mainly in 2000s, this method is introduced to the reliability engineering [9]. Both the methods employ the probability theory. The Bayesian rule is a powerful tool in the credit updating and decision of AI [7]. Of course, the probabilistic concepts are also well known for EMC as the statistical electromagnetics [11]. So it is natural for us to try to build the assessment methodology based on the probability theory.

The objectives of system assessment contain two aspects: one is to find the effects and present the risk/threat warning; the other is to find the weak points and promote the reliability of the system. FTA is utilized as a tool from the perspective of

C. Mao is with the Dipartimento di Elettronica e Telecomunicazioni, Politecnico di Torino, Torino 10129, Italy, and also with the Northwest Institute of Nuclear Technology, Xi'an 710024, China (e-mail: maocongguang@163.com).

F. Canavero is with the Dipartimento di Elettronica e Telecomunicazioni, Politecnico di Torino, Torino 10129, Italy (e-mail: flavio.canavero@polito.it).

the risk assessment [6]. This paper will employ BN to build the assessment method from the view point of system vulnerability and protection. The rest context is organized as follows.

It starts from FTA and introduces the primary notations and ideas applied to the assessment in Section II. Aiming at the shortcomings of FTA, Section III presents the concepts of BNs and discusses how to determine the parameters of the BN model in assessments. The complete methodology framework is given in Section IV, followed by the comparison of FTA and BN. Finally, the further studies needed to be done are presented in Section V.

II. FTA Joint Event Tree Analysis (ETA) for Risk Assessment

A. Primary Concepts

The core task of the FTA is to find all the basic components that could cause the system function to fail and present the promotion suggestions. The general analysis procedure is completed by two steps: the qualitative and the quantitative. The former focuses on the fault tree (FT) modeling and reduction into the collection of the minimal cut sets, i.e., the basic event sets that cause the top events; and the latter mainly calculates the probability of the top event and the importance of the other events.

For building the FT model, first, one system failure result is taken as the top event; then all the secondary events that cause the top event will be analyzed and found; this process is repeated until the root events are found. All the top, secondary (intermediate), and root events are linked and form a hierarchical structure, i.e., the FT model of the system. The whole process and notations have been defined in the relevant standards or handbooks [12].

The mathematical foundation of the traditional FTA is the Boolean algebra. The system and subsystems mainly have two states: success and failure denoted respectively by "0" and "1." The EVENT means Failure. Three typical logic relationships will be used: AND, two secondary events must simultaneously happen, then the upper events happen; OR, only one of two secondary events happens, then the upper event happens; and NOT, the two states of system or subsystem transform from each other. These relationships are described with the Boolean algebra as following:

$$\begin{array}{lcc} \text{AND} & \text{OR} & \text{NOT} \\ 1\cdot 1=1 & 1+1=1 & \\ 1\cdot 0=0\cdot 1=0 & 1+0=0+1=1 & \overline{1}=0 \\ 0\cdot 0=0 & 0+0=0 & \overline{0}=1 \end{array}.$$

After the FT is completed, the root events are assigned the relevant probability values to denote their possibility of happening. Then, the probability of the intermediate and the top event can be calculated.

B. Assessment Procedure

As aforementioned, the reliability analysis for a given system usually conducted at two layers, i.e., the qualitative for FT modeling, and the quantitative for measurement of the performance.

1) Qualitative Analysis: For a system impacted by EMEs, the top event is the system fails to function as its design. Then all the subsystems that could cause the top event should be found and built as intermediate events. Each of the subsystems is analyzed further to find which components are broken down by their electromagnetic stresses (EMSes). The failure components are determined and modeled as the root events. This step often assisted by the fault mode and effects analysis.

What should be noticed in FT modeling is the implication of the idea of the divide-and-rule. Here, the whole system is divided into subsystem thoroughly based on the relatively independent function of the subsystems, or it can be expressed further that the FT is the hierarchical classification of system function. The classification level, to subsystem or components, may be determined according to the practical requirements.

2) Quantitative Analysis: What we concern is limited on the effects caused by the EMEs, so the other factors that cause system failure will be excluded, such as the mechanical fracture or the aging of the structures. For E3, the root events can be determined by the stress–strength Interference (SSI) theory [13], [14]. Set $f(x)$ and $g(y)$ relatively the probability density functions (pdfs) of the electromagnetic strength/threshold of component or subsystem V_i and its electromagnetic stress/load EMS_i , *i* is the code of component in FT C_i. The prerequisite of V_i is $y \geq x$. Thus, the failure probability of component P_{C_i} is

$$P_{C_i} = P(y \geq x) = \int_{y_{\min}}^{y_{\max}} g(y)dy \int_{x_{\min}}^{y} f(x)\,dx \qquad (1)$$

where arguments *x* and *y* are just the physical quantities responsible for the electromagnetic effects.

C. ETA Supplement to FTA

FTA usually focuses the failure of the system itself, where the failure causes are the breakdown of some local components. When FTA is applied to E3, the most initial cause of the system fault is the EME. Genender *et al*. recommend from the viewpoint of risk, if C_i denotes the *i*th risk, its occurring probability $P(C_i)$ is formulated as [6]

$$P(C_i) = P(C_i|S_k)P(S_k) \qquad (2)$$

where $P(S_k)$ is the probability of the *k*th category of the IEMI source S_k, and $P(C_i|S_k)$ is the conditional probability of C_i under the situation of S_k. Since FTA cannot provide more information about environmental factor $P(S_k)$, ETA is adopted to supplement FTA [6].

ETA is also based on the Boolean algebra. However, its logical analysis order is from bottom to top, just contrary to FTA. It starts from the initial event, and determine what results are led to. This process will be iterated until the final desired events are found. All the intentional EMEs are classified considering the following factors: IEMI source categories ($C_{\rm S}$), location ($L_{\rm S}$), and duration of the IEMI attack (*D*). So the probability of the *k*th IEMI scenario $P(S_k)$ is

$$P(S_k) = P(C_S)\cdot\prod_{i=1}^{n} P(L_S(C_S,Z_i))\cdot P(D(C_S,Z_n)) \qquad (3)$$

where Z_n is the final investigated zone and Z_i is the intermediate zone from source to Z_n. After propagation, analyzed by the electromagnetic topology (EMT), this IEMI scenario reaches the system and cause some risk C_i, then the environmental factor

$$P(EME_i) = \sum_{k=1}^{K} P(S_{i,k}). \tag{4}$$

So the risk probability of the component C_i is the sum of all the results caused by the IEMI

$$P(C_i) = \sum_{k=1}^{K} P(C_i|S_k)P(S_k) \tag{5}$$

where *K* is the number of all the possible IEMI scenarios.

D. Summary

The system risk for IEMI is measured by the conditional probability. In the complete risk assessment procedure, three tools are employed: FTA for system model, ETA for environment categorization, and EMT for coupling analysis. The mathematical foundation of FTA and ETA is Boolean algebra and probabilistic theory. The greatest advantage of FTA and ETA is their clear logic.

There are two deficiencies in the application of FTA to E3: One is not involving the environmental analysis, which is remedied by the ETA and EMT. However, for the improvement of protective design, their power about the diagnosis and location of the weak components and critical coupling paths is insufficient. The other limit of FTA is its two-state logic. When coping with the multistate faults, the FT model would have to be treated especially. This is common that a system experiences three states: success; logic upset; and damage of the hardware [10].

So a more powerful tool is needed to inherit the advantages of FTA and conquer its disadvantages.

III. BN for Assessment

Let us once again inspect the interaction process of the system with the EMIs. First, the propagation of EMI is of fluxility, which can be described by the graph model, such as EMT. Second, the interferences and effects are of uncertainty, which can be handled generally with probability theory. Then, what possesses both of the characteristics seems to be the "probabilistic graph."

Fortunately, this theory has been existed in the mathematics society for a long time. However, it is not applied extensively. In 1980s, the important principles connecting the joint probability and the graph model are proved to study the machine inference with uncertainty data, which indicated a new approach was born, named as BNs to commemorate the mathematician T. Bayes. In the application of BN to reliability engineering, the transform method from FT to BN has been promoted [15], which means FT and ET can also modeled by the graph. Thus, at least in the style, three tools FT, ET, and EMT can be unified into a probabilistic graph BN model. This encourages us to build the methodology framework of the system-level vulnerability assessment for the IEMI based on BN [16]–[18]. The train of thought is shown in Fig. 1.

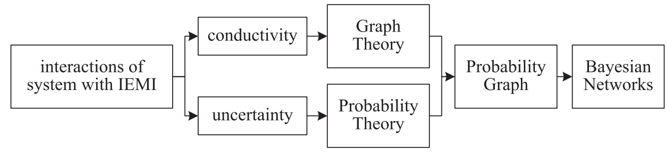

Fig. 1. Derivation of BNs.

A. Primary Concepts

1) About Probability Theory: Because BN is founded on the probability and graph theory, the relative probability formula is introduced first, which will guide our building of assessment methodology.

The probability must conform to three basic axioms as follows:

1) $0 \le P(x) \le 1$;
2) $P(x) = 0$, impossible event; $P(x) = 1$, certain;
3) $P(x \cup y) = P(x) + P(y)$, if $X \cap Y = \varnothing$.

Three rules play an important role in the application.

1) Product formulation for joint probability

The BN theory mainly addresses the factorization of a joint probability distribution, so the product formulation is the core of BN. Given two sets, if probability $P(A) > 0$ and $P(B) > 0$

$$P(A, B) = P(A|B)P(B) = P(B|A)P(B). \tag{6}$$

This rule can be extended to more sets.

2) Total probability formulation

For BN inference from reason to results, the total probability formulation will be used, which is the case of the risk assessment of system for IEMI. If a total set Ω can be divided into a series of subsets $\{B_i\}$, $i = 1, \ldots, I$, $B_i \cap B_{i+1} = \varnothing$ and $\cup B_i = \Omega$ the probability of a subset $A \subset \Omega$ is

$$P(A) = \sum_{i=1}^{I} P(A|B_i)P(B_i). \tag{7}$$

3) The Bayesian formulation

Another major application of BN is to diagnosis the reasons against some results, where the Bayesian formulation is vital. Given the conditions of the total probability formulation, the conditional probability

$$P(B_j|A) = \frac{P(A|B_j)P(B_j)}{\sum_{i=1}^{I} P(A|B_i)P(B_i)}, \quad j = 1, ..., I. \tag{8}$$

2) About BN Theory: A BN is a two-element entirety, $BN = BN\langle G, P\rangle$ [7], one element is the graphic model *G* and the other is the probability *P*, which just constitute two respects, the qualitative structure and the quantitative parameter.

Moreover, the graph model *G* is also a two-element entirety, usually denoted as $G = G\langle V, E\rangle : V = \{X_1, X_2, \ldots, X_n\}$, stand for the random variables and the edges *E* for the dependent relationships between the nodes.

In order to avoid the cycle reasoning, the loops are forbidden in BN. So BN is a specific type of graph, named directed

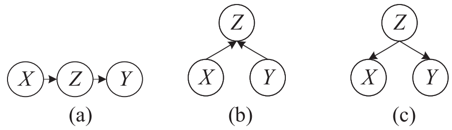

Fig. 2. Three typical connection of BN. (a) Series. (b) Converging. (c) Diverging.

acyclic graph (DAG). In DAG, the conditional independence is represented by the d-separation. Given three variables *X*, *Y*, and *Z* in one BN, and *Z* exclusive *X* and *Y*, if all the paths between *X* and *Y* are blocked by *Z*, then we call *Z* d-separate *X* and *Y*, and denote by $X \perp Y | Z$. Further, if *X* is conditionally independent to *Y*, the conditional probability function $P(x|y) = P(x)$. So the concept of d-separation can reduce the complexity of the joint probability.

BN factorizes a joint probability distribution along the DAG and based on the conditional independence. Since the edges denote the causal relationships, BN is also named casual networks. The start point of one arrow is called the parent vertex and the terminate point called the descendant vertex. Given result variable x_i, all of its parent variables, i.e., causal vertices, are denoted as pa(x_i). Then, based on the concept of d-separation, the joint probability factorized with BN is expressed as [8], [9]

$$P(x_1, x_2, \ldots, x_n) = \prod_{i=1}^{n} P(x_i | pa(x_i)). \tag{9}$$

In addition, there are three typical structures in DAG: serial, converging, and diverging (see Fig. 2). The serial structure can describe the progressive relation, and is similar with the chain rule of the conditional probability. If the probability of events *X*, *Y*, and *Z* is relatively $P(x), P(y), P(z), P(z|x)$, and $P(y|z)$, then $P(x, y, z) = P(x)P(z|x)P(y|z)$. The converging structure represents a number of causes for one result. There are two cases for depending on the logic relationship between *X*, *Y*, and *Z*. If *X* and *Y* are independent and sufficient conditions for *Z*, then $P(x, y, z) = P(z|x)P(x) + P(z|y)P(y)$; if two necessary conditions *X* and *Y* compose the sufficient condition for *Z*, then $P(x, y, z) = P(x)P(y)P(z|x, y)$. The diversion structure stands for several results for one reason. The joint probability $P(x, y, z) = P(z)P(x|z)P(y|z)$.

B. Assessment Procedure

There exists a definite causal relationship in the EME effects (E3). The ambient electromagnetic energy induces responses on conducting structures, the EMSes on the ports breakdown the components or upset the equipments, and the local damages impact whole system functions. Such causal logics are essential to the BN application to E3, which is ensured by the physical disciplines and be used over and again.

So there are two principles that are well worth adopted in our application. On the one hand, the relations between nodes are determined by the physical processes, but not only by the statistical data. On the other hand, the physical computations will be prior to the purely statistical description when the physical equations can hold, which helps to reduce the uncertainty in the assessment results.

Equivalent to the qualitative and quantitative analysis in FT application, there are also two steps in BN analysis: first, building the BN model, and second, assigning the probability values to the vertices and computing, which in BN society are called as *structure learning* and *parameter learning*.

1) Structure Learning—Qualitative Analysis: There are three classes of methods for the BN structure learning [7]: the search and score techniques utilize a preset score function to search all the variables to find the BN structure with the highest score; the constrains-based methods generally test the conditional independence relation with some constrains or principles that have been defined beforehand; and the hybrid both of them. As discussed above, the definite causal logics in the E3 can help us to build the BN structure, which obviously belongs to the second category.

Usually, the causal analysis starts from the reason and terminate at the result. As mentioned above, the EME is the cause and the system effect is the result. So from the exterior of the system into the interior along the EMI and the faults propagation direction, the BN model can be built independently, without the help of other tools. Obviously, this belongs to the second-class modeling method.

The representations of ET, FT, and EMT [6] in BN are depicted through three analysis phases, environments, coupling, and final system effects. Their ideas can be absorbed in BN modeling.

a) Event Tree Analysis: The different categories of the IEMI sources that reach one zone Z_n can be modeled as the converging structure (see Fig. 1), i.e., multireason for one results, and the electromagnetic wave propagates from zone Z_{i-1} to Z_i may be incorporated into EMT. Then, ET starts from the sources and terminated the zone Z_n, which is the space of system located.

b) Electromagnetic Topology: Following ET, EMT starts from the ambient environment of system in zone Z_n, and terminates at the victim components.

EMT model is identical with BN. EMT regards the E3 as the *onion diagram*, i.e., starting from the EME outside the system shielding barrier EMI propagates layer by layer into the victims of components or ports inside the system. Since only the equipment susceptibility is considered, and not the reflection wave from the system, EMT model is also the DAG. The vertices in EMT denote the fields in the free space or the components. The edges in EMT are the propagation paths of EMI.

Here, one issue must be noticed that the failure of components could be caused by two categories of interferences, the radiated and the conducted, which relatively correspond to the radiated and conducted susceptibility. The EMSes of the former are the free fields, and the latter the currents or voltages. This means actually the EMT will stop at the EMSes, which are the direct causes of components failure. Between the ambient environment of system and the local EMSes on components, there exist complex coupling paths depicted by EMT.

c) Fault Tree Analysis: For the system modeling, the BN can be transformed from FT if available [15], or be modeled

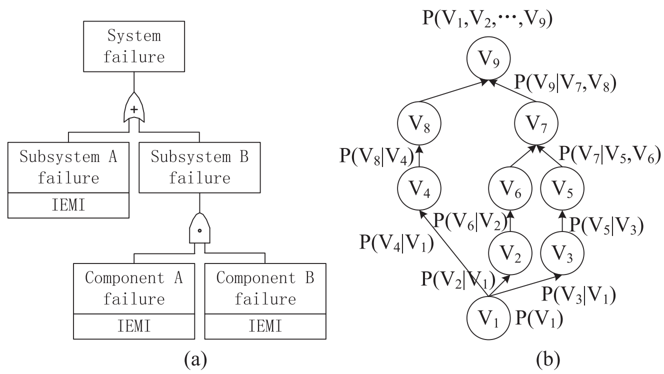

Fig. 3. Comparison of FT [6] and BN model. (a) FT. (b) BN.

directly based on the causal diagram: from lower layer to the upper layer, i.e., from the components to the subsystem/equipment and finally to the whole system, all of them are modeled as vertices of BN, and their dependent relationships the edges.

The examples for the FT and BN are illustrated in Fig. 3. The faults of subsystem A OR B, V_7 OR V_8 in BN, will lead the system (V_9) failure. The subsystem A (V_8) failure could be caused by EMS_1 (V_4). However, the subsystem B (V_7) is supported by components A AND B (V_5 AND V_6), whose EMSes are relatively EMS_2 and EMS_3 (V_2 and V_3). FT includes these EMSes in the EVENTs and stops, whereas the BN can further model the cause of the EMEes, IEMI, as the root nodes (V_1). All the relationships between the vertices are denoted as the conditional probabilities, except the original node V_1. The joint probability $P(V_1, V_2, \ldots, V_9)$ will be formulated in the next section.

What should be noticed is that three tools, ETA, EMT and FTA, employ the causal logic. Only the application angles are different: ETA from the fault effect, EMT from the interference propagation, and FTA from the system function. Since the causal relationship is just the core of BN, there are no obstacles in transforming three kinds of models to BN.

2) Parameter Learning—Quantitative Analysis: Following the BN modeling is the parameter learning, which will present all the parameters and their determination techniques for system vulnerability assessments.

From the perspective of the probability theory, the failure probability of system P_V contains two major factors: the system *V* and EMEs. For comparison with FTA, only IEMI is taken as an example. So joint probability

$$P_V = P_V(V, IEMI). \tag{10}$$

Based on BN, the causal relationship can be expressed by the conditional probability, i.e., the product formulation (6)

$$P_V(V, IEMI) = P_V(V|IEMI)P(IEMI). \tag{11}$$

Actually, it is the radiation susceptibility of the total system. If the system effects need more detailed check and diagnostic, the completed scenarios should be investigated: IEMI produce the ambient environment of system (*AEME*) and further EM stress (*Stres*) on the components, and the components breakdown (*Comp*) lead to the subsystem upset (*Subs*), and ultimately to the function failure of the whole system (see Fig. 1). This physical and logical scenario can be formulated as the joint probability equation and the series of causal relationships

$$\begin{aligned} P_V(V, AEME) = \; & P(V|Subs)P(Subs|Comp) \\ & \times P(Comp, Stres)P(Stres|AEME) \\ & \times P(AEME|IEMI)P(IEMI). \end{aligned} \tag{12}$$

In reliability engineering, 12 is called as *system structural function*. There are three classes of probabilities in the formulation: about system, junction of system and EMSes, and for IEMI. Corresponding to the analysis procedure in structure learning, the determination techniques and the physical sensations of all the parameters are presented as follows.

a) P(IEMI): If IEMI is divided into *k* classes and these subsets S_k pose threats to system, this relationship can be described by the total probability formulation. So the value-assignment method of the probability $P(S_k)$ should abide by the precondition of (7).

First, all the IEMI environments are partitioned into subset S_k, which are exclusive from each other, i.e.,

$$\{IEMI\} = \bigcup_{k=1}^{K} \{S_k\}, \qquad \bigcap_{k=1}^{K} \{S_k\} = \varnothing. \tag{13}$$

Second, every subset is assigned one value $P'(S_k)$ based on the experts' analysis and judge for source categories (C_S), location (L_S), and duration of the IEMI attack (*D*). Finally, all the values are normalized and the new values are taken as the desired probabilities, which are the measurements of the EMEs

$$P(S_k) = P'(S_k)\left(\sum_{k=1}^{K} P'(S_k)\right)^{-1}. \tag{14}$$

The larger of $P(S_k)$ means the greater threats. Two extreme limits 0 and 1, respectively, indicate no threat and threat inevitable existing. Based on the Boolean rule of OR, it is permitted that in FTA $P(S_k) = 1 (k = 1, \ldots, K)$. However, in BN they are forbidden because all the probability values must meet the demand of the basic probability axioms.

b) $P(AEME|IEMI)$ and $P(Stres|AEME)$: From the IEMI source to the system, the electromagnetic wave could have to pass through some barriers, for instance, the buildings. This propagation path can be described by the conditional probability of $P(AEME|IEMI)$. From the threat perspective, the higher coupling efficiency makes the greater probability value.

If several IEMI sources, $S_k (k = 1, \ldots, K)$, simultaneously reach the system, (7) should be employed, which is equivalent to the logic of one result with several causes, i.e.,

$$P(AEME, IEMI) = \sum_{i=1}^{K} P(AEME|S_k)P(S_k). \tag{15}$$

Chosen one category of IEMI, the electromagnetic wave outside the system propagates to the component and induces the EM stress, which can be described by the pdf $g(y)$, and

$$\int_{y_{\min}}^{y_{\max}} g(y)dy = 1, \quad y \in [y_{\min}, y_{\max}]. \tag{16}$$

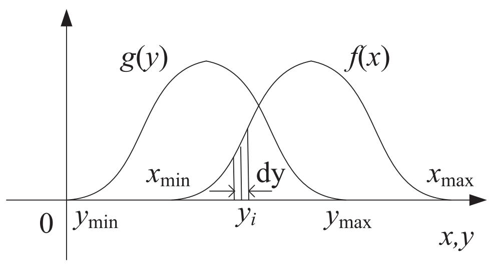

Fig. 4. Failure probability of component in BN.

The stress probability $P(stres|AEME)$ can be assigned values based on the three categories of properties: AEME outside the system, coupling paths, and component susceptibilities. This parameter may be regarded as the explicit measurement of coupling efficiency in the BN model. It indicates no effects, if $P(stres|AEME) = 0$.

Here, if the electromagnetic thresholds of components $f\,(x)$ are available, $P(stres|AEME)$ is proportional to the overlapping portion of the stresses with the strength, i.e.,

$$P(stres|AEME) \propto \int_{x_{\min}}^{y_{\max}} g_k(y)dy, \quad y \in [x_{\min}, y_{\max}]. \tag{17}$$

c) $P(Comp, Stres)$: This probability may be determined by the SSI theory, which has been discussed in Section III-B1.c. However, the physical sensation need be severely deduced according to the BN theorem.

First, the overlaying portion of two functions is divided into *n* intervals with $dx = dy$, and in the *i*th interval, the values of arguments, $x_i = y_i$ (see Fig. 4). Every interval can be regarded as an event with occurring probabilities $\mathrm{F}_i = \mathrm{f}(x_i)\mathrm{d}x$ and $G_i = g(y_i)$ dy. Under the condition of the causal event G_i, all the events with $x_j \leq y_i$ $(j = 1, 2, \ldots, i)$ will happen. So based on the total probability formulation (7), the conditional probability of the component failure

$$P(F_i|G_i) = F'_1 + F'_2 + \cdots + F'_i = \sum_{j=1}^{i} F'_j \tag{18a}$$

and the joint probability of the cause and result

$$P(F_i, G_i) = P(G_i)P(F_i|G_i) = g(y_i)dy \cdot (\sum_{j=1}^{i} f(x_j)dx). \tag{18b}$$

The total probability of the components is the sum of all the discrete events, i.e.,

$$\begin{aligned} P(Comp, Stres) &= \sum_{i=1}^{n} P(F_i, G_i) \\ &= \sum_{i=1}^{n} P(G_i)P(F_i|G_i) \\ &= \sum_{i=1}^{n} g(y_i)dy \cdot \left(\sum_{j=1}^{i} f(x_j)dx\right) \\ &= \int_{x_{\min}}^{y_{\max}} g(y)dy \int_{x_{\min}}^{y} f(x)dx, \quad n \to \infty. \end{aligned} \tag{18c}$$

This result is just the same with the SSI. The deduction process indicates that what the SSI formulation presents is the joint probability $P(Comp, Stres)$, but not the conditional probability $P(Comp|Stres)$.

The event of the component failure is the result of the IEMI and the cause of the system fault. So this factor is the core of E3, and the task of BN is just to help to identify every sensitive component and causal scenario.

d) $P(Subs|Comp)$ and $P(V|Subs)$: These two parameters describe the system structure and are equivalent to FT. The values can be assigned by the similar technique to $P(AEME|IEMI)$. With the probability theory replacing the Boolean algebra, multistate variable can be processed using total probability formulation. For example, if three states are considered, one component node is extended into three, and three edges link the upper subsystem node.

C. System Assessment

1) System Failure Probability: With all the probability values mentioned above, the failure probability of the whole system can be expressed as

$$P_V(V, IEMI) = \sum_{n=1}^{N} \prod_{m=1}^{M} P(V|Subs_{m,n})P(Subs_{m,n}) \tag{19}$$

where *N* is the number of the subsystem that can directly cause the system breakdown and *M* is the number of the redundant subsystem. The failure of subsystems is deduced by its components, so

$$\begin{aligned} P(Subs_{m,n}) = &\sum_{i=1}^{I} \prod_{j=1}^{J} P(Subs_{m,n}|Comp_{m,n,i,j}) \\ &\times P(Comp_{m,n,i,j}) \end{aligned} \tag{20}$$

where the sensation of *i* and *j* are similar to *n* and *m*. The effects of component are caused by its EMSes, so

$$\begin{aligned} P(Comp_{m,n,i,j}) = &\sum_{p=1}^{P} [P(Comp_{m,n,i,j}|Stres_{m,n,i,j,p}) \\ &\times P(Stres_{m,n,i,j,p})]. \end{aligned} \tag{21}$$

The stresses include all the cases induced by all sorts of IEMI sources and the coupling paths, then

$$\begin{aligned} P(Stres_{m,n,i,j,p}) = &\sum_{k=1}^{K} [P(Stres_{m,n,i,j}|EME_{m,n,i,j,p,k}) \\ &\times P(EME_{m,n,i,j,p,k}|S_k)P(S_k)]. \end{aligned} \tag{22}$$

If the category of IEMI is chosen, $P(S_k)$ becomes a constant, i.e.,

$$P(S_k)P(EME_k|S_k) = C_{\mathrm{psk}}. \tag{23}$$

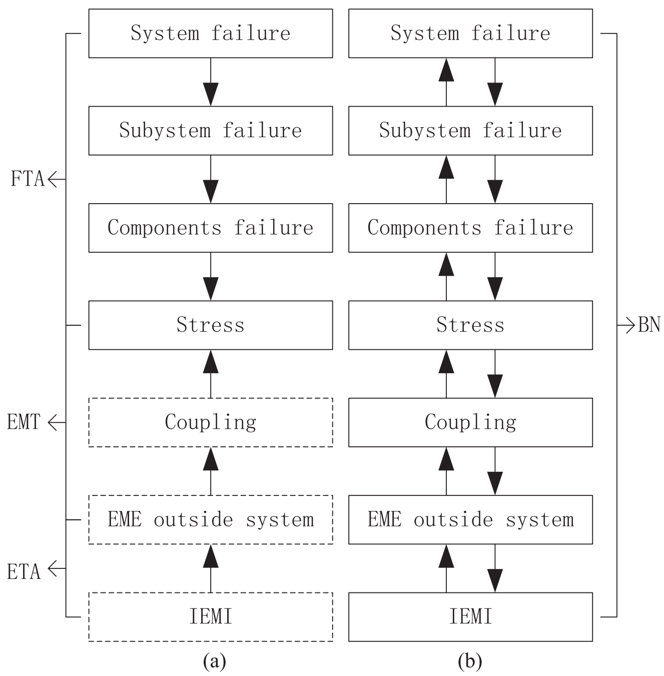

Fig. 5. Comparison of assessment procedures based on FTA and BN. (a) FTA. (b) BN.

Further if the probabilities $P(Subs_{m,n}|Comp_{m,n,i,j})$ and $P(V|Subs_{m,n})$ are only assigned two values 0 and 1, the system failure probability gets almost identical with FT, except $P(Stres_{i,j}) \neq 1$, i.e.,

$$P_V(V, IEMI) = C_{psk} \cdot \sum_{j=1}^{N} \prod_{i=1}^{M} P(Comp_{i,j}|Stres_{i,j})P(Stres_{i,j}). \quad (24)$$

2) System Diagnosis: For the performance certification of protective design, it is critical to find the weak components and the dominant coupling paths.

If the system effect has been found, the probability of the component diagnosed to be the cause can be calculated based on the Bayesian rule (8)

$$P(Comp_{i,j}|V) = \frac{P(V|Comp_{i,j})P(Comp_{i,j})}{P_V(V, IEMI)}. \quad (25)$$

Further, the probability about the coupling mechanism or propagation paths is

$$P(Stres_p|Comp_{i,j}) = \frac{P(Comp_{i,j}|Stres_p)P(Stres_p)}{P(Cpmp_{i,j}|V)}. \quad (26)$$

More detailed introduction is presented in [15].

IV. Methods Comparison

The assessment procedure of FTA and BN are shown in Fig. 5. FTA method employs three models of FT, EMT, and ET. In comparison, BN method can integrate the three tools into one graph model. The environmental factor is directly involved in the system structural function of BN (see 12), while in FT, it is only implicitly contained in the bottom event as the stress probability (see 5). Thus, the importance of environmental factor and the component can be measured equally. Moreover, taking the place of Boolean algebra and logic series with the probability theory and graph, the multistate fault can be easily modeled. Given the specific values, BN can be reduced into FT. What's more beneficial is that BN can conduct the bidirectional inferences, one for risk evaluation and the other for weak components location.

The data play a vital role in the vulnerability assessment, and are acquired by means of the calculation and testing. This implies that the BN method depends on the physical interaction of the electromagnetic wave and systems. On the other hand, under the BN framework, the interaction of system and the EME can be divided into series of nodes including the original EME, various coupling path and mechanisms, fault modes and effects. All the phenomena can be studied independently with different approaches. Since the size of every node is less than the whole system's, the difficulty of calculation and test can be reduced greatly. So BN method is beneficial for the deep study of system-level effects.

As mentioned above, the tests are recommended for the system assessment. If no detailed analysis and local examinations were conducted beforehand, there are two possible kinds of risks for the whole system illumination: no effects and system damages. BN can provide an order from the preanalysis and component sensitivity examination to the complete test or assessment of the system.

V. Summary and Conclusion

The system-level vulnerability for IEMI, as a very comprehensive work, needs many techniques and tools. With the principle of divide-and-rule, this paper devoted itself to set up a methodology framework to integrate various parts of task, such as the analysis, test, computation and data fusion, etc.

The FTA is first utilized and the advantages and disadvantages are pointed out. In order to extend the power of FTA, BN is introduced and the assessment framework for the system-level E3 is set up. Based on the physical process and causal relation, the interaction of systems and EME is modeled into a DAG. The system effects and EME are formulated respectively as the conditional and marginal probabilities.

All of these indicate that BN is capable of inheriting the advantage of FTA and overcoming its limits. So BN method is well worth recommending, although FTA has been applied in extensive fields.

This paper puts the emphasis on the methodology presentation. An application of the method including the test is presented in another paper. The other topics about the common cause failure model, importance analysis, and dynamic model can be seen in [9] and [17].

References

[1] C. E. Baum, "From the electromagnetic pulse to high-power electromagnetics," *Proc. IEEE*, vol. 80, no. 6, pp. 789–817, Jun. l992.

[2] W. R. Graham. (2008, Apr.). Report of the commission to assess the threat to the united states from electromagnetic pulse (EMP) attack. EMP commission USA [Online]. Available: http://www.empcommission.org/docs/A2473-EMP_Commission-7MB.pdf

[3] D. Månsson, R. Thottappillil, and M. Bäckström, “Vulnerability of European rail traffic management system to radiated intentional EMI,” *IEEE Trans. Electromagn. Compat.*, vol. EMC-50, no. 1, pp. 101–109, Feb. 2008.

[4] W. A. Radasky, C. E. Baum, and M. W. Wik, “Introduction to the special issue on high-power electromagnetics (HPEM) and intentional electromagnetic interference (IEMI),” *IEEE Trans. Electromagn. Compat.*, vol. EMC-46, no. 3, pp. 314–321, Aug. 2004.

[5] (2002, Aug.). *Fault Tree Handbook Aerospace Appl. (Version 1.1)* [Online]. Available: http://www.hq.nasa.gov/office/codeq/doctree/fthb.pdf

[6] E. Genender, H. Garbe, and F. Sabath, “Probabilistic risk analysis technique of intentional electromagnetic interference at system level,” *IEEE Trans. Electromagn. Compat.*, vol. EMC-56, no. 1, pp. 200–207, Feb. 2014.

[7] J. Pearl, *Causality: Models, Reasoning Inference*, 2nd ed. New York, NY, USA: Cambridge Univ. Press, Sep. 2009.

[8] T. J. T. Koski and J. M. Noble, “A review of Bayesian networks and structure learning,” *Math. Appl.*, vol. 40, no. 1, pp. 53–103, 2012.

[9] H. Langseth and L. Portinale, “Bayesian networks in reliability,” *Rel. Eng. Syst. Safety*, vol. 92, no. 1, pp. 92–108, Jan. 2007.

[10] “Electromagnetic compatibility (EMC)—Part 1–5: General—High Power Electromagnetic (HPEM) effects on civil systems,” Basic EMC Publication, IEC 61000-1-5, 2004.

[11] R. Holland and R. S. John, *Statistical Electromagnetics*. Philadelphia, PA, USA: Taylor & Francis, 1999.

[12] *Fault Tree Analysis*, 2006, IEC 61025.

[13] M. Modarres, *Risk Analysis in Engineering*. New York, NY, USA: Taylor & Francis, 2006.

[14] Y. Rui and Y. Fu, *Modern Reliability Design* (in Chinese). Beijing, China: National Defence Industrial Press, 200404.

[15] A. Bobbio, L. Portinale, M. Minichino, and E. Ciancamerla, “Improving the analysis of dependable systems by mapping fault trees into Bayesian networks,” *Rel. Eng. Syst. Safety*, vol. 71, no. 3, pp. 249–260, Mar. 2001.

[16] C. Mao and H. Zhou, “Bayesian networks modeling in system-level susceptibility assessments for intentional EMI,” in Proc. Electromagn. Compat. Eur., Wroclaw, Poland, 2010, pp. 502–505.

[17] C. Mao, H. Zhou, Z. Cui, A. Zhai, and B. Sun, “System-level susceptibility analysis for intentional EMI based on Bayesian networks,” in Proc. Progr. Electromagn. Res. Symp. PIERS,Suzhou., China, 2011, pp. 1315–1319.

[18] C. Mao, H. Zhou, J. Fu, B. Sun, and H. Yu, “Introduction to the system-level susceptibility assessments for HEMP and HPEM,” in Proc. Progr. Electromagn. Res. Symp., Beijing, China, 2009, pp. 1746–1750.