MIMO Relay Network with Precoding

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***Abstract*—We study the achievable rate in a MIMO, dual-hop, relay network where source and relay nodes may precode their input signals before transmission. Although an iterative expression for optimal precoders in this scenario is available in the literature, the corresponding achievable rate cannot be obtained analytically. We therefore present approximate expressions for the precoding matrices so as to analytically derive the achievable rate. Our expressions for the precoders provide performance that is very close to the optimum while allowing further analytical investigation of the network system. In particular, we show how our expressions can be successfully used to analyse the trade-offs existing between achievable rate and nodes power consumption.**

I. Introduction

Dual-hop multiple-input multiple output (MIMO) relaying is an efficient cooperative transmission scheme that is able to improve the throughput and the coverage of wireless communication systems. Among the existing relay schemes, amplify-and-forward (AF) is a non-regenerative approach where the information data is not decoded at the relay but data precoding can be performed. Compared to the decode-and-forward approach, AF yields a smaller transmission delay and a better energy efficient performance. In AF relay systems, the precoding design inevitably hinges on the availability of the channel state information (CSI). When perfect CSI concerning the two hops is available at the relay, the optimal precoding matrix has been obtained in presence of fading channels in [1], [2]. Additionally, precoding at both the source and the relay can further improve the network performance if perfect CSI is available at the source too. In this scenario, an iterative algorithm has been proposed in [3], in order to jointly obtain the optimal precoding at the source and the relay. Some recent studies have also addressed the design of the percoding matrix in the presence of partial CSI at the relay [4], or in the presence of a direct link between source and destination [5]. However, in none of the above cases a closed-form expression of the network performance under optimal precoding is available.

In this work we tackle this open problem starting from the fundamental case of a MIMO system with perfect CSI at the source and the relay, and negligible direct source-destination link. In this scenario, we narrow the gap between the optimal precoding design and the analytical tractability of the network performance, by proposing two suboptimal precoding schemes whose maximum achievable date rate is extremely close to the optimum. Then, by assuming that such precoding matrices are employed, we derive an analytically tractable expression of the achievable rate. The obtained rate expression is used to investigate the optimal trade-off existing between the maximum achievable rate and the power consumption of the network nodes.

II. System Model

We consider a MIMO, dual-hop, relay network where the source, the relay and the destination nodes operate in half-duplex mode. We assume that all nodes are equipped with m antennas, although the analysis can be easily extended to any number of antennas. Also, we assume that no direct link exists between source and destination.

We consider that data transmission takes place in two phases, according to the following scheme. In the first phase, the source precodes the input signal vector $\mathbf{x}$ and transmits it towards the relay. The input vector consists of m entries, which are assumed to be i.i.d., zero-mean, circular symmetric, complex Gaussian random variables and such that $\mathbb{E}[\mathbf{x}\mathbf{x}^{\mathsf{H}}] = \mathbf{I}$. The precoding matrix used by the source is denoted by $\mathbf{P}_1$. The signal received at the relay is then given by

$$\mathbf{r} = \sqrt{\alpha_1\rho_1}\mathbf{H}_1\mathbf{P}_1\mathbf{x} + \mathbf{n}_1$$

where $\mathbf{n}_1$ is the noise vector at the relay node whose entries are modeled as complex, i.i.d., zero-mean, Gaussian random variables with variance σ_1^2, α_1 is the path loss, ρ_1 is the source transmit power, and $\mathbf{H}_1 \in \mathbb{C}^{m\times m}$ is the channel matrix between source and relay. Also, the precoding matrix $\mathbf{P}_1$ should be designed in order to meet the normalization constraint

$$\mathop{\mathbb{E}}_{\mathbf{x}}\left[\mathsf{Tr}\{\mathbf{P}_1\mathbf{x}\mathbf{x}^{\mathsf{H}}\mathbf{P}_1^{\mathsf{H}}\}\right] = \mathsf{Tr}\{\mathbf{P}_1\mathbf{P}_1^{\mathsf{H}}\} = 1\,. \tag{1}$$

In the second phase, the relay node forwards to the destination a precoded version of the signal that it has received from the source. Let $\mathbf{H}_2 \in \mathbb{C}^{m\times m}$ be the channel matrix between relay node and destination. Then the signal received at the destination can be expressed as

$$\mathbf{y} \;=\; \sqrt{\alpha_2\rho_2}\mathbf{H}_2\mathbf{P}_2\mathbf{r} + \mathbf{n}_2 \tag{2}$$

where $\mathbf{P}_2$ is an $m\times m$ matrix representing relay precoding, $\mathbf{n}_2$ is the noise vector at the destination whose entries are modeled as complex i.i.d., zero-mean Gaussian random variables with variance σ_2^2, α_2 is the path loss, and ρ_2 is the relay transmit power. For any channel matrices $\mathbf{H}_1$ and $\mathbf{H}_2$, the precoding matrix $\mathbf{P}_2$ should be designed in order to meet the normalization constraint:

$$\begin{aligned}\mathop{\mathbb{E}}_{\mathbf{x},\mathbf{n}_1}\left[\mathsf{Tr}\{\mathbf{P}_2\mathbf{r}\mathbf{r}^{\mathsf{H}}\mathbf{P}_2^{\mathsf{H}}\}\right] &= \sigma_1^2\mathsf{Tr}\left\{\mathbf{P}_2\left(\mathbf{I}+\frac{\alpha_1\rho_1}{\sigma_1^2}\mathbf{T}_1\right)\mathbf{P}_2^{\mathsf{H}}\right\}\\ &= 1\end{aligned} \tag{3}$$

where $\mathbf{T}_1 = \mathbf{H}_1\mathbf{P}_1\mathbf{P}_1^{\mathsf{H}}\mathbf{H}_1^{\mathsf{H}}$ and $\mathbf{I}$ is the identity matrix.

In general, the precoders are functions of the channel matrices. For any choice of the channel matrices and of the precoders satisfying (1) and (3), the rate corresponding to the mutual information $\mathcal{I}(\mathbf{y},\mathbf{x})$ is given by:

$$R(\rho_1,\rho_2)=\frac{B}{2}\underset{\mathbf{H}_1,\mathbf{H}_2}{\mathbb{E}}\log_2\frac{|\mathbf{I}+\frac{\alpha_2\rho_2\sigma_1^2}{\sigma_2^2}\mathbf{H}_2\mathbf{P}_2\mathbf{W}_1\mathbf{P}_2^{\mathsf{H}}\mathbf{H}_2^{\mathsf{H}}|}{|\mathbf{I}+\frac{\alpha_2\rho_2\sigma_1^2}{\sigma_2^2}\mathbf{T}_2|}\,, \tag{4}$$

where B is the signal bandwidth, $\mathbf{T}_2=\mathbf{H}_2\mathbf{P}_2\mathbf{P}_2^{\mathsf{H}}\mathbf{H}_2^{\mathsf{H}}$, $\mathbf{W}_1=\mathbf{I}+\frac{\alpha_1\rho_1}{\sigma_1^2}\mathbf{T}_1$ and the factor $1/2$ accounts for the fact that the nodes work in half-duplex.

Let $\mathbf{H}_1=\mathbf{U}_1\mathbf{\Sigma}_1\mathbf{V}_1^{\mathsf{H}}$ and $\mathbf{H}_2=\mathbf{U}_2\mathbf{\Sigma}_2\mathbf{V}_2^{\mathsf{H}}$ be the singular value decompositions of the channel matrices and let $\mathbf{\Lambda}_1=\mathbf{\Sigma}_1\mathbf{\Sigma}_1^{\mathsf{H}}$ and $\mathbf{\Lambda}_2=\mathbf{\Sigma}_2^{\mathsf{H}}\mathbf{\Sigma}_2$. Then, in [3] it has been shown that the precoders maximizing the above achievable rate are given by

$$\mathbf{P}_1^{\rm opt}=\sigma_1\mathbf{V}_1\mathbf{D}_1$$

and

$$\mathbf{P}_2^{\rm opt}=\frac{\sigma_2}{\sigma_1}\mathbf{V}_2\mathbf{D}_2\left(\mathbf{I}+\frac{\alpha_1\rho_1}{\sigma_1^2}\widetilde{\mathbf{\Lambda}}_1\right)^{-1/2}\mathbf{U}_1^{\mathsf{H}} \tag{5}$$

where $\mathbf{D}_1$ is a diagonal matrix such that

$$\mathbf{D}_1^2=\left[\phi_1\mathbf{I}-\alpha_1\alpha_2\rho_1\rho_2\frac{\sigma_1^2}{\sigma_2^2}\mathbf{\Lambda}^{-1}\right]^+\,,$$

and $\mathbf{D}_2$ is a diagonal matrix such that

$$\mathbf{D}_2^2 = \left[\left(\frac{\alpha_1^2\rho_1^2\widetilde{\mathbf{\Lambda}}_1^2\mathbf{\Lambda}_2^{-2}}{4\sigma_1^4\alpha_2^2\rho_2^2}+\frac{\alpha_1\rho_1\widetilde{\mathbf{\Lambda}}_1\mathbf{\Lambda}_2^{-1}}{\phi_2\sigma_1^2\alpha_2\rho_2}\right)^{\frac{1}{2}} -\left(\mathbf{I}+\frac{\alpha_1\rho_1\widetilde{\mathbf{\Lambda}}_1}{2\sigma_1^2}\right)\frac{\mathbf{\Lambda}_2^{-1}}{\alpha_2\rho_2}\right]^+$$

with $[x]^+=\max(0,x)$. Also, $\widetilde{\mathbf{\Lambda}}_1$ is the matrix of eigenvalues of $\mathbf{H}_1\mathbf{P}_1^{\rm opt}{\mathbf{P}_1^{\rm opt}}^{\mathsf{H}}\mathbf{H}_1^{\mathsf{H}}$, and $\mathbf{\Lambda}$ is the matrix of eigenvalues of $(\mathbf{I}+\frac{\alpha_2\rho_2\sigma_1^2}{\sigma_2^2}\mathbf{H}_2\mathbf{P}_2^{\rm opt}{\mathbf{P}_2^{\rm opt}}^{\mathsf{H}}\mathbf{H}_2^{\mathsf{H}})^{-1}\mathbf{H}_2\mathbf{P}_2^{\rm opt}\mathbf{H}_1\mathbf{H}_1^{\mathsf{H}}{\mathbf{P}_2^{\rm opt}}^{\mathsf{H}}\mathbf{H}_2^{\mathsf{H}}$. We also assume that the elements of $\mathbf{\Lambda}$, $\mathbf{\Lambda}_1$ and $\mathbf{\Lambda}_2$ are ordered in decreasing order. Finally, the parameters ϕ_1 and ϕ_2 are such that $\sigma_1^2\mathsf{Tr}\{\mathbf{D}_1\mathbf{D}_1^{\mathsf{H}}\}=1$ and $\sigma_2^2\mathsf{Tr}\{\mathbf{D}_2\mathbf{D}_2^{\mathsf{H}}\}=1$, respectively. Given the system parameters and the channel matrices $\mathbf{H}_1$ and $\mathbf{H}_2$, the optimal precoders $\mathbf{P}_1^{\rm opt}$ and $\mathbf{P}_2^{\rm opt}$ can be found through an iterative numeric algorithm, as shown in [3].

III. Optimal Precoders Approximation

An analytical expression for the rate (4) yielded by the optimal precoders $\mathbf{P}_1=\mathbf{P}_1^{\rm opt}$ and $\mathbf{P}_2=\mathbf{P}_2^{\rm opt}$ is difficult to obtain due to the non-polynomial functions appearing in their expression. Hence, we propose two suboptimal expressions for the precoders at the source and the relay node providing performance close to the optimum and allowing a simple analytic expression of the achieved rate. Below, we first assume that precoding is only applied by the relay node, i.e., the source precoding matrix is constant and scalar (Section III-A). Then we allow both source and relay node to precode the signals before transmission (Section III-B).

A. Precoding at relay node only

Here we assume that the source precoder $\mathbf{P}_1$ is a scalar and constant matrix. Then, under the power constraint in (1), $\mathbf{P}_1$ can be written as:

$$\mathbf{P}_1=\sqrt{\frac{1}{m}}\mathbf{I}\,.$$

As for $\mathbf{P}_2$, we aim to provide an approximated expression of the optimal precoder in (5) that depends on the number of modes, $1\le k\le m$ to which the relay node allocates power. The number of modes, k, will then be selected so that maximum rate is achieved. To do so we define

$$\mathbf{P}_2^{(k)}=\sqrt{\frac{1}{k\sigma_1^2}}\mathbf{V}_2\mathbf{E}_k\left(\mathbf{I}+\frac{\alpha_1\rho_1}{\sigma_1^2}\widetilde{\mathbf{\Lambda}}_1\right)^{-1/2}\mathbf{U}_1^{\mathsf{H}}$$

where, similarly to (5), $\widetilde{\mathbf{\Lambda}}_1$ is the matrix of eigenvalues of $\mathbf{T}_1=\mathbf{H}_1\mathbf{P}_1\mathbf{P}_1^{\mathsf{H}}\mathbf{H}_1^{\mathsf{H}}$. In the above expression, the $m\times m$ matrix $\mathbf{E}_k$ is given by

$$\mathbf{E}_k=\left[\begin{array}{cc}\mathbf{I}_k & \mathbf{0}\\ \mathbf{0} & \mathbf{0}\end{array}\right] \tag{6}$$

where $\mathbf{I}_k$ is the $k\times k$ identity matrix. It is easy to check that such precoder satisfies (3).

The rate provided by the proposed source and relay precoders can be obtained by substituting the expressions for $\mathbf{P}_1$ and $\mathbf{P}_2^{(k)}$ in (4). We first observe that $\mathbf{T}_1=\mathbf{H}_1\mathbf{H}_1^{\mathsf{H}}/m$, and thus $\widetilde{\mathbf{\Lambda}}_1=\mathbf{\Lambda}_1/m$, and $\mathbf{W}_1=\mathbf{I}+\frac{\alpha_1\rho_1}{m\sigma_1^2}\mathbf{H}_1\mathbf{H}_1^{\mathsf{H}}$. It follows that $\mathbf{P}_2^{(k)}=\sqrt{\frac{1}{k\sigma_1^2}}\mathbf{V}_2\mathbf{E}_k\left(\mathbf{I}+\frac{\alpha_1\rho_1}{m\sigma_1^2}\mathbf{\Lambda}_1\right)^{-1/2}\mathbf{U}_1^{\mathsf{H}}$ and

$$\mathbf{P}_2^{(k)}\mathbf{W}_1{\mathbf{P}_2^{(k)}}^{\mathsf{H}}=\frac{1}{k\sigma_1^2}\mathbf{V}_2\mathbf{E}_k\mathbf{V}_2^{\mathsf{H}}\,.$$

Next, for any $1\le k\le m$, the numerator of (4) is given by

$$\begin{aligned}\left|\mathbf{I}+\frac{\alpha_2\rho_2\sigma_1^2}{\sigma_2^2}\mathbf{H}_2\mathbf{P}_2^{(k)}\mathbf{W}_1{\mathbf{P}_2^{(k)}}^{\mathsf{H}}\mathbf{H}_2^{\mathsf{H}}\right| &=\left|\mathbf{I}+\frac{\alpha_2\rho_2}{k\sigma_2^2}\mathbf{H}_2\mathbf{V}_2\mathbf{E}_k\mathbf{V}_2^{\mathsf{H}}\mathbf{H}_2^{\mathsf{H}}\right|\\ &=\left|\mathbf{I}+\frac{\alpha_2\rho_2}{k\sigma_2^2}\mathbf{\Lambda}_2\mathbf{E}_k\right|\\ &=\prod_{i=1}^k\left(1+\frac{\alpha_2\rho_2}{k\sigma_2^2}\lambda_{2,i}\right)\end{aligned} \tag{7}$$

where $\lambda_{2,i}$ is the i-th diagonal element of $\mathbf{\Lambda}_2$. Similarly, the denominator of (4) is given by

$$\begin{aligned}\left|\mathbf{I}+\frac{\alpha_2\rho_2\sigma_1^2}{\sigma_2^2}\mathbf{T}_2\right| &= \left|\mathbf{I}+\frac{\alpha_2\rho_2\sigma_1^2}{\sigma_2^2}\mathbf{H}_2\mathbf{P}_2^{(k)}{\mathbf{P}_2^{(k)}}^{\mathsf{H}}\mathbf{H}_2^{\mathsf{H}}\right|\\ &= \left|\mathbf{I}+\frac{\rho_2\alpha_2}{k\sigma_2^2}\mathbf{\Lambda}_2\mathbf{E}_k\left(\mathbf{I}+\frac{\rho_1\alpha_1}{m\sigma_1^2}\mathbf{\Lambda}_1\right)^{-1}\right|\\ &= \prod_{i=1}^k\frac{1+\frac{\rho_1\alpha_1}{m\sigma_1^2}\lambda_{1,i}+\frac{\rho_2\alpha_2}{k\sigma_2^2}\lambda_{2,i}}{1+\frac{\rho_1\alpha_1}{m\sigma_1^2}\lambda_{1,i}}\end{aligned} \tag{8}$$

where $\lambda_{1,i}$ is the i-th diagonal element of $\mathbf{\Lambda}_1$. Using the above expressions in (4), we derive the network rate as a function of

ρ_1, ρ_2, and k as

$$\begin{aligned} R^{(k)}(\rho_1,\rho_2) &= \frac{B}{2} \mathop{\mathbb{E}}_{\mathbf{H}_1,\mathbf{H}_2} \log_2 \prod_{i=1}^{k} \frac{(1+\frac{\alpha_1\rho_1}{m\sigma_1^2}\lambda_{1,i})(1+\frac{\alpha_2\rho_2}{k\sigma_2^2}\lambda_{2,i})}{1+\frac{\alpha_1\rho_1}{m\sigma_1^2}\lambda_{1,i}+\frac{\alpha_2\rho_2}{k\sigma_2^2}\lambda_{2,i}} \\ &= \frac{B}{2} \mathop{\mathbb{E}}_{\mathbf{\Lambda}_1,\mathbf{\Lambda}_2} \sum_{i=1}^{k} \log_2 \frac{(1+\frac{\alpha_1\rho_1}{m\sigma_1^2}\lambda_{1,i})(1+\frac{\alpha_2\rho_2}{k\sigma_2^2}\lambda_{2,i})}{1+\frac{\alpha_1\rho_1}{m\sigma_1^2}\lambda_{1,i}+\frac{\alpha_2\rho_2}{k\sigma_2^2}\lambda_{2,i}} \\ &= \frac{B}{2} \sum_{i=1}^{k} \mathop{\mathbb{E}}_{\lambda_{1,i},\lambda_{2,i}} \log_2 \frac{(1+\frac{\alpha_1\rho_1}{m\sigma_1^2}\lambda_{1,i})(1+\frac{\alpha_2\rho_2}{k\sigma_2^2}\lambda_{2,i})}{1+\frac{\alpha_1\rho_1}{m\sigma_1^2}\lambda_{1,i}+\frac{\alpha_2\rho_2}{k\sigma_2^2}\lambda_{2,i}} \end{aligned} \tag{9}$$

Note that the averages that appear in (9) are with respect to the marginal distributions of the i-th ordered eigenvalues of $\mathbf{H}_1$ and $\mathbf{H}_2$. Then, for any given value of power ρ_1 and ρ_2, the maximum rate achieved by these precoders can be obtained by maximizing with respect to k.

B. Precoding at source and relay node

In the case both source and relay precode their transmitted signals, we propose the following expressions for the precoders. As shown later, such expressions provide nearly optimal performance and allow an easy analytic evaluation of the achievable rate. Specifically, for any integer $1 \le k \le m$, we approximate the optimal precoding matrix at the source with

$$\mathbf{P}_1^{(k)} = \sqrt{\frac{1}{k}}\mathbf{V}_1\mathbf{E}_k$$

where $\mathbf{E}_k$ is provided in (6). The approximated relay precoder is given by

$$\mathbf{P}_2^{(k)} = \sqrt{\frac{1}{k\sigma_1^2}}\mathbf{V}_2\mathbf{E}_k\left(\mathbf{I}+\frac{\alpha_1\rho_1}{k\sigma_1^2}\mathbf{\Lambda}_1\right)^{-1/2}\mathbf{U}_1^{\mathsf{H}}\,.$$

The expression for the achievable rate can be obtained by using the above expressions for $\mathbf{P}_1^{(k)}$ and $\mathbf{P}_2^{(k)}$ in (4). In particular, we have:

$$R^{(k)}(\rho_1,\rho_2) = \frac{B}{2} \sum_{i=1}^{k} \mathop{\mathbb{E}}_{\lambda_{1,i},\lambda_{2,i}} \log_2 \frac{(1+\frac{\alpha_1\rho_1}{k\sigma_1^2}\lambda_{1,i})(1+\frac{\alpha\rho_2}{k\sigma_2^2}\lambda_{2,i})}{1+\frac{\alpha_1\rho_1}{k\sigma_1^2}\lambda_{1,i}+\frac{\alpha\rho_2}{k\sigma_2^2}\lambda_{2,i}} \tag{10}$$

where the average is with respect to the marginal distributions of the i-th ordered eigenvalues of $\mathbf{H}_1$ and $\mathbf{H}_2$. As before, the rate expression in (10) should be maximized with respect to k.

IV. Performance evaluation

The expressions for the rate $R^{(k)}(\rho_1,\rho_2)$ in (9) and (10) can be easily computed provided that the marginal distributions of the i-th ordered eigenvalues of $\mathbf{H}_1$ and $\mathbf{H}_2$ are known. An analytic expression for such distributions has been recently derived in [7, Theorem 10], for the cases where the elements of the channel matrix are Gaussian i.i.d. with arbitrary mean, or they are correlated Gaussian with zero mean. These cases encompass the MIMO Rayleigh and Rician channel models: we denote the eigenvalue distributions for the Rayleigh channel by $f_{1,i}(x)$ and for the Rician channel by $f_{2,i}(x)$.

We can then evaluate the rate by computing integrals of the form

$$\mathcal{I}_1 = \int_0^{+\infty} \log_2(1+\gamma x) f_{u,i}(x)\,\mathrm{d}x$$
$$\mathcal{I}_2 = \iint_0^{+\infty} \log_2(1+\gamma_1 x_1+\gamma_2 x_2) f_{u,i}(x_1) f_{v,i}(x_2)\,\mathrm{d}x_1\,\mathrm{d}x_2$$

where $u,v \in \{1,2\}$. Since such integrals cannot be solved in closed form, their computation is carried out numerically.

Next we exploit the expressions we obtained to optimize the performance of the relay network, while accounting for both achievable rate and energy consumption. Specifically, we first maximize the achievable rate subject to a constraint on the transmit power at each hop. Then we minimize the total transmit power while ensuring that the maximum achievable rate is greater than or equal to a given value.

A. Maximizing the achievable rate

Let $\bar{\rho}_1$ and $\bar{\rho}_2$ denote the maximum transmit power at the source and at the relay, respectively. Using (9) or (10), we can solve the following problem

$$\begin{aligned} &\max_k \quad R^{(k)}(\rho_1,\rho_2) \\ &\text{s.t.} \quad \rho_1 \le \bar{\rho}_1 \quad \text{and} \quad \rho_2 \le \bar{\rho}_2\,. \end{aligned} \tag{11}$$

In Figure 1 we show the maximum achievable rate as a function of the maximum transmit power at the source and at the relay ($\bar{\rho}_1 = \bar{\rho}_2 = \bar{\rho}$). The plot compares the performance obtained by the optimal precoders, our approximations, and when no precoding is used. The channels of the two hops are assumed to be Rayleigh distributed. Curves have been obtained for $B = 20$ MHz, $m = 4$, $\sigma_1^2 = \sigma_2^2 = -101$ dBm, and $\alpha_1 = \alpha_2 = -90$ dB. In the plot, we highlighted the values of k for which the achievable rate is maximized. Observe that the approximated precoders in Section III-B perform very close to the optimal for any value of $\bar{\rho}$. Furthermore, the approximated precoder at the relay only provides good performance for high transmit powers (hence SNR), while it approaches the performance obtained by no precoding for low SNR. Indeed, under the latter condition, power should be allocated to the mode corresponding to the largest eigenvalue ($k = 1$). Instead, the source equally distributes power over all antennas. As a consequence, precoding at the relay becomes ineffective. This observation is confirmed by the fact that, as $\bar{\rho}$ increases, the number of modes to be used (k) increases faster in the case of precoding at the relay only than for precoding at both source and relay. Similar results hold for the case of Ricean channels, as shown in Figure 2.

B. Minimizing the total power budget

We now aim to minimize the sum of the transmit powers over the two hops, subject to maximum achievable rate constraints. The problem can be written as:

$$\begin{aligned} &\min_{\rho_1,\rho_2} \rho_1 + \rho_2 \\ &\text{s.t.} \max_k \quad R^{(k)}(\rho_1,\rho_2) \ge \bar{R} \end{aligned} \tag{12}$$

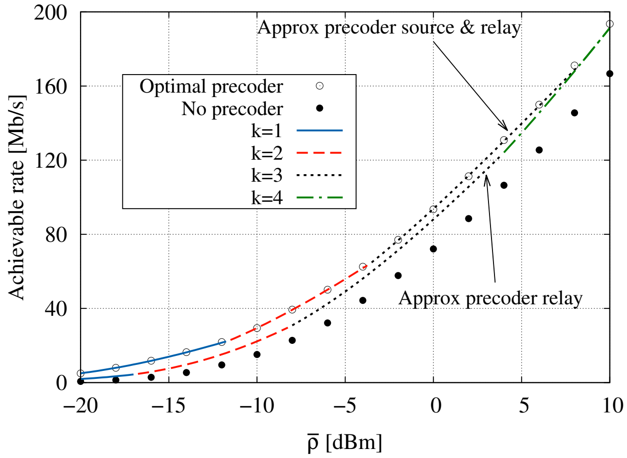

Fig. 1. Maximum achievable rate vs. transmit power at the source and relay ($\bar{\rho}_1 = \bar{\rho}_2 = \bar{\rho}$) in the case of Rayleigh channels. The values of k for which the rate is maximized are reported in the legend.

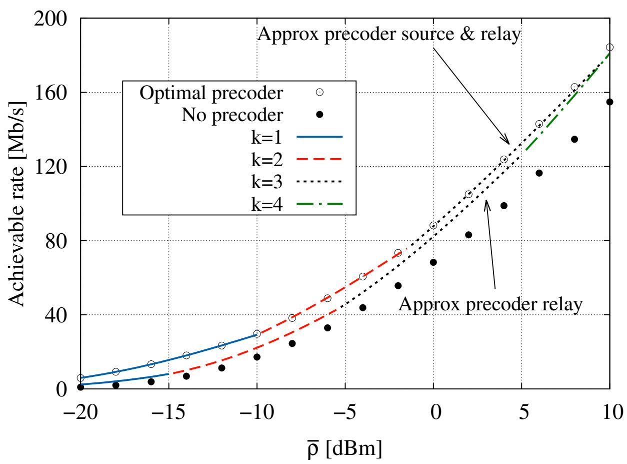

Fig. 2. Maximum achievable rate vs. transmit power at the source and relay ($\bar{\rho}_1 = \bar{\rho}_2 = \bar{\rho}$) in the case of Ricean channels. The values of k for which the rate is maximized are reported in the legend.

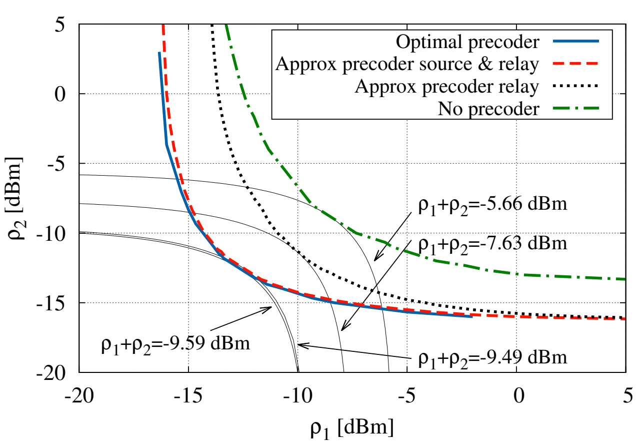

Fig. 3. Power curves each with constant value of $\rho_1 + \rho_2$, and rate curves for $\bar{R} = 20$ Mb/s. The case of Rayleigh channels is considered for different choices of precoding and for no precoding.

where $\bar{R}$ is the target rate.

In Figure 3 we set $\bar{R} = 20$ Mb/s and we investigate the performance as $\rho_1 + \rho_2$ varies. Again, we compare the optimal precoders, our approximations and the no-precoding case. Thick lines show the function $\max_k R^{(k)}(\rho_1, \rho_2) = \bar{R}$ for the different precoding schemes. Thin lines correspond to constant values of the sum of the transmit powers. The contact point between thick and thin lines therefore represents the minimum value of $\rho_1 + \rho_2$ that provides the target rate $\bar{R}$. We can observe that the approximate precoders at both source and relay yield a value of total transmit power that is just 0.1 dBm away from the optimum.

V. Conclusions

We studied a MIMO, dual-hop, relay network where precoding can be performed at both the source and the relay nodes, or at the relay node only. We also assumed that perfect CSI is available at the nodes in charging of signal precoding and that no direct source-destination link exists. Unfortunately, even in such a simple scenario, only an iterative expression for the achievable rate is available under optimal precoding. Thus we first proposed suboptimal precoding matrices whose performance matches that of optimum precoding very tightly. We then derived an analytical expression of the rate and studied the trade-off that exists between achievable rate and nodes power consumption.

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