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# Multivariate Processing of Accelerometric Condition Indicators<sup>\*</sup>

Giovanni Jacazio<sup>\*</sup> Gueorgui Mihaylov<sup>\*</sup> Franco Pellerey<sup>\*\*</sup>

<sup>\*</sup> *Politecnico di Torino, Department of Mechanical and Aerospace Engineering, Turin, 10129 TO, Italy*  
( [giovanni.jacazio@polito.it](mailto:giovanni.jacazio@polito.it), [gueorgui.mihaylov@polito.it](mailto:gueorgui.mihaylov@polito.it) )

<sup>\*\*</sup> *Politecnico di Torino, Department of Mathematical Sciences, Turin, 10129 TO, Italy* ([pellerey@calvino.polito.it](mailto:pellerey@calvino.polito.it))

**Abstract:** An innovative integrated self-learning health monitoring system has been developed and implemented on a fleet of helicopters in actual service. This system improves significantly the efficiency of a previous accelerometric vibrational monitoring tool by means of multivariate third level processing of the accelerometric features. The paper describes the way in which several problems, typical for the monitoring process of a mechanical system, have been treated in this specific case. The applied techniques could be of much more general interest.

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## 1. INTRODUCTION

Failure diagnostics via condition monitoring of mechanical systems is a broad and extremely relevant topic. Different approaches based on the development of specific sensors, model-based and data-driven methods have been applied in various contexts.

This research was initially focused on monitoring the health conditions of helicopters main and tail rotor mechanical power drives. A standard accelerometric health monitoring system has been previously installed on helicopters produced by AgustaWestland. The implementation of this system in actual service has shown that a relatively high number of false alarms is generated, thereby requiring additional troubleshooting workload.

The solution to this critical issue was the development of an innovative multivariate self-learning health monitoring system based on the same accelerometric features, which was able to reduce the rate of false positives. The efficiency of the existing diagnostic system has been improved via **third-level multivariate processing of the condition indicators** without installation of any additional sensor equipment. The multivariate system has been implemented into a complete integrated engineering tool, able to perform both multivariate monitoring and self-learning calibration functions. This article includes some aspects of our research as validation, overall calibration of the system, an example of quadratic filter and the possibilities of fault detection, which were not treated in the previous papers A. Bellazzi (2014a,b).

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The authors believe that some relevant results of this research can be applied in a more general context.

## 2. ACCELEROMETRIC SIGNAL PROCESSING

AgustaWestland provided a large amount of **in service data** collected on a fleet of 372 aircrafts of the same type, over a period of five years and thousands of flight hours in different conditions. A standard accelerometric vibration monitoring method was adopted (see CAA-PARER-2011/01 (2012)). The experimental data set consisted of tens of thousands of measurements of the condition indicators of several mechanical components.

The **first step of the signal processing** consists of the acquisition of the signals provided by a set of accelerometers and the remote storing of the corresponding time and frequency (Fourier) domain spectra.

The **second level signal processing** investigates the local properties of the energy distribution through the spectrum of vibrational modes. It leads to a set of scalar health indicators, which are supposed to detect specific damages. Some “physical” indicators represent the energy of the spectral components corresponding to the main rotational frequency and its multiples, the energy contained in limited bands etc. Other “data driven” indicators are related to local variations, correlations between specific spectral channels, local shape factors, signal standard deviations and kurtosis, signal quality markers (for some more details see CAA-PARER-2011/01 (2012)).

The standard health monitoring method is based on fixed critical thresholds for the values of each condition indicator. Damage alerts are generated when **any** of the indicators exceeds the threshold for certain number of measures, i.e. the method concerns a univariate **interpretation** of the indicators. The AgustaWestland health monitoring team reputed rather satisfactory the diagnostic ability of

this system but, during its in-service exploitation, the high rate of false failure alerts arose as a critical issue.

The efficiency of the existing diagnostic system has been improved via **third level multivariate processing** of the same indicators. The set of  $N$  health indicators has been considered as an  $N$ -dimensional vector state. A monitoring method which combines several multivariate statistical techniques has been developed and implemented in an integrated engineering tool. The integrated control process operates in the following phases:

1. **Anomaly detection** by means of a multidimensional self-learning Shewhart control chart individually calibrated on each component of each aircraft.
2. **Anomaly classification** based on multivariate discriminant models calibrated and validated over the fleet. A vector state judged as anomalous undergoes a set of discriminant methods regarding both the variance and the covariance structure of the calibration data sets.
3. **Evaluation.** For different power drives, different discriminant methods appear as more efficient. A pre-alert status is produced by a suitable combination of discriminant outputs. Such a combination is chosen in order to maximise the efficiency of the integrated control system.

The method is able to distinguish with very high level of statistical confidence true failure situations and false anomaly alerts, if these have been previously observed on an aircraft of the same type. Our research has shown that the univariate interpretation of the health indicators, entails a detectable **loss of relevant information**. The information is partially recovered by multivariate processing of the accelerometric health indicators and this leads to an appreciable **net efficiency gain**.

The research mainly concerned the following set of power drive components, in which true (confirmed by inspection of the power drive) and false alerts were detected: 2nd Stage Pin RH Bearing characterised by 6 health indicators; Tail Rotor Drive System - 2 health indicators; Tail Gear-Box - 12 indicators; Oil cooler bearing - 6 health indicators. Hangar Ball Bearing - 9 health indicators, TTO Pinion - 12 indicators; IGB Pin - 12 indicators etc.

### 3. RECALIBRATION

A general and relevant problem in a health monitoring process of a mechanical system is the definition of its healthy operational regime. The standard approach compares the monitored parameters of the system in certain conditions reputed as **similar** (see for example J. Lacaille1 (2014)). More generally the **comparability** of the operational regimes of a set of “twin” system for health monitoring purposes appears to be a highly non-trivial problem.

Besides the set of component vectors, a record of simultaneous measurements of 16 parameters of the operational condition of each aircraft (an environmental vector state), was available (Engine Torque, Rotor Speed, Roll Angle, Pitch Angle, True Airspeed etc...). The standard health monitoring system performs an accelerometric measurement when the environmental vector state belongs to a **specific prefixed range**. Nevertheless values of the health indicators, which characterise the healthy opera-

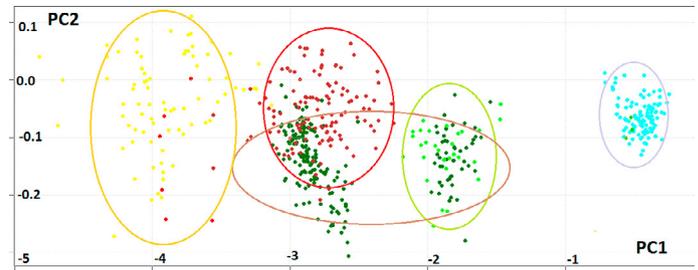


Fig. 1. PCA scores of healthy operational states of TRDS. Different colours represent individual aircrafts.

tional regime of a mechanical component, still vary considerably between individual aircrafts of the same type. The vector states of the same component in healthy regime, on individual helicopters, form neat clusters inside the space of indicators. A striking illustration is given on Fig. 1), where the principal component scores of vector states of a TRDS measured on individual helicopters are displayed in different colours. In this specific case, the individual helicopter clusters spread along the direction determined by the first principal component. This means that by far the most relevant portion of the variance in the data set of healthy operational states can be attributed to differences between aircrafts (see more on PCA in Timm (2002)).

It has been hypothesised that the accelerometric measurements are influenced by the operational regime of the aircraft. In order to test this hypothesis, **canonical correlation analysis** has been applied on the available data set (see Rencher (2002)). The canonical correlation method describes the interconnection between two random vector variables by means of a double set of latent variables (directions in the corresponding state vector spaces).

Many mechanical components are characterised by three or four canonical correlations with considerably high values (over 0,5). This fact led to the construction of the following **multilinear filter**. A linear map  $f : R^{17} \rightarrow R^N$  (where  $N$  is the dimension of the component vector) which provides a “predicted” component vector state  $V_p$  in correspondence to each measured environmental vector state  $V_m$  has been calibrated. The  $k$ -th row of the matrix, which represents this linear map, contains the coefficients of a multiple linear regression of the  $k$ -th component of a state vector over the set of 16 environmental parameters plus an intercept term (see Izenman (2008)). The calibration is done in healthy conditions and the analysis is then performed in terms of **residuals** with respect to the predicted value  $V_r = V_m - V_p$ .

More powerful filters are obtained by applying higher order regression of the health indicators i.e. linear regressions over the set of powers of the environmental parameters of the aircraft. The application of a quadratic filter is illustrated on Fig. 3.

If the reader compares Fig. 1 to Fig. 2 and Fig. 3, will observe that as a consequence of recalibration, scores of healthy operational states measured on different helicopters concentrate (compare the scales of the diagrams) and mix together quite uniformly (see Section 5 below).

The model-based normalization of the parameters of the operational regime gives the possibility to enlarge the

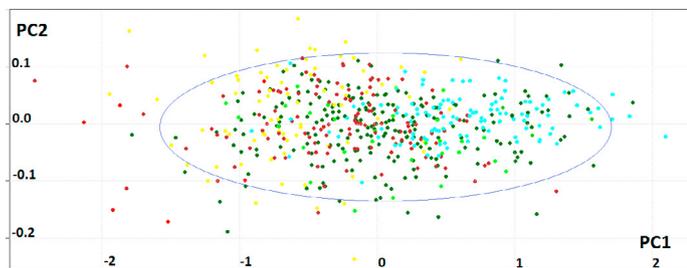


Fig. 2. PCA scores of healthy operational states of TRDS with linear filter. Different colours represent individual aircrafts.

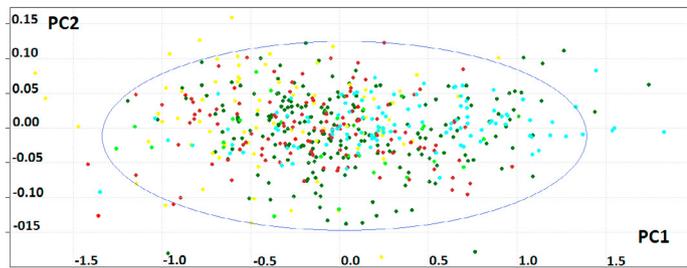


Fig. 3. PCA scores of healthy operational states of TRDS with quadratic filter. Different colours represent individual aircrafts.

ranges of tolerance of the flight condition parameters in which an accelerometric measure can be performed. The advantages of such a procedure for the precision and fidelity of the control process are obvious. A natural limit for enhancement of the tolerance ranges is given by some prefixed acceptable level of validity of the model. In our case such a limit can be imposed on the values of the relevant canonical correlations between the component vector states and the environmental vector states.

More complex functional dependence of the state vectors on the operational regime (on the environmental parameters) can be naturally treated in multivariate context by applying modern manifold learning techniques (see Izenman (2008)). These methods combine notions of multivariate statistics and differential geometry.

#### 4. ANOMALY DETECTION

A problem highlighted by the experts of AgustaWestland is that the healthy operational regime of some power drives on individual helicopters is characterised by high values of the health indicators. Such values would be considered as anomalous if compared to the a priori fixed threshold values. This is a typical problem, which arises in the monitoring process of a set of apparently “twin” systems.

The re-calibration procedures filter the deterministic impact of the flight regime of the aircraft onto the accelerometric measurements. Once filtered the influence of the exploiting condition, the intrinsic variability of the healthy operational states of each mechanical component can be modelled over a random noise process. The extent to which the filtered healthy operational states of each power drive component fit with a multidimensional Gauss distribution has been tested by various multivariate normality tests like Kolmogorov-Smirnoff, Jarque-Bera etc. The distribution

of filtered healthy operational states of each component of a single helicopter can be considered as Normal with very high level of statistical confidence (p-value around  $2 \times 10^{-15}$ ). There is not enough space to illustrate this fact, but the plots of unfiltered vector states are often characterised by asymmetric “tails” and so they visibly differ from the ellipsoidal-shaped clusters expected for normally distributed vectors. Some of the clusters which represent individual aircrafts on Fig.1 are visibly non ellipsoidal.

The set of filtered healthy operational states of a component, measured on several helicopters of the same type, is also normally distributed with roughly the same statistical confidence but with higher variability.

The **first important effect of filtering** is therefore the possibility to apply a standard anomaly detection tool, appropriate for normally distributed populations. The multidimensional Shewhart control chart is based on the statistically relevant Mahalanobis distance (see Izenman (2008)). Control charts aim the evaluation of the likelihood on a single event in the context of a random process. A phenomenon which is very improbable under the hypothesis of being a manifestation of the ordinary regime of the process is judged as a modification, due to not accidental causes (see Shewhart (1931)).

A multivariate self-learning Shewhart control chart, which calibrates itself automatically on the healthy regime of each monitored component of each single aircraft has been implemented. The software implementation computes automatically the upper control limit as a suitable multiple of the sample estimate of the standard deviation of a Gaussian approximation of the Fisher-Snedecor distribution. Any vector state judged “under control” contributes to the real time re-calibration of the parameters of the control chart operating on each component. The anomaly detection process **keeps learning** individually on each component of each aircraft. This anomaly detection method revealed to be very efficient as it eliminates completely the problem described at the beginning of this section.

#### 5. ANOMALY CLASSIFICATION

In many cases the Mahalanobis distance between states corresponding to false alerts and the mean value of the healthy regime exceeded the distance of the true damage states. The self-learning Shewhart control chart is an excellent tool for the detection of anomalous situations, but it is not sufficient for the discrimination of true failure states and anomaly alerts which do not correspond to a failure. For this reason in a second step, called “anomaly classification”, the integrated monitoring system applies in parallel a set of multivariate statistical discrimination tools in order to distinguish true and false anomaly alerts.

The **second important effect of filtering** is that vector states measured on individual helicopters of the same type become **comparable**. The unknown failure modes of an aircraft can be deduced from the observed failure modes of another aircraft in the fleet. For this reason the statistical models included in the discrimination segment of the system, are calibrated and automatically updated over the fleet of aircrafts.

The integrated engineering tool, which implements the control system, calibrates automatically on two levels. While the Shewhart control chart in the anomaly detection segment **learns individually on each aircraft**, the discrimination segment **learns on the monitored fleet**.

The method includes the simultaneous application of a set of discriminant techniques which investigate the structure of the variance, the structure of covariance and the consistency of the accelerometric data sets (see A. Bellazzi (2014a,b)). The analysis has been developed in “geometrical terms”, i.e. directions and distances in a  $N$ -dimensional affine space instead of individual threshold values. The exploited multivariate methods are described in detail in Rencher (2002); Timm (2002); Izenman (2008)

**Principal Component Analysis (PCA)** has been applied in order to investigate the spontaneous clustering of the anomaly type classes.

**Linear Discriminant Analysis (LDA)** detects the directions in the vector space of health indicators which better separate the anomaly type classes.

**Quadratic Discriminant Analysis (QDA)** provides further classification information. A quadratic classifier based on the Mahalanobis distance has been exploited.

**Principal factor models** have been calibrated in order to investigate the covariance structure of the accelerometric data. It has been hypothesised that different health indicators react simultaneously in a correlated way to a true failure. On the contrary false alerts can be interpreted as anomalous measurements not necessarily induced by a consistent reaction of the monitoring system. In terms of projections onto the space generated by the principal factors, one could expect that the projections of the healthy operational cluster and true failure cluster show different characteristic profiles. The direction in which failure states projections spread away from the origin is indicative regarding the correlation modifications introduced by the reaction to a damage. The analysis of the in-service data confirmed the theoretical hypothesis A. Bellazzi (2014b).

Random variables have been interpreted as real projective classes in a vector space. The correlation structure of the data set can be better understood in terms of directions and angles of the state vectors. A minor role is attributed to the modules. Normalised state vectors have been considered and factor models on this set have been calibrated. This study of the projective structure of the data sets provided very interesting results. For some mechanical components the separation between true and false anomaly classes in the subspace generated by the principal factors improves remarkably (see A. Bellazzi (2014b)).

**Consistency based anomaly detection methods** have been also applied. Strong inconsistencies with the linear and the quadratic models which describe the variations accelerometric outputs in terms of flight parameters were considered as manifestations of anomalies. In many cases it has been observed that the 3-4 relevant (see above) canonical correlations between the environmental parameters and the health indicators decrease considerably in presence of anomalous behaviour of the component. This process was more evident in case of true failure than false

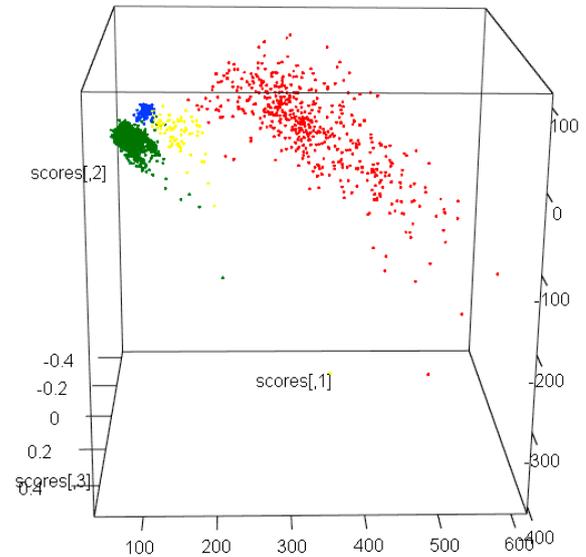


Fig. 4. PCA scores of 2nd Stage Pin RH Bearing vector states (healthy-green, false alert-blue, fault-yellow and failure-red)

anomalies. For components, for which the correlation with the environmental states is particularly high, the canonical correlation method can be considered as a supplementary anomaly detection and classification resource.

Part of these discrimination techniques have been implemented in the integrated engineering tool. In each of the above subspaces (which provide significant dimensional reduction) the tool fixes opportune discriminant conditions between healthy, true and false alert states.

## 6. FAULT DETECTION

The multivariate processing of the indicators, in terms of directions and distances in an affine space, appears more efficient also for fault detection purposes. It has been observed that the true failure states compose well-defined clusters in the space of states. Particularly relevant is the direction in which true failure states spread away from the cluster of the healthy operational regime.

This observation gives the possibility to detect faults and fault-into-failure progressions in an efficient way by exploiting as a criterion the spreading of states in that particular direction. This alternative “direction based” method appears more efficient than the standard univariate processing of the health indicators. A remarkable example is given of Fig. 4. False alerts (blue dots) and fault states (yellow dots) appear at comparable Mahalanobis distances from the cluster of healthy states, but they are positioned in different directions with respect to the healthy cluster.

If the discrimination models in the classification segment are calibrated in a way to consider the fault states as a separate class, the classification of the measured states becomes extremely precise (see Table 1).

On Table 2 is represented the fault-into-failure time progression of a 2nd Stage Pin RH Bearings. The first image represents the PCA scores of the cluster of healthy operational states, the second represents the fault evolution after 69 flight hours, the third - after 150 flight hours and

the last diagram - the situation after 515 flight hours. Establishing the characteristic direction in which vector states corresponding to a true failure spread, can facilitate considerably the definition of protocols for estimating **time to failure** of a mechanical component.

### 7. VALIDATION

Several validation procedures based on splitting of the huge initial data set into calibration and validation data subsets have been applied in order to compare different helicopters of the same type. On Fig. 5 are displayed the results of a validation procedure on a 2nd Stage Pin RH Bearings. In this case the method has been calibrated on seven helicopters and then an eighth helicopter, on which a false alert has been registered, undergoes the

Table 1. Re-classification of 2nd Stage Pin RH Bearing vector states by the integrated method

real \ classified as	normal	false alert	fault	true failure
normal	1864	0	6	0
false alert	0	74	0	0
fault	0	0	75	0
true failure	0	0	0	495

Table 2. Fault-into-failure progression of 2nd Stage Pin Brg. (PCA scores, healthy states-green, fault-blue, failure-red).

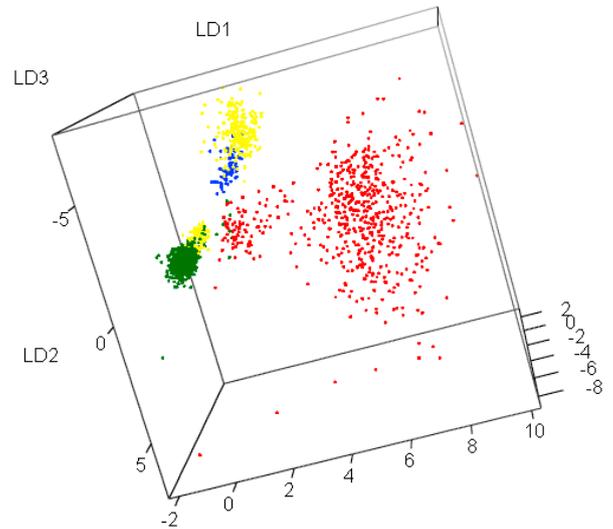
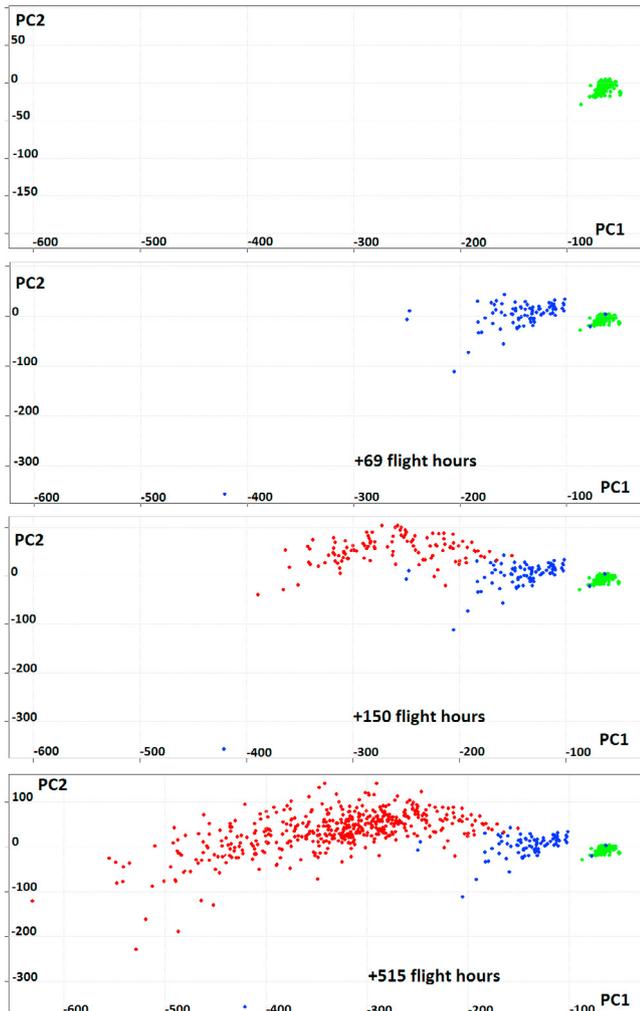


Fig. 5. Validation on 2nd Stage Pin RH Bearing vector states (LDA scores). Calibration set: red, green and blue. Validation set: yellow.

monitoring process. Green dots represent projections of healthy operational states, blue dots - false alerts, red dots - true failure in the calibration data set. Yellow dots represent projections of the validation data set which contain healthy states and false alert states of the same type of the one contained in the calibration set.

The states of the validation set are properly classified by the system. In conclusion, a specific false alert can be identified by means of its multivariate profile. The same remark regards true failure states. When a specific false alert is observed in the fleet, the following false alerts of the same type can be recognised with **very high level of statistical confidence** (the system learns on the fleet).

### 8. OVERALL CALIBRATION

Since the above anomaly detection/discrimination methods involve different mathematical constructions (“distances” and projections on different subspaces of the space of states), in general their simultaneous application do not produce overabundant information. For this reason the integrated exploitation of several discrimination techniques is more efficient than each single method.

The parallel application of the discriminant models in the context of the control process produces a set of pre-alert outputs combined into a vector called pre-alert status. The processing of pre-alert status is done in what we have called the evaluation phase of the control routine. The evaluation and the generation of a failure alert is based on the analysis of the **density of positive pre-failure alerts** over a number of measurements.

For different mechanical components, different types of pre-failure statuses, detect more efficiently true failure situations. Different combinations of pre-alert outputs produce bigger difference between the densities of positive failure indications in case of true failure situations on one side and healthy and false alert states on the other.

The final step of the calibration of the integrated multivariate monitoring system is its overall setting. The first aim of the overall calibration procedure is the choice of the most efficient combination of discriminant outputs. There is a small number of possibilities. For example the alert status can be modified if the control chart status and **all** the discrimination outputs (PCA, LDA, QDA) are positive; the alert status is modified if the control chart status and **any** of the discrimination outputs are positive etc...

The choice of pre-alert status type is based on the evaluation of the density of states classified as true failures in the calibration data set. In order to compute these densities, the available calibration data set undergoes a simulation of a control process, but the evaluation phase is realised using all the possible evaluation conditions. The density of positive alert statuses in the different classes of vector states is computed for each condition. Typically the most efficient combination of discriminant outputs is the one which produces bigger differences between the densities of pre-alerts in true failure and false alert situations.

The integrated control process has been tested on a series of real cases contained in the historical database of AgustaWestland. For the TGB gears and the 2nd Stage Pin RH Bearings the integrated discriminant method judges a state as true failure if **each** discriminant method classifies it as such. With this requirement only 3% of the measured states were miss-classified. For Hangar Ball Bearings a pre-alert is produced if at least one of the discriminant methods gives a positive output. The alert status is modified in 13% of the healthy states, in 28% of the false alerts and in 65% of the true failure states.

Once fixed the most efficient, in terms of difference of densities, combination of discriminant outputs, the critical density of alert statuses is chosen. If in a number of measurements this critical density is exceeded, a failure alert is generated. The previous univariate system generates an alert if the values of a health indicator exceeds the threshold in a proportion of 2/3 in a number of consecutive measurements. The critical density of true failure outputs which generate a failure alarm is an **active parameter** of the integrated tool and is rigorously deduced from the overall validation results. In the example of a Hangar Ball Bearing 1/2 appears to be a suitable density.

The efficiency of the integrated method can be further optimised by modifying the multiple of standard deviations which define the threshold of the Shewhart control chart. The value of that coefficient must be chosen in order to maximise the difference of densities of positive outputs in true and false alert classes. Analogous remark regards a possible “fine tuning” of the PCA and LDA discrimination models. Slight variations of the discrimination boundaries can provoke variations of the proportion of the relevant densities. An appropriate balance between sensibility of the system and the rate of false alerts can be found. Many such tests have been done, but the systematic presentation of the results would require a lot of space.

## 9. CONCLUSION

The integrated control system was finally **tested** over the whole set of accelerometric measurements provided by

AgustaWestland (see Section 2). There were several cases of false alerts of the same type occurred repeatedly on different aircrafts in the fleet and the integrated control system recognised the second (the third etc...) observed false alert as such. In other words **the false failure alerts generated over the same period of time were 50% (or 60% etc...) less then those generated by the standard system.** This was a strong experimental evidence for a net efficiency gain, which would further improve over longer periods because of the self-learning characteristics of the alternative system.

The study has highlighted the advantages of this third-level multivariate approach. The multivariate processing of the health indicators provides more detailed information on the monitored process. Besides the obvious advantages of direct multivariate processing, there is an interesting possibility to define and apply multivariate health monitoring validation protocols which aim to improve the efficiency of each individual health indicator by minimising the overall loss of information. More generally the analysis of the results of this research from the viewpoint of the a posteriori prognostics and health monitoring validation of a diagnostic system is a very interesting task. This is an extremely relevant topic, which concerns the evaluation of the efficiency of the constructed health indicators i.e. how exhaustively they describe the state of the mechanical component. This is an interesting observability problem.

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