



Politecnico di Torino

Dipartimento di Scienza dei Materiali e Ingegneria Chimica

# Modelling and monitoring of the freeze-drying process

Salvatore A. Velardi – Antonello A. Barresi  
salvatore.velardi@polito.it      antonello.barresi@polito.it

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This work has been realised in the framework of:

**Lyo-Pro**  
**Competitive and Sustainable Growth European Project**

**The scientific objective of the proposal is to optimize the freeze-drying process of pharmaceutical proteins on a scientific basis in order to set up efficient and rational freeze-drying diagrams for industrial manufacturing of commercially-used drugs and diagnostic proteins.**

## Freeze-drying (or lyophilization)

**Drying process** whereby water or another solvent is removed from a frozen product by **sublimation**, generally under **vacuum**.

Freeze-drying is the best available technique to dry pharmaceutical proteins reducing the possibility of introduction of immunogenicity or other undesirable changes in the product properties.

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### *Limitations*

- long process duration and high process costs
- optimisation by trial and error runs
- impossibility of direct measure of parameters of interest

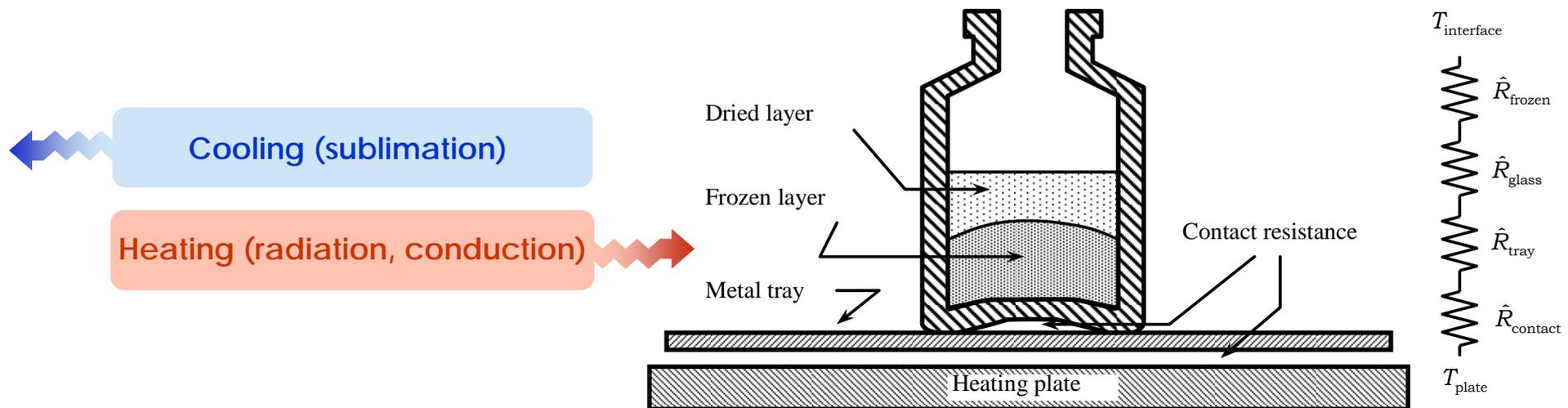
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### *Theoretical modelling*

- help in design, optimization, and control of the process

**Primary  
drying** →

- **Sublimation** of frozen water
- **Condenser** to trap vapour
- Endothermic process, latent **heat must be provided**
- **Moving front** of sublimation
- **Critical parameter:** sublimating front temperature,  $T_i$ 
  - ↳ Collapse/melting
  - ↳ Sublimation speed depend directly on  $T_i$
- **Coupled Heat and mass transfer**





# Modelling

## Bi-dimensional model for vial lyophilisation

### Energy balance dried layer I

$$\rho_{Ie} c_{P,Ie} \frac{\partial T_I}{\partial t} = -c_{P,G} \mathbf{N}_{tot} \cdot \nabla T_I + \nabla \cdot (k_{Ie} \nabla T_I) + \Delta \hat{H}_v \frac{\partial \rho_{sw}}{\partial t} = 0$$

### Energy balance frozen layer II

$$\rho_{II} c_{P,II} \frac{\partial T_{II}}{\partial t} = \nabla \cdot (k_{II} \nabla T_{II})$$

### Material balance vapour

$$\varepsilon_P \frac{\partial \rho_w}{\partial t} = -\nabla \cdot \mathbf{N}_w - \frac{\partial \rho_{sw}}{\partial t}$$

### Material balance inert

$$\varepsilon_P \frac{\partial \rho_{in}}{\partial t} = -\nabla \cdot \mathbf{N}_{in}$$

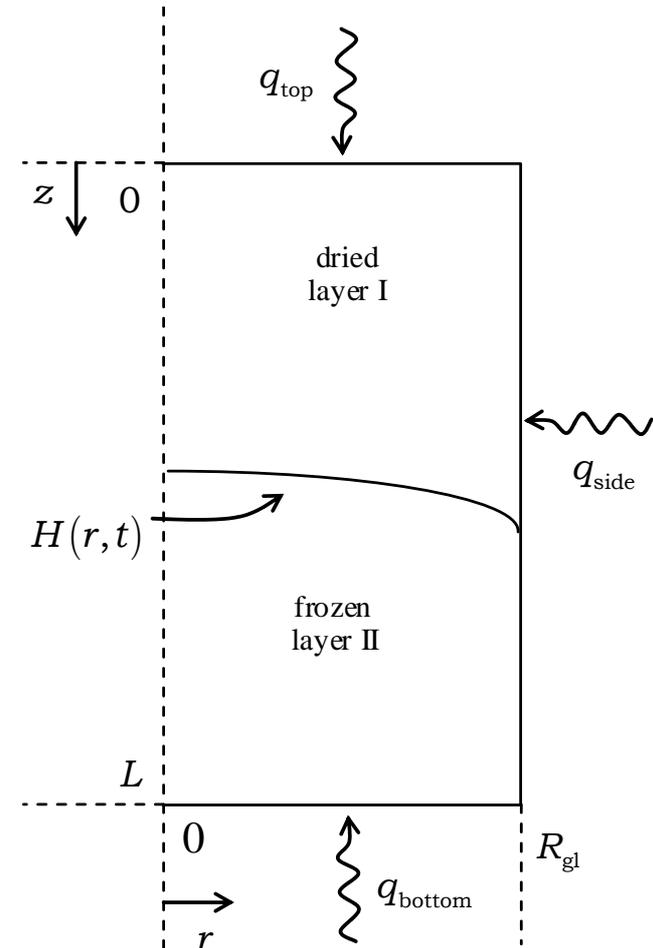
### Material balance at the moving front

$$\mathbf{N}_w \Big|_{z=H(r,t)} = -\mathbf{v} (\rho_{II} - \rho_{Ie})$$

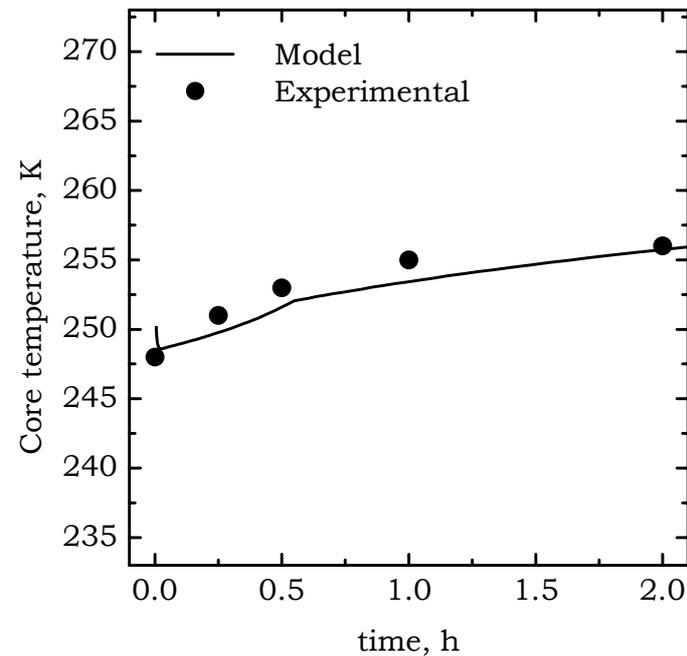
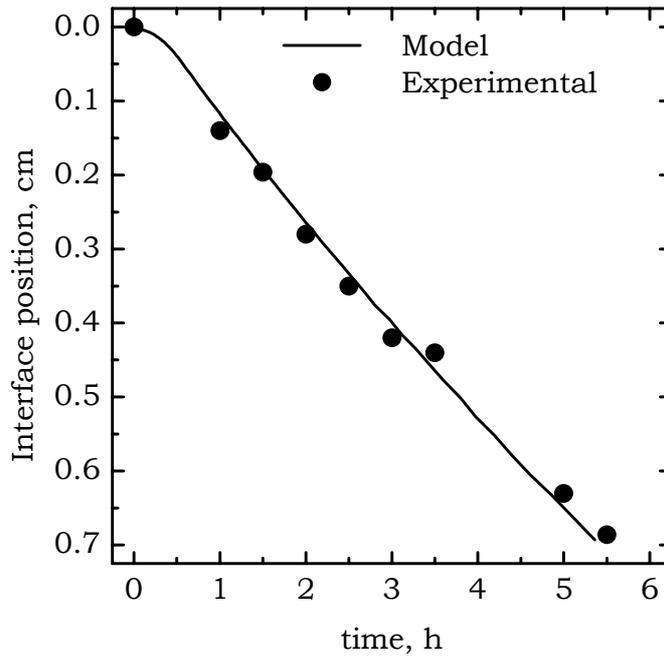
$$\rightarrow \frac{\partial H}{\partial t} = -\frac{1}{\rho_{II} - \rho_{Ie}} \left( N_{w,z} \Big|_{z=H(r,t)} - N_{w,r} \Big|_{z=H(r,t)} \frac{\partial H}{\partial r} \right)$$

### Energy balance at the moving front

$$-k_{II} \nabla T_{II} \Big|_{z=H(r,t)} + k_{Ie} \nabla T_I \Big|_{z=H(r,t)} - \mathbf{N}_w \Big|_{z=H(r,t)} (c_{P,G} T_i + \Delta H_s) - \mathbf{v} (\rho_{II} c_{P,II} T_i - \rho_{Ie} c_{P,Ie} T_i) = 0$$



## A literature case: freeze-drying of skim milk



Comparison between model simulations and experimental results by Wolff et al. (1989). Left hand side: interface position. Right hand side: frozen core temperature

Sample thickness, mm	Primary drying time, min		
	<i>This work</i>	<i>Millman et al. [30]</i>	<i>Mascarenhas et al. [37]</i>
<b>3</b>	13.96	13.77	13.47
<b>6</b>	54.65	54.07	55.26

## Effect of radiation

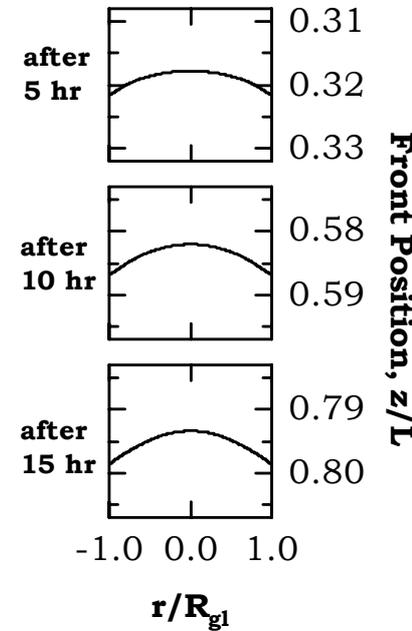
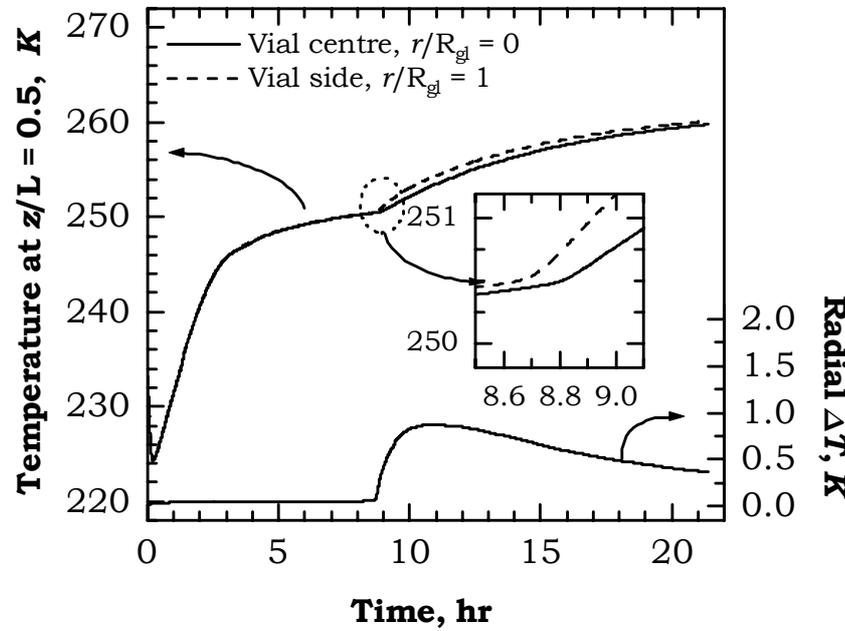
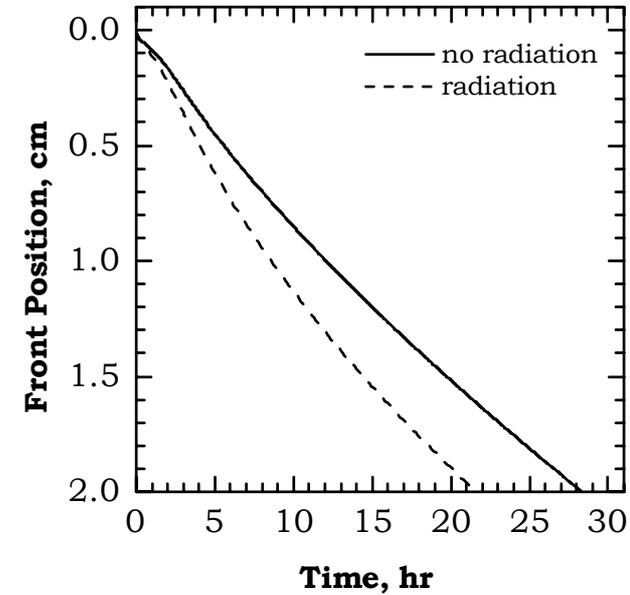
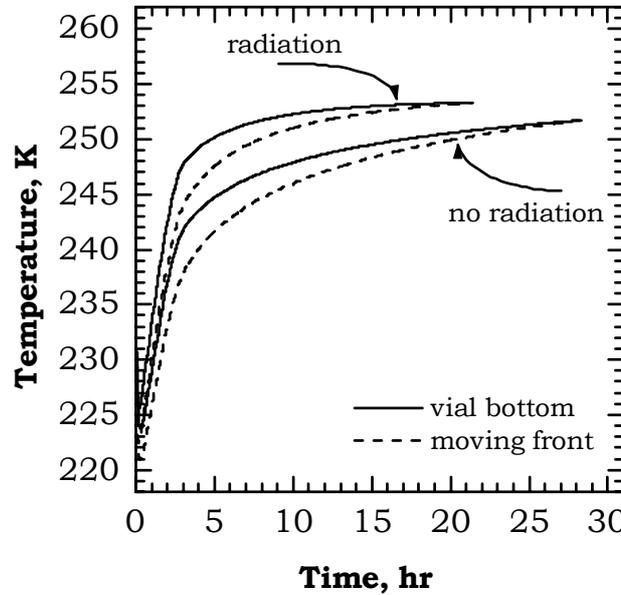
- Lower drying time
- Higher product temperature

*but*

- Radial temperature difference is small
- Moving front curvature is small



*Mono-dimensional approach feasible*



## Mono-dimensional model for vial lyophilisation

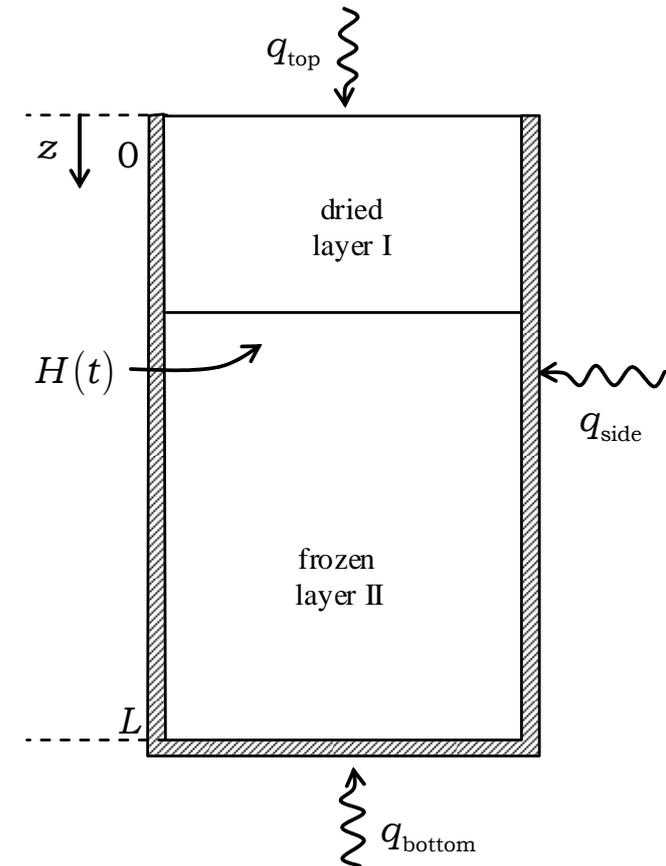
- Mono-dimensional geometry forces the moving front to be **planar**
- **Vial sidewall heat transfer** is accounted

For the vial being in contact with the dried layer I:

$$\frac{\partial T_{I,gl}}{\partial t} = + \left( \frac{\lambda_{gl}}{\rho_{gl} c_{P,gl}} \right) \frac{\partial^2 T_{I,gl}}{\partial z^2} + \left( \frac{h_{I,i}}{\rho_{gl} c_{P,gl}} \frac{2R_{gl,i}}{R_{gl,e}^2 - R_{gl,i}^2} \right) (T_I - T_{I,gl}) - \left( \frac{1}{\rho_{gl} c_{P,gl}} \frac{2R_{gl,e}}{R_{gl,e}^2 - R_{gl,i}^2} \right) \sigma F (T_{I,gl}^4 - T_W^4)$$

For the vial being in contact with the frozen layer II:

$$\frac{\partial T_{II,gl}}{\partial t} = + \left( \frac{\lambda_{gl}}{\rho_{gl} c_{P,gl}} \right) \frac{\partial^2 T_{II,gl}}{\partial z^2} + \left( \frac{h_{II,i}}{\rho_{gl} c_{P,gl}} \frac{2R_{gl,i}}{R_{gl,e}^2 - R_{gl,i}^2} \right) (T_{II} - T_{II,gl}) - \left( \frac{1}{\rho_{gl} c_{P,gl}} \frac{2R_{gl,e}}{R_{gl,e}^2 - R_{gl,i}^2} \right) \sigma F (T_{II,gl}^4 - T_W^4)$$

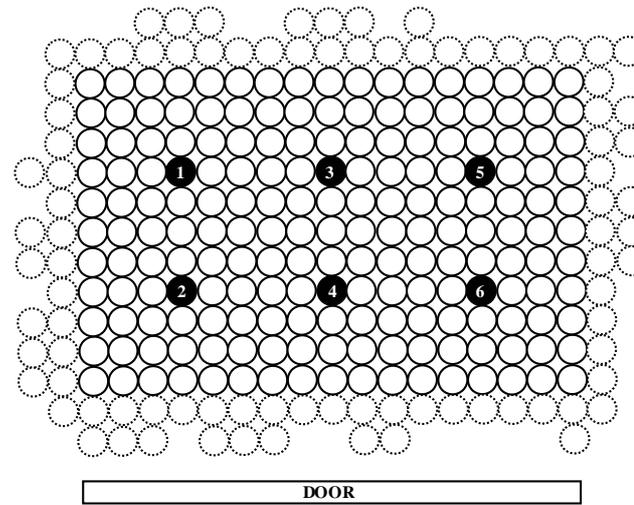


$$\xi^I = \frac{z}{H(t)} \quad \xi^{II} = \frac{z - H(t)}{L - H(t)}$$

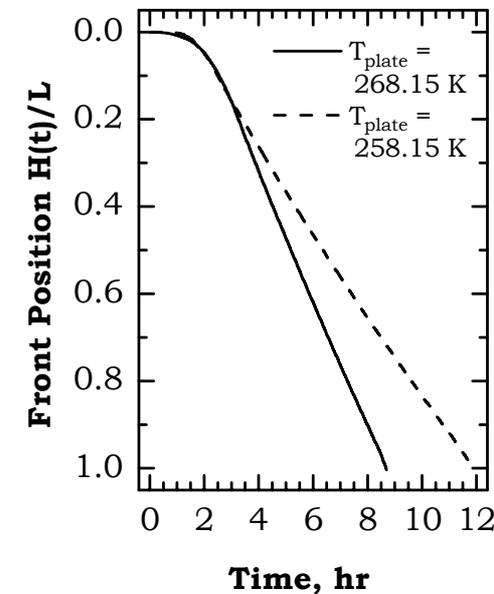
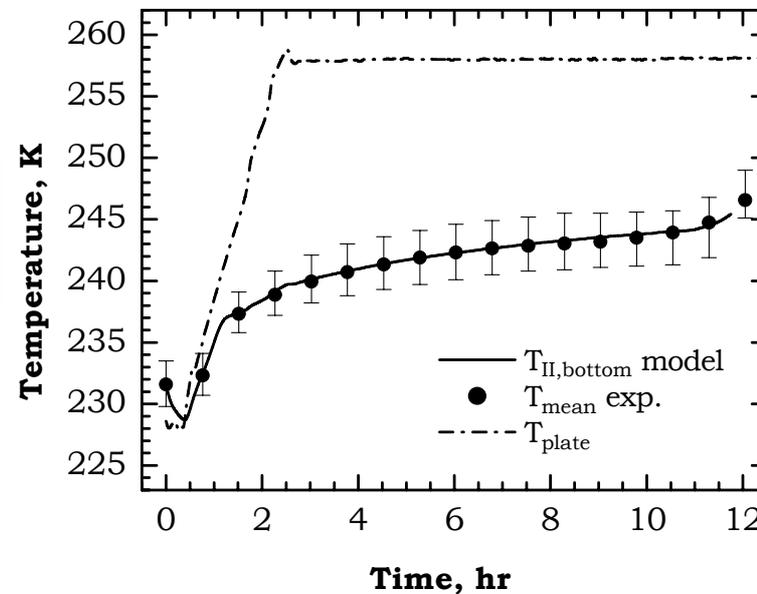
$$0 < z < H(t) \quad H(t) < z < L$$

## Freeze-drying of a 5% bovine serum albumin solution (LAGEP, CPE Lyon)

- Pressure 26 Pa
- 200 vials
- 1 ml BSA solution
- 6 thermocouples measuring bottom product temperature

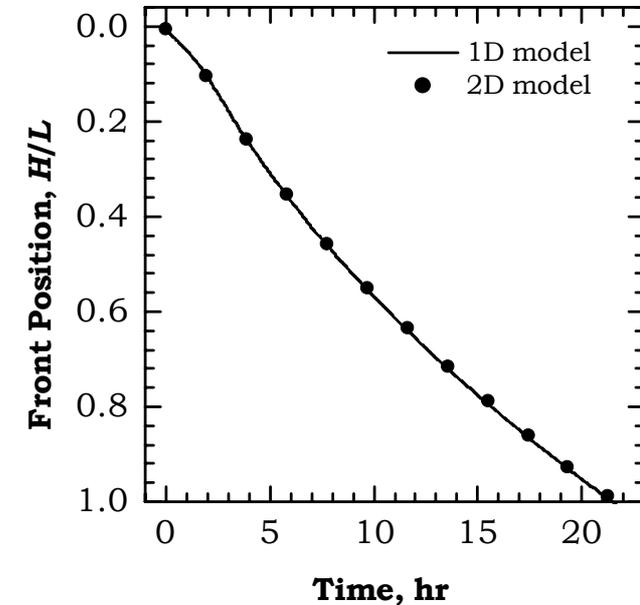
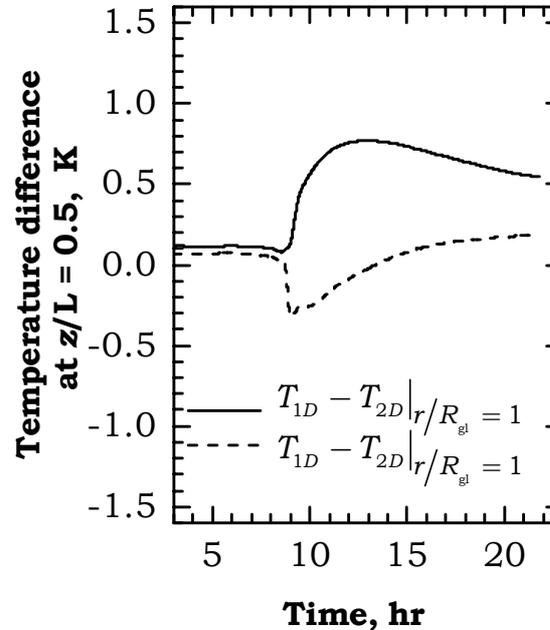
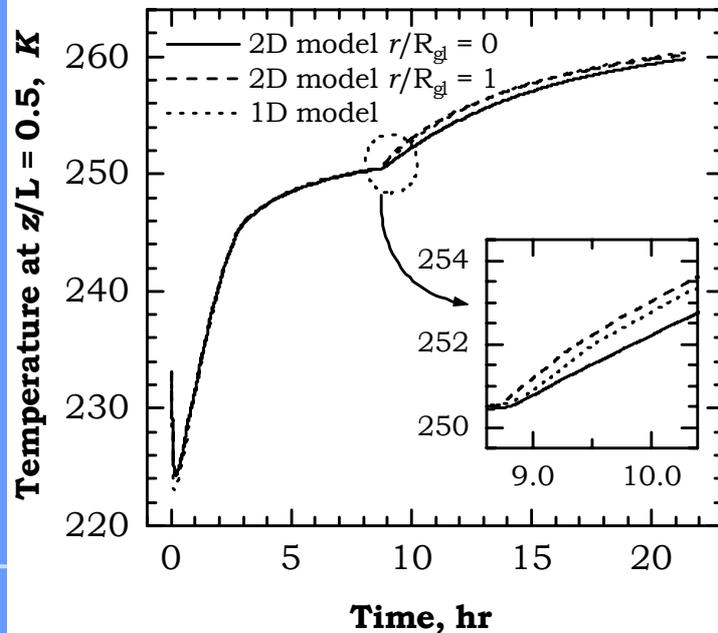


$$T_{\text{plate}} = \begin{cases} 228.15 & 0 < t < 0.5 \text{ hr} \\ 228.15 + 15(t - 0.5) & 0.5 < t < 2 \text{ hr} \\ 258.15 & t > 2 \text{ hr} \end{cases}$$



## Comparison between 1D model and 2D model

- Temperature profiles of the frozen mass are practically coincident
- In the dried layer, predicted 1D temperature is comprised between the two values given by 2D model at extreme radial positions
- Maximum temperature difference between 1D and 2D model  $< 1^\circ\text{C}$
- Moving front evolution practically coincident





# Optimal operating conditions

## Optimal operating conditions (constant $T_{plate}$ )

### At lower pressure

if pressure  $\uparrow$   
heat transfer  $\uparrow$   $\rightarrow$  Drying time  $\downarrow$

### At higher pressure

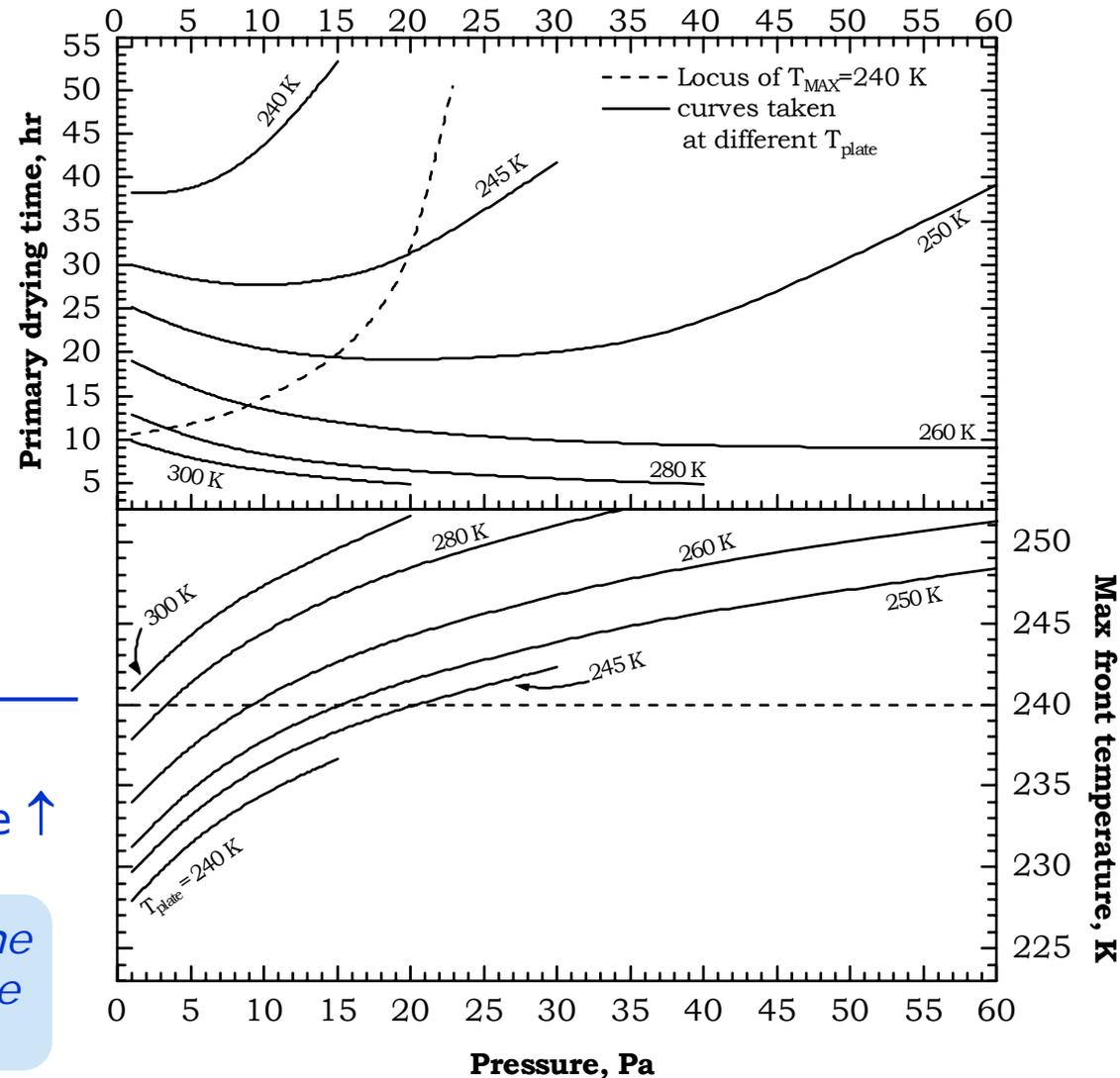
if pressure  $\uparrow$   
mass transfer  $\downarrow$   $\rightarrow$  Drying time  $\uparrow$

**Minimum drying time**

but...

if pressure  $\uparrow$   $\rightarrow$  Max temperature  $\uparrow$

*The operative range is limited by the constraint of maximum temperature that allows safe operation*



## Optimal operating conditions (variable $T_{\text{plate}}$ )

When maximum allowable temperature is approached

Plate temperature is regulated in such a way that  $T_{i,\text{max}}$  is never overcome

$$T_{\text{plate}}(t) = f(T_{i,\text{MAX}}, t, \dots)$$

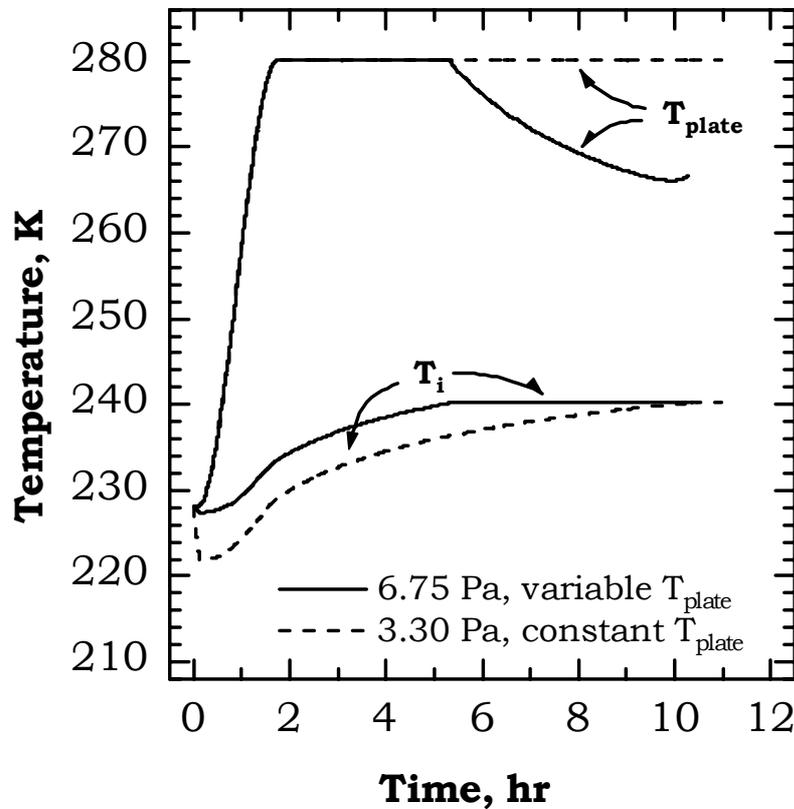
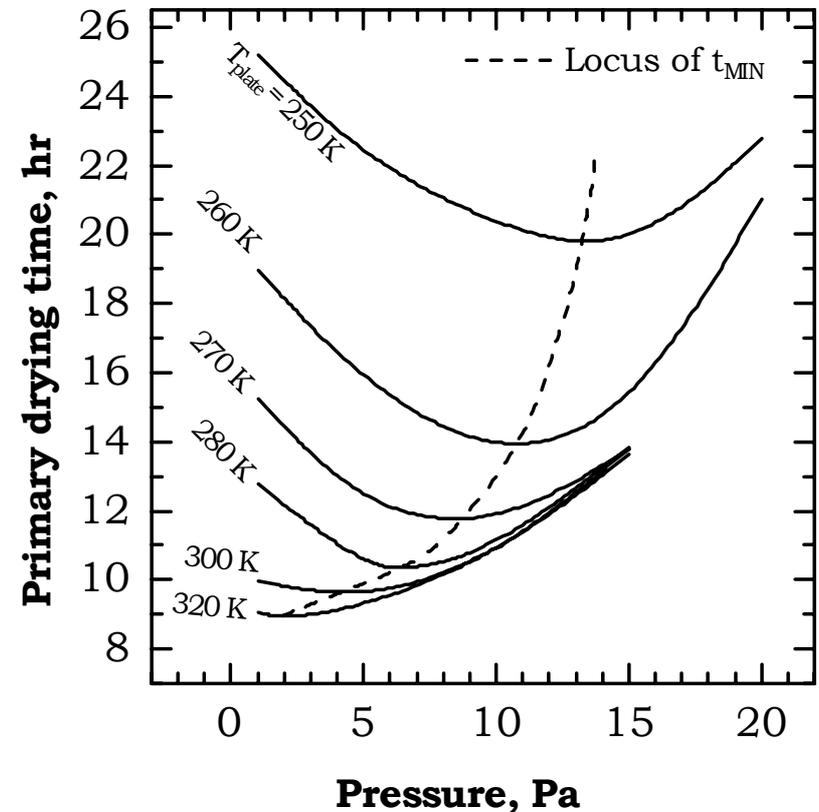


Chart of operating conditions,  $T_{i,\text{max}}$  constraint is always satisfied





# Monitoring and control



## Technological innovation of freeze-drying in Lyo-Pro

A new **analytical balance** that permits to evaluate the mass of the product during the process

Use of the **mass spectrometry** to identify any anomaly in the process

Primary drying should be carried on at a **controlled sublimation temperature** in order to **avoid denaturation of the product.**

**But front temperature can not be directly measured**

Use of **model-based estimators (soft sensors)** and **indirect non-invasive methods** to monitor, control and optimize the process

example:  
**Manometric Temperature Measure**



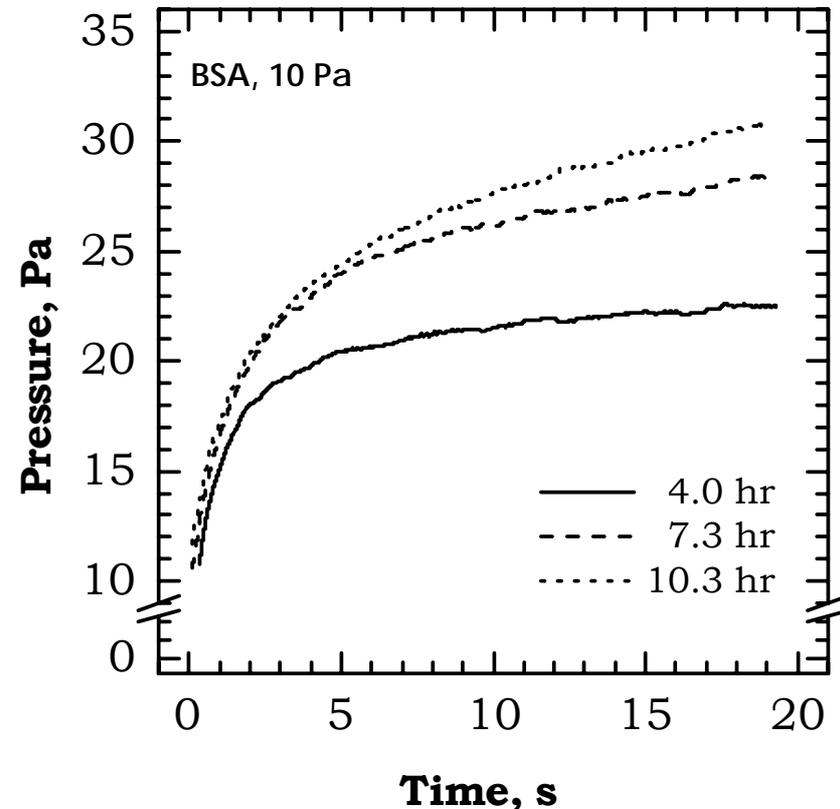
## Manometric Temperature Method (or Pressure Rise Analysis)

- Remote sensing procedure for **determining the temperature of the moving front** at different times during the primary drying stage
- The valve separating chamber and condenser is closed ( $\approx 20$  seconds) and the chamber pressure increases

MTM is currently adopted in some units. Chamber pressure is assumed to reach equilibrium and sublimation T is calculated through thermodynamics

*but*

Effect of "drift" limits MTM test duration (otherwise product can be damaged)



## Modified Manometric Temperature Method

A new approach to the description of the phenomena occurring during the MTM test

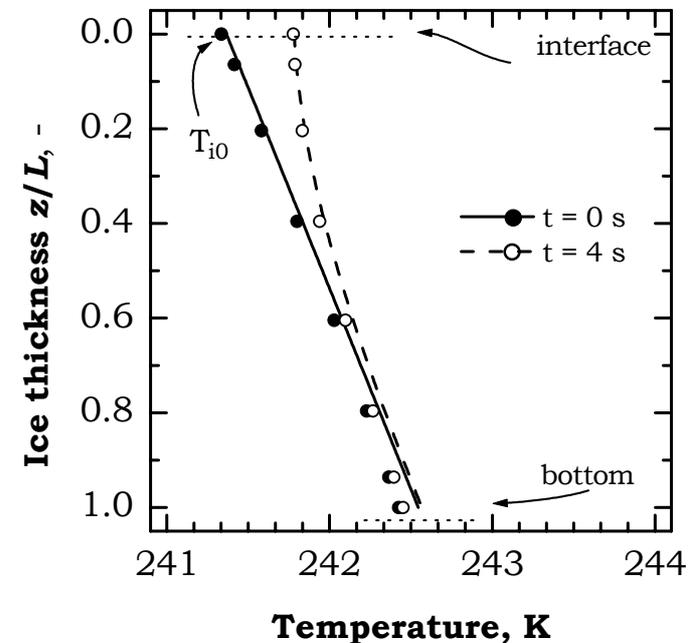
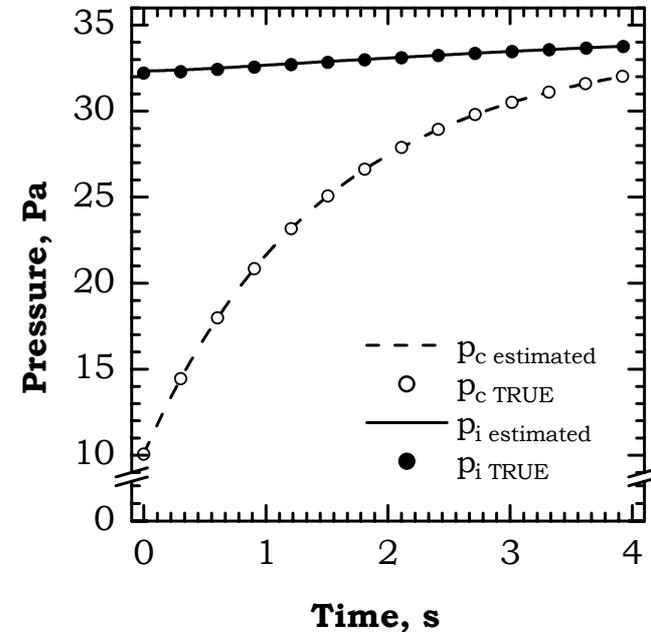
- The new model relates the measured pressure rise dynamics to the front temperature
- Time evolution of the product temperature is considered

$$\rightarrow \begin{cases} \frac{dp_{w,c}}{dt} = \frac{NA}{V} \frac{RT_c}{M_w R_p} (p_i(T_i) - p_{w,c}) \\ \frac{\partial T}{\partial t} = \frac{k_{\text{frozen}}}{\rho_{\text{frozen}} c_{p,\text{frozen}}} \frac{\partial^2 T}{\partial z^2} \end{cases}$$

- Non-linear optimization problem is solved

$$\min_{T_{i0}, R_p} \frac{1}{2} \| p_c(T_{i0}, R_p) - p_{c,\text{measured}} \|_2^2$$

$$\rightarrow \begin{array}{l} \text{Front temperature } T_{i,0} = T|_{z=0} \\ \text{Mass transfer resistance } R_p \end{array}$$



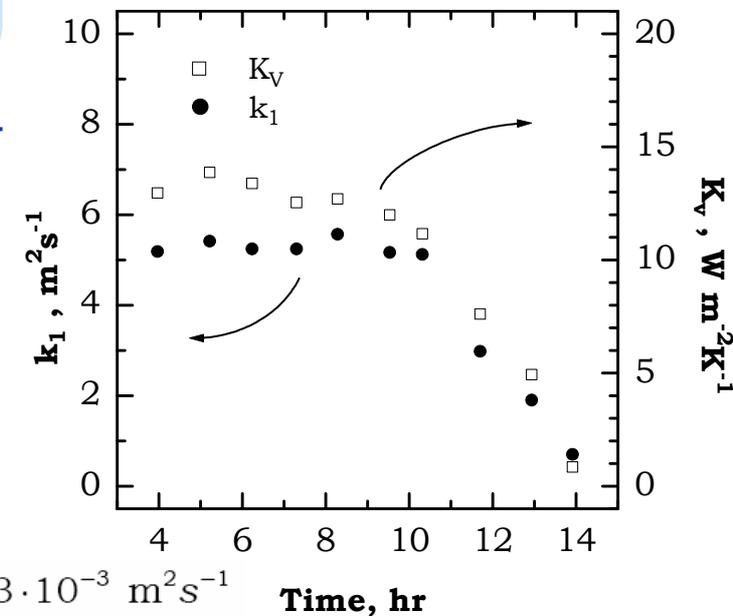
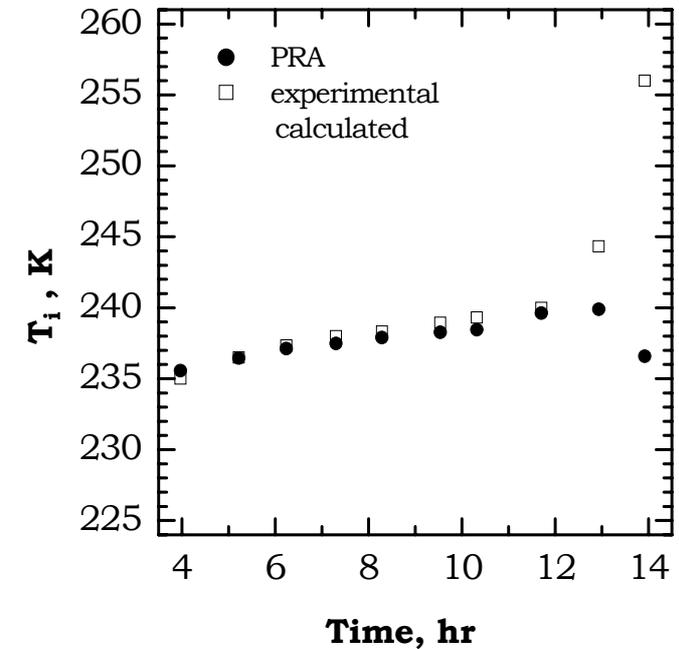
## Modified Manometric Temperature Method

- MTM is repeated several times to provide an estimation of  $T_i$  through all primary drying
- From model equations we can also determine:

$k_1$  mass transfer coefficient  
 $K_v$  heat transfer coefficient at the vial bottom  
 $L$  ice thickness  
 $T(z)$  temperature profile along the frozen mass

- **Limitation:**  
MTM is a global method, vials are considered as a whole


 if there are large heterogeneities between vials, MTM measure is inaccurate (end of primary drying)



$$k_{1,\text{experimental}} = 5.3 \cdot 10^{-3} \text{ m}^2 \text{ s}^{-1}$$

## Control

- Regulatory guidance **do not allow closed loop in manufacturing processes** (in a validated process all cycles must be the same)



**Only monitoring is possible in manufacturing, used as a "process record" to know that cycles are reproducible**

- During cycle development (pilot/lab scale freeze-driers) no regulatory limitations apply, **closing loop is possible**

MTM or soft-sensor  
for estimation of  $T_i$   
can be inserted in a  
feedback loop



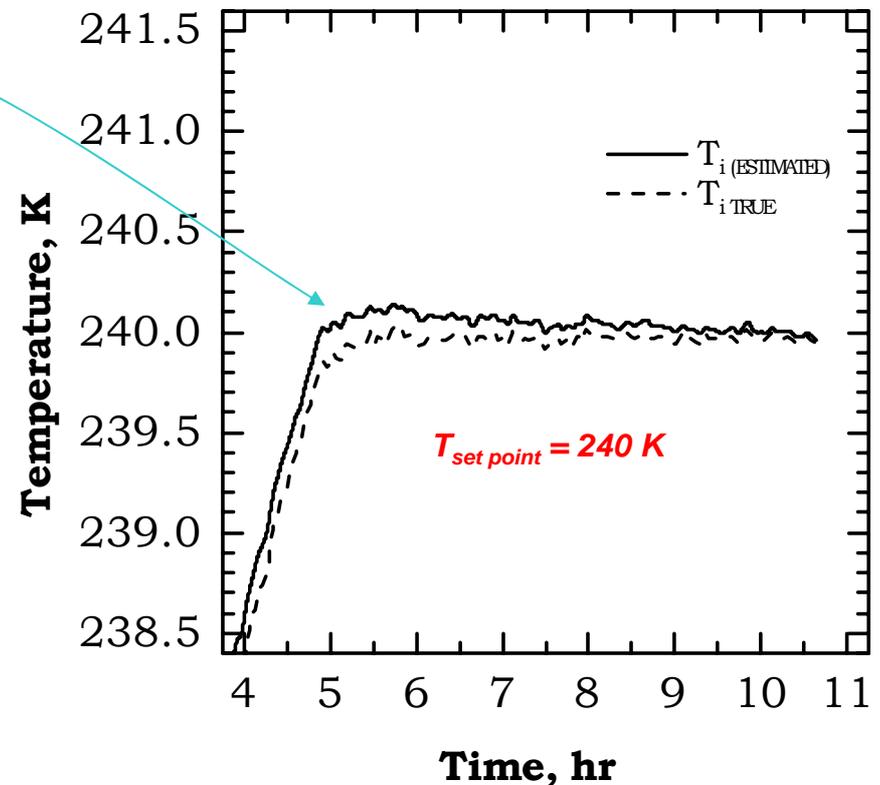
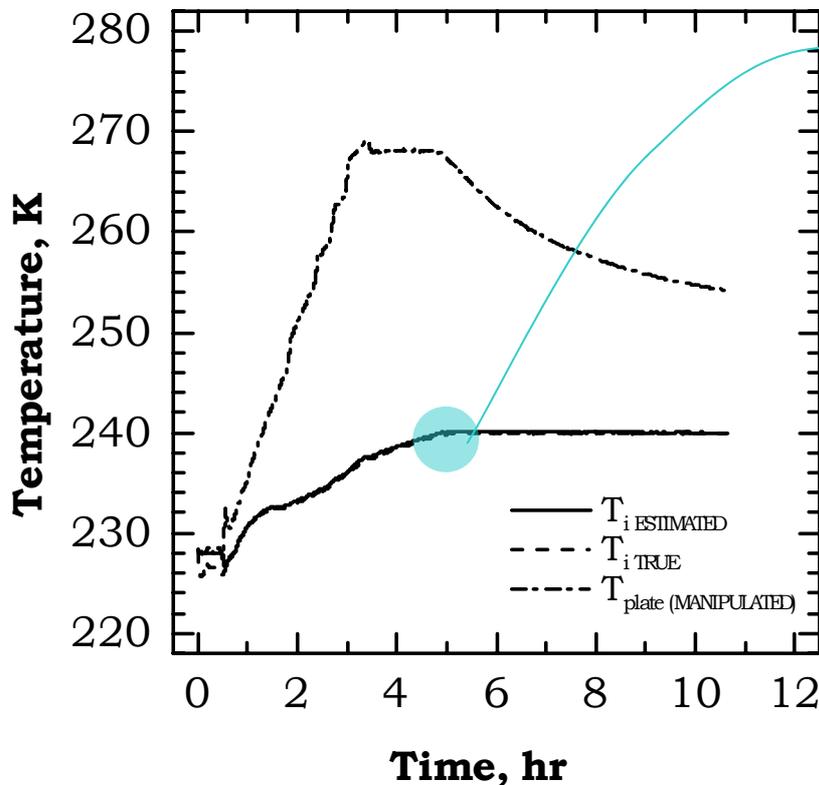
**control the product  
temperature in real  
time preventing  
product degradation,  
maximize heat input**

## Output feedback control (5% BSA solution)

The temperature of the moving front is controlled by manipulating the temperature of the heating plate  $T_{\text{plate}}$

A Proportional-Integral (PI) controller has been implemented

$$T_{\text{plate}}(t) = K(\hat{T}_i(t), T_{i,\text{MAX}}, \dots) \longrightarrow T_{\text{plate}}(t) = -k_P(\hat{T}_i(t) - T_{i,\text{MAX}}) - \frac{1}{k_I} \int (\hat{T}_i(t) - T_{i,\text{MAX}}) dt + T_{\text{plate},0}$$





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### The Lyo-Pro Consortium:

**Politecnico di Torino (Coordinator)**,  
Torino, Italy (leader G. Baldi)  
Dep. of Chemical Engineering and Material  
Science (SMIC) [antonello.barresi@polito.it](mailto:antonello.barresi@polito.it)  
Dep. Electronical Engineering (DELEN)  
[marco.parvis@polito.it](mailto:marco.parvis@polito.it)

**ALFA WASSERMANN S.p.a.**, Bologna, Italy  
(leader G. Viscomi)  
[gcviscomi@alfawassermann.it](mailto:gcviscomi@alfawassermann.it)

**Asymptote Ltd**, Cambridge, UK (leader J.  
Morris) [jmorris@asymptote.co.uk](mailto:jmorris@asymptote.co.uk)

**Institute Nationale de la Recherche  
Agronomique (INRA)**, Thiverval-Grignon,  
France (leader M. Marin)  
[marin@grignon.inra.fr](mailto:marin@grignon.inra.fr)

**Telstar Industrial S.A.**, Terrassa-  
Barcelona, Spain (leader M. Galan)  
[mgalan@telstar.es](mailto:mgalan@telstar.es)

**Laboratoire d'Automatique et de  
Genié des Procédés (LAGEP)**  
Université Claude Bernard Lyon1,  
Lyon, France (leader J. Andrieu)  
[andrieu@lagep.univ-lyon1.fr](mailto:andrieu@lagep.univ-lyon1.fr)

**Utrecht University**, Utrecht, The  
Nederlands (leaders H. Schellekens,  
W. Jiskoot) [w.jiskoot@pharm.uu.nl](mailto:w.jiskoot@pharm.uu.nl)

**bioMerieux**, Marcy Etoile, France  
(leader M. Rapaud, M. Jolivet)  
[michel.rapaud@eu.biomerieux.com](mailto:michel.rapaud@eu.biomerieux.com),  
[michel\\_jolivet@eu.biomerieux.com](mailto:michel_jolivet@eu.biomerieux.com)





**more slides...**

## Bi-dimensional model for vial lyophilisation

- Transient material and energy balances
- Spatial and time evolution of:

➔ *Dried Layer I Temperature*

➔ *Frozen Layer II Temperature*

➔ *Water vapour pressure*

➔ *Inert pressure*

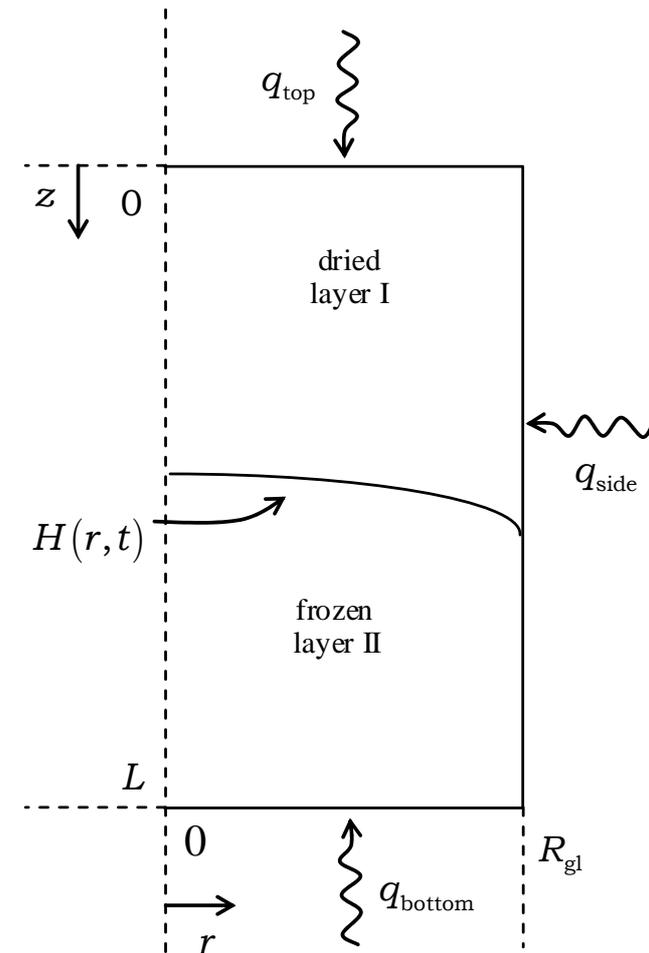
➔ *Bound water concentration*

and moving sublimating interface:

➔ *Position*

➔ *Velocity*

➔ *Temperature*



## Bi-dimensional model for vial lyophilisation

- Moving boundary or Stefan problem

➔ Time changing spatial grid would be required  
Problems arise in time integration



- *Front-fixing* resolution method

➔ Spatial grid fixed in time through a mathematical artifice

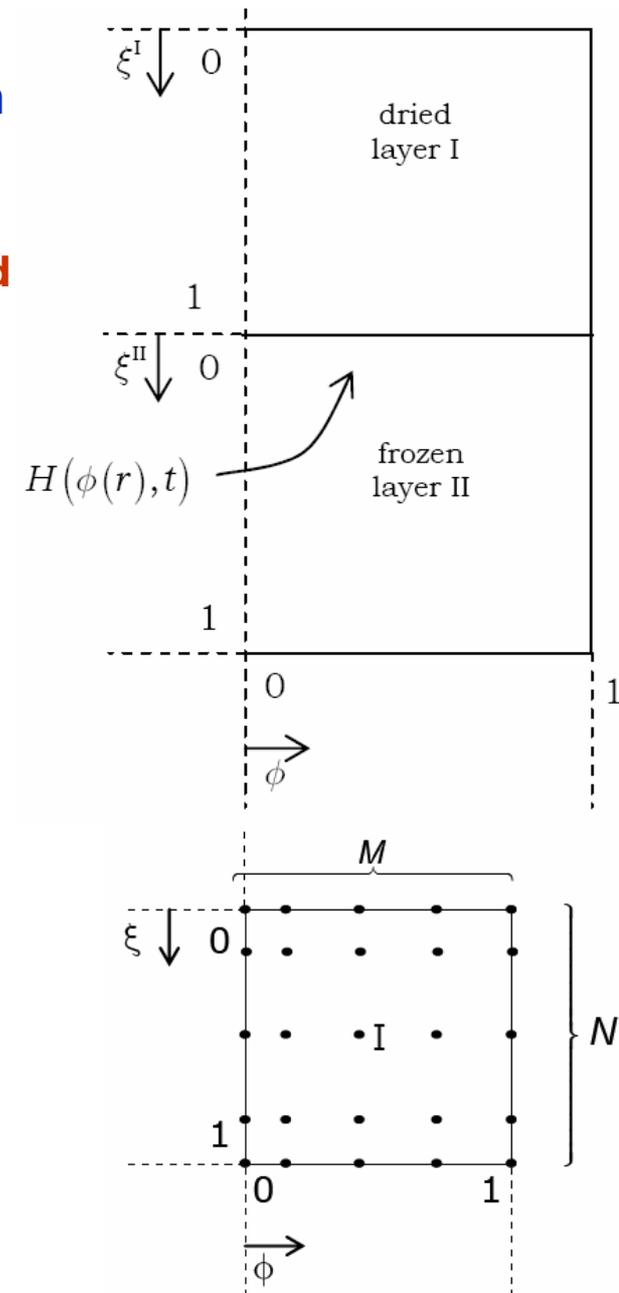
➔ Complex mathematical formulation

$$\xi^I = \frac{z}{H(r,t)} \quad \xi^{II} = \frac{z - H(r,t)}{L - H(r,t)} \quad \phi = \frac{r}{R_{gl}}$$

$$0 < z < H(r,t) \quad H(r,t) < z < L \quad 0 < r < R_{gl}$$

- **Orthogonal collocations**; spatial derivatives are determined via differentiation matrixes

- **Non-uniform N x M grid**

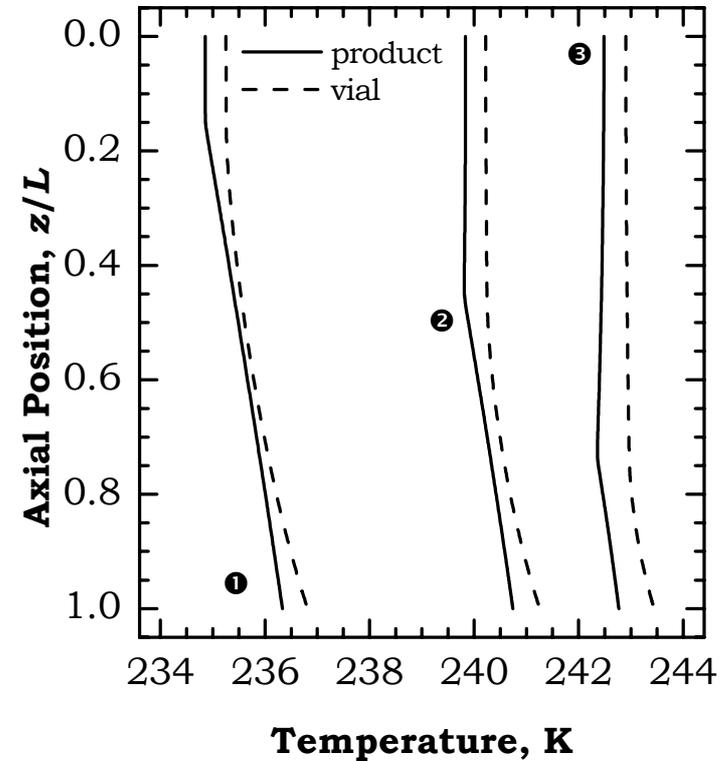
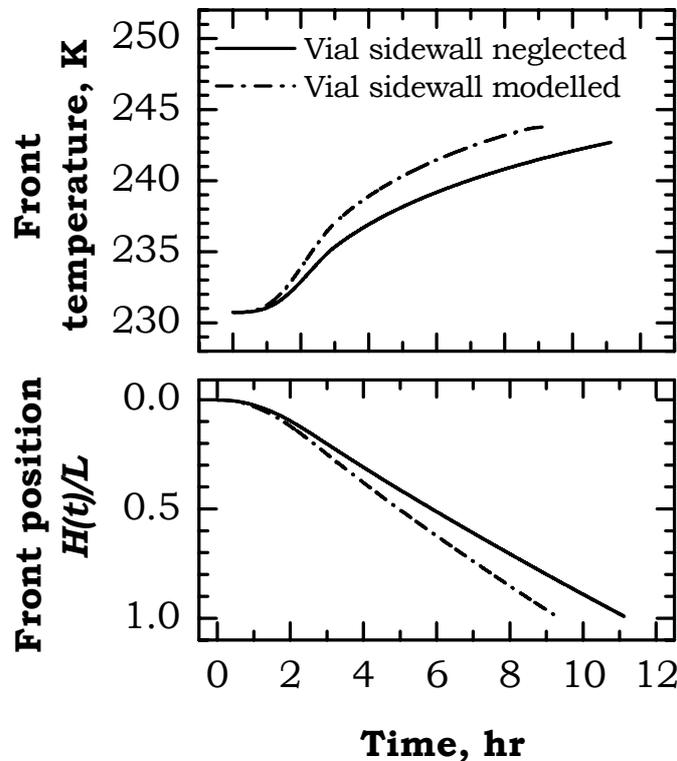


## Heat transfer in the vial sidewall

- Energy provided by the heating plate is exchanged with the product mainly at the bottom and in part at the vial side

**Lower drying time, Higher product temperature**

➔ Analogous effect of radiation on drying time and product temperature, but at a minor extent





# Simplified models for real-time applications

## Simplified models for real-time monitoring of primary drying

Detailed transient models  
of the process require:



**Computational power**

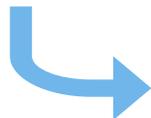
**A large number of parameters, not  
always easily accessible**



**Simplified models have been set up easier to  
implement for real time monitoring and control**

### Main hypothesis

- Pseudo-steady state conditions
- Radiation is neglected



Vials well shielded from edge effects

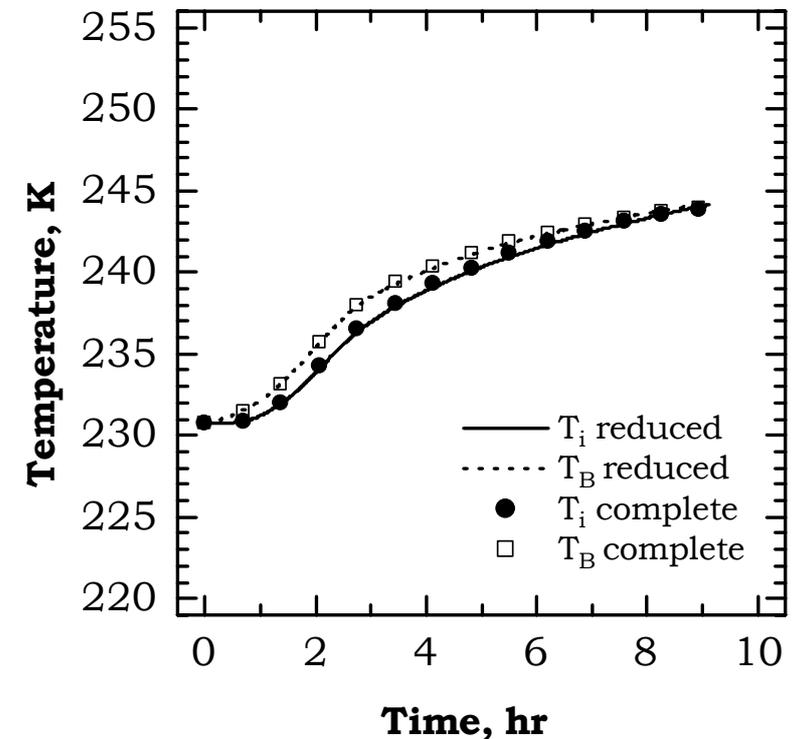
## Simplified model I

- Vial sidewall is not accounted for
- Dynamic behaviour is retrieved by mass balance across the moving front
- Only 1 Ordinary differential equation
- Fast solution

$$\frac{dH}{dt} = \frac{1}{\rho_{II} - \rho_I} \frac{M}{RT_i} \frac{k_1}{H} (p_{w,i}(T_i) - p_{w,c})$$

$$\left( \frac{1}{K_v} + \frac{L - H}{k_{II}} \right)^{-1} (T_{plate} - T_i) = \frac{\Delta H_s M}{RT_i} \frac{k_1}{H} (p_{w,i}(T_i) - p_{w,c})$$

$$T_{II,b} = T_{plate} - \frac{1}{K_v} \left( \frac{1}{K_v} + \frac{L - H}{k_{II}} \right)^{-1} (T_{plate} - T_i)$$



## Simplified model II

The simplified **balances can be integrated *analytically*** in order to get the equations for the temperature profiles along the product and along the vial sidewall

$$\begin{aligned}
 T_I &= -2(1 - a_1)C_3 \cosh(\alpha_1 H \xi) + a_1 C_6 & T_{II} &= -(1 - a_{II}) \left( C_1 e^{-\alpha_{II}(L-H)\vartheta} + C_2 e^{\alpha_{II}(L-H)\vartheta} \right) + a_{II} (C_4 \vartheta + C_5) \\
 T_{I,gl} &= +2a_1 C_3 \cosh(\alpha_1 H \xi) + a_1 C_6 & T_{II,gl} &= +a_{II} \left( C_1 e^{-\alpha_{II}(L-H)\vartheta} + C_2 e^{\alpha_{II}(L-H)\vartheta} \right) + a_{II} (C_4 \vartheta + C_5)
 \end{aligned}$$

$$C_1 - C_2 + C_4 \left( \frac{1}{\alpha_{II}(L-H)} \frac{a_{II}}{1-a_{II}} \right) = \frac{b_2}{\alpha_{II}(1-a_{II})}$$

$$C_1 + C_2 - C_3 \left[ 2 \frac{\alpha_1}{\alpha_{II}} \cosh(\alpha_1 H) \right] + C_5 - C_6 \left( \frac{\alpha_1}{\alpha_{II}} \right) = 0$$

$$C_1 - C_2 + C_3 \left[ 2 \frac{\alpha_1 \alpha_1}{\alpha_{II} \alpha_{II}} \frac{L-H}{H} \sinh(\alpha_1 H) \right] - C_4 \left( \frac{1}{\alpha_{II}} \right) = 0$$

$$C_1 \left( -\frac{c_{II}-\alpha_{II}}{c_{II}} e^{-\alpha_{II}(L-H)} \right) + C_2 \left( \frac{c_{II}+\alpha_{II}}{c_{II}} e^{\alpha_{II}(L-H)} \right) + C_4 \left( 1 + \frac{1}{c_{II}(L-H)} \right) + C_5 = \frac{T_{plate}}{\alpha_{II}}$$

$$C_1 + C_2 - C_3 \left[ 2 \frac{1-\alpha_1}{1-\alpha_{II}} \cosh(\alpha_1 H) \right] - C_5 \left( \frac{\alpha_{II}}{1-\alpha_{II}} \right) + C_6 \left( \frac{\alpha_1}{1-\alpha_{II}} \right) = 0$$

$$C_1 \left( \frac{c_{II,gl}-\alpha_{II}}{c_{II,gl}} e^{-\alpha_{II}(L-H)} \right) + C_2 \left( \frac{c_{II,gl}+\alpha_{II}}{c_{II,gl}} e^{\alpha_{II}(L-H)} \right) + C_4 \left( \frac{L-H+c_{II,gl}}{c_{II,gl}} \right) + C_5 = \frac{T_{plate}}{\alpha_{II}}$$

To complete the model the boundary conditions must be applied, given by the following set of ***linear equations***. The ***analytical solution*** of the system gives the integration constants  $C_1 \dots C_6$ .

The various parameters in the equations are function of  $H$ ,  $T_i$ ,  $K_v$ ,  $k_1$ , geometry and thermal properties of the vial/product.

## Simplified model II

- Vial sidewall is modelled

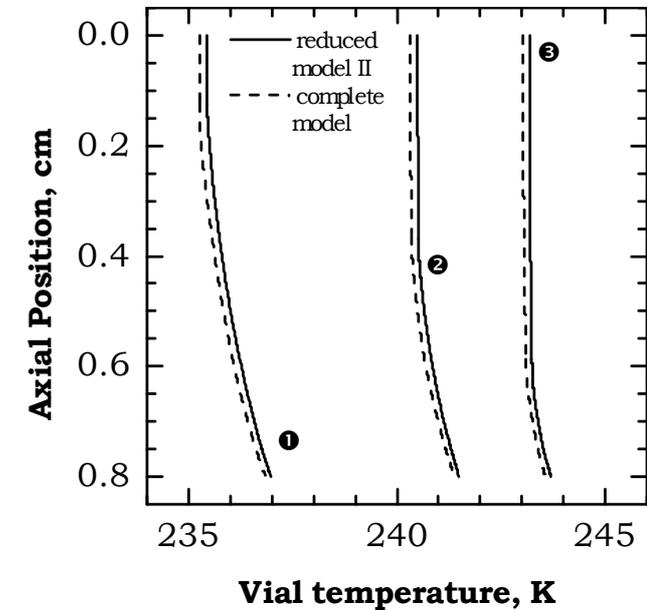
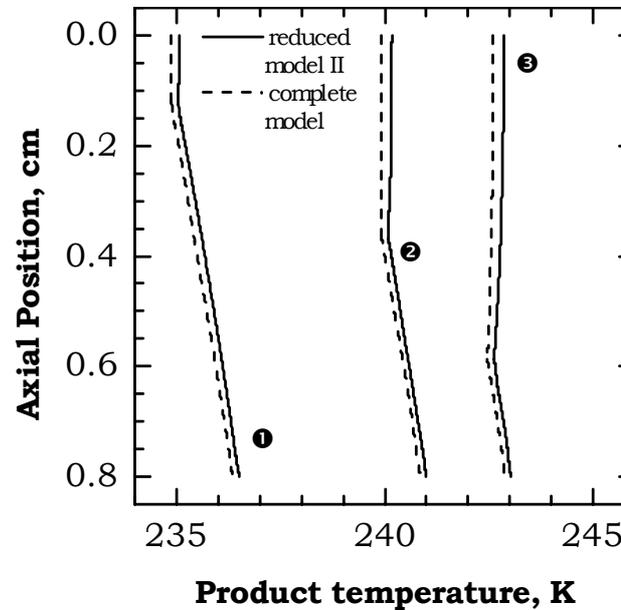
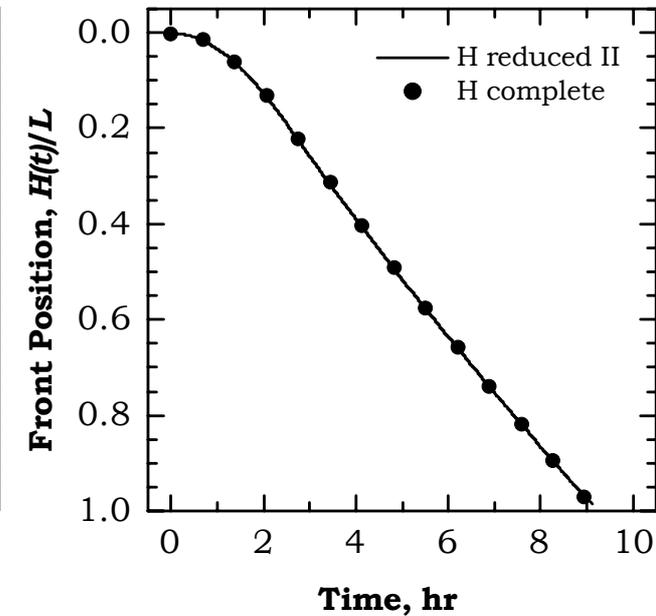
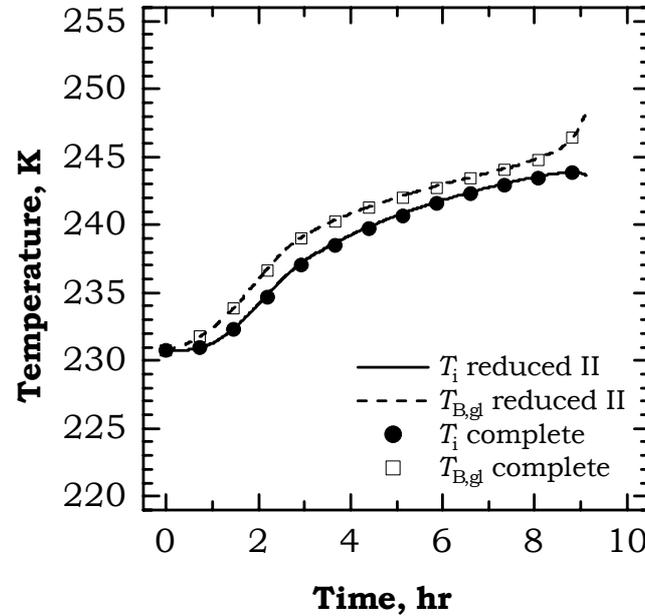
- Dynamics of:

*Front temperature*

*Vial glass temperature*

*Front position*

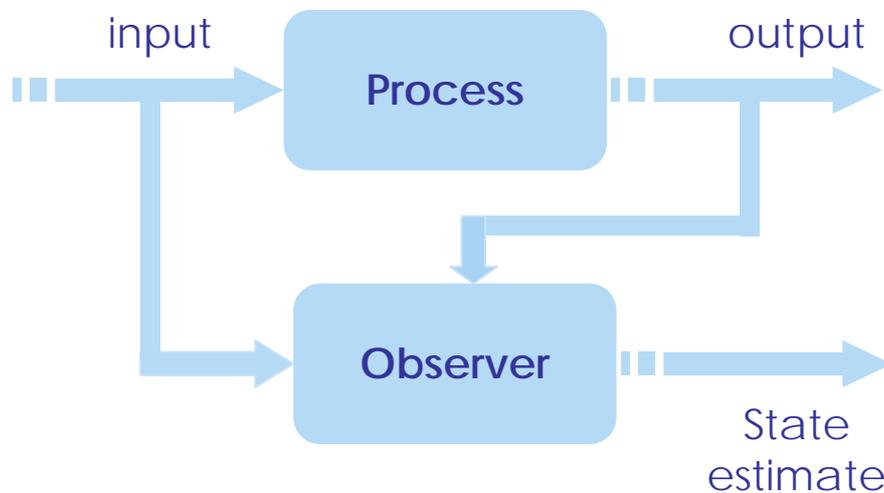
- ①  $t = 2.25$  hr
- ②  $t = 4.5$  hr
- ③  $t = 6.8$  hr



## Soft-sensors (observers)

In many engineering applications it is desirable to have **estimates of hard-to-measure or non-measurable quantities**.

An **observer** combines *a priori* knowledge about the physical system (**mathematical model**) with experimental data (**on-line measurements**) to provide an on-line estimation of the sought quantities.



Process

$$\dot{x} = f(x)$$

$$y = h(x)$$

↓

Observer

$$\dot{\hat{x}} = f(\hat{x}) - K(\hat{y} - y)$$

$$\hat{y} = h(\hat{x})$$

correction

## Soft-sensors (observers)

Primary drying should be carried on at a **controlled sublimation temperature** in order to **avoid denaturation of the product**.

**Problem:** *front temperature can not be directly measured*

From simplified  
models



- Non-linear observers (*Extended Kalman Filter, High Gain Observer*) to estimate  $T_f$  on-line
- unknown heat and mass transfer parameters

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Different typologies of  
observers (EKF, HG)



Different approaches to  
determination of the  
corrective term (gain)

## Improvement of the control system

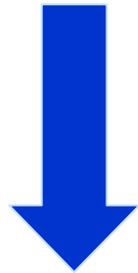
In order to improve both quality and reproducibility some objectives should be fulfilled:

### Objectives

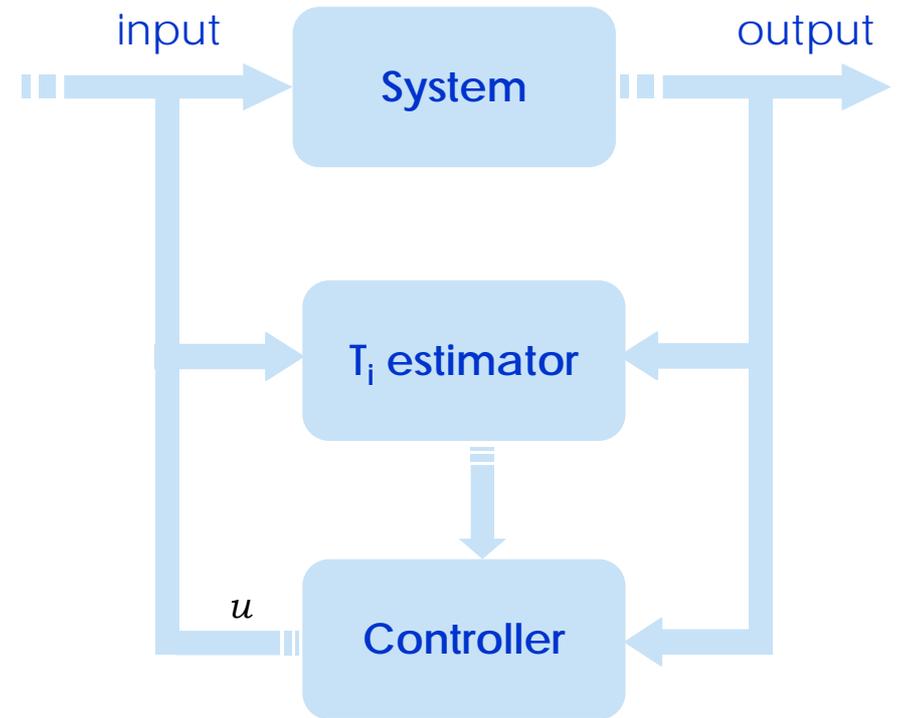
- Control the product **end-use properties**
- Develop a **fault diagnosis** software for controlling the process, detecting problems on-line and preventing large degradation
- Develop a software, based on **remote sensing tools** (soft-sensors, manometric temperature measurements) for the **quality estimation** of the end-use product

## Closed-loop control (for cycle development)

MTM or soft-sensor  
for estimation of  $T_i$   
can be inserted in a  
feedback loop



It is possible to  
control the product  
temperature in real  
time preventing  
product degradation



Estimated  $T_i$

$$u(t) = K(\hat{T}_i(t), p(t))$$

Controller law  
(PI, MPC, ...)

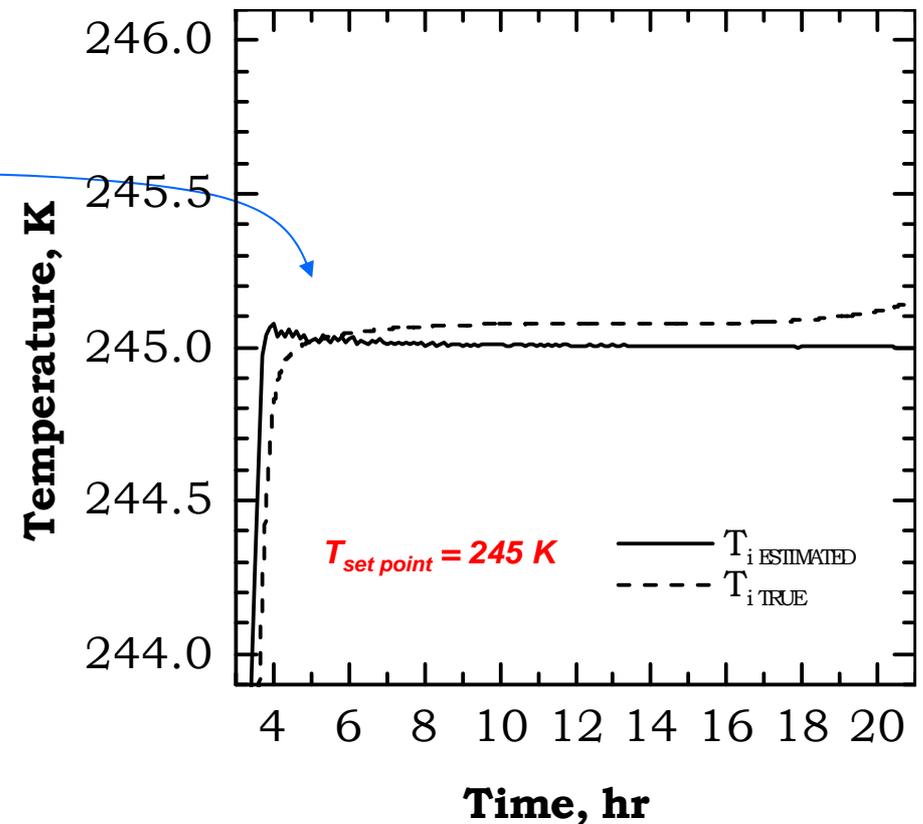
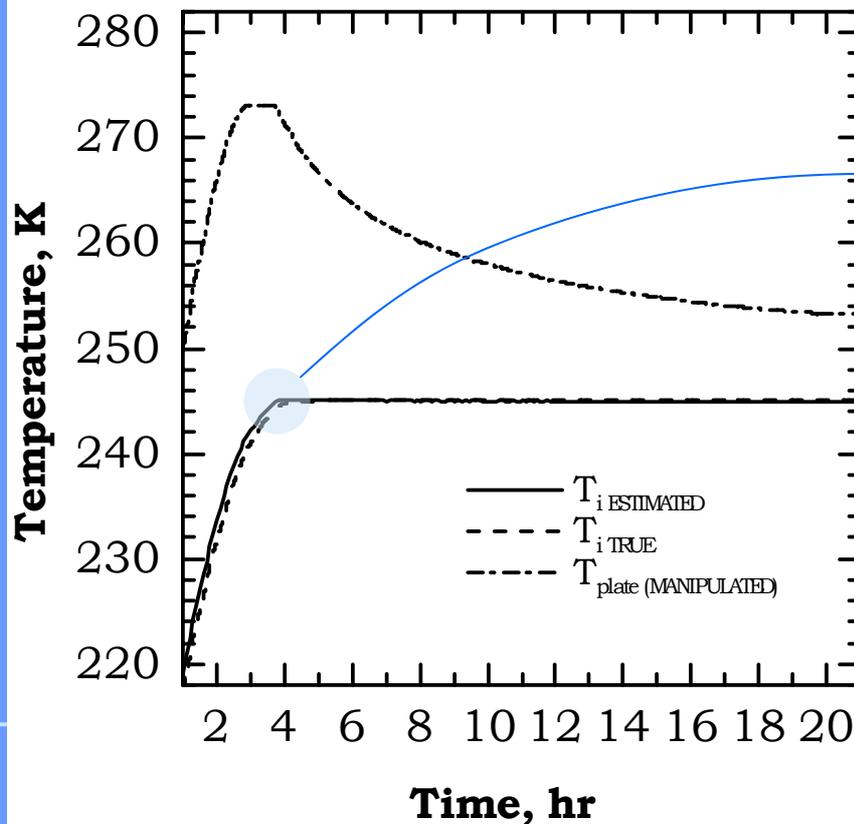
Constraints

## Closed-loop control (skim milk)

The temperature of the moving front is controlled by manipulating the temperature of the heating plate  $T_{plate}$

A conventional Proportional-Integral (PI) controller has been implemented

Controller tuning according to MIN of ISE:  $\min_{K_P, K_I} (ISE) = \min_{K_P, K_I} \int_{t_0}^t (T_{i, predicted}(\tau) - T_{i, MAX})^2 d\tau$



Lyo-Pro  
Competitive and Sustainable Growth European Project

Innovative nucleation technology

The first prototype of freeze-dryer with the nucleation technology  
created by Asymptote and Telstar Industrial

