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# IN-CYLINDER PRESSURE-BASED DIRECT TECHNIQUES AND TIME FREQUENCY ANALYSIS FOR COMBUSTION DIAGNOSTICS IN IC ENGINES.

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## ABSTRACT

In-cylinder pressure measurement and analysis has historically been a key tool for off-line combustion diagnosis in internal combustion engines, but online applications for real-time condition monitoring and combustion management have recently become popular. The present investigation presents and compares different low computing-cost in-cylinder pressure based methods for the analyses of the main features of combustion, that is, the start of combustion, the end of combustion and the crankshaft angle that responds to half of the overall burned mass. The instantaneous pressure in the combustion chamber has been used as an input datum for the described analytical procedures and it has been measured by means of a standard piezoelectric transducer.

Traditional pressure-based techniques have been shown to be able to predict the burned mass fraction time history more accurately in spark ignition engines than in diesel engines. The most suitable pressure-based techniques for both spark ignition and compression ignition engines have been chosen on the basis of the available experimental data. Time-frequency analysis has also been applied to the analysis of diesel combustion, which is richer in events than spark ignited combustion. Time frequency algorithms for the calculation of the mean instantaneous frequency are computationally efficient, allow the main events of the diesel combustion to be identified and provide the greatest benefits in the presence of multiple injection events. These algorithms can be optimized and applied to onboard diagnostics tools designed for real control, but can also be used as an advanced validation tool for refined combustion models.

The presented results on the pressure-based techniques, including a time frequency analysis, have been compared with the numerical outcomes from previously developed two- and three- zone thermodynamic combustion models.

**Keywords:** combustion; in-cylinder pressure; time frequency analysis.

## Highlights:

- Direct pressure-based techniques have been applied successfully to spark-ignition engines.

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- 28 - The burned mass fraction of pressure-based techniques has been compared with that of 2- and 3-zone combustion  
29 models.
- 30 - The time frequency analysis has been employed to simulate complex diesel combustion events.

## 31 1. INTRODUCTION

32 Since the invention of internal combustion (IC) engines, the measured time history of the in-cylinder pressure has been  
33 the principal diagnostic tool for experimenters. The following parameters can be determined through cylinder pressure  
34 analysis: peak pressure and maximum pressure gradient, indicated mean effective pressure, pumping mean effective  
35 pressure, burn duration, the shape of heat release, and regularity of combustion. The accuracy of the results on one hand  
36 depends on the accuracy of each component of the measuring chain, and on the other hand on the correct processing of  
37 the pressure signal. [1]. One of the main features of pressure analysis was its simplicity when the available computational  
38 power was low. A heat release model that still maintains simplicity is described in [2], even though this model is also  
39 capable of considering important effects, such as heat transfer, crevice flow and fuel injection. In recent years, pressure  
40 measurement, by means of highly performing transducers, has still represented one of the most important key parameters  
41 for detailed engine studies and diagnosis. The demand for prompt engine control requires the real-time analysis of  
42 pressure signals for different purposes. A new method that is able to resolve dynamic knock intensity related to each  
43 individual engine cycle, is reported in [3]. The in-cylinder pressure can also be used to obtain a refined closed-loop  
44 function in order to control the low temperature combustion mode in Euro 6-compliant diesel engines under steady-state  
45 and transient conditions [4]. Furthermore, the instantaneous in-cylinder pressure signal, during the compression stroke,  
46 can be used as an alternative to the air mass flow measurement in either naturally aspirated spark ignition or turbocharged  
47 diesel engines. This could avoid the need to install an air mass flow sensor, or could provide an improved exhaust gas  
48 recirculation (EGR) estimate, if the proposed method is used together with an air mass flow sensor [5]. However, digital  
49 filtering must be applied if high quality results are required: the selection of the proper cut-off frequency of a low-pass  
50 filter to remove noise related to measurement is mandatory to preserve only physically meaningful information for a  
51 reliable heat release analysis on a cycle basis [6]. In order to reduce the computational power for real time-applications,  
52 an algorithm that uses the difference pressure instead of the in-cylinder pressure for the calculation of IMEP and MFB50  
53 was developed in [7].

54 The in-cylinder pressure time history is universally measured using piezoelectric pressure transducers, due to their highly  
55 dynamic behavior. However, a piezoelectric sensor only provides the pressure variation with respect to a reference  
56 measurement, that is, only a relative value can be measured. The relative pressure has to be referenced to a known value  
57 by means of a pegging procedure in order to obtain an absolute value [8]. An accurate pegging method consists of the

58 installation of a fast-response piezoresistive pressure transducer that is capable of measuring the absolute pressure in the  
59 intake manifold or runner. The cylinder pressure, measured by the piezoelectric transducer at the induction process bottom  
60 dead center (*BDC*), is set equal to the absolute pressure level at the intake manifold, which is calculated as a time average  
61 value of the piezoresistive transducer trace over a certain interval. Furthermore, a direct and reliable measurement of the  
62 compression top dead center (*TDC*) position and of the thermodynamic loss angle are necessary to obtain an accurate  
63 phasing of the pressure signal with the crank angle [9] and thus to achieve meaningful thermodynamic results from the  
64 pressure measurement. In fact, as far as the *imep* evaluation is concerned, an erroneous adjustment of the *TDC* reference  
65 position has been recognized to be a major error source: an uncertainty of  $1^\circ$  *CA* is expected to provide a change of up to  
66 10% in the calculated value of the *imep* from the pressure-volume indicated diagram [10]. The effects that crank angle  
67 phasing errors can have on the heat release rate and mass fraction burnt have not been clearly defined and quantified [11].  
68 Finally, the measurement noise should be removed from the pressure signal by means of a numerical treatment, which is  
69 usually applied after the data have been averaged over several consecutive cycles. Several different methods can be used  
70 for this numerical treatment, including fast Fourier transform filters, windowing filters and smoothing splines [6]. A  
71 derivative pressure signal coming directly from the pressure transducer is sometimes used without a pressure amplifier  
72 [12]. Particular attention has to be paid to filtering and smoothing operations because the pressure fluctuations, which are  
73 softened by the filters or altered by the interpolating splines, can contain important information for the detection of  
74 combustion [13]. A properly processed pressure signal can be used to provide an evaluation of the main combustion  
75 features, either by means of the direct elaboration of the pressure measurement or by means of an indirect heat release  
76 analysis, which applies the first law of thermodynamics to the chamber contents on the basis of the in-cylinder pressure  
77 time history [14]. Both of these approaches allow the mass fraction of the fuel that has burnt ( $x_b$ ) to be estimated as a  
78 function of time, but the former assumes that  $x_b$  is related to the pressures of the fired and unfired cycles rather than to  
79 the fuel-released chemical energy [15].

80 The most important events related to combustion are the start of combustion (*SOC*), end of combustion (*EOC*) and the  
81 angular position at which half of the fuel has burnt (*MFB50* is the crankshaft angle that corresponds to  $x_b=0.5$ ).  
82 Combustion duration can easily be expressed as the difference between the *EOC* and *SOC* angles, but sometimes an  
83 angular distance between 10% and 90% of  $x_b$  is used as an indicator of the combustion period.

84 One of the first applications of the measured pressure signal used to perform the direct diagnosis of combustion was  
85 illustrated by Marvin in [16]. Ideal compression and expansion phases were considered as simple polytropic evolutions,  
86 and the crankshaft angles at which the actual pressure starts to differ from the ideal compression and the straight expansion  
87 lines in a double logarithmic pressure-volume plane were considered as the *SOC* and *EOC* angles, respectively [17].

88 These can also be evaluated considering the inflection points in the diagrams of the specific heat ratio during the closed  
89 part of the cycle [18].

90 Marvin's graphical method was analytically developed and experimentally validated by Rassweiler and Withrow (*RW*)  
91 in [19]. Their analysis pointed out that the cylinder pressure pattern also depends on the changes in the volume of the  
92 combustion chamber, because of the piston motion, on the heat transfer to the chamber walls and on the mass leakages  
93 through the piston crevices, in addition to the influence exerted by combustion. The effects of volume change, heat  
94 transfer, and mass loss [2] were isolated and analytical formulas were proposed to calculate the *SOC*, the *EOC* and the  
95 burned mass fraction [20]. A refined alternative procedure to that of Rassweiler and Withrow has been developed by  
96 McCuiston, Lavoie and Kaufmann (*MLK*) [21] on the basis of the pressure and the in-cylinder volume time histories. This  
97 procedure works efficiently for gasoline combustion and hypothesizes that the mass of the burned gas is proportional to  
98 the  $pV^\gamma$  quantity, where  $\gamma$  is the constant pressure-to-constant volume specific heat ratio. The *SOC*, *EOC* and the  $x_b$  curve  
99 can easily be determined.

100 Indirect heat release analyses were developed later than direct pressure-based techniques and they stemmed from the  
101 application of the first law of thermodynamics. They have been used to develop ordinary differential equation models of  
102 combustion that allow the evaluation of the heat release rate (*HRR*), from which the burned mass fraction can then be  
103 worked out. Single-zone models define the thermodynamic state of the cylinder charge, in terms of average properties of  
104 a pure phase [2], but do not distinguish between burned and unburned gases. In simpler models, heat transfer and crevices  
105 are neglected and an apparent *HRR*, also referred to as net *HRR*, is calculated [22] as a function of the measured pressures  
106 and volumes. Considering a heat transfer model [23], and also taking the crevice volume into account [2], the gross *HRR*  
107 due to the chemical energy released by the fuel is obtained, and a more accurate  $x_b$  time distribution can be evaluated.  
108 The correlation proposed by Woschni [24], and validated on a huge number of experimental tests, is widely adopted for  
109 the simulation of the heat transfer coefficient, but some constant parameters of this correlation have to be tailored on to  
110 the basis of the considered engine layout and working conditions [25]. The combustion efficiency measured from exhaust  
111 emissions can be used to calibrate these constant parameters, and a physically-consistent final predicted value of  $x_b$  [17]  
112 can thus be obtained from the combustion model.

113 Two-zone models, in which one zone represents the unburned mixture upstream of the flame, and the other simulates the  
114 burned mixture downstream from the flame [25], are more refined and complex than single zone models. The grade of  
115 complexity of the model is further increased in the multi-zone approach [26], in which the burned gases are simulated by  
116 means of different zones, even though this modeling strategy can also be considered reasonable when an enhanced  
117 prediction of engine-out emissions is required. *SOC* is generally easier to determine than *EOC*, because the latter is less  
118 repetitive, due to the inherent difficulty in its localization and the arbitrariness of its practical definition [13]. The energy

conservation equation of the unburned gas mass can be considered for *SOC* evaluation in the context of the *HRR* analysis. Since changes in the sensible internal energy of the charge during the compression phase are only due to work and heat exchanges with the combustion chamber walls, *SOC* can ideally be evaluated as the instant at which the energy conservation mass of the unburned gas fails to be verified [27]. For a practical and effective evaluation, a threshold can be set on the value of the  $x_b$ , e.g. 1%. However, the necessity of larger threshold values in the *SOC* identification process has already been pointed out [28], due to uncertainties that are related to pressure and volume measurements and to the large dispersion of this parameter if ensemble-averaged pressure data are considered. *EOC* can be evaluated in the *HRR* analysis as the crank angle at which  $x_b$  reaches a certain threshold (e.g. 0.99) for single-zone models [29], or as the crank angle at which the theoretical  $x_b$  reaches the experimental combustion efficiency measured from emissions for two- and multi-zone models. Otherwise, since *HRR* expresses the rate at which the fuel releases its chemical energy, a correspondence can be set between *EOC* and the crank angle at which the *HRR* falls below a minimum threshold, e.g. 3% of the maximum *HRR*.

Since accurate heat release analyses are rather complex and time consuming, the direct utilization of the in-cylinder pressure is often the preferred option in order to obtain fast, although approximate, results with the possibility of real-time calculation. Some pressure-based direct approaches, for instance, the polytropic volume method [30], have the aim of maintaining the simplicity and the small computational efforts of the *RW* method, but have been developed to overcome its inherent limitations [31]. Another example is that of the Pressure-Ratio Management (*PRM*) method, developed for naturally aspirated spark-ignition engines, which involves the calculation of the ratio of the fired pressure to the corresponding motored in-cylinder pressure at each crank angle [32]. This methodology, combined with optical analysis, has also been applied to combustion investigation in [33].

The simple *PRM* approach has been modified to obtain the Pressure Departure Ratio (*PDR*) algorithm [32], which is more suitable for diesel propulsion systems. In the *PDR* model, the ratio of the fired to the motored pressure is corrected by means of two constants, which should be calibrated for each engine setup. The *PDR* algorithm is able to predict *MFB50* with adequate accuracy and is suitable for real-time applications, without any preliminary treatment for the pressure data, unlike apparent *HRR* analyses, in which the derivative term of the pressure can be affected negatively by high measurement noise, and moving average techniques should be applied to the original signal in order to remove spurious high frequency oscillations from the *HRR* trace [34]. Other pressure-based direct methods for the real-time calculation of the main combustion parameters are founded on the pressure-difference apparent heat release analysis [7], in which the difference between the actual pressure and motored pressure (the latter is calculated assuming a polytropic law for the compression phase) is used instead of the in-cylinder pressure, since the heat released by the motored pressure is nil [7]. The advantage of these procedures is that the pressure differences are not affected by pegging uncertainties.

150 A simple pressure-based direct procedure, which is founded on the *MLK* technique, relates the *EOC* to the angle at which  
 151 quantity  $pV^{1.15}$  reaches a maximum value [11]. The calculation is started 10 *CA ATDC* and continues up to 10 *CA*, before  
 152 exhaust valve opening by steps of 1 *CA*. *EOC* is set equal to the crank angle that corresponds to the maximum of the  
 153  $pV^{1.15}$  plus 10°*CA*. A low value of the polytropic index is chosen for the expansion phase (the normally adopted values  
 154 are around 1.3, which are significantly higher than 1.15) to ensure reliable results, even in the presence of large pressure  
 155 uncertainties which can make the detection of the maximum point difficult. The addition of 10 *CA* to the angle that  
 156 corresponds to the maximum of  $pV^{1.15}$  allows the low value of the polytropic index to be partly compensated and  
 157 guarantees complete combustion [11].

158 The investigation of the derivative of the pressure signal with respect to time has also been proposed for combustion  
 159 detection. A fourth-order finite difference scheme can be applied to the pressure signal in order to reduce the noise  
 160 measurement effects in the calculus of the pressure derivative [12]. A sudden increase in the  $dp/dt$  signal appears at the  
 161 combustion development stage and this can be used to determine the *SOC*, while *EOC* can be detected on the basis of the  
 162 attenuation of the oscillations in the pressure derivative time history [13]. Time-frequency analysis of the in-cylinder  
 163 pressure derivative has recently been applied to detect combustion [13]. This signal processing technique was introduced  
 164 into the engine field to detect knock and block vibration and for the diagnosis of the injection process [35]. The  
 165 fundamental idea behind time-frequency analysis is to be able to understand and describe how the spectral content of a  
 166 signal changes in time, while classical signal analysis usually dealt with time and frequency separately [36]. In general,  
 167 time-frequency analysis is approached using the spectrogram, which is based on the short-time Fourier transform (*STFT*)  
 168 concept [37], but many other methods have been proposed and applied [38]. Qualitative criteria of the signal frequency  
 169 should allow the existence of combustion to be detected, while time information can serve for combustion localization.

170 In the present work, a comparison has been made between the most popular pressure-based direct methods and single,  
 171 two-zone and three-zone combustion models; reference has been made to different working conditions for both spark-  
 172 ignition and compression-ignition *IC* engines. The time frequency analysis and the spectrogram theory have also been  
 173 applied to both the in-cylinder pressure signal and the difference between fired and motored pressure in diesel engines.  
 174 The mean instantaneous frequency [39] has been calculated from a home-made tool on the basis of the spectrogram, and  
 175 this signal allows detailed information on the development and evolution of diesel combustion to be extracted.

## 176 **2. PRESSURE-BASED DIRECT METHODS FOR COMBUSTION SIMULATION.**

177 Marvin's original work referred to gasoline engines [17]. A typical  $p$ - $V$  diagram of a naturally-aspired spark-ignition  
 178 engine ( $iV=2000\text{ cm}^3$ ) applied to a passenger car [40] is reported in Fig. 1a ( $b_{mep}=790\text{ kPa}$ ,  $n=3300\text{ rpm}$ ) and a portion  
 179 of this diagram is plotted on logarithmic coordinates in Fig. 1b to illustrate the procedure. The dashed line shows actual

180 in-cylinder pressures measured by a piezoelectric pressure transducer, whereas the ideal Otto combustion is represented  
 181 by means of a vertical segment. The compression and the expansion phases are represented as polytropic evolutions,  
 182 according to the  $pV^m=const$  law, and they become straight lines plotted with a solid line in the  $\log p$ - $\log V$  diagram.  
 183 Polytropic exponents equal to  $m_c \approx 1.34$  and  $m_e \approx 1.27$  have been considered for the closer-to-*TDC* portion of the  
 184 compression and expansion phases, respectively, in line with values in [18]. In [33], the following values are proposed:  
 185  $m_c = 1.35$  and  $m_e = 1.25$ . The crankshaft angles at which the actual pressure starts to differ from the ideal compression and  
 186 straight expansion lines can be identified as the *SOC* and *EOC* angles, respectively:  $SOC \approx 348^\circ CA$  and  $EOC \approx 414^\circ CA$   
 187 in Fig. 1b. The combustion duration should be estimated as  $\theta_{comb} = (EOC - SOC)$  and results to be around  $66^\circ CA$  for the  
 188 considered example. Furthermore, Marvin assumed that the pressure rise above the straight compression line was  
 189 proportional to the mass of burned fuel. Therefore,  $x_b$  at a certain point *C* (Fig. 1b) is given roughly by the ratio of length  
 190  $C'A$  to length  $AB$ . Point  $C'$  is the projection, on the top dead center line, of point *C*, which belongs to the actual pressure  
 191 curve. This projection occurs by means of the straight line  $C'C$ , which is parallel to the polytropic compression straight  
 192 line when *C* is located between *SOC* and *TDC* ( $\theta_{TDC} = 360^\circ CA$ ) and is parallel to the polytropic expansion straight line  
 193 when *C* is situated between *TDC* and *EOC*.  
 194 The burned mass fraction curve, determined according to the Marvin procedure ( $x_{b,M}$ ), is reported in Fig. 2 on the basis  
 195 of the data given in Fig. 1b and is compared with the  $x_b$  distribution obtained using a two-zone combustion diagnostic  
 196 model (the *RW* method, to which  $\Delta p_{comb,RW}$  and  $x_{b,RW}$  in Fig. 2 refer, will be discussed in detail later on in this section).  
 197 The agreement between the Marvin distribution and the theoretical curve of the two-zone model is acceptable, with a  
 198 difference of  $\Delta x_b \approx 2.5\%$  between the two curves at  $\theta = 400^\circ CA$ . The most interesting point of the burned mass fraction  
 199 profile is its half value ( $x_b \approx 0.5$ ): Fig. 2 shows that the difference is about  $1.5^\circ CA$  between the *MFB50* values estimated  
 200 by means of the two approaches. None of the abovementioned discrepancies between Marvin's method and the two-zone  
 201 model can be regarded as being only due to the inaccuracy of the Marvin estimation of the burned mass fraction, since  
 202 the  $x_b$  curve of the two-zone model is also affected by some uncertainties, which are primarily due to the calibration  
 203 procedure of the coefficients of the wall heat transfer model. In general, the less the difference between exponents  $m_c$  and  
 204  $m_e$ , the better the performance of the Marvin method in the calculation of the  $x_b$  curve. Marvin's graphical procedure for  
 205 the evaluation of  $x_b$  cannot be extended easily to diesel engines because the Sabathé cycle, which is the general reference  
 206 for compression ignition engines in passenger cars, also involves a combustion phase at constant pressure.



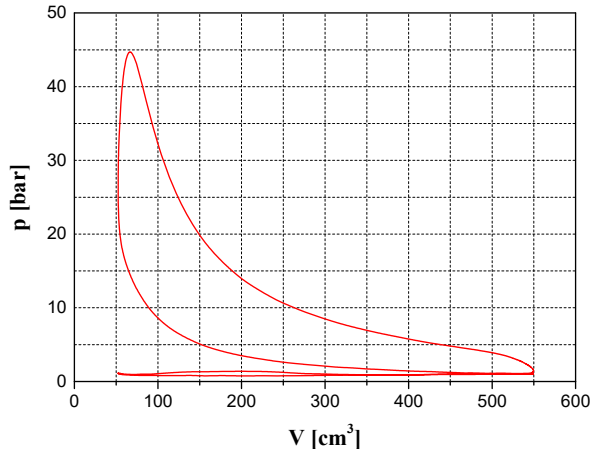


Figure 1a. Indicated  $p$ - $V$  diagram  
(gasoline engine,  $bmeP=790$  kPa,  $n=3300$  rpm)

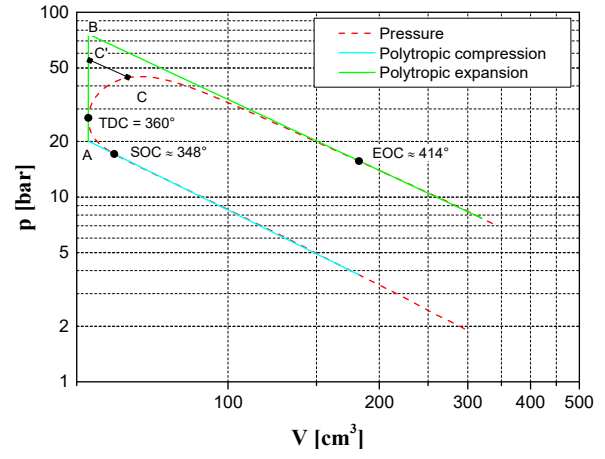


Figure 1b. Enlargement of the indicated diagram in the  
 $\log p$ - $\log V$  plane ( $bmeP=790$  kPa,  $n=3300$  rpm).

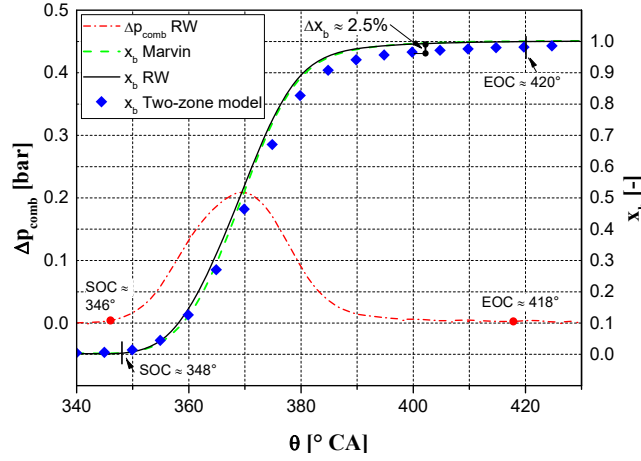


Figure 2.  $x_b$  with the Marvin and RW procedures  
(gasoline engine,  $bmeP=790$  kPa,  $n=3300$  rpm).

The  $RW$  method represents a sort of analytical version of Marvin's graphical procedure. It is based on the hypothesis that the changes in the pressure, due to piston motion and the charge-to-wall heat transfer, can be represented by polytropic processes. In this method, the pressure variation during any crank angle increment  $\Delta\theta=\theta_i-\theta_{i-1}$  consists of two parts: a pressure rise due to combustion ( $\Delta p_{comb,RW}$ ) and a pressure contribution due to volume change and heat exchanges. The term  $\Delta p_{comb,RW}$  can be evaluated as the difference between the actual and polytropic pressures by means of the following formulas, in which the pressure values are measured and the volumes are calculated [19]:

$$\begin{cases} \Delta p_{comb,RW}(\theta_i) = p(\theta_i) - p_{i-1} [V_{i-1}/V(\theta_i)]^{m_c} & \text{if } \theta \leq \theta_{TDC} \\ \Delta p_{comb,RW}(\theta_i) = p(\theta_i) - p_{i-1} [V_{i-1}/V(\theta_i)]^{m_e} & \text{if } \theta \geq \theta_{TDC} \end{cases} \quad (1)$$

217 The *SOC* (lower than  $\theta_{TDC}$ ) and the *EOC* (higher than  $\theta_{TDC}$ ) are the maximum and the minimum crankshaft angles,  
 218 respectively, for which  $\Delta p_{comb,RW}$  becomes virtually nil. The fitting of the  $m_c$  and  $m_e$  polytropic indexes should be carried  
 219 out by means of a least square technique performed over crank angle intervals of  $20^\circ \div 50^\circ CA$  just before the start and  
 220 after the end of combustion, respectively. In particular,  $m_c$  can be worked out over an angular interval located just before  
 221 the high voltage-to-spark instant, whereas  $m_e$  can be calculated over an angular window that starts just after the ( $\theta_{max}+10^\circ$ )  
 222 angle, where  $\theta_{max}$  provides the maximum value of quantity  $pV^{m_e}$ , this being in line with the procedure outlined in [11].  
 223 If reference is made to the data in Fig. 1a ( $m_e=1.27$ ), one obtains  $\theta_{max}+10^\circ \approx 430^\circ CA$ , that is, a higher angle than the *EOC*  
 224 value estimated by means of the Marvin procedure (cf. Fig. 1b).  
 225 Figure 2 reports the  $\Delta p_{comb,RW}$  versus  $\theta$  distribution (solid line) that has been calculated by means of Eq. (1) on the basis  
 226 of the experimental pressure data shown in Fig. 1a. It results that  $SOC \approx 346^\circ CA$  and  $EOC \approx 418^\circ CA$ , and these values are  
 227 very similar to the outcomes of the Marvin graphical procedure.  
 228 In the case of the gasoline engine, the elemental combustion heat ( $\delta Q_{hrr}$ ) released during the  $\Delta\theta$  interval can roughly be  
 229 modeled as a heat adsorbed during an isochoric evolution, according to the following expression:

$$230 \quad \delta Q_{hrr} = Mc_v \Delta T_{comb,RW} \quad (2)$$

231 where  $M$  is the constant mixture mass during combustion. The temperature increase  $\Delta T_{comb}$  can be calculated as

$$232 \quad \Delta T_{comb,RW} = \frac{\Delta p_{comb,RW} V_{TDC}}{M R} \quad (3)$$

233 By substituting Eq. (3) in Eq. (2) and taking into account Eq. (1), one obtains:

$$234 \quad \delta Q_{hrr} \approx \frac{V_{TDC}}{\gamma - 1} \left\{ p(\theta_i) - p_{i-1} \left[ V_{i-1} / V(\theta_i) \right]^m \right\} \quad (4)$$

235 where  $m$  can either be  $m_c$  or  $m_e$  and  $\gamma = c_p / c_v$ . Parameter  $\gamma$  can be considered as a constant property or expressed as a  
 236 function of the temperature. Quantity  $\delta Q_{hrr}$  does not include the heat exchanged with the walls, since it has already been  
 237 included in the  $m$  polytropic exponent. In other words,  $\delta Q_{hrr}$  represents the gross heat release, while the apparent or net  
 238 heat release ( $\delta Q_{ahrr}$ ) can be obtained by replacing polytropic index  $m$  with factor  $\gamma$  in Eq. (4). Furthermore, heat  $\delta Q_{hrr}$   
 239 results to be proportional to  $\Delta p_{comb,RW}$ , in agreement with Marvin's hypothesis. Therefore, the *RW* algorithm establishes  
 240  $x_b$  at the end of the  $i$ -th interval of amplitude  $\Delta\theta$  ( $\theta = i\Delta\theta$ ) as [19]

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$$x_{b,RW}(\theta_i) = \frac{m_b(\theta_i)}{m_{b,tot}} = \frac{\sum_{k=0}^i \Delta p_{comb,RW}(\theta_k)}{\sum_{k=0}^N \Delta p_{comb,RW}(\theta_k)} \quad (5)$$

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$$x_{b,MLK} = \frac{pV^m - p_{SOC}V_{SOC}^m}{p_{EOC}V_{EOC}^m - p_{SOC}V_{SOC}^m} \quad (6)$$

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$$\delta q_{hrr} = dx_b H_i = c_v dT + p dv - \delta q_{ht} \quad (7)$$

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$$dx_b H_i = \frac{m}{m-1} p dv + \frac{1}{m-1} v dp \quad (8)$$

263 If the heat exchanged with the walls is overlooked ( $\delta q_{hr} \approx 0$ ),  $m$  can be replaced by  $\gamma$  in Eq. (6) and one obtains the original  
 264 *MLK* formula: the resulting heat release thus becomes equal to the apparent heat release rate ( $\delta q_{ahrr}$ ). Apparent heat release  
 265 rates are usually 10-20% lower than gross heat release rates [20].  
 266 Eq. (8) can be rewritten as follows [21]

$$267 \quad dx_b = \frac{1}{H_i} \frac{d(pV^m)}{(m-1)V^{m-1}} = \frac{d(y^m)}{(m-1) \frac{H_i}{p_{SOC} V_{SOC}} \left( \frac{V}{V_{SOC}} \right)^{m-1}} \quad (9)$$

268 where the instantaneous volume  $V$  of the combustion chamber and instantaneous pressure have been normalized to their  
 269 values at *SOC* and quantity  $y$  is defined by  $y = (V/V_{SOC})(p/p_{SOC})^{1/m}$ . Eq. (9) can easily be integrated and, for constant  
 270 volume combustion, the following expression can then be obtained ( $p_{SOC} V_{SOC} = MRT_{SOC}$ ):

$$271 \quad y^m(\theta) = 1 + \frac{m-1}{\gamma-1} \frac{H_i}{c_v T_{SOC}} x_b \quad (10)$$

272 Eq. (10) provides a linear expression for  $y^m(\theta)$  as a function of  $x_b$ , and this expression can also be formulated as [21]

$$273 \quad y^m = 1 + [y^m(\theta_{EOC}) - 1] x_b \quad (11)$$

274 where  $y^m(\theta_{EOC})$  corresponds to  $x_b = 1$ . Eq. (11) can be solved, with respect to  $x_b$ , in order to obtain the approximate relation  
 275 for  $x_b$  given by Eq. (6). In short, the *MLK* method can be considered a rough application of the heat release analysis and  
 276 is based on the observation that the heat released by the fuel to the charge in the period from *SOC* to *EOC* is proportional  
 277 to

$$278 \quad Q_{hrr} \propto (p_{EOC} V_{EOC}^m - p_{SOC} V_{SOC}^m) \quad (12)$$

279 Therefore, the heat released during combustion at angle  $\theta$  is proportional to  $pV^m$  and it can be normalized to the value  
 280 expressed by Eq. (12) in order to obtain the  $x_{b,MLK}$  curve. Although Eq. (6) only holds for isochoric combustion, the  
 281 inaccuracies in the determination of the angles at which combustion begins and ends are small when the volume does not  
 282 vary significantly during combustion. Therefore, the simplified *MLK* procedure can be applied to production gasoline  
 283 engines, but is not suitable for diesel engines in which combustion occurs during a significant change in the  $V$  volume.  
 284 Figure 3 plots quantity  $pV^m$ , divided by a reference value, as a function of  $\theta$ , for the pressure and volume data reported in  
 285 Fig. 1a ( $m = m_e \approx 1.27$  since most of the combustion develops in the  $\theta > \theta_{TDC}$  range for the considered case).

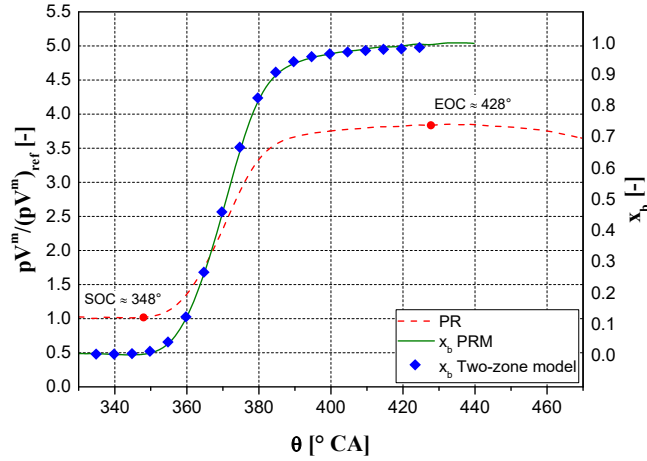


Figure 3.  $x_b$  with the *MLK* procedures (gasoline engine,  $bme_p=790$  kPa,  $n=3300$  rpm).

286 The  $pV^{m_e}/(pV^{m_e})_{ref}$  versus  $\theta$  trace is approximately constant in the compression phase, increases during combustion  
 287 ( $SOC \approx 346^\circ CA$ ) and is again nearly constant for the remainder of the cycle. This is consistent with the results given in  
 288 [41], where  $pV^\gamma$  is nearly constant during the approximately isentropic processes, while it is not constant during a period  
 289 of energy release. A maximum point of  $pV^{m_e}$  is observable in Fig. 3 at  $\theta \approx 424^\circ CA$  and in the *MLK* method this angle  
 290 corresponds to *EOC*, this being in line with other results concerning the meaning of the maximum value of quantity  $pV^{1.15}$   
 291 [11]. The  $x_b$  time histories predicted by means of the *MLK* technique and the two-zone combustion model have also been  
 292 reported in Fig. 3. As can be inferred, the  $x_{b,MLK}$  curve is virtually coincident with the two-zone model burned mass fraction  
 293 distribution: the difference between the *MFB50* points of the two curves is less than  $0.2^\circ CA$ .  
 294 The *PRM* algorithm [32] involves the calculation of the ratio between the fired pressure and the corresponding motored  
 295 cylinder pressure ( $p_{mot}$ ) at each crank angle. The curve  $p_{mot}$  versus  $\theta$  can be measured experimentally, in the absence of  
 296 combustion, when the engine is run by an electrical machine, or can be approximated by means of a polytropic evolution  
 297 (the latter solution is mandatory if the engine is supercharged or turbocharged).

298 The pressure ratio (*PR*) is defined as [33]

299 
$$PR = \frac{p(\theta)}{p_{mot}(\theta)} \quad (13)$$

300 This parameter has a unit value before combustion and becomes higher than one during combustion, reaching a maximum  
 301 value at the end of combustion. The increase in *PR*, with respect to the unit value, is referred to as the modified pressure  
 302 ratio:

303 
$$MPR(\theta) = \frac{p(\theta)}{p_{mot}(\theta)} - 1 \quad (14)$$

304 The maximum value of  $PMR$  is reached when  $PR$  is at a maximum and is called the final  $PR$  value ( $FPR$ ). It usually  
 305 ranges from 2.8 to 4.0 and typically occurs at around  $55^\circ CA$  ATDC for spark-ignition engines [42].  $FPR$  reaches a  
 306 maximum value for stoichiometric mixtures and decreases as the excess of air,  $EGR$  or residual gas are increased;  
 307 therefore, it can also be useful as an indicator of the charge dilution of the combustion system. The  $MPR$ , once it has been  
 308 normalized to  $FPR$ , provides the pressure ratio management procedure for the estimation of  $x_b$  [32]:

$$309 \quad x_{b,PRM} = \frac{MPR(\theta)}{FPR} \quad (15)$$

310 Figure 4a plots the  $p(\theta)$  and  $p_{mot}(\theta)$  signals for the same engine layout and working condition to which Figs. 1-3 refer.  
 311 The  $PR$  ratio is reported in Fig. 4b as a function of  $\theta$  and its maximum value occurs at  $\theta \approx 430^\circ CA$ , that is, around  
 312  $70^\circ ATDC$ . Furthermore, Fig. 4b shows that the  $x_{b,PRM}$  curve is almost coincident with the two-zone model  $x_b$  distribution.  
 313 The direct application of the  $PMR$  technique to diesel engines can result in a burned mass fraction curve that can differ  
 314 from the actual cumulative heat release trace, which is calculated by means of a combustion model. In fact,  $FPR$  occurs  
 315 towards the exhaust valve opening, due to the much higher compression ratios of the diesel engine, and the  $PRM$   
 316 estimation therefore departs from the actual diesel combustion characteristics, even for single fuel injection schedules.  
 317 Furthermore, diesel combustion can consist of discrete heat release events because of multiple injection strategies and  
 318 this represents another main difference from spark ignition engines.

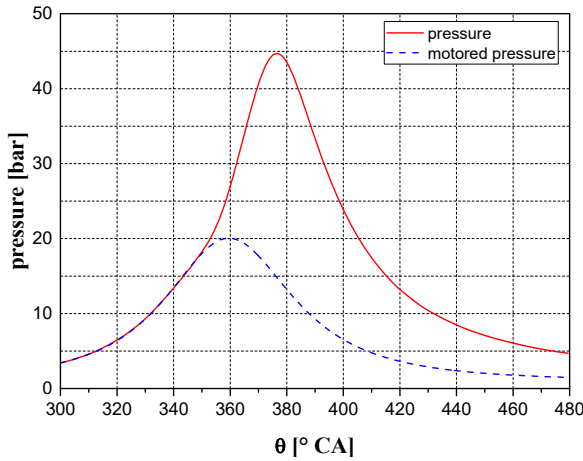


Figure 4a.  $p$  and  $p_{mot}$  angular distributions

(gasoline engine,  $bmeP=790$  kPa,  $n=3300$  rpm).

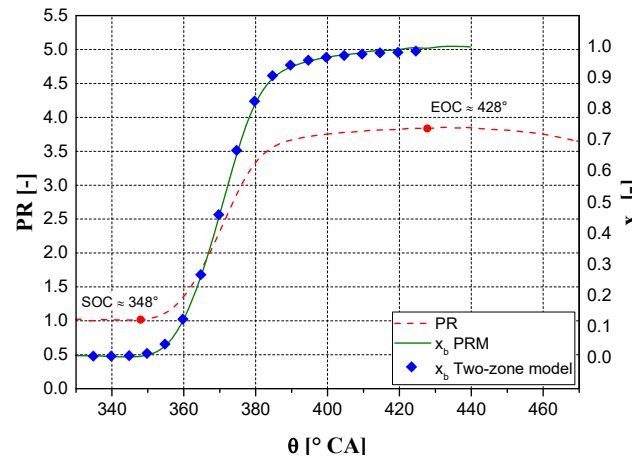


Figure 4b.  $x_b$  with the  $PRM$  procedure

(gasoline engine,  $bmeP=790$  kPa,  $n=3300$  rpm).

319  $MPR$  in diesel engines is usually substituted by the following pressure departure ratio:

$$320 \quad PDR(\theta) = \frac{p(\theta) + C_1}{p_{mot}(\theta) + C_2} - 1 \quad (16)$$

where  $C_1$  is the fired pressure characterization coefficient and  $C_2$  is the motored pressure characterization coefficient [32]. These empirical coefficients are constant for a given engine configuration. The  $PDR$  has an almost zero value before combustion and rises to a maximum value ( $PDR_{max}$ ) which corresponds to  $EOC$ . An estimate of the mass fraction burnt is obtained by normalizing  $PDR$  to its maximum value [32]:

$$x_{b,PDR} = \frac{PDR(\theta)}{PDR_{max}} \quad (17)$$

The  $C_2$  coefficient in Eq. (16) is adjusted so that the  $x_{b,PDR}$  trace matches the results of a heat release model as closely as possible: small changes in the  $C_2$  value cause the curve to pivot around a point. The  $C_1$  constant is then selected to shift the pivotal point as close as possible to the actual  $MFB50$ , because this ensures that small deviations at the extreme ends of the curve will have minimal effects on the prediction of  $MFB50$ .

Figures 5 and 6 compare the performance of the  $RW$ ,  $MLK$  and  $PRM$  methods when applied to the previously considered gasoline engine at  $bmeP=440$  kPa,  $n=2000$  rpm and at  $bmeP=620$  kPa,  $n=2570$  rpm, respectively. The  $x_b$  curves, which have been worked out by means of a calibrated two-zone gasoline combustion model, are used as references to evaluate the different pressure-based direct techniques. Furthermore, Fig. 7 shows a comparison of the same abovementioned methods for a spark-ignited  $CNG$  supercharged engine ( $iV=1242$  cm<sup>3</sup>) at  $bmeP=800$  kPa and  $n=3000$  rpm. The reference heat release curve has been calculated with a two-zone combustion diagnostic model of the considered  $CNG$  engine [43].

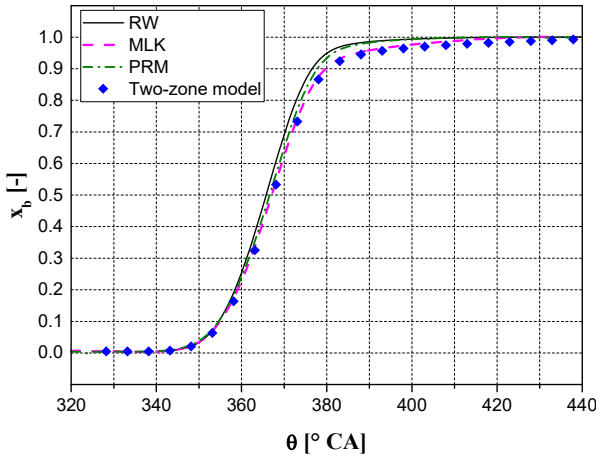


Figure 5. Comparison of the procedures pertaining to  $x_b$   
(gasoline engine,  $bmeP=440$  kPa,  $n=2000$  rpm).

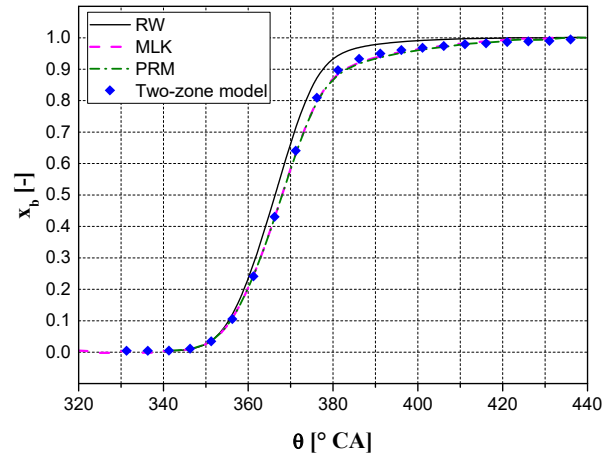


Figure 6. Comparison of the procedures pertaining to  $x_b$   
(gasoline engine  $bmeP=620$  kPa,  $n=2570$  rpm).

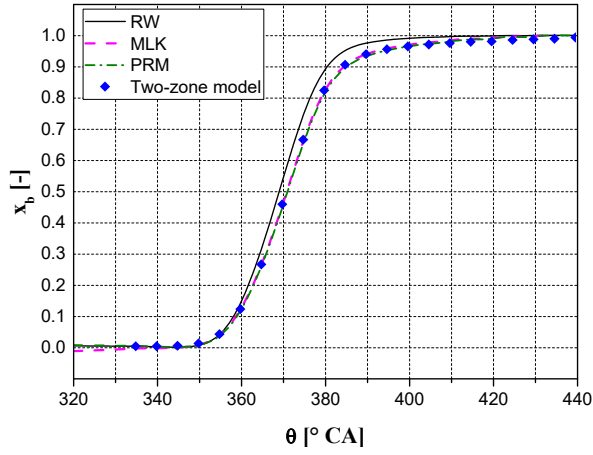


Figure 7. Comparison of the procedures pertaining to  $x_b$   
(CNG engine,  $bme_p=800$  kPa,  $n=3000$  rpm)

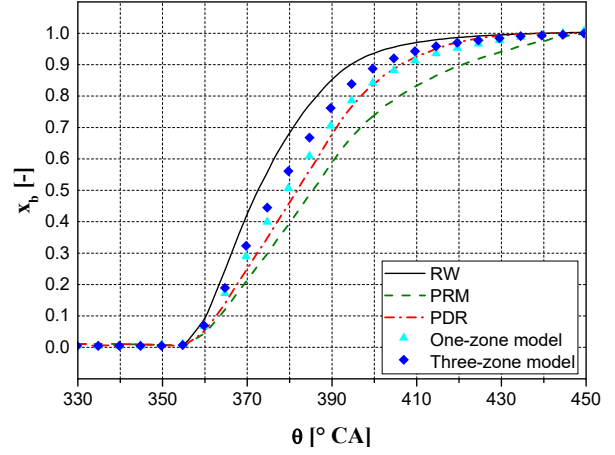


Figure 8. Comparison of the procedures pertaining to  $x_b$   
(diesel engine, single injection event).

339 The *MLK* algorithm generally gives results that are virtually coincident with those of two-zone models, although the *PMR*  
340 method also guarantees very satisfactory performance. The *MLK* and the *PRM* generally give very similar results because  
341 Eqs. (6) and (15) represent the same physical law: in fact, if one divides the numerator and denominator of the right hand  
342 member of Eq. (6) by  $p_{soc} V_{soc}^m$ , the quantity  $p_{soc} V_{soc}^m / V^m$  represents the polytropic pressure (motored pressure) and,  
343 as a consequence, Eq. (15) is obtained. The only slight difference between the two techniques can reside in the evaluation  
344 of the *EOC*, which coincides with the maximum of  $p/p_{mot}$  for the *PMR* procedure and with the maximum of  $pV^m$  for the  
345 *MLK* procedure: these two maxima can occur at different crankshaft angles.

346 Finally, the data in Figs. 8-10 refer to a twin-stage turbocharged diesel engine ( $iV=2000$  cm<sup>3</sup>) [44], fuelled with distinct  
347 injection schedules at different  $bme_p$  and  $n$  conditions. The crankshaft angle based  $x_{b,RW}$ ,  $x_{b,PRM}$  and  $x_{b,PDR}$  distributions  
348 are compared with the  $x_b$  traces derived from the application of single-zone and three-zone diagnostic tools for the  
349 simulation of diesel combustion. In the calculus of  $x_{b,PDR}$ , the calibrated values of  $C_1$  and  $C_2$  were set equal to 1.1 and 1.2,  
350 respectively, for the turbocharged engine setup tested in the current investigation. These values were selected on the basis  
351 of preliminary tests aimed at optimizing the *PDR* method response. As can generally be inferred from Figs. 8-10, the  
352 prediction capability of the considered pressure-based techniques becomes worse when passing from spark ignition to  
353 compression ignition engines. In particular, the accuracy of the pressure based techniques in the prediction of *SOC*, *EOC*  
354 and the  $x_b$  trace deteriorates in diesel engines because the combustion evolution becomes more complex, also because of  
355 the multiple injection events, and requires more sophisticated approaches. The *PDR* method generally guarantees the best  
356 approximation of the  $x_b$  distributions obtained by means of the single zone and three-zone models; both the *SOC* and the  
357 *EOC* are evaluated with satisfactory accuracy for the single (Fig. 8) and the double injection (Fig. 9) strategies. The *RW*  
358 technique gives a better performance than the *PMR* method, which is confirmed to be inadequate for diesel engines, even  
359 in the presence of single injection events.



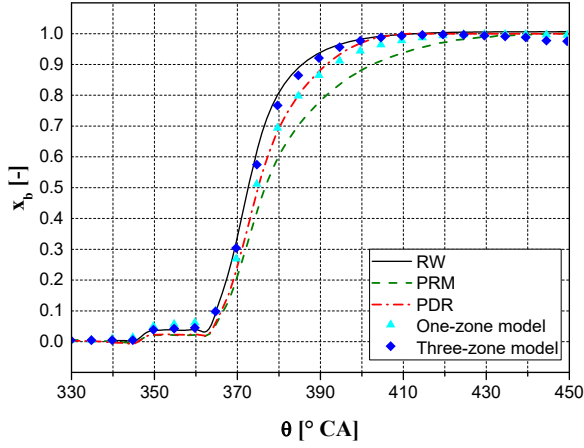


Figure 9. Comparison of the procedures pertaining to  $x_b$   
(diesel engine, double injection event).

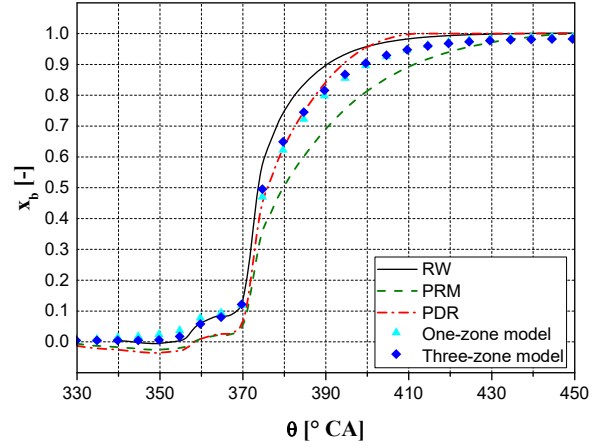


Figure 10. Comparison of the procedures pertaining to  $x_b$   
(diesel engine, triple injection event).

### 3. TIME FREQUENCY ANALYSIS

The Fourier transform is suitable for describing stationary phenomena in the frequency domain, but it does not allow transient phenomena that undergo a time evolution to be analyzed. In fact, Fourier transform coefficients represent the time integral of the product between the considered signal and a complex sinusoidal wave, which is determined perfectly in terms of frequency and phase, but is not localized in the time domain [38]. In other words, the *FFT* points out the contribution of the different harmonic terms to the signal, but the time instants at which each frequency component is relevant are not clarified. Instead, time frequency analysis is used to evaluate the changes, with respect to time, in the frequency spectrum of a transient signal  $f(t)$ . A large number of fast Fourier transforms are realized over different consecutive short time intervals and each *FFT* is referred to the mean instant of the short time interval over which the *FFT* has been performed. In fact, the unsteady signal is assumed to have stationary behavior in each short time interval and a local Fourier spectrum is therefore calculated over this short interval.

From a mathematical point of view, a windowing operation of signal  $f(t)$  is realized in the time-frequency analysis by multiplying  $f(t)$  by a mobile window function  $h(t-\tau)$ , where  $\tau$  is a variable parameter that represents the centre of the window support. The Short Time Fourier Transform (*STFT*) is defined in the following way:

$$\hat{F}_l(\omega, \tau) = \int_{-\infty}^{+\infty} f(t) h(t - \tau) e^{-j\omega t} dt \quad (18)$$

The position of the window in Eq. (18) can be shifted with respect to time by varying  $\tau$ , and different Fourier spectra can then be obtained. Integrating Eq. (18) with respect to  $\tau$ , one obtains

$$\hat{F}(\omega) = \int_{-\infty}^{+\infty} \hat{F}_l(\omega, \tau) d\tau = \int_{-\infty}^{+\infty} \left[ \int_{-\infty}^{+\infty} f(t) h(t - \tau) d\tau \right] e^{-j\omega t} dt \quad (19)$$

378 Function  $\hat{F}_l(\omega, \tau)$  represents a local Fourier transform, and the summation of all of the *STFT* over  $\tau$  values ranging from  
 379  $-\infty$  to  $+\infty$  is equal to  $\hat{F}(\omega)$ . Since  $\hat{F}(\omega)$  is the Fourier transform of a time convolution, it is given by the product of the  
 380 Fourier transform of the signal, namely  $F(\omega)$ , and the Fourier transform of the window, namely  $H(\omega)$ :

$$381 \quad \hat{F}(\omega) = F(\omega) H(\omega) \quad (20)$$

382 When the *STFT* is applied to the analysis of the time variations of a physical signal, the  $\tau$  parameter in Eq. (18) is varied  
 383 by finite steps of amplitude  $\Delta\tau$ . The higher the overlap period of two consecutive positions  $\tau_l$  and  $\tau_{l+1} = \tau_l + \Delta\tau$  of window  
 384  $h$  in the time domain, the more gradual the variation of the Fourier spectrum with respect to time. The extension of the  
 385 time support  $\Delta T_h$  of the window, i.e., the time interval in which the  $h(t)$  window is higher than zero, can be related to the  
 386 sampling circular frequency ( $\omega_s$ ) and to the number of sampling points within the window support ( $N$ ) by the relation  
 387  $\Delta T_h = 2\pi N / \omega_s$ . The overlap period is equal to  $\Delta t_{ov} = s \Delta T_h = 2\pi n / \omega_s$ , where  $n$  is the number of samplings in the overlap period  
 388  $\Delta t_{ov}$  and  $s = n/N$  is the overlap factor. As a result, the temporal resolution of the *SFST* can be worked out as follows:

$$389 \quad \Delta\tau = \Delta T_h - \Delta t_{ov} = \frac{2\pi}{\omega_s} N (1 - s) \quad (21)$$

390 Both a reduction in  $N$  and an increase in  $s$  at fixed sampling frequency  $\omega_s$  induces an improvement in the time resolution,  
 391 that is, a diminution in  $\Delta\tau$ . The minimum value of  $\Delta\tau$  is  $2\pi/N$  and is reached for  $s = N/(N-1)$ .

392 A window function can be characterized in terms of its centre and its semiamplitude. The centre ( $t_0$ ) and the semiamplitude  
 393 ( $\Delta_t$ ) of a window function  $h$  are defined as

$$394 \quad t_0 = \frac{1}{\|h\|_2} \int_{-\infty}^{+\infty} t |h(t)|^2 dt \quad \Delta_t = \frac{1}{\|h\|_2} \left[ \int_{-\infty}^{+\infty} (t - t_0)^2 |h(t)|^2 dt \right]^{1/2} \quad (22)$$

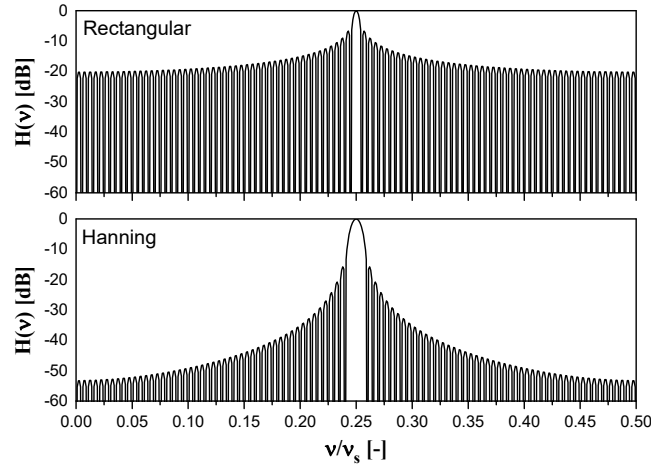
395 where the norm  $\|h\|_2$  represents the energy of the  $h$  function, which is given by

$$396 \quad \|h\|_2 = \int_{-\infty}^{+\infty} |h(t)|^2 dt \quad (23)$$

397 The definitions of centre and semiamplitude extend naturally to the frequency domain for the  $H(\omega)$  function:

$$398 \quad \omega_0 = \frac{1}{\|H\|_2} \int_{-\infty}^{+\infty} \omega |H(\omega)|^2 d\omega \quad \Delta_\omega = \frac{1}{\|H\|_2} \left[ \int_{-\infty}^{+\infty} (\omega - \omega_0)^2 |H(\omega)|^2 d\omega \right]^{1/2} \quad (24)$$

399 where  $\|H\|_2 = \int_{-\infty}^{+\infty} |H(\omega)|^2 d\omega$ .



**Figure 11. Different  $h$  functions and their Fourier transforms.**

400 The selection of the optimum  $h$  window function is a central theme in the time frequency theory. Different shapes of  
 401 window functions have been analyzed and tested by specialists in the field. Eq. (20) shows that the window modifies the  
 402 frequency content of signal  $f(t)$ , which is given by  $F(\omega)$ . A rectangular shaped window ( $h_R$ , cf. Fig. 11) is a simple  
 403 solution, but it is able to alter the spectrum of the original signal to a great extent. As an example, if  $f(t)$  is a sinusoid with  
 404 circular frequency  $\omega = \omega_0$ , the signal, which is windowed by means of a  $h_R$  function with its centre at  $\omega_0$ , features a  
 405 frequency spectrum that can be represented as a band centered on  $\omega_0$  ( $H_R$  is reported in Fig. 11) rather than as a single  
 406 line at  $\omega_0$ . Furthermore, the step transitions that occur in  $h_R$  generate high-frequency harmonic components in the  $\hat{F}(\omega)$   
 407 spectrum. Other window functions have been proposed to partially solve these drawbacks of the rectangular window. One  
 408 popular window function is the Hann filter (cf.  $h_H$  in Fig. 11), which is defined as follows (the here considered window  
 409 has the center at point  $n = (N-1)/2$ ):

$$410 \quad h_H(n) = \frac{1}{2} \left[ 1 - \cos \left( \frac{2\pi n}{N-1} \right) \right] \quad 0 \leq n \leq N-1 \quad (25)$$

411 The  $h_H$  function goes to zero at  $n=0$  and  $n=N-1$ , without any type of discontinuity, and therefore does not introduce any  
 412 spurious high-frequency components in the frequency content of signal  $f$ . The Fourier spectrum of  $h_H$ , that is,  $H_H$ , has  
 413 been illustrated in Fig. 11: a central lobe exists in the Fourier spectrum of the windowed function, with a higher extension  
 414 than the corresponding one in the Fourier spectrum of the  $h_R$  windowed signal, but the number and the maximum values  
 415 of the lateral lobes is reduced compared to the  $h_R$  case.

416 For any window function  $h$ , the amplitude of  $H(\omega)$ , i.e.  $2\Delta_\omega$ , increases as the amplitude of  $h(t)$ , i.e.  $2\Delta_t$ , reduces. The  
 417 Heisenberg principle of indetermination governs the relationship between  $\Delta_\omega$  and  $\Delta_t$  according to the following expression  
 418 [37]:

$$\Delta_t \Delta_\omega \geq \frac{1}{2} \Leftrightarrow \Delta_t \Delta_\nu \geq \frac{1}{4\pi} \quad (26)$$

The two formulations are equivalent because  $\nu = \omega/2\pi$  and hence  $\Delta_\nu = \Delta_\omega/2\pi$ . It is impossible, on the basis of these relations, to simultaneously reduce  $\Delta_\omega$  and  $\Delta_t$  in order to obtain a high resolution of the unsteady signal  $f(t)$  in either the time or the frequency domains. If the frequency resolution is improved,  $\Delta_\omega$  should be reduced, but this leads to an increase in  $\Delta_t$  and therefore to a diminution in the capability of locating events in the time domain: phenomena that are closely coupled become difficult to distinguish. In the particular case of the *FFT*, the window function in Eq. (18) is  $h_{FT}=1$ ,  $\Delta_{t,FT} \rightarrow \infty$  while  $\Delta_{\omega,FT} \rightarrow 0$  because  $H_{FT}$  is the Dirac delta function in the frequency domain: the description is therefore very accurate in the frequency domain, but it is impossible to localize the events with respect to time.

The Gaussian window, i.e.,  $h_G(n) = \exp\left\{-\frac{1}{2} \left[ \frac{n-(N-1)/2}{\sigma(N-1)/2} \right]^2\right\}$ , is the window function that minimizes the product between  $\Delta_t$  and  $\Delta_\omega$ . In fact, it can be proved that the Gaussian window is the only function for which  $\Delta_t \Delta_\omega = 1/2$ . When  $h_G$  is used in Eq. (18), the thus obtained  $\hat{F}(\omega, \tau)$  is named the Gabor transform of the  $f$  function.

### **3.1 Spectrogram and mean instantaneous frequency numerical model**

The time-frequency distribution related to signal  $f(t)$  is a function  $P_f(t, \nu)$ , which is defined as follows [37]:

$$E_f = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} P_f(t, \nu) dt d\nu = \int_{-\infty}^{+\infty} |f(t)|^2 dt \quad (27)$$

where  $E$  represents the energy of signal  $f(t)$ . The  $P_f(t, \nu)$  distribution therefore represents an energy density function with respect to both time and frequency. The Parseval identity allows the energy of the signal to be calculated, starting from the knowledge of the Fourier spectrum ( $F(\omega)$  or alternatively  $F(\nu)$ , which has been considered in what follows):

$$E_f = \int_{-\infty}^{+\infty} |f(t)|^2 dt = \int_{-\infty}^{+\infty} |F(\nu)|^2 d\nu \quad (28)$$

The marginal energy density functions of signal  $f$  are the density functions with respect to either  $t$  or  $\nu$ . These functions are obtained by integrating  $P_f(t, \nu)$  with respect to one of the independent variables:

$$M_f(\nu) = \int_{-\infty}^{+\infty} P_f(t, \nu) dt \quad m_f(t) = \int_{-\infty}^{+\infty} P_f(t, \nu) d\nu \quad (29)$$

The marginal properties are said to hold when the following relations are satisfied:

$$|F(\nu)|^2 = \int_{-\infty}^{+\infty} P_f(t, \nu) dt \quad |f(t)|^2 = \int_{-\infty}^{+\infty} P_f(t, \nu) d\nu \quad (30)$$

442 If the marginal properties are satisfied, Eqs. (27) and (28) are automatically verified, but the opposite is generally not true.  
 443 Therefore, the marginal properties represent a more severe requirement than Eqs. (27) and (28).  
 444 The time-frequency distribution can be interpreted as a density probability function and has been used in the simulation  
 445 code to calculate the mean instantaneous frequency  $\bar{\nu}(t)$  according to the following formula [37]:

$$446 \quad \bar{\nu}(t) = \frac{1}{\int_{-\infty}^{+\infty} P_f(t, \nu) d\nu} \int_{-\infty}^{+\infty} \nu P_f(t, \nu) d\nu \quad (31)$$

447 Eq. (31) gives the baseline harmonic contribution to signal  $f(t)$  at each time instant and is a fundamental relation for many  
 448 engineering applications. The spectrogram  $P_{sp}(t, \nu)$  is the square modulus of  $\hat{F}_l(\omega, \tau)$  and has been selected as the time-  
 449 frequency distribution in the developed code:

$$450 \quad P_{sp}(t, \nu) = \left[ \int_{-\infty}^{+\infty} f(\tau) h(\tau - t) e^{-j2\pi\nu\tau} d\tau \right]^2 = \hat{F}_l(\omega, \tau)^2 \quad (32)$$

451 Function  $P_{sp}$  gives the energy density with respect to time and frequency of the  $f(t)h(t-\tau)$  windowed signal. In fact, the  
 452 following relation holds:

$$453 \quad \begin{aligned} \int_{-\infty}^{+\infty} P_{sp}(t, \nu) d\nu &= \int_{-\infty}^{+\infty} \left\{ \left[ \int_{-\infty}^{+\infty} f(\tau) h(\tau - t) e^{-j2\pi\nu\tau} d\tau \right] \left[ \int_{-\infty}^{+\infty} f^*(\tau') h^*(\tau' - t) e^{j2\pi\nu\tau'} d\tau' \right] \right\} d\nu = \\ &= \int_{-\infty}^{+\infty} e^{-j2\pi\nu(\tau - \tau')} d\nu \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} [f(\tau) f^*(\tau') h(\tau - t) h^*(\tau' - t) e^{-j2\pi\nu\tau}] d\tau d\tau' \end{aligned} \quad (33)$$

454 where  $f^*$  and  $h^*$  are the complex conjugates of functions  $f$  and  $h$ , respectively. Furthermore, since the representation of the  
 455 Dirac delta function in the frequency domain is generally given by

$$456 \quad \delta(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{j\omega t} d\omega = \int_{-\infty}^{+\infty} e^{j2\pi\nu t} d\nu \quad (34)$$

457 it is possible to obtain:

$$458 \quad \begin{aligned} \int_{-\infty}^{+\infty} P_{sp}(t, \nu) d\nu &= \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} [f(\tau) f^*(\tau') h(\tau - t) h^*(\tau' - t) \delta(\tau' - \tau)] d\tau d\tau' = \\ &= \int_{-\infty}^{+\infty} |f(\tau)|^2 |h(\tau - t)|^2 d\tau \neq |f(t)|^2 \end{aligned} \quad (35)$$

459 Finally, by developing a symmetric procedure for quantity  $\int_{-\infty}^{+\infty} P_{sp}(t, \nu) dt$ , the following expression can be obtained:

$$460 \quad \int_{-\infty}^{+\infty} P_{sp}(t, \nu) dt = \int_{-\infty}^{+\infty} |F(\nu')|^2 |H(\nu' - \nu)|^2 d\nu' \neq |F(\nu)|^2 \quad (36)$$

461 The marginal properties are not verified by Eqs. (36) and (35); they could be satisfied if  $h(\tau-t)=\delta(\tau-t)$  and  
 462  $H(v'-v)=\delta(v'-v)$  simultaneously, but, on the basis of the Heisenberg principle, this is not possible. In other words, the  
 463 windowing operation alters the signal and leads to spurious contributions, due to the square of  $|h|$  and  $|H|$ . This is why the  
 464 integrals of  $P_{sp}$ , with respect to  $t$  or  $v$ , are not equal to  $|F(v)|^2$  and  $|f(t)|^2$ , respectively. However, if  $\int_{-\infty}^{+\infty} |h(\tau-t)|^2 d\tau = 1$ ,  
 465 integral  $\int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} P_{sp}(t, v) dt dv$  becomes equal to energy  $E$  of signal  $f(t)$ , and the spectrogram can therefore be confirmed to  
 466 be a feasible choice for the  $P_f(t, v)$  function in Eq. (31) because it does not alter the energy of the signal, although the  
 467 marginal properties are not satisfied.  
 468 Eqs. (18), (21)-(25), (31) (32), together with energy consistency condition  $\int_{-\infty}^{+\infty} |h_H(\tau-t)|^2 d\tau = 1$ , which makes Eqs. (27)  
 469 and (28) hold, have been implemented in the developed Matlab numerical tool to perform the combustion time frequency  
 470 analyses. In this code,  $f(t)$  can be either the in-cylinder pressure signal  $p(t)$  or the pressure difference basis of a preliminary  
 471 campaign of *STFT* tests on  $p(t)$ :  $N=50$ ,  $s=0.96$ ,  $\omega_s=120$  kHz and, from Eq. (21),  $\Delta\tau \approx 170 \mu s$ .

### 472 **3.2 Diesel combustion characterization by means of time-frequency analysis.**

473 Figures 12, 14, 16 and 17 plot the injected mass flow-rate ( $G_{inj}$ ), the cylinder pressure (in the upper part), the *HRR*, the  $x_b$   
 474 and the mean instantaneous frequency crankshaft angle-based distributions (in the bottom part) at different engine  
 475 working conditions for the same diesel engine layout to which Figs. 8-10 refer. Besides, Figs. 13 and 15 report the 3D  
 476 graphs of spectrogram  $P_{sp}$ , normalized to its maximum value, for two distinct engine working conditions;  $P_{sp}$  has been  
 477 calculated by means of Eq. (32), in which  $f:=p$ ,  $h:=h_H$  and  $t$  is replaced by  $\theta$ . The plotted  $G_{inj}$  data were measured at the  
 478 hydraulic rig, in the absence of combustion, under the same injector working conditions as those of the engine tests,  
 479 whereas the cylinder pressure time histories were measured in the engine at the dynamometer cell. The *HRR* and the  $x_b$   
 480 plotted traces have been calculated by means of a three-zone diesel combustion model on the basis of the measured  $p$  and  
 481  $G_{inj}$  histories. It is worth observing that the measured  $G_{inj}$  can differ significantly from the actual injected flow-rate time  
 482 history measured at the dynamometer cell, especially for small injection events, because the temperature levels of the fuel  
 483 are much higher at the dynamometer cell than at the hydraulic test rig. Finally, the mean instantaneous frequency has  
 484 been evaluated through Eq. (31), in which function  $f(t)$  has been replaced by  $p(\theta)$ . The reported diagrams allow the pattern  
 485 of the  $\bar{v}(\theta)$  signal to be interpreted on the basis of the other physical variable distributions according to a learning  
 486 procedure.  
 487 Figures 12 and 13 refer to a pilot-main injection schedule (two injection pulses are present in the  $G_{inj}$  trace) performed at  
 488  $bme_p=500$  kPa and  $n=2000$  rpm. Fig. 13 shows that  $P_{sp}$  is only higher than zero in the  $320^\circ \div 450^\circ$  CA  $\theta$  range. Furthermore,

489 the spectrogram indicates that the frequency content of the considered combustion event is lower than 4 kHz. In Fig. 12,  
490 the first maximum point ( $P_1$ ), which occurs along the  $\bar{v}(\theta)$  distribution, is caused by the injection of fuel into the  
491 combustion chamber. The subsequent decrease in  $\bar{v}$  can be ascribed to fuel evaporation and the *SOC* is located in  
492 correspondence to the minimum  $P_2$  point. The possible discrepancies between the phasing of  $P_2$  and the beginning of the  
493 *HRR* trace can be ascribed to the inaccuracy of the combustion model when simulating the start of combustion. The time  
494 frequency analysis can therefore also be applied to validate and possibly improve the prediction capability of combustion  
495 diagnostic tools. The increase in  $\bar{v}$ , which is detected after  $P_2$ , is a consequence of the combustion development. A first,  
496 more pronounced peak, related to the premixed combustion phase of the pilot injection, can be detected ( $P_3$ ) and this is  
497 followed by a second one ( $P_4$ ), which is due to a smaller, late combustion event related to the pilot injected fuel. The  
498 intensity of the  $P_4$  peak is also reduced because the piston has already started its downstroke after *TDC*; if this phenomenon  
499 is taken into account, the late combustion event of the pilot injected fuel seems to be underestimated in the *HRR* trace  
500 obtained when the model is used.

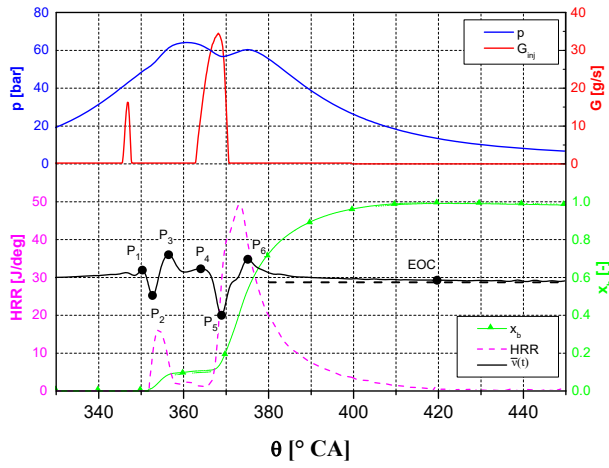


Figure 12. Time frequency analysis  
(diesel engine,  $bmeP=500$  kPa,  $n=2000$  rpm).

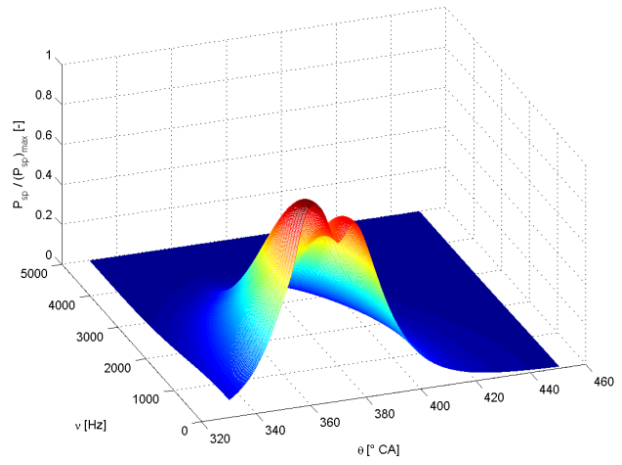


Figure 13. Normalized spectrogram of  $p$   
(diesel engine,  $bmeP=500$  kPa,  $n=2000$  rpm).

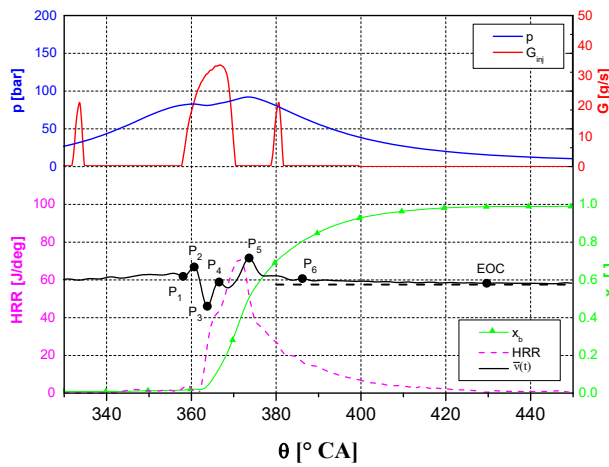


Figure 14. Time frequency analysis of  $p$   
(diesel engine,  $bmeP=800$  kPa,  $n=2500$  rpm).

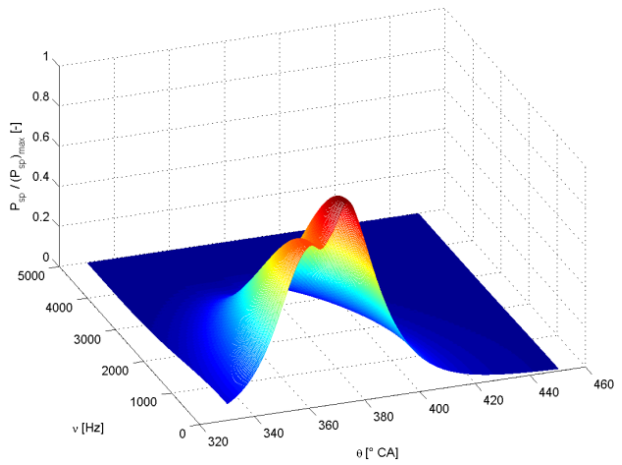


Figure 15. Normalized spectrogram of  $p$   
(diesel engine,  $bmeP=800$  kPa,  $n=2500$  rpm).

501 The diminution in the  $\bar{\nu}$  values, which occurs within the  $365\div368^\circ CA$   $\theta$  range, is induced by the evaporation of the main  
502 injected fuel and by the combustion chamber expansion due to the piston downstroke. The  $SOC$  of the main injected fuel  
503 is placed at about  $\theta \approx 368^\circ CA$ , and a minimum point ( $P_3$ ) can be observed in the  $\bar{\nu}$  curve. The consequent  $\bar{\nu}$  increase is  
504 characterized by the presence of two phases: a first phase (up to  $375^\circ CA$ ), in which the frequency grows at a higher rate,  
505 and a second phase (from  $375^\circ CA$  to  $P_6$ ), in which the slope of  $\bar{\nu}$  with respect to  $\theta$  becomes lower. It can be stated, on the  
506 basis of the  $HRR$  trace, that the former phase is related to the premixed combustion of the main injected fuel and the latter  
507 phase refers to its diffusive combustion regime. The  $EOC$  is evaluated as the time instant at which the  $\bar{\nu}$  curve matches a  
508 horizontal line, which is represented by a dashed horizontal line in Figs 14-17. The  $EOC$  prediction of the time frequency  
509 analysis is physically consistent with the pattern of the  $x_b$  and  $HRR$  curves at that angle.  
510 Figs 14 and 15 refer to  $bmeP=800$  kPa and  $n=2500$  rpm under a pilot-main-after triple injection schedule (cf. the  $G_{inj}$   
511 trace). In Fig. 14, the  $SOC$  of the pilot injection occurs at  $P_1$ , which represents the minimum just before the first distinct



512 maximum; the increase in the mean instantaneous frequency, due to pilot combustion, is soon overbalanced by both the  
 513 cylinder volume expansion after *TDC* and by the evaporation of the main injected fuel; a maximum point of  $\bar{v}$  therefore  
 514 occurs in  $P_2$ . The following minimum  $\bar{v}$  point, i.e.  $P_3$ , indicates the beginning of the main combustion after the  
 515 evaporation of part of the fuel injected in the main pulse; the  $P_4$  peak is related to the premixed combustion part of the  
 516 main injected fuel and also to a possible residual pilot combustion, whereas  $P_5$  refers to the diffusive combustion of the  
 517 main injected fuel and corresponds to a slope variation in the *HRR* trace. Furthermore, the after-injection causes peak  $P_6$   
 518 in the  $\bar{v}$  distribution (also perceivable in the *HRR* trace) and, finally, the  $\bar{v}$  curve becomes almost horizontal at  $\theta \approx 430^\circ CA$ ,  
 519 where the *HRR* trace is practically null. Fig. 15 reports the 3D spectrogram that corresponds to the data in Fig. 14: two  
 520 bumps can be noticed in the  $P_{sp}(v, t)$  graph, as in Fig. 13, even though the number of injection shots is different (three  
 521 shots in Fig. 15 and two shots in Fig. 13). In both cases, the earlier peak corresponds to the piston at *TDC* (it has been  
 522 verified that this peak is also present in the spectrogram of the motored pressure), while the other bump is phased with  
 523 the main combustion *HRR* peak.

524 Figure 16 reports a pilot-pilot-main injection case at  $bme_p=200$  kPa and  $n=1500$  rpm. The closest-to-main pilot injection  
 525 is referred to as pilot 1, whereas the more distant pilot injection is referred to as pilot 2. Since the timing of the pilot 2  
 526 injection occurs early during the piston compression stroke, the pilot 2 injected fuel exhibits a two-stage ignition: the  
 527 *HRR* dashed curve becomes different from zero at  $\theta \approx 350^\circ CA$ , but the sharp increase in *HRR* occurs later, at  $\theta \approx 356^\circ CA$ .  
 528 Correspondingly, in the curve of the mean instantaneous frequency,  $P_2$  and  $P_3$  are the *SOC* and the maximum  $\bar{v}$  point  
 529 related to the cool flames, whereas  $P_4$  and  $P_5$  (related to the first peak in the *HRR* trace) are analogous to  $P_2$  and  $P_3$  for  
 530 the hot flames. The pilot 1 *SOC* is phased with  $P_6$ , while  $P_7$  represents the maximum  $\bar{v}$  point due to pilot 1 combustion  
 531 and is related to the second peak in the *HRR* trace. The *SOC* of the main injected fuel occurs at  $P_8$ , and the development  
 532 of the main combustion leads to  $P_9$ , which is related to the third and maximum peak in *HRR*. The end of combustion  
 533 occurs at  $\theta \approx 420^\circ CA$ , an angle at which both *HRR* and  $x_b$  are practically constant.

534 Finally, Fig. 17 refers to a pilot-pilot-main-after quadruple injection event ( $bme_p=500$  kPa and  $n=1500$  rpm). Even though  
 535 the overall combustion evolution is complex and rich in events, the mean instantaneous frequency distribution allows the  
 536 main features of the different combustion events to be identified. The *SOC* of the pilot 2, pilot 1 and main injected fuel  
 537 quantities are phased with  $P_1$ ,  $P_3$  and  $P_6$ , respectively, and related to the local minima of the  $\bar{v}$  curve. The local maximum  
 538  $P_2$  refers to pilot 2 combustion, while  $P_4$  is related to both the pilot 2 and pilot 1 injected fuel masses (part of the pilot 2  
 539 injected fuel burns together with the pilot 1 injected fuel) and  $P_5$  concerns a late combustion event of the pilot 1 injected  
 540 fuel. The local maxima  $P_7$  and  $P_8$  of the  $\bar{v}-\theta$  distribution correspond, respectively, to a premixed phase of the main

541 combustion and to a diffusive phase of both the main combustion and after combustion. The after combustion is  
 542 responsible for the weak decrease in  $\bar{v}$  that occurs after  $P_8$ , and the *EOC* is located close to  $\theta \approx 400^\circ CA$ .

543 The time frequency analysis can also be applied to the difference,  $\Delta p_{comb}$ , between  $p(\theta)$  and motored pressure  $p_{mot}(\theta)$   
 544 instead of to the  $p(\theta)$  signal. This is justified by the fact that  $\Delta p_{comb}(\theta)$  roughly represents the  $p(\theta)$  portion, which should  
 545 be more closely related to the combustion events, whereas  $p_{mot}(\theta)$  is closely related to the chamber volume variation that  
 546 is due to piston motion. Figs. 18 and 19 report  $\bar{v}$  calculated on the basis of  $\Delta p_{comb}$  ( $f := \Delta p_{comb}$  in spectrogram  $P_{sp}$ ) for the  
 547 same engine working conditions as Figs. 14 and 17, respectively. The main advantages of the time frequency analysis  
 548 applied to  $\Delta p_{comb}$  are the amplification of the peaks in the  $\bar{v}$  distribution and a general reduced uncertainty in the  
 549 determination of *EOC*, compared to the  $p$  analysis. In fact, the  $\theta$  value at which the  $\bar{v}$  curve matches the horizontal line  
 550 pattern at the end of combustion results to be more clearly identified for the  $\Delta p_{comb}$  case than for the  $p$  case. On the other  
 551 hand, the knowledge of the motored pressure time history is required for the calculus of  $\Delta p_{comb}(\theta)$ . However, the  
 552 determination of *SOC* is always easier, and virtually corresponds to the crank angle at which  $p(\theta)$  and  $p_{mot}(\theta)$  start to  
 553 differ, whereas the two pressure traces will never result in the same values when combustion extinguishes because  $p(\theta)$   
 554 will always remain higher. In addition, the suitable  $p_{mot}(\theta)$  signal cannot be experimentally measured in a turbocharged  
 555 engine. In fact, because of the difference in intake manifold pressure between firing and motoring tests, the  $\Delta p_{comb}$  signal,  
 556 obtained as the difference between the experimental measurements of  $p(\theta)$  and  $p_{mot}(\theta)$ , would be higher than zero before  
 557 *SOC*. Therefore, in the case of turbocharged engines, the proper motored trace, which is useful for the  $\Delta p_{comb}$  calculus,  
 558 has to be calculated starting from a firing cycle and considering a polytropic evolution from an angular position where  
 559 combustion has not yet started. Finally, by comparing Fig 14 with Fig. 18 and Fig 17. with Fig 19, it can be observed that  
 560 the trends of the mean instantaneous frequency distributions pertaining to  $p(\theta)$  and  $\Delta p_{comb}$  are really quite similar.

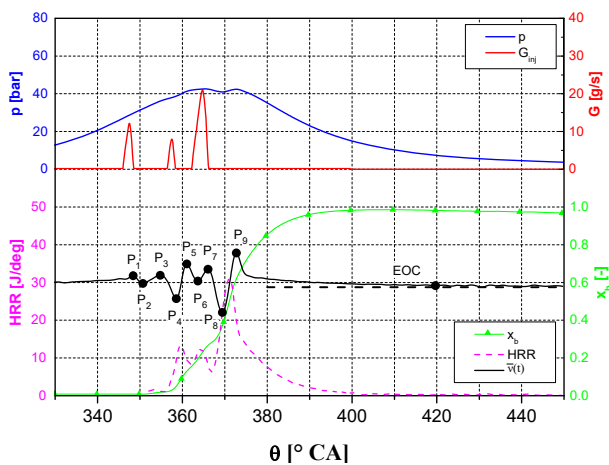


Figure 16. Time frequency analysis of  $p$   
 (diesel engine,  $bmep=200$  kPa,  $n=1500$  rpm ).

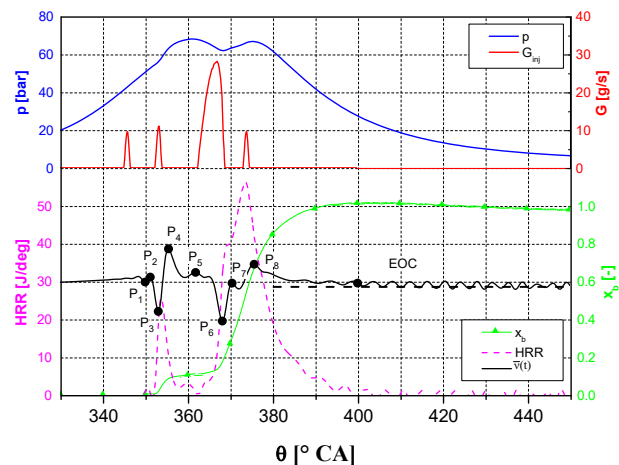


Figure 17. Time frequency ana  
 (diesel engine,  $bmep=500$  kPa,  $n=$

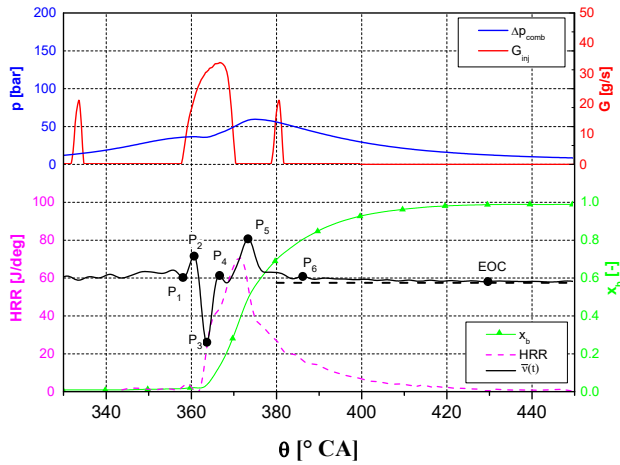


Figure 18. Time frequency analysis of  $\Delta p_{comb}$   
(diesel engine,  $bmep=800$  kPa,  $n=2500$  rpm).

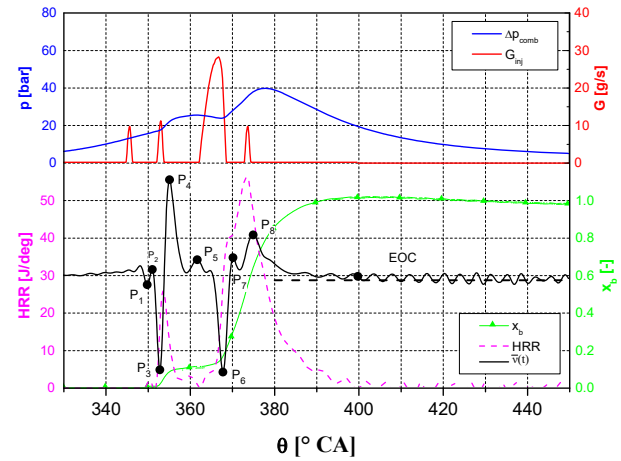


Figure 19. Time frequency analysis  
(diesel engine,  $bmep=500$  kPa,  $n=2500$  rpm).

#### 4. CONCLUSIONS.

The most popular in-cylinder pressure-based direct techniques have been analyzed mathematically, and the hypotheses on which each technique is founded have been pointed out. Different typologies of engines, i.e., gasoline, methane and diesel engines, and different  $bmep$  and  $n$  working conditions have been considered in order to perform a significant comparison of the tested pressure-based direct methods. The predictions obtained using these methods have been compared with the numerical outcomes of two-zone and three-zone combustion models of spark-ignition and compression-ignition engines, respectively.

The start and end of combustion, for spark ignition engines fuelled with either gasoline or methane, are generally identified with satisfactory precision by the pressure-based methods. The simulation of the burned mass fraction time history is also accurate: the best performance has been achieved with the *MLK* procedure, which accurately reproduces the  $x_b$  traces obtained with the two-zone combustion model, but the *PMR* technique also provides very satisfactory results. In particular, the *PMR* and *MLK* procedures have been proved to be based on the same physical law and this justifies the very similar performance that is obtained when these methods are used.

As far as diesel combustion is concerned, the accuracy of the pressure-based direct techniques in the prediction of *SOC*, *EOC* and the  $x_b$  trace generally deteriorates, compared to spark-ignition engines, and the results can become worse as the number of injections increases, because the combustion evolution is more complex, and more sophisticated approaches are required. The only acceptable technique has proved to be the *PDR* method: both the *SOC* and the *EOC* can be evaluated

579 with adequate accuracy for single and double injection strategies. On the other hand, an accurate prediction of the burned  
580 mass time history with pressure-based direct techniques seems to be impracticable for diesel engines.

581 The time frequency analysis of the in-cylinder pressure, in spite of not being able to provide a complete  $x_b$  curve,  
582 represents a useful means of investigation for diesel engines because it can offer a great deal of information on the  
583 combustion evolution as well as on the initial and final instants of the overall process. A home-made numerical tool has  
584 been developed to perform *STFT* tests on the in-cylinder pressure. The mean instantaneous frequency versus crankshaft  
585 angle distribution, which is obtained from the spectrogram of the in-cylinder pressure, allows the *SOC*, the fuel  
586 evaporation phase and the combustion stage pertaining to each injection shot to be identified clearly. Furthermore, the  
587 ignition delays of both cool and hot flames as well as the timings of both premixed and diffusive combustion can be  
588 localized accurately during the piston compression and expansion strokes.

589 The application of the time frequency analysis to the pressure difference between the in-cylinder pressure and the motored  
590 pressure leads to better results than in the case of the in-cylinder pressure signal. In particular, it leads to amplified peaks  
591 in the  $\bar{v}$  distribution and to less uncertainty in the determination of *EOC* than in the case of the in-cylinder pressure.

## 592 5. NOMENCLATURE.

593	$b_{mep}$	brake mean effective pressure
594	$BDC$	bottom dead center
595	$c$	constant specific heat of the polytropic evolution
596	$c_v$	specific heat at constant volume
597	$dp/dt$	pressure derivative with respect to time
598	$E$	energy of the signal
599	$EOC$	end of combustion
600	$f(t)$	time function
601	$F(\omega)$	Fourier transform of function $f$
602	$FFT$	fast Fourier transform
603	$\hat{F}_l(\omega, \tau)$	short time Fourier transform
604	$\hat{F}(\omega)$	integral of the short time Fourier transform with respect to time
605	$FPR$	final pressure ratio
606	$G_{inj}$	injected mass flow-rate
607	$h(t)$	window function

608	$H(\omega)$	Fourier transform of the window function
609	$imep$	indicated mean effective pressure
610	$iV$	engine displacement
611	$\tau$	parameter that defines the position of the window function in the time domain
612	$H_i$	lower heating value
613	$HRR$	heat release rate
614	$IC$	internal combustion engines
615	$m$	marginal function in the time domain; polytropic exponent
616	$m_b$	mass of burned fuel
617	$m_{b,tot}$	total mass of burned fuel
618	$m_c, m_e$	exponents of the polytropic compression and expansion phases
619	$M$	mass of the burning mixture; marginal function in the frequency domain
620	$MFB50$	angle at which 50% of the combustion mixture has burned
621	$MLK$	McCuiston, Lavoie and Kaufmann (procedure)
622	$MPR$	modified pressure ratio
623	$n$	engine speed; number of points in the overlap zone during the shift of the window function
624	$N$	number of intervals in the combustion process; number of points of the window function
625	$P$	time-frequency distribution function
626	$P_1 \div P_9$	noteworthy points along the mean instantaneous frequency time distribution
627	$P_{sp}$	spectrogram
628	$p$	in-cylinder instantaneous pressure
629	$PDR$	pressure departure ratio (algorithm)
630	$PR$	pressure ratio
631	$PRM$	pressure ratio management (procedure)
632	$R$	elastic constant of the gas
633	$RW$	Rassweiler and Withrow (procedure)
634	$SOC$	start of combustion
635	$STFT$	short time Fourier transform
636	$t$	time
637	$t_0$	center of the window function in the time domain
638	$TDC$	top dead center

639	$v$	specific volume
640	$V$	instantaneous volume of the combustion chamber
641	$x_b$	burned gas mass fraction
642	$y$	dimensionless quantity in the <i>MLK</i> algorithm
643	$\gamma$	ratio between the principal specific heats
644	$\delta(t)$	Dirac delta function
645	$\delta q$	heat per unit mass
646	$\delta Q$	heat
647	$\Delta_t$	amplitude of the window function in the time domain
648	$\Delta_v$	amplitude of the window function in the frequency domain
649	$\Delta_\omega$	amplitude of the window function in the circular frequency domain
650	$\Delta p$	pressure difference
651	$\Delta T$	temperature difference
652	$\Delta T_h$	time support of the window function
653	$\Delta t_{ov}$	overlap time interval between consecutive window function positions
654	$\Delta \tau$	time resolution of the short time Fourier transform
655	$\Delta \theta$	crankshaft angle increment
656	$\nu$	frequency
657	$\bar{\nu}(t)$	mean instantaneous frequency
658	$\theta$	crankshaft angle
659	$\Theta$	overall duration
660	$\omega$	circular frequency
661	$\omega_0$	center of the window function in the circular frequency domain
662	$\omega_s$	sampling circular frequency
663		
664	<u>Subscripts</u>	
665	<i>ahrr</i>	apparent heat release rate
666	<i>comb</i>	combustion
667	<i>EOC</i>	at end of combustion
668	<i>f</i>	relative to function <i>f</i>

669	<i>hrr</i>	heat release rate
670	<i>ht</i>	heat transfer
671	<i>max</i>	maximum
672	<i>mot</i>	motored
673	<i>SOC</i>	at start of combustion
674	<i>TDC</i>	corresponding to top dead center

## 675 **6. REFERENCES.**

- 676 [1] Kuratle, R. and Märki, B. Influencing Parameters and Error Sources During Indication on Internal Combustion  
677 Engines. SAE Technical Paper 920233, 1992, doi:10.4271/920233.
- 678 [2] Gatowski JA, Balles EN, Chun KM., Nelson FE. Ekchian JA and Heywood JB, Heat Release Analysis of Engine  
679 Pressure Data. SAE Technical Paper 841359, 1984, doi:10.4271/841359.
- 680 [3] Galloni E. Dynamic knock detection and quantification in a spark ignition engine by means of a pressure based  
681 method”, *Energy Conversion and Management*, 64 (2012), pp. 256-262.
- 682 [4] Carlucci AP, Laforgia D, Motz S, Saracino R, Wenzel SP. Advanced closed loop combustion control of a LTC  
683 diesel engine based on in-cylinder pressure signals. *Energy Conversion and Management*, 77 (2014), pp. 193-207.
- 684 [5] Desantes JM, Galindo J, Guardiola C, Dolz V. Air mass flow estimation in turbocharged diesel engines from in-  
685 cylinder pressure measurement. *Experimental Thermal and Fluid Science*, 34 (2010), pp. 37-47.
- 686 [6] Payri F, Olmeda P, Guardiola C, Martin J. Adaptive determination of cut-off frequencies for filtering the in-cylinder  
687 pressure in diesel engines combustion analysis. *Applied Thermal Engineering*, 31 (2011), pp. 2869-2876.
- 688 [7] Chung J, Oh S, Min K, Sunwoo M. Real-time combustion parameter estimation algorithm for light-duty diesel  
689 engines using in-cylinder pressure measurement. *Applied Thermal Engineering* 60 (2013) 33-43.
- 690 [8] Antonopoulos AK, Hountalas DT. Effect of instantaneous rotational speed on the analysis of measured diesel engine  
691 cylinder pressure data. *Energy Conversion and Management*, 60 (2012), pp. 87-95.
- 692 [9] Rocco V. Dynamic T.D.C. and Thermodynamic Loss Angle Measurement in a D. I. Diesel Engine. SAE Technical  
693 Paper 851546, 1985, doi:10.4271/851546.
- 694 [10] Rocco V. D.I. Diesel Engine In-Cylinder Pressure Data Analysis Under T.D.C. Setting Error. SAE Technical Paper  
695 930595, 1993, doi:10.4271/930595.
- 696 [11] Brunt MFJ and Emtage AL. Evaluation of Burn Rate Routines and Analysis Errors. SAE Technical Paper 970037,  
697 1997, doi:10.4271/970037.
- 698 [12] Bueno AV, Velasquez JA, Milanez LF. A new engine indicating measurement procedure for combustion heat release  
699 analysis. *Applied Thermal Engineering* 29 (2009) 1657–1675, doi:10.1016/j.applthermaleng.2008.07.023.

700 [13] Lujan JM, Bermudez V, Guardiola C, Abbad A. A methodology for combustion detection in diesel engines through  
701 in-cylinder pressure derivative signal. *Mechanical Systems and Signal Processing* 24 (2010) 2261–2275,  
702 doi:10.1016/j.ymssp.2009.12.012.

703 [14] Savva NS, Hountalas DT. Evolution and application of a pseudo-multi-zone model for the prediction of NO<sub>x</sub>  
704 emissions from large-scale diesel engines at various operating conditions. *Energy Conversion and Management*, 85  
705 (2014), pp. 373-388.

706 [15] Rakopoulos CD, Michos CN. Development and validation of a multi-zone combustion model for performance and  
707 nitric oxide formation in syngas fueled spark ignition engine. *Energy Conversion and Management* 49 (2008) 2924–  
708 2938.

709 [16] Marvin CF. Combustion time in the engine cylinder and its effects on engine performance. *NACA Tech Report* 276,  
710 1927.

711 [17] Heywood JB. Internal combustion engine fundamentals. New York: McGraw-Hill; 1988.

712 [18] Shehata MS. Cylinder pressure, performance parameters, heat release, specific heats ratio and duration of  
713 combustion for spark ignition engine. *Energy*, 35(12) (2010), pp. 4710-4725.

714 [19] Rassweiler GM and Withrow L. Motion Pictures of Engine Flames Correlated with Pressure Cards. SAE Technical  
715 Paper 380139, 1938, doi:10.4271/380139.

716 [20] Brunt MFJ, Rai H and Emtage AL. The Calculation of Heat Release Energy from Engine Cylinder Pressure Data.  
717 SAE Technical Paper 981052, 1998, doi:10.4271/981052.

718 [21] McCuiston FD, Lavoie GA and Kauffmann CW. Validation of a Turbulent Flame Propagation Model for a Spark  
719 Ignition Engine. *SAE Transactions*, vol. 86 (1977), pp. 200-223.

720 [22] Krieger RB and Borman GL. The computation of apparent heat release for internal combustion engines. ASME  
721 paper 66-WA/DGP-4 in Proc. Diesel Gas Power, ASME, 1966.

722 [23] Hohenberg GF. Advanced Approaches for Heat Transfer Calculations. SAE Technical Paper 790825, 1979,  
723 doi:10.4271/790825.

724 [24] Woschni GA. Universally Applicable Equation for the Instantaneous Heat Transfer Coefficient in the Internal  
725 Combustion Engine. SAE Technical Paper 670931, 1967, doi:10.4271/670931.

726 [25] Guezennec Y and Hamama W. Two-Zone Heat Release Analysis of Combustion Data and Calibration of Heat  
727 Transfer Correlation in an I. C. Engine. SAE Technical Paper 1999-01-0218, 1999, doi:10.4271/1999-01-0218.

728 [26] Finesso R and Spessa E. A real time zero-dimensional diagnostic model for the calculation of in-cylinder tempera-  
729 tures, HRR and nitrogen oxides in diesel engines. *Energy Conversion and Management*, 79 (2014), pp. 498-510.



730 [27] Baratta M, d'Ambrosio S, Spessa E, Vassallo A. Cycle-resolved detection of combustion start in SI engines by  
731 means of Different in-cylinder pressure data reduction techniques. In: Spring technical conference of the ASME  
732 internal combustion engine division; 2006.

733 [28] Baratta M, d'Ambrosio S, Spessa E, Vassallo A. Analysis of cyclic variability in a Bi-fuel engine by means of a  
734 'cycle-resolved' diagnostic technique. In: Fall technical conference of the ASME ICED; 2005.

735 [29] Bayraktar H. Theoretical investigation of flame propagation process in an SI engine running on gasoline-ethanol  
736 blends. *Renewable Energy* 32 (2007) 758–771.

737 [30] Thurnheer T, Soltic P. The polytropic volume method to detect engine events based on the measured cylinder  
738 pressure. *Control Engineering Practice* 20 (2012) 293–299. doi:10.1016/j.conengprac.2011.11.005.

739 [31] Ball JK, Raine RR, Stone CR. Combustion analysis and cycle-by-cycle variations in spark ignition engine  
740 combustion - Part 1: an evaluation of combustion analysis routines by reference to model data. *Proc. Instn Mech.*  
741 *Engrs, Part D, Journal of Automobile Engineering*, Vol 212, 381-399, 1998. DOI: 10.1243/0954407981526046

742 [32] Asad U, Zheng M. Fast heat release characterization of a diesel engine. *International Journal of Thermal Sciences*  
743 47 (2008) 1688–1700. doi:10.1016/j.ijthermalsci.2008.01.009.

744 [33] Irimescu A, Marchitto L, Merola SS, Tornatore C, Valentino G. Evaluation of different methods for combined  
745 thermodynamic and optical analysis of combustion in spark ignition engines. *Energy Conversion and Management*,  
746 87 (2014), pp. 914-927.

747 [34] Baratta M, Misul D. Development and assessment of a new methodology for end of combustion detection and its  
748 application to cycle resolved heat release analysis in IC engines. *Applied Energy* 98 (2012) 174–189.  
749 <http://dx.doi.org/10.1016/j.apenergy.2012.03.020>.

750 [35] Payri F, Luján JM, Guardiola C, Rizzoni G. Injection diagnosis through common-rail pressure measurement. *Proc.*  
751 *Instn Mech. Engrs, Part D, Journal of Automobile Engineering*, Vol 220, 347-357, 2006. DOI:  
752 10.1243/09544070JAUTO34.

753 [36] Samimy B, Rizzoni G. Engine Knock Analysis and Detection Using Time-Frequency Analysis. SAE Technical  
754 Paper 960618, 1996, doi:10.4271/960618.

755 [37] Cohen L. Time-Frequency Signal Analysis. McGraw Hill, 1995.

756 [38] Feng Z, Liang M, Chu F. Recent advances in time-frequency analysis methods for machinery fault diagnosis: a  
757 review with application examples. *Mechanical Systems and Signal Processing*, 38 (2013), pp. 165-205.

758 [39] Stankovic L, Djurovic I, Stankovic S, Simeunovic M, Djukanovic S, and Dakovic M. Instantaneous frequency in  
759 time-frequency analysis; enhanced concepts and performance of estimation algorithms. *Digital Signal Processing*,  
760 35 (2014), pp. 1-13.

761 [40] Catania AE, d'Ambrosio S, Mittica A and Spessa E. Experimental investigation of fuel consumption and exhaust  
762 emissionsof a 16V pent-roof engine fueled by gasoline and CNG. SAE Technical Paper No. 2001-01-1191.

763 [41] Harrington JA. Application of a new combustion analysis method in the study of alternate fuel combustion and  
764 emission characteristics. Symposium on Future Automotive Fuels, 1975, GM Research Lab, Michigan, USA.

765 [42] Sellnau M, Matekunas FA, Battiston PA, Chang CF, Lancaster DR. Cylinder-Pressure-Based Engine Control Using  
766 Pressure-Ratio-Management and Low-Cost Non-Intrusive Cylinder Pressure Sensors. SAE Technical Paper No.  
767 2000-01-0932.

768 [43] d'Ambrosio S, Spessa E, Vassallo A, Ferrera M, Peletto C. Experimental investigation of fuel consumption, exhaust  
769 emissions and heat release of a small-displacement turbocharged *CNG* engine. SAE Technical Paper 2006-01-0049.

770 [44] d'Ambrosio S, and Ferrari A. Potential of multiple injection strategies in improvement of diesel engine emissions  
771 and performance. Part 1: double pilot injection. Submitted to *Applied Energy* for publication.