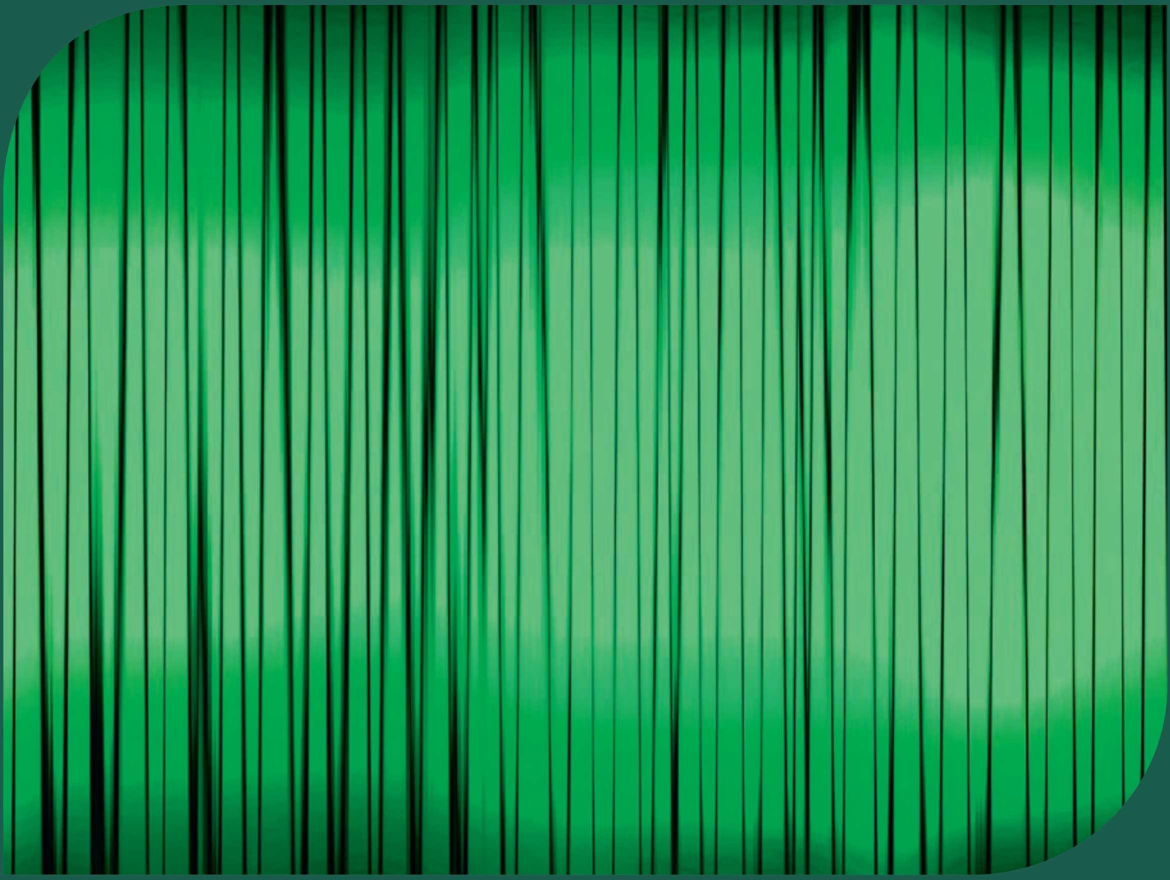
SciTech Publishing
an imprint of the IET

ISMB
Istituto Superiore Mario Boella

U.R.S.I.



The Wiener-Hopf Method in Electromagnetics

Vito G. Daniele and Rodolfo S. Zich

MARIO BOELLA SERIES ON ELECTROMAGNETISM IN INFORMATION & COMMUNICATION

The Wiener-Hopf Method in Electromagnetics

Mario Boella Series on Electromagnetism in Information and Communication

Piergiorgio L. E. Uslenghi, PhD – Series Editor

The Mario Boella series offers textbooks and monographs in all areas of radio science, with a special emphasis on the applications of electromagnetism to information and communication technologies. The series is scientifically and financially sponsored by the Istituto Superiore Mario Boella affiliated with the Politecnico di Torino, Italy, and is scientifically co-sponsored by the International Union of Radio Science (URSI). It is named to honor the memory of Professor Mario Boella of the Politecnico di Torino, who was a pioneer in the development of electronics and telecommunications in Italy for half a century, and a vice president of URSI from 1966 to 1969.

Published Titles in the Series

Introduction to Wave Phenomena
by Akira Hirose and Karl Lonngren

Scattering of Waves by Wedges and Cones with Impedance Boundary Conditions
by Mikhail Lyalinov and Ning Yan Zhu

Complex Space Source Theory of Spatially Localized Electromagnetic Waves
by S. R. Seshadri

The Wiener-Hopf Method in Electromagnetics
by Vito Daniele and Rodolfo Zich

Forthcoming Titles

Higher Order Numerical Solution Techniques in Electromagnetics
by Roberto Graglia and Andrew Peterson (2015)

Slotted Waveguide Array Antennas
by Sembiam Rengarajan and Lars Josefsson (2015)

The Wiener-Hopf Method in Electromagnetics

ISMB Series

Rodolfo Zich
Instituto Superiore de Mario Boella

Vito Daniele
Polytechnic of Torino

SciTech Publishing
an imprint of the IET

theiet.org

Published by SciTech Publishing, an imprint of the IET.
www.scitechpub.com
www.theiet.org

10 9 8 7 6 5 4 3 2 1

ISBN 978-1-61353-001-6 (hardback)
ISBN 978-1-61353-031-3 (PDF)

Typeset in India by MPS Limited
Printed in the US by Integrated Books International
Printed in the UK by CPI Group (UK) Ltd, Croydon

Contents

Preface

In 1931, Wiener and Hopf (1931) invented a powerful technique for solving an integral equation of a special type. By introducing the Laplace transform of the unknown, the integral equation was rephrased in terms of a functional equation in a suitably defined complex space. The solution method of the latter is very ingenious indeed. It is based on a sophisticated procedure exploiting some properties of the analytic functions and it stands as one of the most important mathematical inventions for obtaining analytical solutions of very difficult problems.

In electromagnetic geometries, a fundamental approach due to Jones (1952a) applies the Laplace transforms directly to the partial differential equations, and the complex variable functional equations are obtained directly without having to formulate an integral equation. Jones's approach has been adopted systematically by Noble (1988) in his book on the Wiener-Hopf technique. Noble's work presents many applications of the Wiener-Hopf technique in a systematic way and is fundamental for readers interested in this powerful method. Unfortunately, this book was written many years ago (the first edition was in 1958); in the meantime, many scientists have devoted efforts to studying the Wiener-Hopf technique and have achieved important developments.

The main purpose of this book is to provide students and scientists of diffraction phenomena with a comprehensive treatment of the Wiener-Hopf technique, including its latest developments. In particular, these developments illustrate the wide range of possible applications of this method. In practice, it is now possible to solve all canonical diffraction problems involving geometrical discontinuities using the Wiener-Hopf technique, which has definitively established it as the most general and powerful analytical method for this purpose.

A great number of problems can be effectively approached using the W-H technique (Fig. 1). Shown in the figure are geometrical structures that can be considered equivalent to a (uniform or nonuniform) waveguide in which semi-infinite geometrical discontinuities have been introduced. These discontinuities may be also modified in the transversal or longitudinal direction of the waveguide, thus augmenting considerably the number of possible problems that can be effectively studied by this technique. It must be observed that most of these problems are very important and that often there are no alternative approaches available for solving them efficiently, even numerically. Some general remarks about the W-H techniques are necessary before delving into specific problems in detail.

First of all, no W-H problem is simple to study. For instance, for a given electromagnetic problem that perhaps may be formulated in terms of W-H equations, it could be

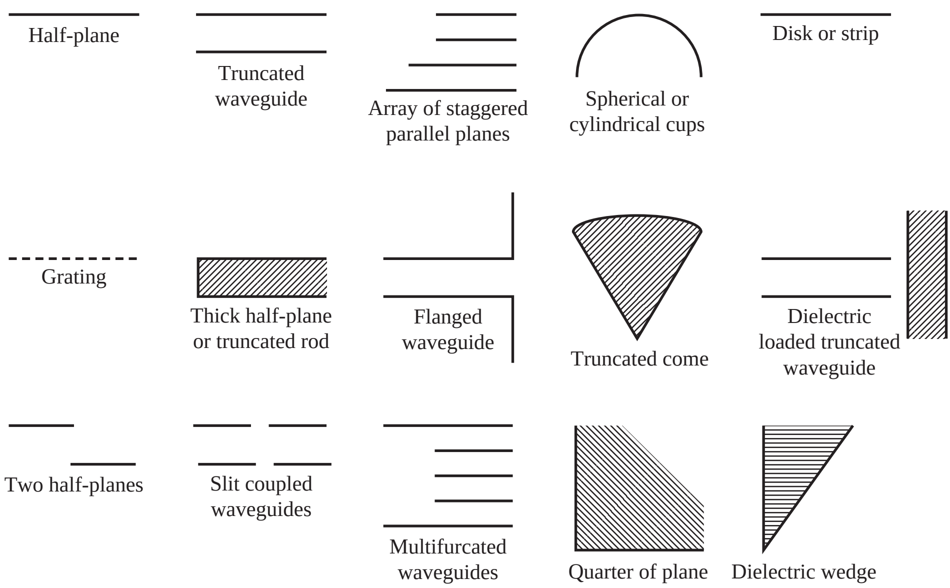


Fig. 1: A few examples of W-H geometries

quite difficult to obtain these equations. In the literature, many problems are formulated in terms of functional equations that, even though equivalent to Wiener-Hopf equations, do not present the so-called standard forms considered in this book. We emphasize that it is important to formulate the problems in terms of standard W-H equations because it provides a uniform methodology to obtain exact or approximate solutions in a systematic way. The key function containing all the information in the standard Wiener-Hopf equations is the W-H kernel. It is generally a matrix $\mathbf{G}$ function of a complex variable α. It follows that the first step of the W-H technique is to find $\mathbf{G}(\alpha)$ for a specific geometry. Sometimes this is a difficult task requiring a profound knowledge both of the formulation of electromagnetic problems and of the underlying physical concepts.

The central problem in solving the standard W-H equations is conceptually very simple: the factorization of the matrix $\mathbf{G}(\alpha)$. This problem constitutes a very beautiful mathematical problem that in the past has become a cult activity for many students. However, even though this problem has been extensively studied in the past, up to now a method to factorize a general $n \times n$ matrix (chapter 4) was not known. Fortunately, several approximate factorization techniques have recently been developed. In particular, the reduction of the factorization problem to the solution of Fredholm integral equations of the second kind constitutes a powerful tool that provides efficiently the approximate factorized matrices of $\mathbf{G}(\alpha)$.

Once the factorization of $\mathbf{G}(\alpha)$ is achieved, new efforts are necessary to extract solutions. In fact, even if formal solutions may be obtained, a long and difficult elaboration is always required to make them effective from the physics and engineering points of view.

The W-H technique involves complex and cumbersome algebraic manipulations. Nowadays these manipulations do not constitute a serious impediment because powerful algebraic manipulator codes are readily available. In particular, all the results in this book were obtained by intensive use of the computing software MATHEMATICA.

Concerning the overall philosophy of the subject presentation, this book has been written for readers primarily interested in the fundamental concepts and possible applications of the presented method. For this reason, the considered arguments are often only delineated and not discussed in great mathematical depth. The W-H technique requires the knowledge and use of many advanced topics of complex analysis, whose exposition might discourage readers who are interested primarily in application aspects. Of course, the best way to render the mathematical tools appealing is to present them only in as much detail as is required for the specific applications. We tried to follow this principle, but it was sometimes impossible. Therefore, we divided the book into two parts. The first part (chapters 1–6) is devoted to the mathematical aspects of the W-H technique, whereas the second part (chapters 7–10) presents applications that we hope illustrate the beauty, aims, and power of the theory. In particular, in the applications we often emphasized only the first and more difficult step of the W-H technique: the deduction of the matrix kernel $\mathbf{G}(\alpha)$ of the problem. In fact, this is the step that in some sense lacks of a general methodology. It is the intensive presentation of the deduction of $\mathbf{G}(\alpha)$ in different problems that provides the useful tools and the practice needed for solving new problems.

The Wiener-Hopf equations studied in this book are substantially one dimensional. It is possible to introduce multidimensional W-H equations (Meister & Speck, 1979) and generalize the concept of factorization that constitutes the fundamental tool that distinguishes the W-H equations from other integral equations. In particular, two works by Radlow (1961, 1964) attempted to solve two fundamental diffraction problems[1] by factorizing kernels defined in two-dimensional space. In these cases, the factorization method needs function-theoretic tools employing analytical functions with two complex variables. The involved analytical difficulties may easily lead to errors, and as a consequence unfortunately Radlow's solutions are incorrect. To date, the only way to solve multidimensional W-H equations appears to be the use of the moment method. Even though approximate, this kind of solution is very powerful; some examples will be considered in chapter 8.

In this book we consider only time harmonic fields with a time dependence specified by the factor $e^{j\omega t}$ (electrical engineering notations), which is omitted throughout, and where the imaginary unit is indicated with j. Conversely, in applied mathematics the factor $e^{j\omega t}$ is usually replaced by $e^{-i\omega t}$. This means that in the natural domain the change $j \Rightarrow -i$ transforms the engineering notation into applied mathematics notation (and vice versa). However, in the spectral domain, usually the same notations are used in both engineering and applied mathematics. In fact, regarding for example the Fourier transforms, the following definitions are the most frequently used in the literature:

$$F_e(\alpha) = \int_{-\infty}^{\infty} f_e(x) e^{j\alpha x} dx, \quad F_a(\alpha) = \int_{-\infty}^{\infty} f_a(x) e^{i\alpha x} dx$$

where the subscript e means engineering and the subscript a means applied mathematics. Consequently, in the spectral domain on the real axis we have

$$F_a(\alpha) = F_e(-\alpha)$$

and j is replaced by $-i$ (and vice versa).

[1] The diffraction problems studied by Radlow are the diffraction by a quarter-plane and the diffraction by a right-angle dielectric wedge.

For example, let us consider in the natural domain the propagation factor that is defined in electrical engineering notation by

$$f_e(x) = e^{-jkx}$$

with the propagation constant k defined by

$$k = \beta - ja, \quad a \geq 0$$

The same propagation factor in applied mathematics notation is written

$$f_a(x) = e^{ik_a x}$$

with $k_a = \beta + ia$.

In the Laplace domain, on the real axis, we have

$$F_e(\alpha) = \int_0^\infty f_e(x) e^{-jkx} e^{j\alpha x} dx = \frac{j}{\alpha - k}$$

which in applied mathematics notations is written

$$F_e(\alpha) = \frac{j}{\alpha - k} \Rightarrow F_a(\alpha) = \frac{-i}{-\alpha - k_a} = \frac{i}{\alpha + k_a}$$

Analytic continuations define the previous functions in the whole complex plane α. This means that the Laplace Transforms are defined for every value of α by

$$F_e(\alpha) = \frac{j}{\alpha - k}, \quad F_a(\alpha) = \frac{i}{\alpha + k_a}$$

In the following we will define plus $F_+(\alpha)$ and minus $F_-(\alpha)$ (section 1.1). Notice that a plus (or minus) function in the electrical engineering notation is also a plus (or minus) function in the applied mathematics notation. The only difference between the two is given by the location of the singularities. For example, $F_e(\alpha)$ and $F_a(\alpha)$ are plus functions both with engineering and applied mathematics notation. However, $F_e(\alpha) = \frac{j}{\alpha - k}$ has a singularity at $\alpha = k = \beta - ja$, whereas $F_a(\alpha) = \frac{i}{\alpha + k_a}$ has it at $\alpha = -k_a = -\beta - ia$. The notation and definitions presented in this preface will be used throughout the book.

In the 80 years since the seminal 1931 paper by Wiener and Hopf, an enormous amount of work has been performed using their powerful function-theoretic method and its further extensions. It would not be possible to reproduce all that work in detail within a single volume. Therefore, we simply report many results without proof, referring the interested reader to the bibliographical sources for additional details. Similarly, we list many applications of the method to electromagnetic boundary-value problems, often just providing the results without the detailed derivations that readers may find in the original publications.

Foreword

The Mario Boella series offers textbooks and monographs in all areas of radio science, with a special emphasis on the applications of electromagnetism to information and communication technologies. The series is scientifically and financially sponsored by the Istituto Superiore Mario Boella affiliated with the Politecnico di Torino, Italy, and is scientifically cosponsored by the International Union of Radio Science (URSI). It is named to honor the memory of Professor Mario Boella of the Politecnico di Torino, who was a pioneer in the development of electronics and telecommunications in Italy for half a century and was vice president of URSI from 1966 to 1969.

This advanced research monograph is devoted to the Wiener-Hopf technique, a function-theoretic method that has found applications in a variety of fields, most notably in analytical studies of diffraction and scattering of waves. It contains a compendium of the research work of Professor Vito G. Daniele of the Politecnico di Torino, who is a foremost international authority on the Wiener-Hopf method. Professor Daniele has teamed with his colleague and coauthor, Professor Rodolfo S. Zich, past rector of the Politecnico di Torino and current president of the Istituto Superiore Mario Boella, in writing this monograph.

It is hoped that this work will be well received by scientists, engineers, and applied mathematicians and will serve as a benchmark reference in the field of theoretical electromagnetism for the foreseeable future.

Piergiorgio L. E. Uslenghi
Series Editor
Chicago, January 2014