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Wiener-Hopf Formulation of an Unaligned PEC Wedge over a Stratification

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***Abstract*— This paper deals with the problem of evaluating the electromagnetic field of a perfect electrical conductor (PEC) wedge over stratified media. The particular case wherein one face of the wedge is perpendicular to the direction of stratification has been previously considered and solved by using the generalized Wiener-Hopf technique [1]. In this paper the directions of the two faces of the wedge are arbitrary. We formulate the problem in terms of generalized Wiener-Hopf equations (GWHE) and we propose a solution method based on the reduction of the GWHE to Fredholm integral equations of second kind.**

I. Introduction

In this paper we consider the problem constituted by the evaluation of the electromagnetic field in the physical structure shown in Fig.1. A plane wave with direction $\varphi=\varphi_o$ ($0\leq\varphi_o \leq\Phi_a$) is incident on a PEC wedge located in the half-space y>0. The faces of the PEC wedge are defined by $\varphi=\Phi_a$ (face a) and $\varphi=\pi-\Phi_b$ (face b). The half-space y<0 is constituted by a stratification of media with permittivity ε_i and permeability μ_i ($i=0,1,\ldots$). For the sake of simplicity the direction of the incident plane wave is assumed to be parallel to the plane (x,y). The skew incidence case does not introduce any conceptual difficulty but doubles the number of equations to be solved. The literature on similar problems is apparently very poor. Particular cases of a wedge immersed in a stratified medium were studied in [2-3] by using the Uniform Theory of Diffraction (UTD). However the application of this method is limited to edges not close to the stratified regions.

In this paper we propose the procedure to solve the problem via the so-called axial spectra typically encountered in angular problems studied via generalized Wiener-Hopf equations (GWHE) [4-10]: $V_+(\eta)$, $I_+(\eta)$, $V_{\pi+}(\eta)$, $I_{\pi+}(\eta)$ see section 2 for definitions. Using the axial spectra it is possible to evaluate the fields everywhere in the space domain using GTD and UTD. This paper introduce how to deduce the GWHE and how to solve them using Fredholm factorization [11-14] via numerical discretization [15]. In this work, the notation and the terminology of [1] are applied.

I. Fundamental Definitions

With reference to Fig.1, we consider time harmonic electromagnetic fields with a time dependence specified by the

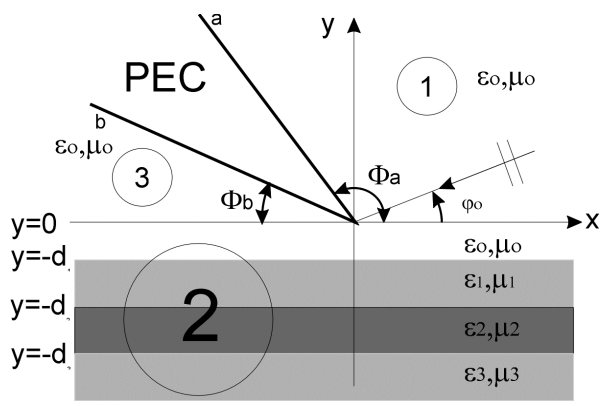

Fig.1 The physical structure of the problem

factor $e^{j\omega t}$ which is omitted. Cartesian coordinates (x,y,z) as well as polar coordinates (ρ,φ,z) are used. The incident field is constituted by the E-polarized plane wave with longitudinal component $E_z^i = E_o e^{jk_o\rho\cos(\varphi-\varphi_o)}$ where $k_o=\omega\sqrt{\mu_o\varepsilon_o}$.

The following transforms (1) and (2) in the η-plane assume a fundamental role.

$$\begin{cases} V_+(\eta,\varphi)=\int_0^\infty E_z(\rho,\varphi)e^{j\eta\rho}d\rho \\ I_+(\eta,\varphi)=\int_0^\infty H_\rho(\rho,\varphi)e^{j\eta\rho}d\rho \end{cases}, \quad (y>0, 0\leq\varphi\leq\Phi_e) \quad (1)$$

$$\begin{cases} v_\eta(y)=\int_{-\infty}^\infty E_z(x,y)e^{j\eta x}dx \\ i_\eta(y)=\int_{-\infty}^\infty H_x(x,y)e^{j\eta x}dx \end{cases}, \quad (y<0) \quad (2)$$

In particular we define the spectral unknowns: $V_+(\eta)= V_+(\eta,0)$, $I_+(\eta)= I_+(\eta,0)$, $V_{\pi+}(\eta)= V_+(\eta,\pm\pi)$, $I_{\pi+}(\eta)= I_+(\eta,\pm\pi)$, $V_-(\eta)= V_{\pi+}(-\eta)$, $I_-(\eta)=-I_{\pi+}(-\eta)$.

From here on, all these quantities will be called the spectra. The axial spectra are the spectral unknowns (1) and (2) evaluated along $\varphi=0$ and $\varphi=\pi$ directions.

In Fig.1 we distinguish three regions: the angular region 1 ($y>0$, $0\leq\varphi\leq\Phi_a$), the stratified region 2 ($y<0$) and the angular region 3 ($y>0$, $\pi-\Phi_b\leq\varphi\leq\pi$).

II. THE GWHE AND SOLUTION PROCEDURE

We observe that the axial spectra $V_+(\eta)$, $I_+(\eta)$, $V_{\pi+}(\eta)$, and $I_{\pi+}(\eta)$ provide the electromagnetic field everywhere as already done in [1]. The system of GWHE is constituted by three equations and are given in terms of the axial spectra and the spectra on the face a and b. The first equation of this system relates the spectral quantities in section $y=0$

$$Y(\eta)v_\eta(0) = -i_\eta(0) \tag{3}$$

where $Y(\eta)$ is obtained from circuital consideration presented in [1]. Considering the definition of the axial spectra, (3) is rewritten as

$$Y(\eta)(V_{\pi+}(-\eta)+V_+(\eta)) - I_{\pi+}(-\eta)+I_+(\eta) = 0 \tag{4}$$

Substituting $\eta \to -\eta$ in (4), taking into account equation (11) of [1] for the angular region and enforcing that the voltage spectrum is vanishing on the face a of the wedge, the spectral unknowns satisfy the following equation:

$$Y_c(\eta)V_+(\eta) - I_+(\eta) = -I_+(-m_a,\Phi_a) = -I_{a+}(-m_a) \tag{5}$$

where $Y_c(\eta)=\xi/\omega\mu_o$, $\xi(\eta)=\sqrt{(k_o^2-\eta^2)}$, $m_a=-\eta\cos\Phi_a+\xi\sin\Phi_a$. Similarly for region 3 we get the equation

$$Y_c(\eta)V_{\pi+}(\eta) + I_{\pi+}(\eta) = -I_+(-m_b,\pi-\Phi_b) = I_{b+}(-m_b) \tag{6}$$

where $m_b=-\eta\cos\Phi_b+\xi\sin\Phi_b$.

In order to solve the GWHE (5) and (6) together with (4) we need to highlight the source term characterized by poles in the spectra due to the geometrical optic field contributions. For illustrative purposes, we consider that $\varphi_o>\pi/2>\Phi_a/2$, $\Phi_a>\pi/2$. Taking into account simple geometrical considerations, in this case only the function $I_{a+}(-m)$, is not conventional. Consequently only in (5) we need to extract the source term. As discussed in [1], (5) can be rewritten in the not homogeneous form:

$$Y_c V_+(\eta) - I_+(\eta) = -I_{a+}^{d}(-m) + \frac{R_m}{m-k\cos(\Phi_a-\varphi_o)} \tag{7}$$

where the new unknown $I_{a+}^{d}(-m)$ is conventional and $R_m=2j\sin(\Phi_a-\varphi o)E_o/Z_o$. Although the system GWHE (7), (6) and (4) cannot be solved in closed form, the reduction to Fredholm integral equation (FIE) is always possible. FIE are simply numerically solvable by yielding efficient semi-analytical solution of the GWHE. To simplify this approach, it can be convenient to resort to suitable mappings. For the GWHE (7) and (6), we introduce respectively the complex planes α and β through:

$$\eta = -k\cos\left[\frac{\Phi_a}{\pi}\arccos(-\frac{\alpha}{k})\right],\ \eta = -k\cos\left[\frac{\Phi_b}{\pi}\arccos(-\frac{\beta}{k})\right].$$

The procedure to reduce the GWHE to Fredholm equations is discussed in several papers [12-14]. By using cumbersome analytical manipulations that for reason of space are here omitted we get a system of coupled FIEs in the unknowns $V_+(\eta)$ and $V_{\pi+}(\eta)$ that can be solved by numerical quadrature. Alternatively to the Fredholm formulation, the GWHE can be reduced to difference equations in the w-plane by introducing the mapping $\eta=-k\cos w$, useful to obtain analytical continuation for the approximate axial spectra. The complete procedure and numerical validation will be presented at the conference and proposed in [16].

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