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Chemical and mechanical instabilities in high energy heavy-ion collisions

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Abstract. We investigate the possible thermodynamic instability in a warm and dense nuclear medium where a phase transition from nucleonic matter to resonance-dominated Δ-matter can take place. Such a phase transition is characterized by both mechanical instability (fluctuations on the baryon density) and by chemical-diffusive instability (fluctuations on the isospin concentration) in asymmetric nuclear matter. Similarly to the liquid-gas phase transition, the nucleonic and the Δ-matter phase have a different isospin density in the mixed phase. In the liquid-gas phase transition, the process of producing a larger neutron excess in the gas phase is referred to as isospin fractionation. A similar effects can occur in the nucleon-Δ matter phase transition due essentially to a Δ^- excess in the Δ-matter phase in asymmetric nuclear matter. In this context, we study the hadronic equation of state by means of an effective quantum relativistic mean field model with the inclusion of the full octet of baryons, the Δ-isobar degrees of freedom, and the lightest pseudoscalar and vector mesons. Finally, we will investigate the presence of thermodynamic instabilities in a hot and dense nuclear medium where phases with different values of antibaryon-baryon ratios and strangeness content may coexist. Such a physical regime could be in principle investigated in the future high-energy compressed nuclear matter experiments where will make it possible to create compressed baryonic matter with a high net baryon density.

1. Introduction

One of the very interesting aspects in nuclear astrophysics and in the heavy-ion collisions experiments is a detailed study of the thermodynamical properties of strongly interacting nuclear matter away from the nuclear ground state.

The new accumulating data from x-ray satellites provide important information on the structure and formation of compact stellar objects. Concerning the structure, these data are at first sight difficult to interpret in a unique and selfconsistent theoretical scenario, since some of the observations indicate rather small radii and other observations indicate large values for the mass of the star.

On the other hand, the information coming from experiments with heavy ions in intermediate- and high-energy collisions is that the EOS depends on the energy beam but also sensibly on the electric charge fraction Z/A of the colliding nuclei, especially at not too high temperature [1, 2, 3, 4]. Moreover, the study of nuclear matter with arbitrary electric charge fraction results to be important in radioactive beam experiments and in the physics of compact stars.

In this article, we to study a hadronic equation of state (EOS) at finite temperature and density by means of a relativistic mean-field model with the inclusion $\Delta(1232)$-isobars [5, 6, 7] and by requiring the Gibbs conditions on the global conservation of baryon number and net electric charge. Transport model calculations and experimental results indicate that an excited state of baryonic matter is dominated by the Δ resonance at the energies from the BNL Alternating Gradien Synchrotron (AGS) to RHIC [8]. Moreover, in the framework of the nonlinear Walecka model, it has been predicted that a phase transition from nucleonic matter to Δ-excited nuclear matter can take place and the occurrence of this transition sensibly depends on the Δ-meson coupling constants [9, 10].

The main goal of this paper is to show that, for asymmetric warm and dense nuclear medium, the possible Δ-matter phase transition is characterized by mechanical and chemical-diffusive instabilities. Similarly to the liquid-gas phase transition [11], chemical instabilities play a crucial role in the characterization of the phase transition and can imply a very different electric charge fraction Z/A in the coexisting phases during the phase transition. In this context we show that a hadron phase transition of the baryon-antibaryon plasma can take place in the physical region of finite net baryon density ($\rho_0 \leq \rho_B < 2\rho_0$, where ρ_0 is the nuclear saturation density) and temperature ($T \simeq 120 \div 150$ MeV).

2. Hadronic equation of state

Let us briefly to introduce the adopted nuclear equation of state (EOS) to describe the interaction in the many-particles system involved in high energy heavy ion collisions.

The relativistic mean-field model is widely successful used for describing the properties of finite nuclei as well as hot and dense nuclear matter [4, 5, 12]. The Lagrangian for the self-interacting octet of baryons (p, n, Λ, Σ^+, Σ^0, Σ^-, Ξ^0, Ξ^-) can be written as [5]

$$\begin{aligned}\mathcal{L}_{\rm octet} &= \sum_k \overline{\psi}_k\,[i\,\gamma_\mu\,\partial^\mu - (M_k - g_{\sigma k}\,\sigma) - g_{\omega k}\,\gamma_\mu\,\omega^\mu - g_{\rho k}\,\gamma_\mu\,\vec{t}\cdot\vec{\rho}^{\,\mu}]\,\psi_k \\ &+\frac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2) - \frac{1}{3}a\,(g_{\sigma N}\,\sigma)^3 - \frac{1}{4}\,b\,(g_{\sigma N}\,\sigma^4) \\ &+\frac{1}{2}\,m_\omega^2\,\omega_\mu\omega^\mu + \frac{1}{4}\,c\,(g_{\omega N}^2\,\omega_\mu\omega^\mu)^2 + \frac{1}{2}\,m_\rho^2\,\vec{\rho}_\mu\cdot\vec{\rho}^{\,\mu} \\ &-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}\vec{G}_{\mu\nu}\vec{G}^{\mu\nu}\,, \end{aligned} \tag{1}$$

where the sum runs over the full octet of baryons, M_k is the vacuum baryon mass of index k, the quantity $\vec{t}$ denotes the isospin operator that acts on the baryon.

In regime of finite values of temperature and density, a state of high density resonance matter may be formed and the $\Delta(1232)$-isobar degrees of freedom are expected to play a central role [6, 8]. In particular, the formation of resonances matter contributes essentially to baryon stopping, hadronic flow effects and enhanced strangeness. The Lagrangian density concerning the Δ-isobars (Δ^{++}, Δ^+, Δ^0, Δ^-) can be expressed as [13]

$$\mathcal{L}_\Delta = \overline{\psi}_{\Delta\,\nu}\,[i\gamma_\mu\partial^\mu - (M_\Delta - g_{\sigma\Delta}\sigma) - g_{\omega\Delta}\gamma_\mu\omega^\mu]\psi_\Delta^\nu\,, \tag{2}$$

where ψ_Δ^ν is the Rarita-Schwinger spinor for the Δ-isobars (Δ^{++}, Δ^+, Δ^0, Δ^-). Due to the uncertainty on the meson-Δ coupling constants, we limit ourselves to consider only the coupling with the σ and ω meson fields ($x_{\sigma\Delta} = g_{\sigma\Delta}/g_{\sigma N}$, $x_{\omega\Delta} = g_{\omega\Delta}/g_{\omega N}$), more of which are explored in the literature. The adopted coupling constants are the same of Ref. [6].

The ρ_i^B and ρ_i^S are the baryon density and the baryon scalar density, respectively. They are given by

$$\rho_i^B = \gamma_i \int \frac{{\rm d}^3k}{(2\pi)^3}[n_i(k) - \overline{n}_i(k)]\,, \tag{3}$$

$$\rho_i^S = \gamma_i \int \frac{{\rm d}^3k}{(2\pi)^3}\,\frac{M_i^*}{E_i^*}\,[n_i(k) + \overline{n}_i(k)]\,, \tag{4}$$

where $n_i(k)$ and $\overline{n}_i(k)$ are the fermion particle, antiparticle distributions function, given by

$$n_i(k) = \frac{1}{\exp(E_i^*(k) - \mu_i^*)/T + 1}\,, \tag{5}$$

$$\overline{n}_i(k) = \frac{1}{\exp(E_i^*(k) + \mu_i^*)/T + 1}\,. \tag{6}$$

The effective chemical potentials μ_i^* are given in terms of the chemical potentials μ_i by means of the following relation

$$\mu_i^* = \mu_i - g_{\omega i}\,\omega - g_{\rho i}\,t_{3i}\,\rho\,, \tag{7}$$

where t_{3i} is the third component of the isospin of i-th baryon. The baryon effective energy is defined as $E_i^*(k) = \sqrt{k^2 + {M_i}^{*2}}$, where the effective mass of the ith baryon is defined as

$$M_i^* = M_i - g_{\sigma i}\sigma\,. \tag{8}$$

The thermodynamical quantities can be obtained from the grand potential Ω_B in the standard way. More explicitly, the baryon pressure $P_B = -\Omega_B/V$ and the energy density ϵ_B can be written as

$$\begin{aligned}
P_B &= \frac{1}{3}\sum_i \gamma_i \int \frac{\mathrm{d}^3k}{(2\pi)^3}\,\frac{k^2}{E_i^*(k)}\,[f_i(k) + \overline{f}_i(k)] - \frac{1}{2}\,m_\sigma^2\,\sigma^2 \\
&- \frac{1}{3}a\,(g_{\sigma N}\,\sigma)^3 - \frac{1}{4}\,b\,(g_{\sigma N}\,\sigma^4) + \frac{1}{2}\,m_\omega^2\,\omega^2 \\
&+ \frac{1}{4}\,c\,(g_{\omega N}\,\omega)^4 + \frac{1}{2}\,m_\rho^2\,\rho^2\,,
\end{aligned} \tag{9}$$

$$\begin{aligned}
\epsilon_B &= \sum_i \gamma_i \int \frac{\mathrm{d}^3k}{(2\pi)^3}\,E_i^*(k)\,[f_i(k) + \overline{f}_i(k)] + \frac{1}{2}\,m_\sigma^2\,\sigma^2 \\
&+ \frac{1}{3}a\,(g_{\sigma N}\,\sigma)^3 + \frac{1}{4}\,b\,(g_{\sigma N}\,\sigma^4) + \frac{1}{2}\,m_\omega^2\,\omega^2 \\
&+ \frac{3}{4}\,c\,(g_{\omega N}\,\omega)^4 + \frac{1}{2}\,m_\rho^2\,\rho^2\,.
\end{aligned} \tag{10}$$

Finally, from a phenomenological point of view, we can take into account the meson (boson) particle degrees of freedom by adding their one-body contribution to the thermodynamical potential, that is, the contribution of a quasi-particle Bose gas with an effective chemical potential μ_j^* and an effective mass m_j^* for the j-meson, which contain the self-consistent interaction of the meson fields [4]. Following this working hypothesis, we can evaluate the pressure P_M, the energy density ϵ_M and the particle density ρ_j^M of mesons as

$$P_M = \frac{1}{3}\sum_j \gamma_j \int \frac{\mathrm{d}^3k}{(2\pi)^3}\,\frac{k^2}{E_j(k)}\,g_j(k)\,, \tag{11}$$

$$\epsilon_M = \sum_j \gamma_j \int \frac{\mathrm{d}^3k}{(2\pi)^3}\,E_j(k)\,g_j(k)\,, \tag{12}$$

$$\rho_j^M = \gamma_j \int \frac{\mathrm{d}^3k}{(2\pi)^3}\,g_j(k)\,, \tag{13}$$

where $\gamma_j = 2J_j + 1$ is the degeneracy spin factor of the j-th meson ($\gamma = 1$ for pseudoscalar mesons and $\gamma = 3$ for vector mesons), the sum runs over the lightest pseudoscalar mesons (π,

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K, η, η') and the lightest vector mesons (ρ, ω, K^*, ϕ), considering the contribution of particle and antiparticle separately. In Eqs.(11)-(13) the function $g_j(k)$ is the boson particle distribution given by

$$g_j(k) = \frac{1}{\exp\{(E_j^*(k) - \mu_j^*)/T\} - 1}\,, \tag{14}$$

where $E_j^*(k) = \sqrt{k^2 + {m_j}^{*2}}$. The corresponding antiparticle distribution will be obtained with the substitution $\mu_j^* \to -\mu_j^*$. Moreover, the boson integrals are subjected to the constraint $|\mu_j^*| \le m_j^*$, otherwise Bose condensation becomes possible. We have verified that such a condition is never realized in the considered range of temperature and density.

Finally, the total pressure and energy density will be

$$P = P_B + P_M\,, \tag{15}$$
$$\epsilon = \epsilon_B + \epsilon_M\,. \tag{16}$$

3. Stability conditions and phase transition

We are dealing with the study of a multi-component system at finite temperature and density with two conserved charges: baryon (B) number and zero net strangeness (S) number ($r_S = \rho_S/\rho_B = 0$). For what concern the electric charge (Q), we work in symmetric nuclear matter with a fixed value of $Z/A = 0.5$ and we do not consider fluctuation in the electric charge fraction, due to the high temperature regime. Therefore, the electric charge results to be separately conserved in each phase during the phase transition.

The chemical potential of particle of index i can be written as

$$\mu_i = b_i\,\mu_B + s_i\,\mu_S\,, \tag{17}$$

where b_i and s_i are, respectively, the baryon and the strangeness quantum numbers of i-th hadronic species.

For such a system, the Helmholtz free energy density F can be written as

$$F(T, \rho_B, \rho_S) = -P(T, \mu_B, \mu_S) + \mu_B\rho_B + \mu_S\rho_S\,, \tag{18}$$

with

$$\mu_B = \left(\frac{\partial F}{\partial \rho_B}\right)_{T,\rho_S}\,, \quad \mu_S = \left(\frac{\partial F}{\partial \rho_S}\right)_{T,\rho_B}\,. \tag{19}$$

In a system with N different particles, the particle chemical potentials are expressed as the linear combination of the two independent chemical potentials μ_B and μ_S and, as a consequence, $\sum_{i=1}^N \mu_i\rho_i = \mu_B\rho_B + \mu_S\rho_S$.

Assuming the presence of two phases (denoted as I and II, respectively), the system is stable against the separation in two phases if the free energy of a single phase is lower than the free energy in all two phases configuration. The phase coexistence is given by the Gibbs conditions

$$\mu_B^I = \mu_B^{II}\,, \qquad \mu_S^I = \mu_S^{II}\,, \tag{20}$$
$$P^I(T, \mu_B, \mu_S) = P^{II}(T, \mu_B, \mu_S)\,. \tag{21}$$

At a given baryon density ρ_B and at a given zero net strangeness density $r_S = \rho_S/\rho_B = 0$, the chemical potentials μ_B are μ_S are univocally determined by the following equations

$$\rho_B = (1 - \chi)\,\rho_B^I(T, \mu_B, \mu_S) + \chi\,\rho_B^{II}(T, \mu_B, \mu_S)\,, \tag{22}$$
$$\rho_S = (1 - \chi)\,\rho_S^I(T, \mu_B, \mu_S) + \chi\,\rho_S^{II}(T, \mu_B, \mu_S)\,, \tag{23}$$

where $\rho_B^{I(II)}$ and $\rho_S^{I(II)}$ are, respectively, the baryon and strangeness densities in the low density (I) and in the higher density (II) phase and χ is the volume fraction of the phase II in the mixed phase ($0 \leq \chi \leq 1$).

An important feature of this conditions is that, unlike the case of a single conserved charge, baryon and strangeness densities can be different in the two phases, although the total ρ_B and ρ_S are fixed.

For such a system in thermal equilibrium, the possible phase transition can be characterized by mechanical (fluctuations in the baryon density) and chemical instabilities (fluctuations in the strangeness number). As usual the condition of the mechanical stability implies

$$\rho_B \left(\frac{\partial P}{\partial \rho_B}\right)_{T,\,\rho_S} > 0\,. \tag{24}$$

By introducing the notation $\mu_{i,j} = (\partial\mu_i/\partial\rho_j)_{T,P}$ (with $i,j = B,S$), the chemical stability can be expressed with the following conditions [6]

$$\mu_{B,B} > 0\,, \quad \mu_{S,S} > 0\,. \tag{25}$$

In addition to the above conditions, for a process at constant P and T, it is always satisfied that

$$\rho_B\,\mu_{B,B} + \rho_S\,\mu_{S,B} = 0\,, \tag{26}$$

$$\rho_B\,\mu_{B,S} + \rho_S\,\mu_{S,S} = 0\,. \tag{27}$$

Whenever the above stability conditions are not respected, the system becomes unstable and the phase transition take place. The coexistence line of a system with one conserved charge becomes in this case a two dimensional surface in (T, P, r_S) space, enclosing the region where mechanical and diffusive instabilities occur.

4. Antibaryon-baryon and strangeness phase transition

By increasing the temperature and the baryon density during the high energy heavy ion collisions, a multi-particle system may take place and the formation antiparticles become much more relevant [7, 14, 15].

In analogy with the liquid-gas case, we are going to investigate the existence of a possible phase transition in the nuclear medium by studying the presence of instabilities (mechanical and/or chemical) in the system.

As already observed, during a phase transition with two conserved charges, the strangeness fraction $r_S = \rho_S/\rho_B$ is not locally conserved in the single phase but only globally conserved. Therefore, during the compression of the system, the appearance of particles with strangeness could, in principle, shift the diffusive instability region to negative (or positive) values of r_S, even if the system is prepared with a zero net strangeness. Such a feature has no counterpart in the liquid-gas phase transition and it turns out to be very relevant in order to properly determine the instability region through the binodal phase diagram.

Taking into account that Eq. (27) becomes in this case

$$\left(\frac{\partial \mu_B}{\partial r_S}\right)_{T,P} + r_S \left(\frac{\partial \mu_S}{\partial r_S}\right)_{T,P} = 0\,, \tag{28}$$

the chemical stability condition is satisfied if

$$\left(\frac{\partial \mu_S}{\partial r_S}\right)_{T,P} > 0 \quad \text{or} \quad \begin{cases} \left(\dfrac{\partial \mu_B}{\partial r_S}\right)_{T,P} < 0\,, & \text{if } r_S > 0\,, \\[2ex] \left(\dfrac{\partial \mu_B}{\partial r_S}\right)_{T,P} > 0\,, & \text{if } r_S < 0\,. \end{cases} \tag{29}$$

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Let us report some numerical results by considering the hadronic EOS discussed here. First of all, we observe that the condition of mechanical and chemical stability is not satisfied in a large range of temperature and density, therefore, the system goes to a phase transition. In this context, let us remark that the Δ-isobar degrees of freedom play a crucial role in the presence of the instability conditions of the EOS and in the nature of the phase transition from nucleonic matter to a resonance-dominated Δ-matter.

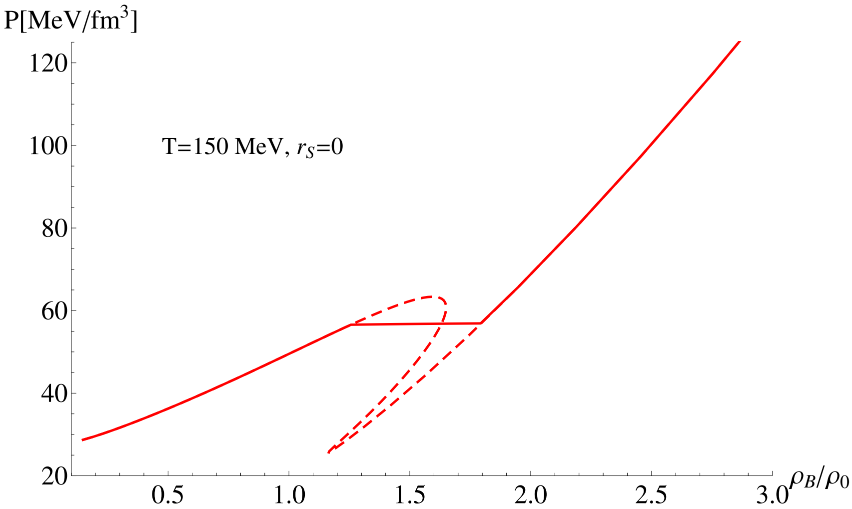

Figure 1. Pressure as a function of baryon density at fixed temperature. The continuous (dashed) lines correspond to the solution obtained with (without) the Gibbs construction.

To better clarify this matter of fact, in Fig. 1, we report the pressure as a function of baryon density (normalized to the nuclear saturation density ρ_0) at fixed temperature and zero net strangeness. In this case the condition of mechanical instability (29) is not satisfied. It is possible to verify that also chemical-diffusion instability takes place and the system goes to a phase transition. Let us remark that, although the system has globally zero net strangeness, the mixed phase is realized with two phases with different values of baryon-antibaryon and strangeness content.

In Fig. 2, we report the antibaryon-barion ratio R (left panel) and the strangeness fraction Y_S (right panel) for particle and antiparticle as a function of baryon density. The continuous (dashed) lines correspond to the solution obtained with (without) the Gibbs construction. During the phase transition (between the two vertical dashed lines reported in the left panel) a strong variation in the antibaryon-barion ratio takes place with a significant increase of antibaryon ($\bar{B}$) particles. This feature is reflected in the strangeness fraction with a stronger increase in positive values of Y_S due to antibaryon $\bar{B}$ (antihyperon) particles with respect to negative values due to baryons B (hyperons). Also meson M and antimeson ($\bar{M}$) particles undergo to fluctuation in the strangeness fraction but this feature appears to be less pronounced.

This matter of fact appears more explicitly evident in Fig. 3, where the antiproton-proton ratio (left panel) and kaon-antikaon ratio (right panel) are reported as a function of baryon density at $T = 150$ MeV. Also in this particular case, let us point out the abrupt variation of the ratios during the phase transition region.

In conclusion, similarly to the liquid-gas phase transition in a warm and low density nuclear matter, a pure hadronic phase transition also can occur at higher temperatures and densities due to the presence of both mechanical and chemical instabilities. During the phase transition the two hadronic phases have a different baryon density and different strangeness fraction in

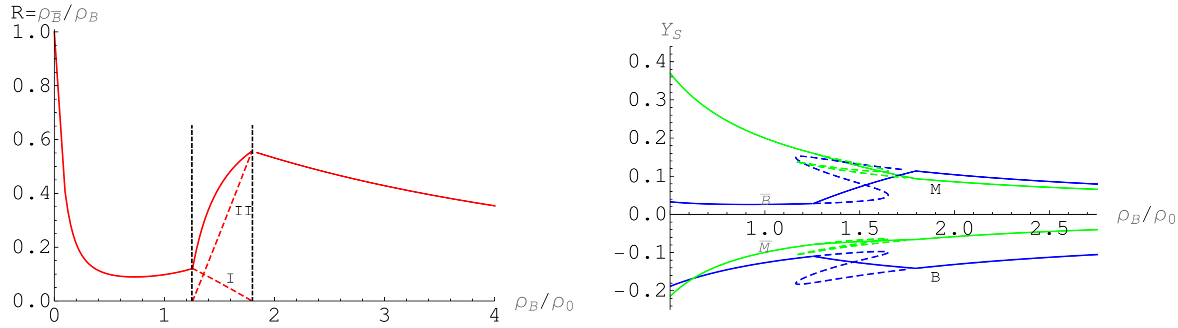

Figure 2. Left panel: Antibaryon-baryon ratio (R) versus baryon density at fixed temperature $T = 150$ MeV. The vertical dashed lines indicate the beginning and the end of the phase transition. The dashed lines with the symbols I and II give the values of R in the two coexisting phases. Right panel: Strangeness fractions (Y_S) of baryons (B), mesons (M) and their antiparticles as a function of baryon density in the pure hadronic phase and in the mixed phase at fixed temperature and zero net strangeness. The continuous (dashed) lines correspond to the solution obtained with (without) the Gibbs construction.

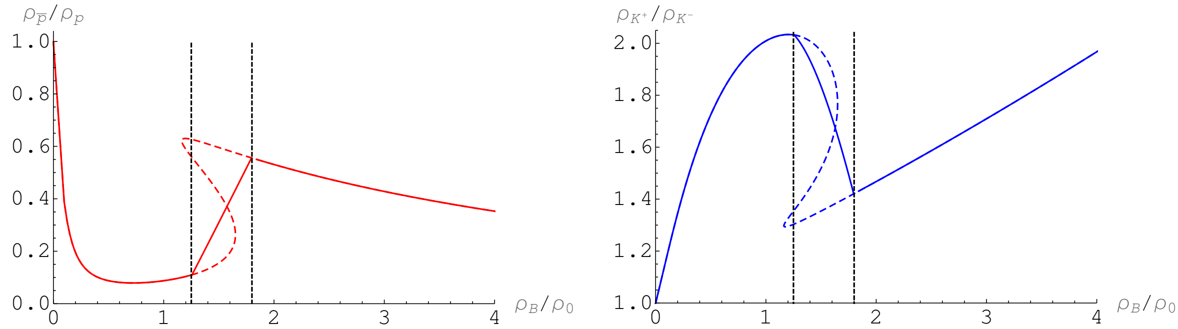

Figure 3. Antiproton-proton $\overline{p}/p$ (left panel) and kaon to antikaon K^+/K^- (right panel) ratios as a function of baryon density. The continuous (dashed) lines correspond to the solution obtained with (without) the Gibbs construction.

the mixed phase, although the global strangeness is fixed to zero. This matter of fact appears to be strictly connected to a strong variation of the antibaryon to baryon ratio during the phase transition. As a consequence, in the mixed phase, an increase of antihyperons (positive strangeness) with respect to hyperons (negative strangeness) is compensated by a suppression of strange mesons (kaon particles with positive strangeness) with respect to stange antimesons (antikaons particle with negative strangenss). This feature could be phenomenological very relevant in order to identify such phase transition in the future compressed baryonic matter experiments like CBM-FAIR experiment at GSI Darmstadt and NICA at JINR Dubna

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