A data-driven approach to nonlinear braking control

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Carlo Novara[1], Simone Formentin[2], Sergio M. Savaresi[2], Mario Milanese[3],

***Abstract*— In modern road vehicles, active braking control systems are crucial elements to ensure safety and lateral stability. Unfortunately, the wheel slip dynamics is highly nonlinear and the on-line estimation of the road-tire conditions is still a challenging open research problem. These facts make it difficult to devise a braking control system that is reliable in any situation without being too conservative. In this paper, we propose the *Data-Driven Inversion Based Control* (D^2-IBC) approach to overcome the above issues. The method relies on a two degrees of freedom architecture, with a linear controller and a nonlinear controller in parallel, both designed using only experimental data. The effectiveness of the proposed approach is shown by means of an extensive simulation campaign.**

I. INTRODUCTION

Anti-lock Braking System (ABS), have recently become a standard for cars [13]. Among all, wheel slip controllers have attracted great interest as they allow to recast the ABS design issue into a more classical feedback regulation problem.

In the general case, the wheel slip dynamics is strongly nonlinear and uncertain, mainly due to the lack of knowledge of the road-tire conditions, whose on-line estimation is still a challenging open research problem [16], [24]. Therefore, to ensure that safety is really guaranteed in any possible condition, braking systems must be fine tuned directly on the vehicle, by means of long (and expensive) road tests. This problem becomes even more complicated when dealing with motorcycles or electric vehicles, which have different dynamic properties and possibly also different actuation architectures (e.g., regenerative braking) [15], [2].

The above discussion has justified the production of a considerable number of contributions on robust (but conservative) control of braking systems, see, e.g., [25], [28]. As far as the authors are aware, all the existing methods strongly rely on a simplicistic physical description of the system dynamics, that is the well-known quarter car model denoting a single corner of the vehicle [26]. It follows that such approaches suffer from two main drawbacks: (i) the effects due to the other dynamics of the vehicle, including the coupling among the vehicle axes, are neglected. This choice might be a problem for those vehicles where the load transfer effect is significant, like the two-wheeled ones; (ii) since the system cannot adapt to different road conditions, but must be acceptable for all possible working points, the existing robust approaches generally turn out to be very conservative.

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In this work, our aim is to solve the braking control design problem *using experimental data only* - to avoid undermodeling issues - and such that *the braking system adapts to different operating conditions without the need to estimate the friction curve*. To this goal, we introduce the *Data-Driven Inversion Based Control* (D^2-IBC) method, a nonlinear fully data-driven design technique. The controller obtained by means of this method relies on two different degrees of freedom, to share the advantages of nonlinear controllers - whose action depends on the operating conditions - and the peculiarities of linear systems (which allow us to guarantee zero steady-state error, effective noise rejection and other important properties).

Direct design of feedback controllers from data is not a new idea in the systems and control literature. The first contribution in this field dates back to the Ziegler and Nichols approach [29] in 1942. Since then, many different approaches have appeared, ranging from adaptive methods [14] to iterative off-line methods [10], [12], until noniterative off-line techniques [1], [9], [7], [18].

However, the D^2-IBC approach is different from the existing data-driven philosophies in that it relies on a completely different architecture (a nonlinear controller and a linear controller in parallel) and gives effective stability and performance guarantees [19]. The nonlinear controller is designed through the NIC (Nonlinear Inversion Control) approach presented in [20], the linear controller is designed using a suitably modified version of the VRFT (Virtual Reference Feedback Tuning) method [1]. The design of both the controllers is carried out from data and is based on system inversion, hence the name D^2-IBC.

A fully data-driven approach for braking control system has been already presented in [5]. Notice that, unlike the method presented herein, the method in [5] is based on the empirical design of the nonlinear compensator parameter and is valid only for systems with stable zero dynamics. For completeness, the performance of the approach in [5] is compared with that of D^2-IBC in the simulation section.

II. PROBLEM STATEMENT

Consider the single-corner system depicted in Figure 1. This system is usually considered as representative of the dynamics of a quarter car during braking maneuvering. Notice that such a system description will be used only in this section to describe the application at hand and its main critical aspects, whereas it will never be employed in the rest of the paper for control design. In fact, the whole design procedure is not based on any model structure assumption, but on experimental data only.

The physical model of the single-corner system is given by

$$\begin{cases} J\dot{\omega} = rF_x - T_b, \\ m\dot{v} = -F_x, \end{cases} \tag{1}$$

where ω [rad/s] is the angular speed of the wheel, v [m/s] is the longitudinal speed of the vehicle body, T_b [Nm] is the braking torque, F_x [N] is the longitudinal tire-road contact force and $J\,[\mathrm{kg\,m^2}]$, m [kg] and r [m] are the rotational inertia of the wheel, the quarter-car mass and the wheel radius, respectively.

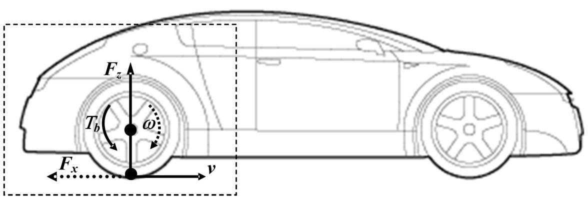

Fig. 1. Schematic view of the single-corner system.

The nonlinear behavior of the system is hidden in the expression of F_x, which depends on the state variables v and ω. The most general expression of F_x depends on a large number of features of the road, tire, and suspension. Generally, it can be well approximated by $F_x = F_z\mu(\lambda, \eta)$, where F_z is the vertical force at the tire-road contact point; $\lambda = (v - \omega r)/v$ is the longitudinal slip when braking, and η is a set of parameters which characterize the shape of the static function $\mu(\lambda, \eta)$ [22]. From now on, for simplicity, the dependency of $\mu(\lambda, \eta)$ on η is omitted, and the tire-road friction conditions are indicated with $\mu(\lambda)$.

By employing the expression of F_x introduced above and substituting $\dot{\lambda} = -\frac{r}{v}\dot{\omega} + \frac{r\omega}{v^2}\dot{v}$ and $\omega = \frac{v}{r}(1-\lambda)$ into the first of (1), it yields

$$\begin{cases} \dot{\lambda} = -\frac{1}{v}[\frac{(1-\lambda)}{m} + \frac{r^2}{J}]F_z\mu(\lambda) + \frac{r}{vJ}T_b, \\ m\dot{v} = -F_z\mu(\lambda). \end{cases} \tag{2}$$

In the ideal situation where the road conditions are perfectly known at each time instant, the goal of a brake-by-wire system would be to regulate the slip λ to make it correspond to the peak of the current (dynamic) friction curve. Unfortunately, estimating such a curve on-line from data is still an open research problem, see e.g. [16], [24]. Therefore, the reference slip for a brake-by-wire system is usually selected as a constant (low) value. This choice is further motivated by the observation that high values of the longitudinal slip correspond to low lateral adherence, that is, to unsafe maneuvering. Even with this reference selection, the lack of knowledge about the shape of the friction curve still remains a critical problem, as this feature significantly affects the dynamics of the system.

In what follows, the D^2-IBC approach is introduced to address the above control problem. The motivation (and the goal) of the D^2-IBC method is to design a wheel slip controller from experimental measurements without the need of identifying the dynamic tire-road interaction and, at the same time, without being too conservative. Notice that, since the approach is entirely based on data, also the vehicle dynamics neglected in the single-corner system could now be taken into account in any real vehicle setup.

III. THE D^2-IBC APPROACH

A. Notation

A column vector $x \in \mathbb{R}^{n_x \times 1}$ is denoted as $x = (x_1, \ldots, x_{n_x})$. A row vector $x \in \mathbb{R}^{1 \times n_x}$ is denoted as $x = [x_1, \ldots, x_{n_x}] = (x_1, \ldots, x_{n_x})^\top$, where $\top$ indicates the transpose.

A discrete-time signal (i.e. a sequence of vectors) is denoted with the bold style: $\boldsymbol{x} = (x_1, x_2, \ldots)$, where $x_t \in \mathbb{R}^{n_x \times 1}$ and $t = 1, 2, \ldots$ indicates the discrete time; $x_{i,t}$ is the ith component of the signal $\boldsymbol{x}$ at time t.

A regressor, i.e. a vector that, at time t, contains n present and past values of a variable, is indicated with the bold style and the time index: $\boldsymbol{x}_t = (x_t, \ldots, x_{t-n+1})$.

The ℓ_p norms of a vector $x = (x_1, \ldots, x_{n_x})$ are defined as

$$\|x\|_p \doteq \begin{cases} \left(\sum_{i=1}^{n_x} |x_i|^p\right)^{\frac{1}{p}}, & p < \infty, \\ \max_i |x_i|, & p = \infty. \end{cases}$$

The ℓ_∞ norm is also used to denote the absolute value of a scalar: $\|x\|_\infty \equiv |x|$ for $x \in \mathbb{R}$.

The ℓ_p norms of a signal $\boldsymbol{x} = (x_1, x_2, \ldots)$ are defined as

$$\|\boldsymbol{x}\|_p \doteq \begin{cases} \left(\sum_{t=1}^{\infty} \sum_{i=1}^{n_x} |x_{i,t}|^p\right)^{\frac{1}{p}}, & p < \infty, \\ \max_{i,t} |x_{i,t}|, & p = \infty, \end{cases}$$

where $x_{i,t}$ is the ith component of the signal $\boldsymbol{x}$ at time t. These norms give rise to the well-known ℓ_p Banach spaces.

B. D^2-IBC setting

Consider a nonlinear discrete-time SISO system in regression form:

$$y_{t+1} = g(\boldsymbol{y}_t, \boldsymbol{u}_t, \boldsymbol{\xi}_t) \tag{3}$$

$$\begin{aligned} \boldsymbol{y}_t &= (y_t, \ldots, y_{t-n+1}) \\ \boldsymbol{u}_t &= (u_t, \ldots, u_{t-n+1}) \\ \boldsymbol{\xi}_t &= (\xi_t, \ldots, \xi_{t-n+1}) \end{aligned}$$

where $u_t \in U \subset \mathbb{R}$ is the input, $y_t \in \mathbb{R}$ is the output,$\xi_t \in \Xi \subset \mathbb{R}^{n_\xi}$ is a disturbance including both process and measurement noises, and n is the system order. U and Ξ are compact sets. In particular, $U \doteq [\underline{u}, \overline{u}]$ accounts for input saturation.

Suppose that the system (3) is unknown, but a set of measurements is available:

$$\mathcal{D} \doteq \{\tilde{u}_t, \tilde{y}_t\}_{t=1-L}^{0} \tag{4}$$

where $\tilde{u}_t$ and $\tilde{y}_t$ are bounded for all $t = 1 - L, \ldots, 0$. The tilde is used to indicate the input and output samples of the data set (4).

Let $\mathcal{Y}^0 \subseteq \mathbb{R}^n$ be a set of initial conditions of interest for the system (3) and, for a given initial condition $\boldsymbol{y}_0 \in \mathcal{Y}^0$, let $\mathcal{Y} \equiv \mathcal{Y}(\boldsymbol{y}_0) \subseteq \ell_\infty$ be a set of output sequences of interest.

The aim is to control the system (3) in such a way that, starting from any initial condition $\boldsymbol{y}_0 \in \mathcal{Y}^0$, the system output sequence $\boldsymbol{y} = (y_1, y_2, \ldots)$ tracks any reference sequence

$\boldsymbol{r} = (r_1, r_2, \ldots) \in \mathcal{Y}(\boldsymbol{y}_0)$. The set of all possible disturbance sequences is defined as $\varXi \doteq \{\boldsymbol{\xi} = (\xi_1, \xi_2, \ldots) : \xi_t \in \Xi, \forall t\}$.

To accomplish this task, we use the feedback control structure depicted in Figure 2, where S is the system (3), K^{nl} is a nonlinear controller, K^{lin} is a linear controller, $r_t \in Y$ is the reference, and $Y \subset \mathbb{R}$ is a compact set where the output sequences of interest lie.

K^{nl} is used to stabilize the system (3) around the trajectories of interest, while K^{lin} allows us to further reduce the tracking error (especially in steady-state conditions). K^{nl} is designed through the NIC (Nonlinear Inversion Control) approach presented in [20], K^{lin} is designed using a suitably modified version of the VRFT (Virtual Reference Feedback Tuning) method [1], [4]. As shown in Sections III-C and III-D, the design of both the controller is based on system inversion. Sufficient conditions for the stability of this control system are derived in [19].

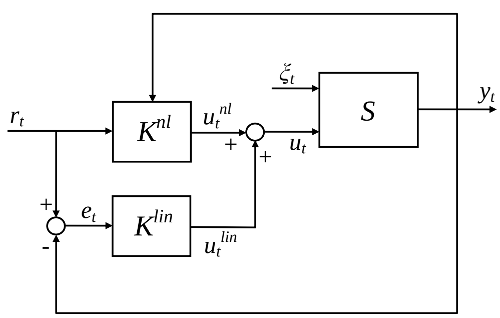

Fig. 2. Feedback control system.

C. Nonlinear controller

The nonlinear controller design is based on the NIC method presented in [20]. The first step of this method is to identify from the data (4) a model for the system (3) of the form

$$\begin{aligned} \hat{y}_{t+1} &= f(\boldsymbol{y}_t, \boldsymbol{u}_t) \equiv f(\boldsymbol{q}_t, u_t) \\ \boldsymbol{q}_t &= (y_t, \ldots, y_{t-n+1}, u_{t-1}, \ldots, u_{t-n+1}) \end{aligned} \tag{5}$$

where u_t and y_t are the system input and output, and $\hat{y}_t$ is the model output. For simplicity, the model is supposed of the same order as the system but this choice is not necessary: all the results presented in the paper hold also when the model and system orders are different. Indications on the choice of the model order are given in [20].

A parametric structure is taken for the function f:

$$f(\boldsymbol{q}_t, u_t) = \sum_{i=1}^{N} \alpha_i \phi_i(\boldsymbol{q}_t, u_t) \tag{6}$$

where ϕ_i are basis functions and α_i are parameters to be identified. The basis function choice is in general a crucial step, [27], [11], [21]. In the present D^2-IBC approach, polynomial functions are used, with the following motivations: (i) polynomials have been proved to be effective approximators in a huge number of problems; (ii) as shown in [20], they allow a "fast" controller evaluation.

The identification of the parameter vector $\alpha \doteq (\alpha_1, \ldots, \alpha_N)$ can be carried out by means of convex optimization, [8].

Once a model of the form (5) has been identified, the command action u_t^{nl} of the controller K^{nl} is obtained by the on-line inversion of this model. In the NIC approach, the following optimization problem is solved to perform such an inversion:

$$\begin{aligned} u_t^{nl} \;=\; & \arg\min_{\mathfrak{u} \in U} J(\mathfrak{u}) \\ & \text{subject to } \mathfrak{u} \in U. \end{aligned} \tag{7}$$

The objective function is given by

$$J(\mathfrak{u}) = \frac{1}{\rho_y}\left(r_{t+1} - f(\boldsymbol{q}_t, \mathfrak{u})\right)^2 + \frac{\mu}{\rho_u}\mathfrak{u}^2 \tag{8}$$

where $\rho_y \doteq \|(\tilde{y}_{1-L}, \ldots, \tilde{y}_0)\|_2^2$ and $\rho_u \doteq \|(\tilde{u}_{1-L}, \ldots, \tilde{u}_0)\|_2^2$ are normalization constants computed from the data set (4), and $\mu \geq 0$ is a design parameter, allowing us to determine the trade-off between tracking precision and command activity. See [20] for indications on the choice of μ. This inversion technique is similar to the one in [17], where a Set Memebrship model is used instead of (6).

Note that the objective function (8) is in general non-convex. Moreover, the optimization problem (7) has to be solved on-line, and this may require a long time compared to the sampling time used in the application of interest. In order to overcome these two relevant problems, the method presented in [20] can be used, allowing a very efficient computation of the optimal command input u_t^{nl}.

The nonlinear controller K^{nl} to use in the feedback system of Figure 2 is fully defined by the control law (7).

D. Linear controller

The linear controller K^{lin} is defined by the extended PID (Proportional Integral Derivative) control law

$$u_t^{lin}(\theta) = u_{t-1}^{lin}(\theta) + \sum_{i=0}^{n_\theta} \theta_i e_{t-i} \tag{9}$$

where $e_t = r_t - y_t$ is the tracking error, n_θ is the controller order and the θ_i's denote the controller parameters. For $n_\theta = 1$ and $n_\theta = 2$, the standard PI and PID controller are selected, respectively.

Notice that in the considered setting, where most of the information about the system is that inferrable from data, finding a good control-oriented model of the error system, i.e. the system describing the relationship between u_t^{lin} and y_t for the nonlinear loop with S and K^{nl}, is not an easy task. Therefore, in this paper, the Virtual Reference Feedback Tuning (VRFT) method originally developed in [1] and recently extended in [9] is employed. Specifically, the linear controller K^{lin} is found from data as the controller minimizing the mismatch between the closed-loop system and a user-defined ideal model M. Notice that the VRFT strategy already proved its effectiveness in some real-world applications, e.g. [23], [6].

IV. D^2-IBC FOR BRAKING SYSTEMS

In this section, we show how the issues in braking control, namely the nonlinear characteristics of the tire-road dynamics and the uncertainty about the unmodeled parts, can be overcome using the D^2-IBC approach proposed in the previous sections. The model (2) is considered as the unknown "true" system to control, where the Pacejka model (see e.g. [22]) is used in the longitudinal tire-road contact force. The goal is to control the "true" system in such a way λ tracks suitable reference values.

A. Controller design

A set of data, to use for control design, has been generated from the "true" system, assumed to have a quarter-car mass of 250 kg and a moment of inertia for the wheel of 1 kgm^2. Different road and vehicle conditions have been considered:

- road: dry or wet (these two situations correspond to different parameter values of the Pacejka model);
- braking torque T_b: random or chirp signal with amplitude 370 Nm and mean $M_{T_b} \in \{120, 200, 280\}$;
- initial vehicle speed (i.e. the speed of the vehicle just before the braking action starts): $v_0 \in \{14, 28\}$ m/s.

For each combination of these road/vehicle conditions, 1000 input-output samples have been obtained by simulation of the "true" system (2), using a sampling time $T_s = 0.005$ s. The design set is thus composed of 24000 data (the total number of combinations is 24):

$$\begin{array}{l} \mathcal{D} \doteq \cup_{i=1}^{24} \mathcal{D}_i^e \\ \mathcal{D}_i^e \doteq \{\tilde{u}_t, \tilde{y}_t\}_{t=-999}^{0} \end{array} \tag{10}$$

where $\tilde{u}_t = T_b(tTs)$, $\tilde{y}_t = \lambda(tTs)+\xi_t$ and ξ_t is a white Gaussian noise with zero mean and a noise to signal standard deviation ratio of 0.03.

From the data set (10), several controllers have been designed:

1) NL1: Designed following the approach of Section (III-C). The basis functions have been generated as products of univariate polynomials with degree 4, yielding a set of 91 functions with maximum degree 8. Then, the optimization problem has been solved, where η has been chosen on the basis of the trade-off between accuracy and sparsity: a satisfactory accuracy has been obtained whit about 60 functions, while a degradation accuracy has been observed for lower numbers of functions.
2) NL2: Designed using the flatness-based approach of [3]. This approach relies on the inversion of a so-called phenomenological model. The parameters of this model have been chosen through a trial and error procedure in order to obtain the best tracking performance considering the identification data set.
3) PI1: Designed following the approach of Section (III-D), posing $\tilde{\boldsymbol{\delta u}} = \tilde{\boldsymbol{u}} - \tilde{\boldsymbol{u}}^{nl}$, where $\tilde{\boldsymbol{u}}^{nl}$ is the output sequence provided by the controller NL1. The reference model M as been selected as a first order system with a unitary steady-state gain and a bandwidth of 18 rad/s, i.e. about 2.86 Hz. This is a reasonable choice for the application at hand, as it yields an efficient braking action.
4) PI2: Designed following the approach of Section (III-D), posing $\tilde{\boldsymbol{\delta u}} = \tilde{\boldsymbol{u}} - \tilde{\boldsymbol{u}}^{nl}$, where $\tilde{\boldsymbol{u}}^{nl}$ is the output sequence provided by the controller NL2. The reference model for PI2 is the same used for PI1.
5) PI3: Designed following the approach of Section (III-D), posing $\tilde{\boldsymbol{\delta u}} = \tilde{\boldsymbol{u}}$. The reference model used for PI1 and PI2 has been used also for PI3.
6) iPI: Parallel connection of NL2 and PI2 (according to the approach proposed in [5]).
7) D^2-IBC: Parallel connection of NL1 and PI1 (corresponding to the controller K in Section III).

Other controllers have been designed similarly, considering a PID structure for the linear part instead of a PI one. However, no significant improvements have been observed in comparison with the controllers using a simpler PI structure. These PID-based controllers are thus not considered in the reminder of the paper.

B. Performance assessment

The designed controllers have been tested through extensive simulations. Different road and vehicle conditions have been considered:

- road: dry or wet (these two situations correspond to different parameter values of the Pacejka model);
- reference: step signal with amplitude $r \in \{5\%, 10\%\}$;
- quarter-car mass: $m \in \{200, 250, 300\}$ kg;
- initial vehicle speed (i.e. the speed of the vehicle just before the braking action starts): $v_0 \in \{8, 21, 33\}$ m/s (corresponding to $\{30, 75, 120\}$ km/h).

For each combination of these road/vehicle conditions, 7 simulations have been performed where, in each test, one of the above 7 controllers is applied to the "true" system. In these simulations, the feedback output (i.e. λ_t) has been corrupted by a white Gaussian noise with zero mean and a noise to signal standard deviation ratio of 0.03.

To evaluate the controller performance, two indexes have been considered: the RMS (Root Mean Square) tracking error and the maximum overshoot $\hat{s}$ of the output. For a signal of length L, these quantities are defined as

$$\begin{array}{l} RMS \doteq \left\|\boldsymbol{r}^M - \boldsymbol{y}\right\|_2 / \sqrt{L} \\ \hat{s} \doteq \left\|(\boldsymbol{y} - y_1) / (r - y_1)\right\|_\infty - 1 \end{array}$$

where $\boldsymbol{y} = (y_1, \ldots, y_L)$ is the output sequence and $\boldsymbol{r}^M = (r_1^M, \ldots, r_L^M)$ is the output of the reference model M (as explained in Section III-D, M is the model to which the controlled system should be similar).

Notice that the two above indeces give complementary information about the performance of a braking control system. The RMS tracking error is a measure of the average behavior of the closed-loop system and is related to model mismatch. Instead, the maximum overshoot $\hat{s}$ of the output denotes the quality of the transient response, which is critical in terms of safety. As a matter of fact, a step response with low RMS but large overshoot would still temporarily lead the vehicle to a condition with low lateral adherence.

This fact could easily affect the stability of the vehicle and therefore needs to be avoided.

The obtained results are shown in Tables I and II for all the considered road/vehicle conditions and for the controllers 3-7. The bold style indicates a "bad" performance, i.e. an RMS value significantly larger than those obtained by the other controllers or an overshoot larger than 15%. The output signals observed in some of the 216 simulations are shown in Figures 3-4. In particular, Figure 3 illustrates the case of a reference λ on a wet road in the linear zone of the friction curve, where linear and nonlinear controllers yield comparable results. Instead, Figure 4 depicts the case of a reference slip close to the peak of the friction curve. Notice that, in this situation, the behavior of the system becomes strongly nonlinear and a linear controller (or a nonlinear controller without proper guarantees) may yield unacceptable performance. The results obtained in a μ-split test (i.e. a situation where the friction coefficient changes while the vehicle is traveling) are finally shown in Figure 5. Also in this case, the D^2-IBC approach significantly outperforms the other solutions.

The results provided by the first two controllers (i.e. NL1 and NL2) are not shown since these controllers do not include an integrator and thus cannot give a high precision in steady-state. It can be noted that the steady-state performance of the NL1 controller could have been improved using design data recorded in steady-state. However, this operation is not needed since, as discussed in Section III, the steady state performance can be obtained by simply adding the linear controller.

From the results summarized in the tables and figures, it can be concluded that the D^2-IBC controller is the only one to provide a high performance for all the road/vehicle conditions, proving to be able to deal with all the nonlinearities involved in the dynamics of a braking system. As expected, the tracking accuracy of any linear controller alone is fair or even good in certain conditions but quite low in other conditions. A similar comment holds for the iPI controller which, for large reference values, yields a large overshoot. This is not surprising, as NL2 is tuned empirically on a different data set, and therefore it might work well in some conditions (see [5]), but no guarantees can be given on the performance of the step response. An interesting fact is that the D^2-IBC controller is quite robust with respect to the road conditions, the quarter-car mass and the involved input signals. Recall that the input signal types used to obtain the design data are completely different from those occurring in the control testing.

V. CONCLUSIONS

In this paper, we proposed a data-driven approach based on system inversion, called D^2-IBC, for active braking control system design. The method relies on a two degrees of freedom scheme, with a nonlinear controller and an extended PID, both designed using experimental data, running in parallel. Through an extensive simulation study, we demonstrated that the nonlinear controller is able to stabilize the system around a given road-tire condition (without explicitely estimating it), whereas the linear controller takes the steady-state tracking error and the quality of the model-matching reponse into account. The achieved results was also shown to outperform simple linear control and intelligent PID control, especially for reference slip values close to the peak of the friction curve, where the behavior of the system is highly nonlinear. Future work will be dedicated to the validation of the proposed approach on a real vehicle setup.

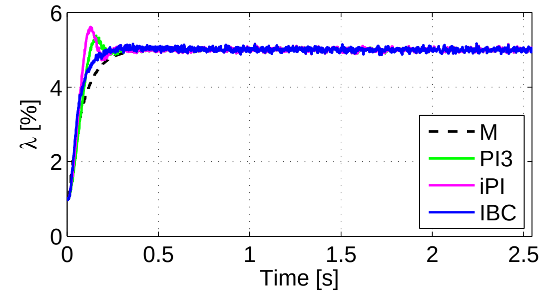

Fig. 3. Control performance. Wet road, $\lambda^{ref} = 5\%$, $m = 250$ kg, $v_0 = 21$ m/s.

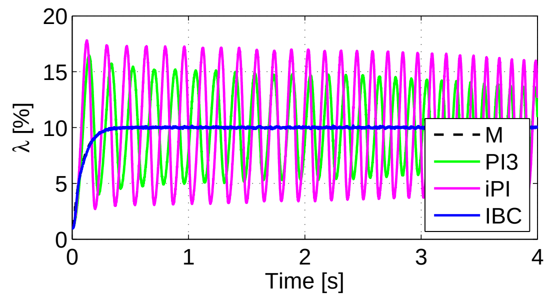

Fig. 4. Control performance. Wet road, $\lambda^{ref} = 10\%$, $m = 300$ kg, $v_0 = 120$ m/s.

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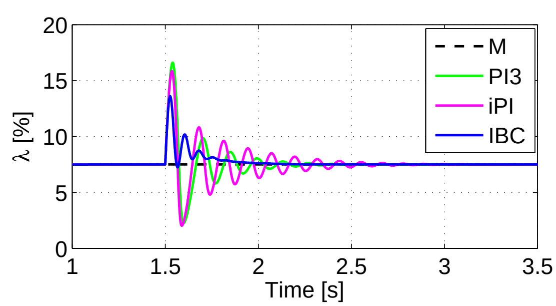

Fig. 5. μ-split (change at 1.5 s from dry to wet road). $\lambda^{ref} = 7.5\%$, $m = 250$ kg, $v_0 = 33$ m/s.

TABLE I

TRACKING RESULTS FOR DRY ROAD. BOLD: “BAD” PERFORMANCE (I.E. “LARGE” RMS OR $\hat{s} > 15\%$).

	λ^{ref}	5%									10%								
	mass (kg)	200			250			300			200			250			300		
	speed (m/s)	8	21	33	8	21	33	8	21	33	8	21	33	8	21	33	8	21	33
RMS	PI1	**0.016**	**0.008**	0.006	**0.018**	**0.009**	0.007	**0.020**	**0.011**	**0.008**	**0.026**	0.012	0.009	**0.032**	**0.016**	0.012	**0.038**	**0.019**	0.014
	PI2	0.003	0.001	0.001	0.004	0.002	0.002	0.005	0.003	0.002	0.017	0.011	0.011	0.013	0.008	0.008	0.011	0.007	0.006
	PI3	0.002	0.001	0.001	0.003	0.001	0.001	0.004	0.002	0.002	0.019	0.011	0.011	0.014	0.009	0.008	0.012	0.007	0.007
	iPI	0.004	0.002	0.002	0.003	0.002	0.001	0.002	0.001	0.001	**0.024**	**0.016**	**0.017**	0.019	0.012	0.012	0.016	0.009	0.009
	D^2-IBC	0.002	0.001	0.001	0.004	0.002	0.002	0.007	0.003	0.003	0.011	0.005	0.004	0.016	0.008	0.006	0.021	0.010	0.008
$\hat{s}$	PI1	0.000	0.047	0.065	0.000	0.043	0.064	0.000	0.041	0.063	0.000	0.030	0.030	0.000	0.030	0.030	0.000	0.029	0.029
	PI2	0.057	0.069	0.069	0.051	0.068	0.069	0.035	0.068	0.069	**0.279**	**0.378**	**0.464**	**0.186**	**0.290**	**0.363**	**0.146**	**0.214**	**0.277**
	PI3	0.066	0.079	0.123	0.056	0.069	0.069	0.053	0.068	0.069	**0.457**	**0.550**	**0.631**	**0.336**	**0.435**	**0.518**	**0.248**	**0.350**	**0.425**
	iPI	0.058	0.069	0.069	0.054	0.068	0.069	0.047	0.068	0.069	**0.299**	**0.399**	**0.474**	**0.206**	**0.298**	**0.390**	**0.142**	**0.237**	**0.309**
	D^2-IBC	0.000	0.066	0.068	0.000	0.064	0.067	0.000	0.059	0.067	0.000	0.032	0.031	0.000	0.031	0.031	0.000	0.031	0.031

TABLE II

TRACKING RESULTS FOR WET ROAD. BOLD: “BAD” PERFORMANCE (I.E. “LARGE” RMS OR $\hat{s} > 15\%$).

	λ^{ref}	5%									10%								
	mass (kg)	200			250			300			200			250			300		
	speed (m/s)	8	21	33	8	21	33	8	21	33	8	21	33	8	21	33	8	21	33
RMS	PI1	**0.010**	0.004	0.004	**0.013**	0.006	0.005	**0.015**	**0.007**	0.006	0.009	0.004	0.004	0.015	0.007	0.006	**0.020**	0.009	0.008
	PI2	0.004	0.002	0.002	0.003	0.002	0.002	0.003	0.002	0.001	**0.043**	**0.045**	**0.049**	**0.037**	**0.037**	**0.041**	**0.030**	**0.030**	**0.035**
	PI3	0.004	0.002	0.002	0.003	0.002	0.002	0.003	0.001	0.001	**0.044**	**0.043**	**0.048**	**0.036**	**0.036**	**0.040**	**0.030**	**0.030**	**0.033**
	iPI	0.006	0.003	0.003	0.005	0.002	0.002	0.004	0.002	0.002	**0.055**	**0.059**	**0.063**	**0.048**	**0.052**	**0.055**	**0.041**	**0.044**	**0.048**
	D^2-IBC	0.005	0.002	0.002	0.003	0.001	0.001	0.002	0.001	0.001	0.004	0.002	0.001	0.003	0.001	0.001	0.006	0.003	0.002
$\hat{s}$	PI1	0.000	0.066	0.066	0.000	0.065	0.065	0.000	0.064	0.064	0.019	0.031	0.044	0.003	0.031	0.040	0.000	0.032	0.030
	PI2	0.073	**0.156**	**0.222**	0.055	0.093	0.137	0.054	0.069	0.084	**0.847**	**0.855**	**0.873**	**0.735**	**0.768**	**0.794**	**0.638**	**0.683**	**0.734**
	PI3	0.117	**0.243**	**0.340**	0.070	**0.164**	**0.239**	0.064	0.116	**0.189**	0.001	0.001	0.001	**0.964**	**0.932**	**0.933**	**0.851**	**0.860**	**0.876**
	iPI	0.061	**0.150**	**0.248**	0.059	0.114	**0.167**	0.055	0.070	0.107	**0.816**	**0.842**	**0.871**	**0.712**	**0.756**	**0.792**	**0.629**	**0.690**	**0.725**
	D^2-IBC	0.091	0.098	0.107	0.038	0.069	0.069	0.011	0.068	0.069	0.023	0.035	0.032	0.018	0.034	0.032	0.011	0.035	0.032

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