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Method for calculating the application factor K_A in gears subjected to variable working conditions

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Abstract

The present paper aims to propose a method, in ISO Standard environment, in order to calculate the K_A application factor when the nominal torque is not defined. A procedure is presented, based on the Miner damage rule, in order to process a given load spectrum and to calculate a value of the equivalent tangential force including variable amplitude effects, useful for bending and pitting life calculation. A practical case is presented, based on a recorded vehicle mission, referring to the input gear of a transfer box for industrial truck. A comparison is also given with the ISO Standard results.

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Keywords: gear; application factor; variable loads; cumulative damage.

1. Introduction

Several methods can be found in literature in order to predict the load capacity of spur and helical gear drives. The conventional design procedure compares the gear teeth strength with a calculated stress value, in both bending and pitting cases. The most common approach used in calculation of these stress values, as widely described in ISO Standard 6336 [1-5] and in AGMA Standard 2001-D04 [6], involves the use of influence factors, derived from results of research and field service [7].

Influence factors may be distinguished between two categories, factors which are determined by gear geometry or which have been established by convention and factors which account for several influences and which are dealt with as independent of each other; in this last group is included the application factor K_A. The factor K_A adjusts the nominal load F_t in order to compensate for incremental gear loads from external sources. These additional loads are largely dependent on the characteristics of the driving and driven machines, as well as the masses and stiffness of

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the system, including shafts and couplings used in service.

For applications such as marine gears and others subjected to cyclic peak torque (torsional vibrations) and designed for infinite life, the application factor can be simply defined as the ratio between the peak cyclic torques and the nominal rated torque. The nominal rated torque is defined by the rated power and speed. It is the torque used in the load capacity calculations.

If the gear is subjected to a limited number of known loads in excess of the amount of the peak cyclic torques, this influence may be covered directly by means of cumulative fatigue or by means of an increased application factor, representing the influence of the load spectrum.

ISO Standard Part 6 [5] proposes a procedure for the calculation of an equivalent load in order to obtain the application factor value K_A; this factor, defined as the ratio between the equivalent torque and the nominal one, may give a first estimation of the service life already in the design phase.

The present paper aims to propose a method, in ISO Standard environment, in order to calculate the K_A factor when the nominal torque is not defined, but above all when the gear has been already designed at least from the cinematic point of view and specific variable amplitude loading data are given.

In particular, in this work a procedure is presented based on the Miner damage rule, as in ISO Standard, in order to process a given load spectrum and to calculate a value of the equivalent tangential force that includes the variable amplitude effects; this force value is useful for bending and pitting calculation of the service life. This procedure is applied to the input gear of a transfer box for industrial truck. Variable amplitude loads refer to a vehicle mission recorded during different routes (highway, mixed route, mountain road, ..): both measured torque and cycles number are available to be processed.

Obtained results in terms of K_A factor have been compared to those calculated by means of the classical ISO Standard procedure.

2. State of the art: ISO Standard procedure for calculating the application factor K_A under variable loads

ISO Standard part 6 [5] proposes a calculation method of service life of gears under variable load [8]. In particular, in addition to the classical tables (usually available for components) where an application factor K_A is indicated for working characteristics of both driving and driven machines, it describes a method for a design case where the Woehler-damage line is simplified by ignoring all damage which occurs at stresses below some limit stress corresponding to the fatigue limit of a material for pitting and bending strength.

The application factor K_A for a given load spectrum, in terms of transmitted torque, is simply defined as the ratio between the equivalent torque T_{eq} and the nominal torque T_n as:

$$K_A = \frac{T_{eq}}{T_n} \tag{1}$$

As already quoted, the application factor K_A has to be determined for tooth root brakeage (bending) and for pitting resistance; the equivalent torque is defined by the damage Miner rule where the slope p of the Woehler-damage line and the number of load cycles N_{Lref} at the reference point (endurance limit cycles) have to be known.

By resuming, the procedure described in [5] needs the material Woehler-damage curves (slope p and N_{Lref}) and an hypothesis of nominal torque.

The equivalent torque T_{eq} is defined by the following equation:

$$T_{eq} = \left(\frac{n_1 T_1^p + n_2 T_2^p + \ldots + n_i T_i^p}{n_1 + n_2 + \ldots + n_i} \right)^{\frac{1}{p}} \tag{2}$$

where $i=1, 2, \ldots$ is the number of bin, n_i is the number of cycles for bin i, T_i is the torque for bin i.

The focus point in the procedure is that the number of bins to be used in equation (2) cannot be predetermined.

A torque T_i in the bin i can be replaced by a torque T_j in a new bin ($j=i+1$), so that the damage caused by the torque T_i is the same as that caused by the torque T_j expressed by the following equation:

$$T_i^p n_i = T_j^p n_j \tag{3}$$

The load bins have to be denoted as (T_i, n_i) and numbered in descending order of torque, where T_1 is the highest torque. Then the cycles n_1 at torque T_1 are equivalent in terms of damage to a larger number of cycles n_{1a}, at lower torque T_2, where:

$$n_{1a} = n_1 \left(\frac{T_1}{T_2} \right)^p \tag{4}$$

If $n_{2e} = n_2 + n_{1a}$, then the bins 1 and 2 can be replaced by a single bin (T_2, n_{2e}) and so on.
This procedure has to be stopped when n_{ie} reaches the endurance limit cycles N_{Lref}.

3. Method for calculating the application factor K_A of a component under variable loads

The method proposed in the present paper for determining the application factor K_A in the case of variable amplitude loads aims to be a completion of the ISO Standard [5] procedure when the gear has been already designed at least from the cinematic point of view.

The heart of the method involves the use of a fatigue curve of the component instead of the Woehler-damage curve of the material.

So, being the pitch diameters of the engaging gears known, it is possible to calculate the tangential forces (involved in ISO Standard basis equations) corresponding to each torque level.

The way of thinking that is the basis of this method allows also the specific calculation of the service life of gears, even if details about this procedure are here omitted for sake of brevity.

It may also be utilised when the nominal torque value is unknown, as an example when experimental data are available about loading conditions of the transmission.

As ISO Standard [5], this method utilises the Mine damage rule and the corresponding exponent p of Woehler-damage curves (slope p) for both bending and pitting cases.

The substantial difference is that the number of bins to be used in the Miner damage rule refers to the loading blocks that really are damaging the gears. In other words, being the amount of damage depending on the stress level, it is necessary to calculate the tangential force level F_{tD} for which the damage entity can be considered as zero.

This force limit F_{tD}, belonging to the fatigue curve of the gear (slope p), simply corresponds to the endurance limit cycles N_{Lref}.

The procedure will be described in detail in the following, referring only to the bending case for sake of brevity.

Analogous relationships may be obtained for pitting.

According to [1], [3], the tooth root stress σ_F is the maximum tensile stress at the surface in the root and it may be calculated by the following equation:

$$\sigma_F = \sigma_{FO} K_A K_V K_{F\alpha} K_{F\beta} \tag{5}$$

where K_A and K_V are respectively application and dynamic factors, $K_{F\beta}$ and $K_{F\alpha}$ face and transverse load factors, σ_{FO} is the nominal tooth stress expressed by:

$$\sigma_{FO} = \frac{F_t}{b m_n} Y_F Y_S Y_\beta Y_B Y_{DT} \tag{6}$$

where F_t is the nominal tangential load, b is the facewidth, m_n is the normal module, Y_F, Y_S, Y_β, Y_B, Y_{DT} are respectively form, stress correction, helix angle, rim thickness and deep tooth factors.

Equation (5) may be expressed in a compact form as:

$$\sigma_F = \frac{F_t}{bm_n} K_A A \tag{7}$$

where:

$$A = K_V K_{F\alpha} K_{F\beta} Y_F Y_S Y_\beta Y_B Y_{DT} \tag{8}$$

The permissible bending stress σ_{FP}, following [3] (method B), is given by:

$$\sigma_{FP} = \sigma_{F\lim} Y_{NT} \frac{Y_{ST} Y_{\delta relT} Y_{RrelT} Y_X}{S_{F\min}} \tag{9}$$

where σ_{Flim} is the nominal stress number for bending from reference test gears, Y_{ST} is the stress correction factor, Y_{NT} is the life factor for tooth root stress expressed as a function of the the number of load cycles N_L, S_{Fmin} is the minimum required safety factor for tooth root stress, $Y_{\delta relT}$, Y_{RrelT}, Y_X are respectively relative notch sensitivity, relative surface and size factors.

Also equation (9) may be expressed in a compact form as:

$$\sigma_{FP} = \sigma_{F\lim} Y_{NT} B \tag{10}$$

where:

$$B = \frac{Y_{ST} Y_{\delta relT} Y_{RrelT} Y_X}{S_{F\min}} \tag{11}$$

This procedure makes the tooth root stress corresponding to the permissible bending stress as:

$$\sigma_F = \sigma_{FP} \tag{12}$$

So, by substituting equations (7) and (10) respectively in equation (12), the following relationship may be obtained:

$$\frac{F_t}{bm_n} K_A A = \sigma_{F\lim} B Y_{NT} \tag{13}$$

Equation (13) provides the basis of the procedure, provided that the number of endurance limit cycles N_{Lref} is known. For the case of bending stress, N_{Lref} generally corresponds [1-5] to 3×10^6, so:

$$\frac{F_t}{bm_n} K_A A = \sigma_{F\lim} B \left[\frac{3\times10^6}{N_L}\right]^{\exp} \tag{14}$$

where *exp* means the general term for slope *p* of the material fatigue curve. If N_L coincides with the number of endurance limit cycles $N_{Lref} = 3\times10^6$, also the tangential force F_t coincides with the load level for which the damage entity can be considered as zero, so $F_t = F_{tD}$; in this way, the F_{tD} force value can be easily obtained:

$$F_{tD} = \frac{\sigma_{F\lim} b m_n B}{A} \tag{15}$$

Once F_{tD} is known for that gear, the Miner procedure may be run, by considering only the fatigue cycles that really are damaging the gear. Rearranging equation (14) in order to calculate the application factor K_A, it gives:

$$F_t K_A N_L^{\exp} = \frac{b m_n}{A}\sigma_{F\lim} B\left[3\times10^6\right]^{\exp} \tag{16}$$

or, in a more compact form:

$$\left[F_t K_A\right]^{\frac{1}{\exp}} N_L = C_F \tag{17}$$

where:

$$C_F = \left[\frac{b m_n}{A}\sigma_{F\lim} B\right]^{\frac{1}{\exp}}\left(3\times10^6\right) \tag{18}$$

Then, the equivalent tangential force F_{eq} can be obtained by the following equation:

$$F_{teq} = \left[\frac{\sum_{i=1}^{L} n_{iw} F_{ti}^{\frac{1}{\exp}}}{\sum_{i=1}^{L} n_{iw}}\right]^{\exp} \tag{19}$$

where only the tangential force levels F_{ti} (respectively running for n_{iw} cycles) higher than F_{tD} are taken into account. The numbers of cycles n_{iw} means both a number of cycles (as in equation (2)) or, depending on the available data, an already weighted (subscript *w*) number of cycles, as described in the following paragraph related to a practical case.

Finally, the application factor K_A in the case of variable amplitude loads can be obtained:

$$K_A = \frac{F_{teq}}{F_{tD}} \tag{20}$$

4. Practical case: input gear of a transfer box for industrial truck

The practical case developed in the present work is the application factor K_A calculation of the input gear (gear 1, number of teeth z_1 = 36) (see Figure 1) of the transfer box for industrial truck in on road conditions. Material has been chosen as hardening steel.

Experimental data are available on this gear in terms of torque level and cycles number (root number of the transfer box input shaft). In other words, for a 100 km standard distance for each route (Borgo Taro – Pontremoli, Torino – Aosta, … and so on), a mission of the vehicle has been run and the corresponding reference parameters have been measured in different driving conditions (see Table 1), following an established percent division (7% Motorway, 70% Mixed Route and Mountain, 23% Urban route).

It can be noted than positive values for torque refer respectively to engine driving condition, while negative values to engine braking conditions.

Data of Table 1 have been therefore processed to obtain a weighted mission (see Table 2), for which only global values of cycles number are available to make easier the following procedure.

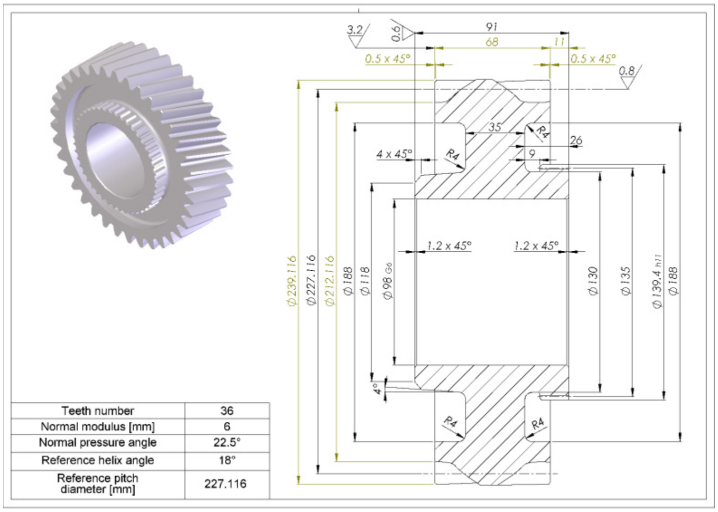

Fig. 1.Gear 1 geometry.

5. Results and discussion

Table 3 reports the coefficients values related to gear 1 obtained following the ISO Standard formula (method B) [1]. The dynamic factor has been calculated referring to the maximum speed of the vehicle (90 km/h). The exponent of the Woehler-damage line indicated as p in paragraph 2 corresponds to the general exponent indicated as *exp* in paragraph 3.

Coefficients have been obtained following the indications of the manufacturer about material, surface finish, and so on.

Table 4 reports steps and corresponding calculations for obtaining the application factor K_A of gear 1, following the ISO Standard procedure described in paragraph 3, on the basis of the vehicle experimental data. In particular, the first column shows the bin number (i=1, 2, ...), columns two and three respectively the corresponding torque T_i and tangential force F_{ti} values for gear 1, the fourth column shows the weighted number of cycles n_{iw} (being the procedure applied to a mission), the fifth one reports the equivalent cycles from row above n_{iaw}, the fifth one the total number of cycles n_{iew} and the last one shows the switch s_w. As it can be observed, bin 6 (shadowed row) corresponds to splitting limited and infinite service life (switch s_w passes from 0 to 1).

Table 1. Vehicle mission.

	Roots number of the transfer box input shaft							
	7% Motorway			70% Mixed Route and Mountain				23% Urban route
T_i [Nm]	Borgo Taro	Torino	Piacenza	Pontremoli	Alba	Susa	Pontremoli	Torino

	Pontremoli	Aosta	Alessandria	BivioAlbiano	PianaCrixia	Moncenisio	Fornovo	
-7410	0	0	0	0	0	0	820	0
-6063	0	0	0	0	8344	628713	4512	2968
-4716	0	0	0	3202	2785047	42710569	7629682	21765
-3368	0	823	0	62669	6443907	4065052	12670931	209244
-2021	29297009	5850	0	5682721	23984154	8355316	18490329	5456655
-674	33835673	33765228	18841413	56881807	30587415	4781348	22650063	53325433
674	3417698	70881279	104752505	17109155	7571964	2203311	5508630	21437318
2021	41098368	23273995	8543788	18909925	6780116	2069436	6088321	13098247
3368	21716592	1770478	0	25401541	21278784	5281190	21399614	19110412
4716	6389	823	0	4177910	21714466	9385653	23580717	11240778
6063	0	0	0	568897	7515029	47981135	10000954	4698332
7410	0	0	0	220636	302015	406005	150707	1607168
8757	0	0	0	59009	64297	153268	14111	724190
10105	0	0	0	10521	19633	0	6071	193414
11452	0	0	0	2440	982	0	656	9893
12799	0	0	0	762	0	0	246	1484
14147	0	0	0	152	0	0	0	0

Table 2. Total weighted mission.

T_i [Nm]	Roots number of the transfer box input shaft			
	7% Motorway	70% Mixed Route and Mountain	23% Urban route	Total Mission
-7410	0	205	0	143,5
-6063	0	160392,3	2968	112957,2
-4716	0	13282125	21765	9302493
-3368	274,3333	5810640	209244	4115593
-2021	9767620	14128130	5456655	11828455
-674	28814105	28725158	53325433	34389448
674	59683827	8098265	21437318	14777237
2021	24305384	8461950	13098247	10637338
3368	7829023	18340282	19110412	17781624
4716	2404	14714687	11240778	12885828
6063	0	16516504	4698332	12642169
7410	0	269840,8	1607168	558537,2
8757	0	72671,25	724190	217433,6
10105	0	9056,25	193414	50824,6
11452	0	1019,5	9893	2989,04
12799	0	252	1484	517,72
14147	0	38	0	26,6

Table 3. Coefficients for bending strength calculation for gear 1[1].

b [mm]	K_V	$K_{F\beta}$	$K_{F\alpha}$	Y_F	Y_S	Y_β	Y_B	Y_{DT}	σ_{Flim} [N/mm^2]	S_{Fmin}	Y_{ST}	$Y_{\delta relT}$	Y_{RrelT}	Y_X	N_{Lref}	exp
60	1.066	1.198	1.013	1.325	2.039	0.850	1.472	1	525	1	2	0.996	1.043	0.990	3×10^6	0.115

Even if at this point the equivalent torque T_{eq} could be obtained by equation 2 (first 6 bins of Table 4), the application factor K_A has been simply calculated by dividing torque values respectively of bins 6 and 7, being unknown the nominal torque value in this practical case: K_A = 1.222. So, only bins sequence and switch values are taken into account in the procedure. For this calculation, column three (F_t values) has not been utilized.

Then, following the procedure object of this work, F_{tD} force value (F_{tD} = 88946 N) has been calculated by means of equation (15). Referring to weighted data of Table 2, it can be noted that in this case only positive torque values have been taken into account, being the most damaging for gear 1. As it can be observed from data of Table 4, F_{tD} force value stands between bins 7 and 8, so only four bins (8-11) have been excluded from the calculation of F_{teq} and from K_A. From equation (19) the equivalent force has been obtained (F_{teq} = 99553 N) and finally from equation (20) also the application factor K_A of gear 1: K_A = 1.119.

By varying other parameters, as an example the gear facewidth, it is possible to tune as one likes the application factor K_A.

Table 4. Loads data and ISO Standard sequence for K_A calculation.

i	T_i [Nm]	F_{ti} [N]	n_{iw}	n_{iaw}	n_{iew}	s_w
1	14147	213552	27	64,8		0
2	12799	193204	518	1540	582,8	0
3	11452	172871	2989	13516	4529	0
4	10105	152537	50825	2E+05	64341	0
5	8757	132189	2E+05	2E+06	4E+05	0
6	7410	111856	6E+05	1E+07	2E+06	0
7	6063	91522	1E+07	2E+08	3E+07	1
8	4716	71189	1E+07	5E+09	3E+08	1
9	3368	50841	2E+07	4E+11	5E+09	1
10	2021	30507	1E+07	6E+15	4E+11	1
11	674	10174	3E+07		6E+15	1

6. Conclusions

The method described in the present paper aims to complete the ISO Standard calculation procedure for as concerns the application factor K_A in the case of variable amplitude loads.

In particular, a case of industrial vehicle subjected to real working conditions has been presented.

The substantial difference between ISO Standard and the present work is the use respectively of a Woehler-damage line of the material and a specific fatigue curve of the component.

As a consequence of that, ISO Standard method may be used already in the design phase, while the present method needs the knowledge of almost the cinematic data of the gear.

Nevertheless, being the calculation specific for a gear, the real amount of damage may be obtained for both application factor K_A and service life determination.

Results show that ISO Standard procedure provides an application factor value a little more conservative than that specifically obtained.

It may be finally observed that the method presented in this paper allows to monitor the variation of the application factor K_A on the basis of the given values for gear coefficients, as an example for facewidth.

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