Sparse noise-compliant synthesis of scattering macromodels via resistance extraction

# Sparse Noise-Compliant Synthesis of Scattering Macromodels via Resistance Extraction

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***Abstract*—The construction of black-box macromodels, either from frequency responses or via model order reduction, has become a standard practice in Computer-Aided Design flows of digital, analog, mixed-signal and radio-frequency systems. In order to be proficiently used, such macromodels need to be synthesized into equivalent circuits that are compatible with common circuit solvers. In this work, we propose a classical synthesis approach based on resistance extraction, which is here revisited and suitably modified to achieve sparsity, or equivalently, to produce a minimum number of basic circuit elements in the final netlist. The proposed approach produces circuit equivalents that are compatible with any type of circuit simulation, including transient and noise analysis.**

## I. Introduction

Black-box macromodeling techniques have become very popular in the Computer-Aided Design (CAD) flows of digital, analog, mixed-signal and Radio-Frequency (RF) systems [1]. These methods construct reduced-order circuit equivalents of possibly complex interconnect networks, and more generally of any Linear and Time Invariant (LTI) structure, starting from many possible different characterizations. For instance, when the system is known as a set of large-scale descriptor or state-space equations arising from an extraction or discretization process, many Model Order Reduction (MOR) approaches are available to remove redundant states and reduce circuit complexity [2]–[4]. When instead the system is known from its external frequency or time domain responses, curve fitting techniques [5]–[8] can be used to derive low-order rational approximations of the system transfer matrix, which is then easily realized in state-space form. In both scenarios, macromodel passivity can be checked and enforced, either a priori [4], [9] or through some postprocessing stage [10].

Macromodels are almost invariably black-box approximate representations of the original system behavior. The corresponding state-space equations are characterized by matrix entries that are not related to the underlying physics (propagation, loss, and dispersion to name a few), but that are computed through a mathematical process that does not take this physics into account. This fact should be considered when macromodels are synthesized into equivalent networks made of primitive circuit elements (resistors, capacitors, inductors, dependent sources, etc.) for use in off-the-shelf circuit solvers. A good macromodel realization will produce correct transient and AC responses, but the circuit elements that are embedded in the macromodel netlist should be considered just as basic building blocks that translate a mathematical expression into a language that the solver understands. This fact explains why thermal noise analyses based on macromodels usually fail, by producing incorrect noise spectral densities. In fact, the basic mechanism for noise generation is implemented in circuit solvers by adding suitable noise sources to lossy elements, i.e., resistors [11]. When these resistors do not represent physical components, the associated noise sources that are automatically considered by the solvers are meaningless and lead to incorrect results.

In this work, we propose a noise-compliant synthesis, that produces equivalent netlists for a given passive macromodel in scattering representation. The approch is based on the classical idea of resistance extraction [12], [13], which decomposes the system as an interconnection of an augmented lossless part plus a set of resistors. We show that, even if such resistors are "artificial" components, the resulting noise analysis performed by SPICE will be correct. With respect to earlier noise-compliant synthesis approaches [14], the proposed technique exploits sparsity to reduce netlist complexity, which scales only linearly with the number of macromodel states. Moreover, there is no need of an explicit additional noise companion synthesis as in [15].

## II. Formulation

We consider as a starting point a given $p$-port macromodel expressed in regular state-space form

$$\dot{\boldsymbol{x}}(t) = \mathbf{A}\boldsymbol{x}(t) + \mathbf{B}\boldsymbol{u}(t), \quad (1a)$$

$$\boldsymbol{y}(t) = \mathbf{C}\boldsymbol{x}(t) + \mathbf{D}\boldsymbol{u}(t), \quad (1b)$$

with $\mathbf{A} \in \mathbb{R}^{n\times n}$, $\mathbf{B} \in \mathbb{R}^{n\times p}$, $\mathbf{C} \in \mathbb{R}^{p\times n}$ and $\mathbf{D} \in \mathbb{R}^{p\times p}$, where $\boldsymbol{x}(t) \in \mathbb{R}^n$ collects internal state variables, and where $\boldsymbol{u}(t), \boldsymbol{y}(t) \in \mathbb{R}^p$ denote input and output vectors. Throughout this paper, we consider macromodels in scattering representation normalized to some common port reference resistance $R_{\rm ref}$, so that $\boldsymbol{u}(t) = \boldsymbol{a}(t)$ collects the incident scattering waves at the interface ports, and $\boldsymbol{y}(t) = \boldsymbol{b}(t)$ the corresponding reflected waves. The transfer matrix of (1)

$$\mathbf{H}(s) = \mathbf{D} + \mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B} \quad (2)$$

thus coincides with the scattering matrix of the macromodel.

The state-space matrices in (1) can be derived from an identification process based on rational curve fitting of frequency samples, or through application of a MOR technique to a set of state-space equations of larger size. In any of these two cases, we assume that the state-space system (1) is passive. Therefore, the following alternative equivalent conditions hold:

- the transfer matrix $\mathbf{H}(s)$ is Bounded Real (BR); this is guaranteed if $\mathbf{A}$ is strictly stable, i.e., the eigenvalues of $\mathbf{A}$ have strictly negative real part, and if

$$\sigma_i \leq 1\,, \quad \forall \sigma_i \in \sigma\{\mathbf{H}(\mathrm{j}\omega)\} \quad \forall \omega \in \mathbb{R}\,, \tag{3}$$

  where $\sigma\{\}$ denotes the set of singular values of its matrix argument;
- the state-space matrices satisfy the Bounded Real Lemma (BRL) condition

$$\exists \mathbf{P} = \mathbf{P}^\mathsf{T} > 0 : \\ \begin{bmatrix} \mathbf{A}^\mathsf{T}\mathbf{P} + \mathbf{P}\mathbf{A} + \mathbf{C}^\mathsf{T}\mathbf{C} & \mathbf{P}\mathbf{B} + \mathbf{C}^\mathsf{T}\mathbf{D} \\ \mathbf{B}^\mathsf{T}\mathbf{P} + \mathbf{D}^\mathsf{T}\mathbf{C} & -(\mathbf{I} - \mathbf{D}^\mathsf{T}\mathbf{D}) \end{bmatrix} \leq 0\,, \tag{4}$$

  which can be restated in the following equivalent form

$$\exists \mathbf{P} = \mathbf{P}^\mathsf{T} > 0, \quad \exists \mathbf{L}, \mathbf{W} : \\ \begin{cases} \mathbf{A}^\mathsf{T}\mathbf{P} + \mathbf{P}\mathbf{A} + \mathbf{C}^\mathsf{T}\mathbf{C} &= -\mathbf{L}\mathbf{L}^\mathsf{T}\,, \\ \mathbf{P}\mathbf{B} + \mathbf{C}^\mathsf{T}\mathbf{D} &= -\mathbf{L}\mathbf{W}\,, \\ \mathbf{I} - \mathbf{D}^\mathsf{T}\mathbf{D} &= \mathbf{W}^\mathsf{T}\mathbf{W}\,, \end{cases} \tag{5}$$

  where also the additional matrices $\mathbf{L} \in \mathbb{R}^{n \times \rho}$ and $\mathbf{W} \in \mathbb{R}^{\rho \times p}$, with $\rho \leq p$, are unknowns which can be determined numerically as discussed in [12].

In case the model is obtained by a curve fitting scheme, it is assumed that its passivity has been checked and enforced through one of the many existing algorithms [10]. Conversely, in case the model is obtained from a MOR scheme, it is assumed that a passivity-preserving method such as [4], [9] has been employed, or that model passivity has been checked or enforced a posteriori.

Let us assume that a triplet of matrices $\{\mathbf{P}, \mathbf{L}, \mathbf{W}\}$ that solve (5) has been found. We define

$$\mathbf{A}_L = \mathbf{A} \tag{6a}$$

$$\mathbf{B}_L = \begin{bmatrix} \mathbf{B} & -\mathbf{P}^{-1}(\mathbf{C}^\mathsf{T}\mathbf{D}_{12} + \mathbf{L}\mathbf{D}_{22}) \end{bmatrix} \tag{6b}$$

$$\mathbf{C}_L^\mathsf{T} = \begin{bmatrix} \mathbf{C}^\mathsf{T} & \mathbf{L} \end{bmatrix} \tag{6c}$$

$$\mathbf{D}_L = \begin{bmatrix} \mathbf{D} & \mathbf{D}_{12} \\ \mathbf{W} & \mathbf{D}_{22} \end{bmatrix}\,, \tag{6d}$$

where the block columns formed by $\mathbf{D}_{12}$ and $\mathbf{D}_{22}$ are determined to form the orthonormal complement to the first $p$ columns $(\mathbf{D}; \mathbf{W})$, so that

$$\mathbf{I} - \mathbf{D}_L^\mathsf{T}\mathbf{D}_L = 0 \tag{7}$$

(orthonormality of the first $p$ columns follows from (5)). One can show that the $(p+\rho)$-port scattering system defined by the state-space matrices $\{\mathbf{A}_L, \mathbf{B}_L, \mathbf{C}_L, \mathbf{D}_L\}$ with transfer function

$$\mathbf{H}_L(s) = \mathbf{D}_L + \mathbf{C}_L(s\mathbf{I} - \mathbf{A}_L)^{-1}\mathbf{B}_L \\ = \begin{bmatrix} \mathbf{H}_{11}(s) & \mathbf{H}_{12}(s) \\ \mathbf{H}_{21}(s) & \mathbf{H}_{22}(s) \end{bmatrix} \tag{8}$$

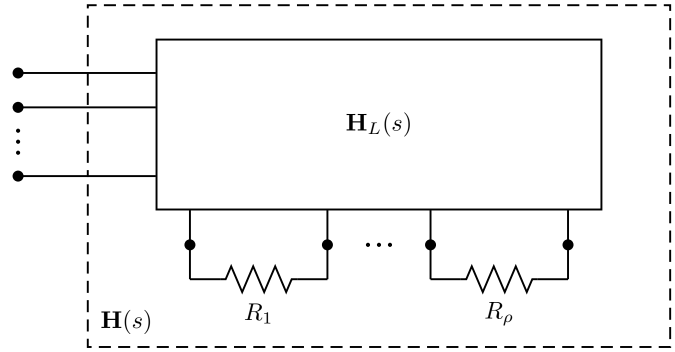


Fig. 1. Graphical illustration of synthesis by resistance extraction. All external resistances $R_i = R_{\mathrm{ref}}$, and $\mathbf{H}_L(s)$ is lossless.

is passive and lossless, i.e., it satisfies the following property

$$\mathbf{I} - \mathbf{H}_L^\mathsf{T}(-\mathrm{j}\omega)\mathbf{H}_L(\mathrm{j}\omega) = 0 \quad \forall \omega \in \mathbb{R} \tag{9}$$

and, equivalently, verifies the BRL (5) in its lossless form

$$\begin{cases} \mathbf{A}_L^\mathsf{T}\mathbf{P}_L + \mathbf{P}_L\mathbf{A}_L + \mathbf{C}_L^\mathsf{T}\mathbf{C}_L &= 0\,, \\ \mathbf{P}_L\mathbf{B}_L + \mathbf{C}_L^\mathsf{T}\mathbf{D}_L &= 0\,, \\ \mathbf{I} - \mathbf{D}_L^\mathsf{T}\mathbf{D}_L &= 0\,, \end{cases} \tag{10}$$

with the same $\mathbf{P}_L = \mathbf{P}$. Moreover, since the upper left block of $\mathbf{H}_L(s)$ equals the original transfer function $\mathbf{H}_{11}(s) = \mathbf{H}(s)$, it is immediate to conclude that closing the last $\rho$ ports of $\mathbf{H}_L(s)$ into the reference resistance $R_{\mathrm{ref}}$ of the adopted scattering representation leads to a realization of $\mathbf{H}(s)$. Figure 1 provides a graphical illustration.

Since $\mathbf{H}_L(s)$ is lossless, it can be realized as an equivalent network that does not include resistors, hence the common denomination of this synthesis as *resistance extraction* [12], [13]. Classical approaches are usually targeted towards a synthesis of $\mathbf{H}_L(s)$ based on physically realizable circuit elements, in this case inductors, capacitors, and ideal transformers. This approach would require two additional constraints:

1) the model is reciprocal;
2) the original state-space realization is *internally passive*, i.e., both (4) and (5) are satisfied with $\mathbf{P} = \mathbf{I}$.

Condition 1 holds in most practical applications, but is not explicitly required in our proposed approach. Condition 2 instead is usually not verified, and a suitable coordinate change in the state-space is required in order to convert the given realization into an internally passive one [16]. Although this process is not really critical (it is sufficient to apply a similarity transformation $\mathbf{T}$ where $\mathbf{T}$ is computed, e.g., as the Cholesky factorization of $\mathbf{P}$), it invariably produces state matrices that are full. Therefore, the complexity of the equivalent network that synthesizes $\mathbf{H}_L(s)$ is $O((n+p+\rho)^2)$.

Our main objectives in this work are instead:

a) minimize the number of circuit elements;
b) make sure that the realized circuit, when run by standard circuit solvers of the SPICE class, can be used also for noise analysis, in addition to AC and transient simulations.

Objective a) is easy to achieve. In fact, we observe from (6) that $\mathbf{A}_L = \mathbf{A}$, which is the dynamic matrix of the original macromodel. We have therefore the freedom to choose the original state-space realization so that $\mathbf{A}$ is sparse: this sparsity will be preserved also in $\mathbf{H}_L(s)$. All results in this work are based on macromodels in pole-residue form

$$\mathbf{H}(s) = \mathbf{D} + \sum_{k=1}^{m} \frac{\mathbf{R}_k}{s - p_k}\,, \tag{11}$$

for which a sparse (quasi-diagonal) realization is obtained with

$$\mathbf{A} = \text{blkdiag}\left\{\ldots, q_k, \ldots, \begin{bmatrix} \sigma_k & \omega_k \\ -\omega_k & \sigma_k \end{bmatrix}, \ldots\right\}, \tag{12}$$

where $q_k$ are real poles and $\sigma_k \pm \mathrm{j}\omega_k$ are complex conjugate poles, and where each term is repeated $\text{rank}(\mathbf{R}_k)$ times [17]. The number of nonvanishing coefficients in $\{\mathbf{A}_L, \mathbf{B}_L, \mathbf{C}_L, \mathbf{D}_L\}$ therefore scales as $O(n(2p + 2\rho + 1))$, i.e., only linearly in the macromodel order.

Objective b) is also straightforward. Noise compliance of a circuit synthesis, here intended as the property of a given netlist to be correctly analyzed by standard circuit solvers in order to produce consistent noise spectra, is guaranteed if all circuit elements that generate noise are equipped with a proper noise model in the simulator. The only such elements are resistors $R$, characterized by associated noise (voltage) sources with spectral density $\bar{V}^2(\omega) = 4K_bTR$, where $K_b$ is the Boltzmann constant and $T$ is the operating temperature. In our approach, we explicitly extract resistances as depicted in Fig. 1. The complementary part $\mathbf{H}_L(s)$ does not generate noise since lossless, contributing to the noise spectral densities and cross-spectra at the $p$ output ports only via a "filtering" or "spectral shaping" process. Since $\mathbf{H}_L(s)$ must not generate noise, we are free to use any circuit element for its realization, as far as the simulator assumes it as a "cold" (noiseless) device. Dependent sources are thus allowed. Therefore, we realize $\mathbf{H}_L$ as in [1], [4], with each state encoded as the voltage across a unit capacitor, and with all entries in the state matrices (6) realized by suitably connected dependent sources. This process is standard and not further commented here.

Comparing the proposed synthesis with a direct realization of $\mathbf{H}(s)$ via capacitors and dependent sources [1], [4], we gain noise compliance at the price of a slightly increased circuit complexity. In fact, the resistance extraction process forces us to synthesize the lossless subsystem $\mathbf{H}_L(s)$, which has additional $\rho$ internal ports, which in the worst case can reach $\rho = p$. The leading complexity scales however only linearly with the number of states $n$ for both approaches, which is a significant advancement with respect to the reactance extraction approach of [14], for which complexity is $O(n^2)$.

## III. Numerical results

The proposed macromodel synthesis is applied to a packaging interconnect with $p = 4$ ports. The structure was first characterized via a full-wave electromagnetic solver from 100 MHz to 30 GHz. The resulting sampled scattering responses were then processed by a rational fitting engine based on Vector Fitting [5], [8], and the resulting pole-residue form (11) was realized into a Gilbert state-space system [17], with a quasi-diagonal $\mathbf{A}$ as in (12), and with dynamic order $n = 144$. Finally, macromodel passivity was enforced based on the Hamiltonian perturbation algorithm [10].

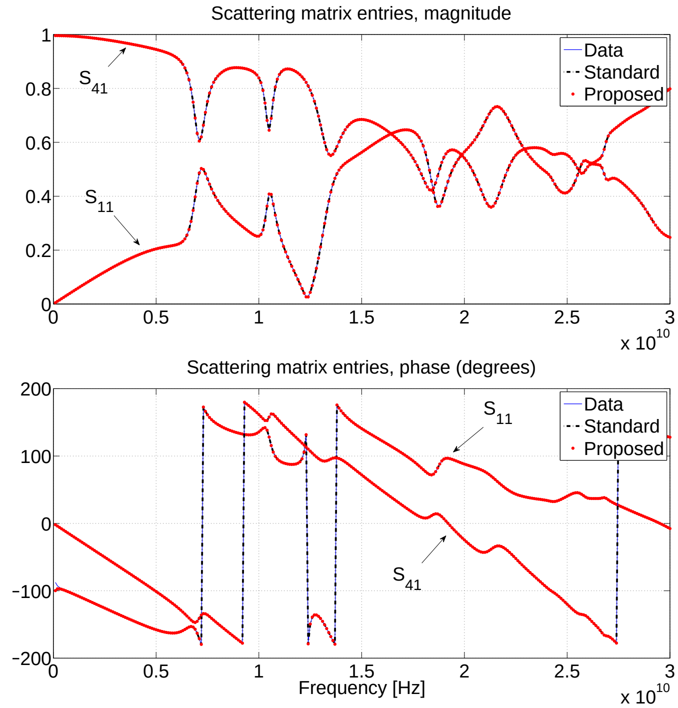


Fig. 2. Selected scattering responses of a 4-port packaging interconnect. Comparison between raw data from field solver (thin black solid line) and synthesized macromodels via standard (blue dash-dotted line) and proposed resistance extraction (red dashed line) approaches.

The resulting passive macromodel was then synthesized in two different equivalent circuits. The first, based on [1], [4], is a direct translation of the state-space equations into a netlist including capacitors and dependent sources. The second is the proposed approach, which first extracts resistances as discussed in Sec. II by constructing the lossless subsystem $\mathbf{H}_L(s)$, which is then realized into a netlist following the same above procedure. The last $\rho$ ports of $\mathbf{H}_L(s)$ are finally closed on $R_{\text{ref}} = 50\Omega$ resistors, which are declared as "noisy" to the circuit simulator.

Figure 2 demonstrates the accuracy of the macromodel netlists. A few selected scattering responses obtained by a SPICE frequency sweep on the two macromodels are compared to the original frequency responses from the field solver. There is no visible difference between models and data, so both macromodel implementations are very accurate. Figure 3 reports instead the results of a set of SPICE noise analysis, under various different termination conditions. For each panel, the same simulation testbench is used for computing output (voltage) noise spectral densities for both macromodel implementations. In addition, reference noise spectra are also obtained by replacing the macromodels with an S-parameter

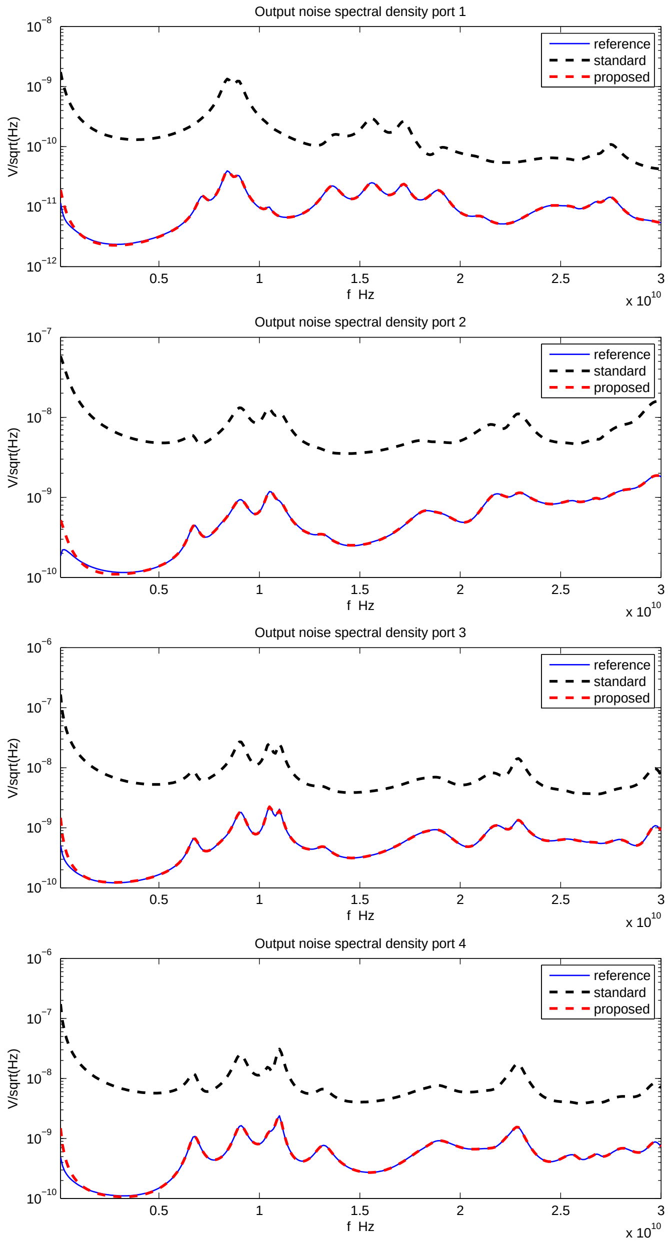


Fig. 3. Selected (voltage) noise spectral densities of a 4-port packaging interconnect obtained using raw scattering data from field solver (thin black solid line) and synthesized macromodels via standard (blue dash-dotted line) and proposed resistance extraction (red dashed line) approaches.

block defined through the raw scattering samples from the field solver. The results show that only the noise compliant synthesis by resistance extraction matches the reference noise spectra, whereas the direct synthesis produces incorrect results. This was in fact expected, since the direct synthesis employs dependent sources that contribute to the internal losses of the macromodel (thus producing correct frequency responses in AC analyses), but cannot produce the corresponding noise contribution in noise analyses, since the simulator is not able to associate to them a thermal noise model.

## IV. Conclusion

We have presented a sparse noise compliant synthesis of passive state-space scattering macromodels. The approach builds on the classical Darlington synthesis by resistance extraction, which is here adapted and optimized in order to reduce the number of its circuit elements. Numerical results show that the noise spectra computed by SPICE solvers are correct, as expected, although the employed circuit elements (which include dependent sources) are not at all related to the topology and the physics of the underlying structure. This noise compliance is achieved with only a moderate increase in circuit complexity compared to optimally sparse (but not noise-compliant) realizations. In summary, the proposed synthesis can be safely adopted to produce efficient macromodel netlists for universal use in circuit simulators.

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