

POLITECNICO DI TORINO

SCUOLA INTERPOLITECNICA DI DOTTORATO

Dottorato in Computer and Control Engineering

Tesi di Dottorato

**Stochastic programming for City
Logistics: new models and
methods**



Luca Gobbato

Tutore
prof. G. Perboli

Coordinatore del corso di dottorato
prof. M. Sonza Reorda

February 2014

Acknowledgements

First and foremost, I am deeply indebted to my advisor Professor Guido Perboli. I truly appreciate his guidance, encouragement, patient, and support through my doctorate study. His teaching and influence have been and will be immensely valuable to me. Last but not least, this collaboration has allowed me to find one of my dearest friends.

I would also like to thank to all members of Operations Research and Optimization (ORO) group and all my friends of Laboratory 8 at Politecnico di Torino. In particular, Professor Roberto Tadei represents a model to imitate in the research. While, Mauro Baldi and Francesca Perfetti have built a working environment lovable and full of laughter. This allowed us to express our best every day, overcoming the difficulties.

I have to give a grateful thought also to Professor Teodor Gabriel Crainic and Walter Rei, who welcomed me at CIRRELT research center in Montreal and gave me the opportunity to work with them.

I thank the Scuola InterPolitecnica di Dottorato (SIPD) for its generous financial support of this research.

I want to express my gratitude to my family and to all my friends that brighten my weekends: Massimo, Virginia, Laura, Michele, Enrico and Marcello.

Finally, special thanks to Sara Guagenti, the love of my life. Words can never express my feelings to Sara for her love and sacrifice. It is her love, support, and encouragements accompanying me in walking through the last eleven years of my life.

Contents

| | |
|---|----|
| Acknowledgements | II |
| 1 Introduction | 1 |
| 2 City Logistics | 6 |
| 2.1 The concept of City Logistics | 6 |
| 2.2 Planning issues | 10 |
| 3 Emerging problems in the City Logistics supply chain | 13 |
| 4 The Capacity Planning Problem under Uncertainty | 20 |
| 4.1 Problem description and literature review | 21 |
| 4.1.1 Literature review | 23 |
| 4.2 The stochastic VCSBPP model | 24 |
| 4.3 A lower bound for the SVCSBPP | 26 |
| 4.4 Heuristic based on progressive hedging | 26 |
| 4.4.1 Scenario decomposition of the SVCSBPP | 27 |
| 4.4.2 Obtaining consensus among subproblems | 32 |
| 4.4.3 Termination criteria | 36 |
| 4.4.4 Phase 2 of the meta-heuristic | 36 |
| 4.4.5 Parallel implementation | 37 |
| 4.5 Computational results | 38 |
| 4.5.1 Instance set | 40 |
| 4.5.2 PH algorithm validation | 43 |
| 4.5.3 Impact of uncertainty | 48 |

| | | |
|----------|---|-----------|
| 4.5.4 | Solution analysis | 51 |
| 4.5.5 | Sensitivity analysis | 53 |
| 4.5.6 | Effect of the capacity planning | 55 |
| 5 | The Multi-Handler Knapsack Problem under Uncertainty | 62 |
| 5.1 | Problem description and literature review | 63 |
| 5.1.1 | Literature review | 64 |
| 5.2 | Problem formulation | 65 |
| 5.3 | The deterministic approximation | 67 |
| 5.4 | A two-stage stochastic model with fixed recourse | 72 |
| 5.5 | Experimental plan | 74 |
| 5.5.1 | Instance set | 75 |
| 5.6 | Computational results | 77 |
| 5.6.1 | Calibration of the model | 77 |
| 5.6.2 | Impact of uncertainty | 78 |
| 5.6.3 | Comparison of the two-stage program and the approximation | 79 |
| 5.6.4 | Usage of the approximated model as a decision tool | 81 |
| 6 | The Multi-Path Travelling Salesman Problem with stochastic travel times | 84 |
| 6.1 | Problem description and literature review | 85 |
| 6.1.1 | Literature review | 85 |
| 6.2 | Problem formulation | 87 |
| 6.2.1 | A sub-tour elimination based two-stage model with recourse for the mpTSPs | 88 |
| 6.2.2 | A flow-based two-stage model with recourse for the mpTSPs | 89 |
| 6.3 | Heuristic based on progressive hedging | 91 |
| 6.3.1 | Scenario decomposition of the mpTSPs | 91 |
| 6.3.2 | Defining the overall capacity plan | 96 |
| 6.3.3 | Strategies for penalty adjustments | 97 |
| 6.3.4 | Implementation of the methodology | 98 |
| 6.4 | Computational results | 99 |
| 6.4.1 | Instance set | 100 |

| | | |
|----------|--|------------|
| 6.4.2 | Progressive Hedging validation | 104 |
| 6.4.3 | Impact of uncertainty | 107 |
| 7 | Conclusions | 113 |

List of Tables

| | | |
|-----|--|-----|
| 4.1 | Comparison of RP solutions from CPLEX and PH | 44 |
| 4.2 | Comparison of LB and PH solutions | 47 |
| 4.3 | Average computational times for PH algorithm with respect to parallel threads | 48 |
| 4.4 | <i>EVPI</i> and <i>VSS</i> comparison | 50 |
| 4.5 | Structure of solutions | 52 |
| 4.6 | Sensitivity of capacity-planning solutions | 54 |
| 4.7 | Sensitivity of capacity-planning solutions | 54 |
| 5.1 | <i>EVPI</i> comparison | 78 |
| 5.2 | <i>VSS</i> comparison | 78 |
| 5.3 | Optimality gap and solution likelihood of the deterministic approximation | 80 |
| 5.4 | Optimality gap and solution likelihood for the profit correlation rules | 80 |
| 5.5 | Optimality gap and solution likelihood for the maximum profit oscillations | 81 |
| 5.6 | Optimality gap with fixed first stage decision | 82 |
| 5.7 | Optimality gap with fixed first stage decision for the profit correlation rules | 82 |
| 5.8 | Optimality gap with fixed first stage decision for the maximum profit oscillations | 83 |
| 6.1 | Comparison of the results between CPLEX and the PH algorithm | 106 |
| 6.2 | Full instance set results: <i>EVPI</i> and <i>VSS</i> values comparison | 111 |

List of Figures

| | | |
|-----|--|-----|
| 2.1 | Relationship between freight transportation features (white blocks) and negative effects (gray blocks) | 8 |
| 2.2 | Urban freight supply chain model and relationships with the stakeholders | 9 |
| 3.1 | A representation of the two-echelon distribution with city-freighter vehicles (green vehicles). Yellow vehicles are the urban vehicles that bring freight to satellite facilities (green triangles) from UCCs (black squares) around the city. Dotted lines means that vehicles are empty. | 15 |
| 4.1 | Average relative optimality gap (%) with respect to the number of parallel threads used by CPLEX | 45 |
| 4.2 | Speed-up of PH algorithm for different numbers of parallel threads . . | 49 |
| 4.3 | Demand trends for the Monte Carlo simulation | 59 |
| 4.4 | Monte Carlo simulation for a testing instance | 60 |
| 4.5 | Monte Carlo simulation for a realistic instance | 61 |
| 5.1 | Scenario tree generation | 75 |
| 6.1 | Subdivision in neighborhoods and admissible areas (grey) for each distribution strategy | 102 |
| 6.2 | Distribution of central (dark gray circle) and suburban (ligh gray circle) speed sensors in the city of Turin in Italy. | 104 |
| 6.3 | Sensitivity of the first stage objective function (solid line) and the total objective function (dotted line) with respect to the number of scenarios | 106 |

6.4 Comparison of the first-stage Expected Value Problem solution (a) and the Recourse Problem solution (b). Figure (c) shows the common arcs (solid line), arcs used only in the Expected Value Problem solution (long-dotted line) and arcs used only in Recourse Problem one (short-dotted line). 112

Chapter 1

Introduction

The need for mobility that emerged in the last decades led to an impressive increase in the number of vehicles as well as to a saturation of transportation infrastructures. This is particularly true in urban areas whose population has been steadily growing since the 50s. Consequently, traffic congestion, accidents, transportation delays, and polluting emissions are some of the most recurrent concerns transportation and city managers have to deal with (Dimitrakopoulos and Demestichas, 2010). However, just building new infrastructures, as new roads or high capacity highways, might be not sustainable because of their cost, the land usage, which usually lacks in metropolitan regions, and their negative impact on the environment (Crainic et al., 2009b). Therefore, a different way of improving the performance of transportation systems while enhancing travel safety has to be found in order to make people and good transportation operations more efficient and cost effective and support their key role in the economic development of either a city or a whole country. The concept of City Logistics (CL) (Taniguchi et al., 2001) is being developed to answer to this need. Indeed, CL focus on reducing the number of vehicles operating in the city, controlling their dimension and characteristics, and reducing the number of empty vehicles (Benjelloun and Crainic, 2008). CL solutions do not only improve the transportation system but the whole logistics system within an urban area, trying to integrate interests of the several stakeholders (e.g. carriers, shippers, public authorities and citizens) of the *Supply Chain* (SC).

This global view challenges stakeholders in the extension of urban SCs (Seuring

and Muller, 2008), which consider economic aspects (e.g. cost, performance and customer satisfaction) related to carriers and shippers as well as the environmental sustainability and the efficiency of transport that contribute to preserving the quality of life of citizens. However, more complex SCs require a higher level of cooperation and coordination. Coordination, in particular, becomes a key factor for the successful of the new solutions proposed in CL projects. On the other hand, CL challenges researchers to develop planning models, methods and decision support tools for the optimization of the structures and the activities of the transportation system. In particular, this leads researchers to the definition of strategic and tactical problems belonging to well-known problem classes, including network design problem, *vehicle routing problem* (VRP), *traveling salesman problem* (TSP), *bin packing problem* (BPP), which typically act as sub-problems of the overall CL system optimization. When long planning horizons are involved, these problems become stochastic and, thus, must explicitly take into account the different sources of uncertainty that can affect the transportation system. Various sources and type of uncertainty may be defined in the urban system (Klibi et al., 2010), such as time (i.e. service time at customers, the travel time due to traffic congestions) and demand (i.e. the revenue associated with the shipping of the orders, the volume associated with the flows). Due to these reasons and the large-scale of CL systems, the optimization problems arising in the urban context are very challenging. Their solution requires investigations in mathematical and combinatorial optimization methods as well as the implementation of efficient exact and heuristic algorithms. However, contributions answering these challenges are still limited number.

This work contributes in filling this gap in the literature in terms of both modeling framework for new planning problems in CL context and developing new and effective heuristic solving methods for the two-stage formulation of these problems.

Three stochastic problems are proposed in the context of CL: the *stochastic variable cost and size bin packing problem* (SVCSBPP), the *multi-handler knapsack problem under uncertainty* (MHKP_u) and the *multi-path traveling salesman problem with stochastic travel times* (mpTSP_s).

The SVCSBPP (Crainic et al., 2014a) arises in supply-chain management, in which companies, that want to be competitive in the market and fulfill customers' requirements in a cost-efficient and flexible manner, outsource the logistics activities

to a *third-party logistic firm* (3PL). The procurement of sufficient capacity, expressed in terms of vehicles, containers or space in a warehouse for varying periods of time to satisfy the demand plays a crucial role. The **SVCSBPP** focuses on the relation between a company and its logistics capacity provider and the tactical-planning problem of determining the quantity of capacity units to secure for the next period of activity. The **SVCSBPP** is the first attempt to introduce a stochastic variant of the *variable cost and size bin packing problem* (VCSBPP) considering not only the uncertainty on the demand to deliver (Crainic et al., 2014b), but also on the renting cost of the different bins and their availability.

A large number of real-life situations can be satisfactorily modeled as a **MHKP_u** (Perboli et al., 2014), in particular in the last mile delivery. Last mile delivery may involve different sequences of consolidation operations, each handled by different workers with different skill levels and reliability. The improper management of consolidation operations can cause delay in the operations reducing the overall profit of the deliveries. Thus, given a set of potential logistics handlers and a set of items to deliver, characterized by volume and random profit, the **MHKP_u** consists in finding a subset of items which maximizes the expected total profit. The profit is given by the sum of a deterministic profit and a stochastic profit oscillation, with unknown probability distribution, due to the random handling costs of the handlers.

The **mpTSP_s** (Tadei et al., 2014; Gobbato et al., 2014) arises mainly in City Logistics applications. Cities offer several services, such as garbage collection, periodic delivery of goods in urban grocery distribution and bike sharing services. These services require the planning of fixed and periodic tours that will be used from one to several weeks. However, the enlarged time horizon as well as strong dynamic changes in travel times due to traffic congestion and other nuisances typical of the urban transportation induce the presence of multiple paths with stochastic travel times. Given a graph characterized by a set of nodes connected by arcs, **mpTSP_s** considers that, for every pair of nodes, multiple paths between the two nodes are present. Each path is characterized by a random travel time. Similarly to the standard TSP, the aim of the problem is to define the Hamiltonian cycle minimizing the expected total cost.

These planning problems have been formulated as two-stage integer stochastic programs with recourse (Birge and Louveaux, 1997). Discretization methods

are usually applied to approximate the probability distribution of the random parameters using a finite-dimension scenario tree (Mayer, 1998). The resulting approximated program becomes a deterministic linear program with integer decision variables of generally very large dimensions, beyond the reach of exact methods. Therefore, heuristics are required.

For the \mathbf{MHKP}_u , we apply the extreme value theory and derive a deterministic approximation, while for the $\mathbf{SVCSBPP}$ and the \mathbf{mpTSP}_s we introduce effective and accurate heuristics based on the *progressive hedging* (PH) ideas, initially proposed by Rockafellar and Wets (1991) for stochastic convex programs. The PH mitigates the computational difficulty associated with large problem instances by decomposing the stochastic program by scenario. When effective heuristic techniques exist for solving individual scenario, that is the case of the $\mathbf{SVCSBPP}$ and the \mathbf{mpTSP}_s , the PH further reduces the computational effort of solving scenario subproblems to optimality by means of a commercial solver. In particular, we propose a series of specific strategies to accelerate the search and efficiently address the symmetry of solutions, including an aggregated consensual solution, heuristic penalty adjustments, and a bundle fixing technique. Yet, although solution methods become more powerful, combinatorial problems in the CL context are very large and difficult to solve. Thus, in order to significantly enhance the computational efficiency, these heuristics implement parallel schemes.

With the aim to make a complete analysis of the problems proposed, we perform extensive numerical experiments on a large set of instances of various dimensions, including realistic setting derived by real applications in the urban area, and combinations of different levels of variability and correlations in the stochastic parameters. The campaign includes the assessment of the efficiency of the meta-heuristic, the evaluation of the interest to explicitly consider uncertainty, an analysis of the impact of problem characteristics, the structure of solutions, as well as an evaluation of the robustness of the solutions when used as decision tool.

The numerical analysis indicates that the stochastic programs have significant effects in terms of both the economic impact (e.g. cost reduction) and the operations management (e.g. prediction of the capacity needed by the firm). The proposed methodologies outperform the use of commercial solvers, also when small-size instances are considered. In fact, they find good solutions in manageable computing

time. This makes these heuristics a strategic tool that can be incorporated in larger decision support systems for CL.

This thesis is organized as follows. The Chapter 2 describes the concept of CL, introducing the economic and social aspects considered and the planning issues arising in the urban system.

The introduction of some CL initiatives in the urban SC is discussed in Chapter 3. In particular, we explore the need of new planning models for strategic and tactical decisions.

In Chapter 4, we present a capacity planning application in freight transportation and introduce the **SVCSBPP**. After introducing the problem, we formulate a stochastic two stage model and propose a meta-heuristic based on the PH algorithm. In particular, we show how the model can be decompose in deterministic subproblems and describe the strategies adopted in the algorithm to deal with the existence of multiple equivalent solutions. Then, we present new instance sets for tackling the problem partially derived by real parcel delivery applications and extensive computational results.

In Chapter 5, we present the **MHKP_u** and the deterministic approximation for the stochastic problem. The accuracy of this deterministic approximation is tested against the two-stage stochastic program of the **MHKP_u**.

In Chapter 6, we present the two-stage stochastic program of the **mpTSP_s**, where tour design makes up the first stage, while the best paths to use is selected in the second stage. Then, Concorde is combined to the PH algorithm in order to deal with new real instances derived from the speed sensor network of Turin, a medium-sized city in Italy. Extensive computational results are presented to qualify the algorithm and the impact of the uncertainty on the decisions.

Conclusions and future developments of the research activity are reported in Chapter 7.

Chapter 2

City Logistics

In this chapter, we recall the fundamental concepts of CL (Section 2.1). In particular, Section 2.2 describes the main planning issues related to this system, which challenges researchers to develop appropriate models that incorporate the sources of uncertainty of the urban context.

2.1 The concept of City Logistics

Before the 90s', the urban transportation was not managed by public authorities. The only type of interventions from the public authorities were restrictive measures to deal with emergencies (e.g. pricing strategies, regulations of parking area, limited traffic zones). Only in 90s' and, in particular, at the beginning of the 21th century, the urban traffic problem became relevant and first studies tried to define some common policies for the freight transportation in the urban context. One of first studies is *COST Action 321: Urban Goods Transport* that involved twelve European countries. Nowadays, the continuous developments of the urban area are changing the living conditions such as growth in population, increased of urbanization and living standards as well as expansion of Just-In-Time delivery and the growth of home delivery services. Due to the combination of these factors the urban area, on one side, requires large quantities of goods and services for commercial and domestic use, but, on the other side, is affected by a deterioration of the quality of life caused by increasing congestion and pollution. These issues become even more critical

if we consider that, the global proportion of urban population increased from 13 per cent in 1900, to 29 per cent in 1950, 49 per cent in 2005, and 58 per cent in 2013. Seventy per cent of the global population is expected to live in urban areas by 2050 (OECD, 2012). For these reasons, in the last decades, CL has emerged to provide a solution to the negative impacts of freight transportation. Taniguchi et al. (1999, 2001) defined the CL as "the process for totally optimizing the logistics and transport activities by private companies in urban areas while considering the traffic environment, the traffic congestion and energy consumption within the framework of a market economy".

A variety of negative effects can be targeted in the urban freight transportation including to economic, environmental and social factors (see Figure 2.1). One of the main economic factors that is considered is the transportation cost. This cost correspond to the total cost of the vehicles traveling in the city and it is also used to calculate other indicators, such as the pollution or in some cases the noise. The air pollution and the greenhouse gas (GHG) emission need to be reduced in order to mitigate the environmental impacts of the freight transportation. The result of the combustion of diesel fuel is the production of toxic gaseous emissions which include Carbon Monoxide, Carbon Dioxide, Nitrogen Oxides and Particulate Matter. Another factor that has to be considered as an environmental aspect is the traffic noise. Urban freight vehicles produce a significant amount of noise in a city affecting the human health and the city comfort. It is not only produced by the engine, but also by the (un)loading of freight. When considering the social aspects, the safety of the city and the congestion of the streets are the most important effects to be reduced. Citizens do not participate directly to the freight transportation but they divide the same transportation network. With increasing freight transportation activity the problem of safety and accidents become significant in urban areas where pedestrians might be thought to be particularly at risk from freight vehicles. The latter, in fact, frequently stop on-street to load and unload, blocking the flow and causing congestion.

These negative effects and the inefficient activities related to the freight-vehicles movements are the main reasons to carry out urban freight analysis and optimization. In fact, CL focuses on the improving the efficiency of urban freight transportation reducing traffic congestion and lessening environmental impacts without

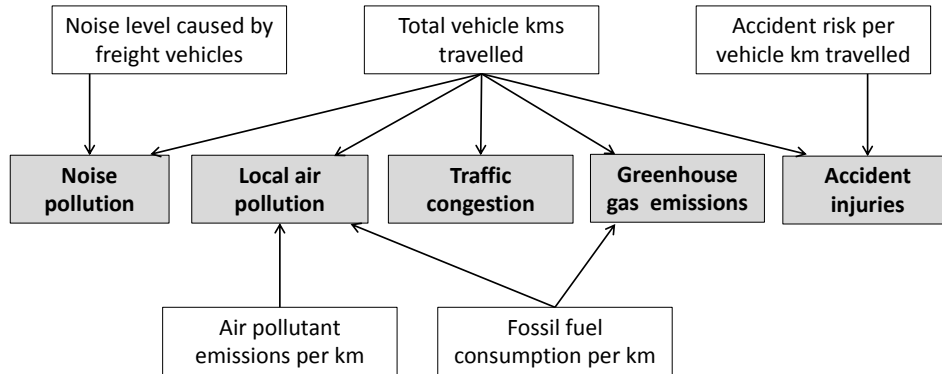


Figure 2.1: Relationship between freight transportation features (white blocks) and negative effects (gray blocks)

penalizing the city social and economic activities. This holistic view leads CL initiatives to consider the transportation system as a whole. Moreover, they incorporate a row of activities between different actors, from production, commerce and supply of different clients and inhabitants, who appear in form of inner urban goods transport, or distribution of interurban freights, fulfilling a substantial contribution to economy, city life and operation. The frame for CL is given by local and regional economy, the transport infrastructure, the surrounding environment, legal and regulatory conditions.

[Taniguchi et al. \(2001\)](#) identified four groups of key stakeholders of the city: shippers, carriers, citizens and public authorities. All these stakeholders follow different goals and have therefore different perspectives, too. Public authorities and citizens share some of their problem views regarding externalities like accidents, congestion, noise, air pollution caused by freight vehicles. These impacts are felt to reduce the quality of life and the urban environment substantially. Shippers and carriers have a completely different point of view. They have the goal to deliver/receive goods as cheap as possible to maximize their own profits within a given regulatory framework and a given transport infrastructure. Their priorities are therefore to remove costly obstacles, which hinder them to deliver faster and cheaper without taking into consideration externalities. The congestions caused by freight vehicles by loading/delivering goods into/on vehicles, are often substantially negatively contributing to air pollution and noise in sensitive living areas.

When we consider the urban freight SC (Boerkamps et al., 2000), stakeholders can be grouped by the impacts of their decisions have on the SC . In particular, the stakeholders are closely connected to at least one component of the SC, but loosely connected to whole SC (see Figure 2.2). For example, carriers can plan the route in order to avoid the congestion, but they cannot affect the traffic flow. Shippers and citizens represent the demand for urban freight transport. Thus, they are strictly related to the spatial organization (e.g. where people live and work, where facilities are located, and where freight are produced and consumed) and trade activities. On the contrary, carriers and public authorities are involved only in the transportation components of the SC. The transport links demand and supply of transportation services. Vehicle fleet, human resources and infrastructural provisions are important supply aspects. Decisions in the transport market result in traffic flows, which are made on the multimodal infrastructure network.

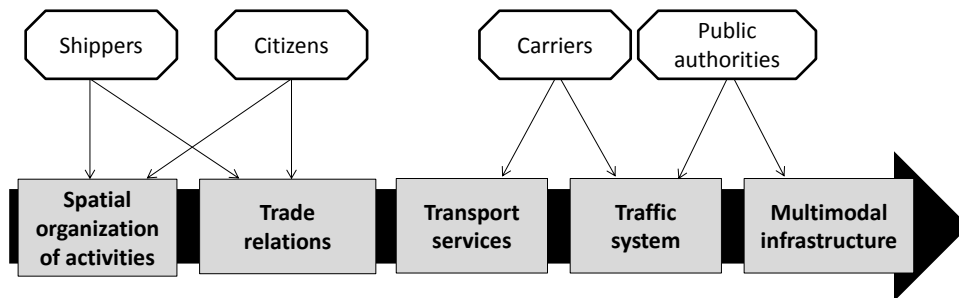


Figure 2.2: Urban freight supply chain model and relationships with the stakeholders

In attempting to reduce the scale of negative impacts and to optimize the decisions at each level of the urban freight SC, a range of initiatives of CL have been proposed and are actually implemented in several cities by stakeholders in order to alter urban freight operations. Some of these initiatives will only address a single impact, while others will address several impacts at the same time. In general, several CL solutions focuses on the consolidation of freight in *urban consolidation centers* (UCCs) (Benjelloun and Crainic, 2008; McKinnon et al., 2010), which is a logistic platform located in a strategic node of the city, where the freight to deliver can be consolidated and then performed by environmental-friendly vehicles. This initiative needs both the consolidation of loads of different shippers and carriers within the

same delivery vehicles and the coordination of the resulting freight operations within the city. We describe this topic and the impact of UCCs in the urban SC in the Chapter 3.

Finally, we report the most recent CL initiatives and projects, organized for the main goal. Other fundamental concepts, issues, trends, and challenges of CL may be found in [Taniguchi et al. \(2001\)](#), [Russo and Comi \(2004\)](#), [Benjelloun and Crainic \(2008\)](#) and the proceeding books of the City Logistics conferences available through the [Institute of City Logistics \(1999\)](#) web-site. The latter, in particular, results in the state-of-the-art of city logistics concepts and implementations.

- Researching future urban freight transportation requirements and strategies: [CITY FREIGHT Consortium \(2005\)](#), [CITYLOG Consortium \(2010\)](#), [BESTUFS Consortium \(2008\)](#);
- Investigating the feasibility of new logistics concepts for urban distribution and supply ([CITYBOX Consortium, 2002](#));
- Focusing on the application of intelligent transport systems for urban freight transport ([eDRUL Consortium, 2004](#); [SMARTFREIGHT Consortium, 2008](#));
- Concerned with freight terminals serving urban areas ([FV-2000 Consortium, 1999](#));
- Investigating changing modal split and encouraging rail use ([IDIOMA Consortium, 2001](#); [PROMIT Consortium, 2009](#));
- Urban freight demonstration projects intended to improve freight efficiency and reduce energy use ([CITY PORTS Consortium, 2005](#); [CIVITAS I Consortium, 2006](#); [START Consortium, 2009](#); [PIEVERDE Consortium, 2014](#)).

2.2 Planning issues

In urban context, where space is limited and infrastructure expansion can be enormously expensive, importance of proper planning becomes essential. Similarly to any complex transportation system, CL systems require planning decisions at strategic, tactical and operational levels ([Benjelloun and Crainic, 2008](#)). Strategic decisions

have a long term effect, tactical decisions have an effect over the medium term (monthly or quarterly), and operational decisions have an immediate short-term effect. One more level assumes a key role in the urban system: the real-time.

Individual levels differ by the impact they have on future activities:

Strategic level involves the highest level of management and requires large capital investments over long term horizon. Strategic decisions determine infrastructure aspects (e.g. location of UCCs), freight distribution networks, vehicles and technological aspects and general development policies and strategies. Corresponding decisions are complex and of high risk and uncertainty. These decisions have a key role in the planning process. In fact, they constrain the activities and the decisions made at tactical and operational levels.

Tactical level defines, over a medium term horizon, the usage of the available resources in order to improve the system's performance. In particular, it concerns the efficient and effective use of transportation infrastructure and the alignment of operations according to strategic objectives. Tactical decisions includes the acquisition and replacement of resources (e.g. planning the capacity for freight deliveries), the definition of periodic tours used for the entire planning horizon (e.g. tours of vehicles for the garbage collection service or bike sharing relocation) and costs and performance analysis. In fact, tactical level controls and evidences the strengths and the weaknesses of the strategic decisions. On the other hand, decisions made here limit the activities of operational and real-time management level.

Operational level concerns short term decisions, day-to-day operations and the plan of next day activities. The time factor and a detailed representation of the system, facilities and vehicles are essential at these level. The decisions are made in a very dynamic environment. Thus, anticipate the future (e.g. the congestion, the transportation demand) is fundamental. Here, logistic service provider plan the routing, assigning the transportation request to the transportation resources, and the handling operations.

Real-time level reacts on differences between information known at the moment of planning decision and the actual state of the transportation system. Here,

the activities are directly related to operational decisions and aims to mitigate the effects of poor quality and reliability of operational planning. Errors in the forecasting of the future may lead to inferior customer service or higher costs to adjust the planning (e.g. rent one more vehicle for the day at the premium cost).

More in general, planning means a certain level of look-ahead skills. The forecasting of random events assume a key role in the planning process. The variation of the demand to be shipped or the costs of operations over the horizon of the planning levels, as well as delays at customer locations due to traffic conditions are only some examples of the uncertainty sources in the urban context. They constitutes a particularly important element to consider, as it may be a strong impact on decisions and the efficiency of the urban freight transportation service. This information may not be known at the initial planning stage, and/or may change during the plan execution. This means that these events must be included in the decision process at all levels.

Stochastic programming (SP) (Birge, 1982) can be used to model optimization problems that involve uncertainty. The most widely applied and studied SP models are two-stage linear programs. Here the decision maker takes some action in the first stage, after which a random event occurs affecting the outcome of the first stage decision. A recourse decision can then be made in the second stage that compensates for any bad effects that might have been experienced as a result of the first-stage decision. The optimal policy from such a model is a single first-stage solution and a collection of recourse decisions defining which second-stage action should be taken in response to each random outcome.

Solution approaches to SP models are driven by the type of probability distributions governing the random parameters. A common approach to handling uncertainty is to define a small number of scenarios to represent the future. In this case it is possible to compute a solution to the SP problem by solving a deterministic equivalent linear program. These problems are typically very large scale problems, and so, much research effort in the SP community has been devoted to developing algorithms that exploit the problem structure, in particular in the hope of decomposing large problems into smaller more tractable components (Ruszczynski, 1997).

Chapter 3

Emerging problems in the City Logistics supply chain

In this chapter, we explore the impacts of *urban consolidation centers* (UCCs) and environmental-friendly vehicles on the urban freight SC. They affect the "Transport services" and the "Traffic system" blocks (see Figure 2.2). In particular, we discuss the need of new strategic and planning problems for the resulting SC.

A major problem tackled by CL solutions is the inefficient utilization of freight vehicles, which contributes significantly to congestion and pollution emissions. Among the various CL initiatives, we investigate the opportunity to use UCCs in the urban system. UCCs are platforms located close to the urban area (e.g. close to the access of the city or ring highways) with the functions of collecting and dispatching goods. They link the city to regional, national and international hubs, and, thus, must be easily accessible and integrated with other logistics platforms. UCCs receive large trucks of logistics companies and smaller vehicles. Logistics companies drop their loads and avoid the need to enter congested urban areas and thereby saving time and costs, while the local distribution is performed by often using environmentally friendly vehicles such as electric and gas-powered light vehicles. Here, the freight to deliver to the city is stored, sorted, and consolidated (de-consolidated) in an integrated way. The UCCs may so be viewed as an intermodal centralized platform with enhanced functionality that efficiently coordinates the supply and the demand to and from the city. They represent the first step toward a better CL organization

and a more efficient freight movements. By improving the lading factor of vehicles making final deliveries in congested locations, UCCs reduce the total distance traveled by delivery vehicles in urban areas, as well as reducing GHG emissions and local air quality pollutants associated with these journeys (Browne et al., 2011).

Example of the realization of UCCs can be Padova one in Italy or Brema one in Germany. According to a study done by Gonzalez-Feliu et al. (2010) emissions of GHG in 2008 were reduced of 68% respect to 2003 thanks to the installation of a UCC called Cityporto. Another example is the UCC in Nijmegen which the deliveries from multiple suppliers for the retailer and delivering the goods at the retailer time requirements. Van Rooijen and Quak (2009) show that the UCC, serving 98 retailers, leads to a reduction of the number of trucks and also the number of kilometers.

When a single level of consolidation is considered and the freight distribution starts at an UCC and arrives directly to customers in the urban area, the system is called single-tier CL system. Most CL projects in Europe and Japan involved some form of single-tier system, mostly considering a single UCC and a limited number of shippers and carriers. However, this system does not appear interesting for large urban areas or for cities with strong transportation issues. Indeed, UCCs results in extra costs (e.g. need the support of public authorities), delays in the transportation and limited benefits in term of gas emissions and pollutions. If the aim of UCCs is to minimize the number of trucks in the urban areas, then heavy trucks should be used in order to consolidate on the same vehicle as many orders as possible. This implies that there will be large trucks moving within the urban areas, performing long routes and causing congestion.

One of the trends is to substitute traditional single-tier systems with two-tier ones. (Crainic et al., 2004) proposes an extension of UCCs with a net of smaller facilities closer to the city center respect to UCCs called satellite platforms. Figure 3.1 represents a two-tier distribution system. UCCs (black square in the figure) form the first level of the system and are located on the outskirts of the city (black circle). The second level is constituted of satellites, where the freight, coming from the UCCs or from external points, is transshipped again into vehicles adapted for utilization in dense urban areas. Existing facilities can be used as satellites. Indeed, satellites offer limited logistics services and storing of freight.

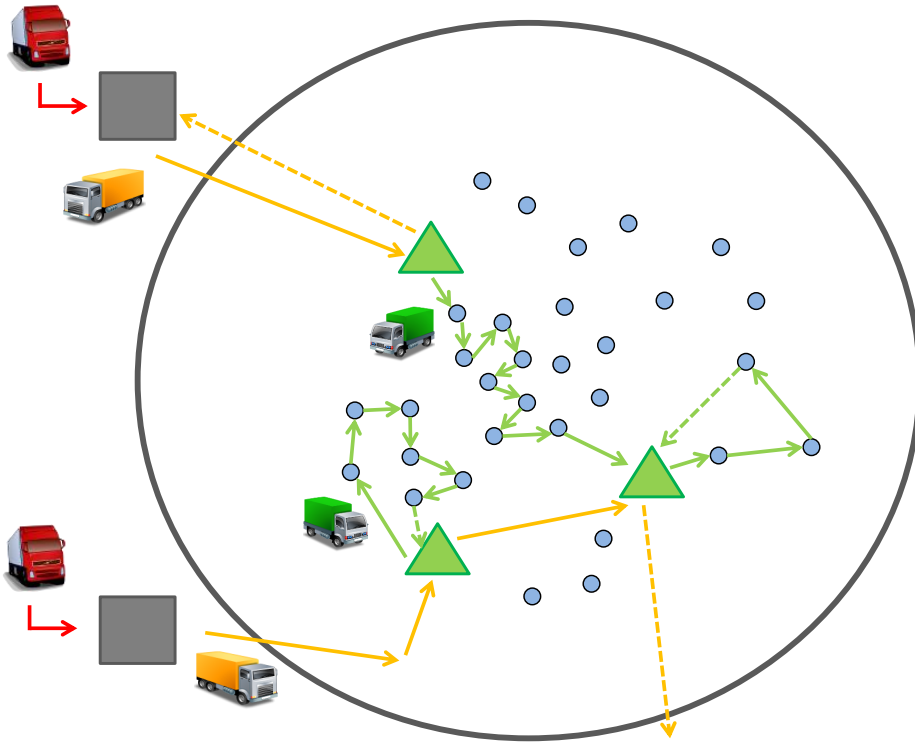


Figure 3.1: A representation of the two-echelon distribution with city-freighter vehicles (green vehicles). Yellow vehicles are the urban vehicles that bring freight to satellite facilities (green triangles) from UCCs (black squares) around the city. Dotted lines means that vehicles are empty.

Two types of vehicles are involved in a two-tier distribution system: urban-trucks (yellow trucks in the figure) and city-freighters (green trucks in the figure). Freight is moved by urban trucks from UCCs where it has been already consolidated, not directly to consumer destinations, but to satellites. When it is possible, they use streets specially selected to facilitate the access to satellites and reduce the impact on the traffic congestion. Here, freight is transshipped into city-freighters. Both the vehicles are supposed to be environmentally friendly, but, especially for city-freighters that move only in the city areas, they must be environment-friendly vehicles, such as electric-vehicles. [Van Mierlo et al. \(2003\)](#) report that electric vehicles, in average, has more than three times lower environmental impact compared to a diesel trucks and twice as low impact compared to liquefied petroleum gas trucks.

This extension of previous system allows more benefits in terms of congestion and emissions, in particular when small and environmental-friendly vehicles are used inside the city. Although, the system increases the costs for the additional transshipment operations, these costs will be compensated by the consolidation of the freights, the decrease of empty trips (e.g. reverse logistics) and by the economy of scale. However, two-tier distribution also needs of more complex cooperation and planning decisions. In fact, the transshipment needs integration of different companies requirements and a reliable planning of vehicle routing to deliver shipments on time. Several planning and operative problems arise in this context. The following list briefly describes the main problems that stakeholders and decision makers of CL system must address.

Satellite location problem As inbound flows to UCCs tend to be consolidated in full trucks and outbound flows tend to be in smaller units, the number of UCCs has a large effect on transport efficiency. Moreover, the number of UCCs and satellites to be realized depends primarily on the size and structural characteristics of the urban system or the production chain approach, especially in the case of perishable categories (e.g. food, pharmaceuticals).

Within literature there is a whole stream of research on facility location, which mainly deals with the number and location of UCCs and satellites. Environmental aspects of supply chain design and facility location in particular have recently received considerable attention (Li et al., 2008; Diabat and Simchi-Levi, 2009; Wang et al., 2011; Mallidis et al., 2012). However, only a few papers concerning the location of satellites under uncertainty are currently available (Ricciardi et al., 2002; Snyder et al., 2007; Tadei et al., 2009). For the first time, Baldi et al. (2012b) addressed the congestion effects inside the transshipment facilities, leading to a strategic planning location-allocation problem that explicitly considers the fixed cost of locating the satellite, the transportation cost and the cost from handling operations. In fact, they integrated in a comprehensive model the two levels of a two-echelon network (i.e. the design and the management levels).

Two-echelon vehicle routing In two-tier freight distribution system, (1) freight arrives at an UCC where it is consolidated into urban trucks, which, given a

departure time and a first-level route, (2) travel to one or several satellites. At a satellite, (3) freight is transferred to city-freighters, which (4) perform a second-level route to serve the designated customers and (5) return to a satellite (or a depot) for their next cycle of operations.

The two-echelon vehicle routing problems optimizes the routes of urban trucks and city freighters, as well as coordinates the accesses to satellites. The literature on this problem in CL contexts is still somewhat limited. Crainic et al. (2009a, 2010); Perboli et al. (2011); Crainic et al. (2013a) introduced the problem, related issues and formal mathematical models. They also proposed heuristic methods for the problem. Here, authors supposed that the demand is known a priori. Thus, the demand consolidated at the first level defines the number of city freighters to use in the second-level distribution. This hypothesis is hard to be met in real application. Crainic et al. (2012a) proposed for the first time a two-echelon CL tactical planning that focuses on the uncertainty related to the variation of volume of demand to be dealt with the day after. Several topics are still open: effective solution methods for the tactical problem and the extension of the model integrating uncertainty related to the transportation network (e.g. transportation cost, congestion, travel and service time).

Capacity and fleet management The introduction of a UCC as additional block in the urban freight SP expects strict compliance of logistics service providers since storage space of UCCs is limited and, in the case of satellites, even not available (or available only for a very short time). City-freighters can travel along any street in the city-center area to perform the required distribution activities, but they are vehicles of relatively small capacity.

Efficient freight distribution requires the management of the capacity within warehouses of UCCs and satellites, as well as the design of different city freighter types (e.g. capacity, power-train, functionality) within a given CL system and the planning of the fleet of vehicles available for the next period of activity. Logistics capacity planning is major challenge in supply chain management (Monczka et al., 2008) that involves tactical planning decisions.

When tactical decisions are made, no detailed knowledge on the future demand and, thus, the real needs in terms of loads to be moved or stored, as well as the availability and costs of capacity is available.

Here, the planning addresses the needs for sufficient capacity to store (e.g. space in a warehouse) and move (e.g. dimension of the fleet of vehicles) freight to meet demand in the next cycle of its activities. When logistic operations are outsourced to logistics providers, this process becomes more critical and results in a medium-term contract that ensures sufficient capacity for the planning horizon and a cost-effective logistics service. Chapter 4 introduces the procurement process for logistics capacity and its mathematical formulation, the **SVCSBPP**.

Handling operators management In the satellites different sequences of consolidation operations are done by different workers in order to transship freight from urban trucks and city freighters. The different skill levels and reliability of the handling operators can cause delay in the activities reducing the overall profit. More in detail, the single orders can be managed by several handlers (e.g. third-party logistics providers or sub-contractors), whose costs affect the net profit of the item itself. The large number of possible handler cost scenarios and the difficulty to measure the associated handler costs require the planning of transshipment operations for the future activities.

The capacity, in this case, becomes unique, being the actual capacity of the satellite. Chapter 5 addresses the problem of planning transshipment operations in a multiple handler context with unknown items profit.

Satellite and vehicle replenishment A limited activities (e.g. load sorting, vehicle replenishment or cross-docking transshipment) are performed at the satellites. Moreover, intermediate storage is allowed at satellites only for a very small time. This point is fundamental for the actual implementation of the two-echelon distribution in the urban freight SC, since satellites do not need special infrastructures and functions have to installed, but existing facilities can be used (e.g. underground parking slots or municipal bus depots, or spaces like city squares). Thus, no high additional costs have to be sustained

for satellite activities (Crainic et al., 2004).

Satellites usually operate according to a vehicle-synchronization model. Urban vehicles and environmental-friendly vehicles meet at satellites at appointed times with only short waiting times being permitted. Here, in order to manage the transshipment of freight and control the arrival time, the uncertainty on traffic congestion have to be explicitly included in the decision process. We address this problem in Chapter 6, where we propose the **mpTSP_s**.

Chapter 4

The Capacity Planning Problem under Uncertainty

In this chapter, we introduce the **SVCSBPP** together with the stochastic model, a lower bound and the solving strategy. After defining the problem in a formal way, we propose a two-stage stochastic formulation that explicitly takes into account more sources of uncertainty arising in capacity planning problems. We then present a lower bounds that can be easily solved by mean of commercial *mixed-integer programming* (MIP) solvers and an heuristic based on the PH algorithm for the **SVCSBPP**. The accuracy and the efficiency of the latter is demonstrated through extensive computational experiments. A large number of instance sets for the **SVCSBPP** are introduced, partially covering realistic parcel delivery applications with more than 10000 items. The instance sets are designed to challenge the proposed methodology and provide insight into the impact of uncertainty on capacity planning solutions.

This chapter is organized as follows. Section 4.1 discusses the connection between the capacity planning problem and contract procurement with logistic providers. Section 4.2 recalls the formulation of the problem; a lower bound and the heuristic solution method are introduced in Sections 4.3 and 4.4, respectively. Section 4.5 presents the experimental plan and analyzes the computational results.

4.1 Problem description and literature review

In this section, we analyze the process of contract procurement between a major retail firm and a third-party logistics service provider (3PL) and its relevance to the capacity planning problem.

The tactical capacity planning problem we address is relevant in many contexts, e.g., manufacturing firms desiring to secure transportation capacity to bring in their resources or to distribute their products, wholesalers and retailers planning for transportation and storage capacity to support their procurement and sales processes, and logistics service providers securing capacity contracts with carriers for long-distance, regular shipments. Manufacturing and whole/retail distribution firms may negotiate directly with carriers and owners/managers of storage space but, very often, they do business with a logistic service provider. Consequently, in order to simplify the presentation, but without loss of generality, we describe the problem within the context of the process of contract procurement between a major retail firm and a third-party logistics (capacity) service provider (3PL).

In the contemporary economic and business environment, firms are engaged in a continuous procurement process (Aissaoui et al., 2007; Rizk et al., 2008), and engage in various collaborations with its supply-chain partners. Such inter-firm alliances yield several benefits, including a reduction in inefficiencies, total cost, and financial risks. The greatest advantages result from the outsourcing of logistics activities to a 3PL (Marasco, 2008). The overall supply-chain process may then be summarized as follows. The firm regularly orders products from suppliers in a given geographical region, according to current inventories, short-term forecast demand, estimated lead times, and specific procurement and inventory policies (Bertazzi and Speranza, 2005; Bertazzi et al., 2007). The suppliers are instructed to deliver their goods to a consolidation center (Crainic and Kim, 2007; Bertazzi and Speranza, 2012). The 3PL then consolidates the goods into containers and ensures their shipping, by consolidating these containers with those of other customers into the slots it secured on long-haul carriers, e.g., ships and trains (Chen et al., 2001; Crainic et al., 2013b), which generally operate according to a fixed schedule, e.g., twice a week.

A key factor in this process is the procurement of sufficient capacity, that we express in the following in terms of *bins*, at different locations in the network and

for varying periods of time, to satisfy the demand. This entails negotiations with the 3PL to book the necessary capacity *a priori*, before operations start, at the best rate (Ford, 2001). The results of these negotiations often take the form of medium-term contracts specifying both the capacity to be used, the *quantity* and the *type* of bins, and the additional services to be performed (storage, transportation, bin operations, etc.) for a given planning horizon, e.g., a semester or a year. These contracts guarantee a regular volume of business to the 3PL (e.g., a fixed number of containers to ship every week for the next semester), which ensures a cost-effective service to the firm.

We refer to the costs of bins selected in advance as *fixed costs* because they are fixed by the contract and thus represent the specific rates offered by the 3PL for bins of different sizes. The values of the fixed costs are in practice influenced by several factors, including bin size, bin type (e.g., thermal or refrigerated containers), physical handling operations required, the time period for which the bin is to be used, and the economic characteristics of the departure location (e.g., access rules and costs). The result of negotiations and the scope of the capacity planning problem then is a tactical plan defining these quantities for the firm at each location, given the proposed bin types and costs and an estimation of the demand over the planning horizon. The plan thus specifies the set of bins of particular volumes and fixed costs to be made available at each location to ship the estimated items.

Given the time lag that usually exists between the signing of the contracts and the logistics operations, the negotiations are performed under uncertainty, without all the necessary information concerning the *demand*, expressed as a number of items of variable sizes to be shipped or stored at the particular locations. At each application of the plan, variations in the demand may thus yield numbers and sizes of items that differ from the estimation used at negotiation. These variations then require further negotiations with the 3PL and an adjustment of the plan when the booked capacity is not sufficient. Extra capacity (additional bins available at the shipping date) must then be purchased, generally at a much higher cost (the so-called spot-market value) than the fare negotiated initially. Moreover, while the 3PL generally ensures the planned capacity for the firm, the extra capacity may not be available when needed. The extra capacity must therefore be considered stochastic as well.

4.1.1 Literature review

While different variants of the stochastic bin packing problem are present in the literature, the strategic and tactical version of the **SVCSBPP** is recent. [Crainic et al. \(2014b\)](#) introduced the *variable cost and size bin packing problem with stochastic items* (VCSBPPSI), a stochastic variant of the VCSBPP, that considers not only the renting cost of the different bins, but also their availability. Given a certain number of bin types, their costs when we book them in advance and the costs if we rent them when needed as well as the demand of goods to deliver, the VCSBPPSI decides the capacity planning in terms of bins rented in advance in order to minimize the total cost.

Other variants of stochastic bin packing problem arise mainly when strategic and tactical problems must be solved and, in particular, when one has to plan the capacity of his fleet ([Crainic et al., 2012b](#)) or stochastic costs and profits are present in the problem ([Perboli et al., 2012, 2014](#)). In ([Coffman Jr. et al., 1980](#); [Lueker, 1983](#); [Rhee and Talagrand, 1993a,b](#)) the source of uncertainty is the item volume and strong hypotheses on the probability distribution of the random terms are usually done. Recently, [Peng and Zhang \(2012\)](#) have studied a more general stochastic variant with both item volumes and bin capacities are uncertain, while [Fazi et al. \(2012\)](#) have recently studied the Stochastic VCSBPP, with the addition of time constraints.

The mostly part of the literature on packing problems focuses on introducing uncertainty on the arrival of the items, defining the so-called on-line versions of the classic packing problems ([Zhang, 1997](#); [Seiden, 2000](#); [Iwama and Taketomi, 2002](#); [Seiden et al., 2003](#); [Han et al., 2010](#); [Epstein et al., 2011](#)). In particular, a lot of studies focused on the on-line version of the Bin Packing Problem, i.e. the variant of the Bin Packing where the items come one after the other and no knowledge (or a limited one) on the volume of the next items is given to the decision maker. The researchers studied policies for loading the items in the different bins giving results on their absolute or asymptotic behavior.

Regarding the solving strategy, numerous algorithms for solving *stochastic mixed-integer programs* (SMIPs) ([Kall and Wallace, 1994](#)) are based on decomposition by

time stages of the scenario tree (e.g. vertical decomposition) by complete scenarios (e.g. horizontal decomposition). PH algorithm is a horizontal decomposition approach and has emerged as an effective method for solving stochastic linear, non-linear, and linear SMIPs (Silva and Abramson, 1994; Crainic et al., 2011a; Ryan et al., 2013; Watson et al., 2014).

PH is a natural algorithm for solving large-scale SMIPs. By decomposing the *recourse problem* (RP) according to scenario, and iteratively solving penalized versions of the sub-problems, the PH gradually enforces implementability. However, non-convergence of solution is possible in the case of non-convex optimization problems such as the **SVCSBPP**. Integer decision variables render SP problems non-convex and significantly increase the difficulty of solution. For some smaller problem instances, standard MIP solvers can be used (Crainic et al., 2014b) to directly solve the RP. However, standard MIP solvers fail to consistently solve even individual scenario sub-problems in realistic applications in the context of CL. In fact, solving scenario problems separately, PH is particularly appropriate when there exist good, fast heuristics for generating solutions of individual scenarios.

4.2 The stochastic VCSBPP model

We propose a two-stage stochastic programming formulation, where the first stage concerns the selection of bins, and the second stage concerns the acquisition of extra capacity when the actual demand information is revealed.

Let \mathcal{T} be the finite set of bin types, which are defined according to the volume and fixed cost associated with the bins that are available at the first stage. For $\tau \in \mathcal{T}$, let V^τ and f^τ be respectively the volume and fixed cost associated with bins of type τ . We define \mathcal{J}^τ to be the set of available bins of type τ and $\mathcal{J} = \cup_{\tau} \mathcal{J}^\tau$ to be the set of available bins at the first stage.

Let set Ω be the sample space of the random event, where $\omega \in \Omega$ defines a particular realization. Let vector ξ contain the stochastic parameters defined in the model, and $\xi(\omega)$ be a given realization of this random vector. Let the first-stage variables be $y_j^\tau = 1$ if bin $j \in \mathcal{J}^\tau$ is selected and 0 otherwise. The two-stage model

of the **SVCSBPP** may then be formulated as

$$\min_y \sum_{\tau \in \mathcal{T}} \sum_{j \in \mathcal{J}^\tau} f^\tau y_j^\tau + E_\xi[Q(y, \xi(\omega))] \quad (4.1)$$

$$\text{s.t. } y_j^\tau \in \{0,1\}, \quad \forall \tau \in \mathcal{T}, j \in \mathcal{J}^\tau, \quad (4.2)$$

where $Q(y, \xi(\omega))$ is the extra cost paid for the capacity that is added at the second stage, given the tactical capacity plan y and the vector $\xi(\omega)$. The objective function (4.1) then minimizes the sum of the total fixed cost of the tactical capacity plan and the expected cost associated with the extra capacity added during the operation, while constraints (4.2) impose the integrality requirements on y .

To formulate $Q(y, \xi(\omega))$, we consider the following stochastic parameters in $\xi(\omega)$: $\mathcal{K}^\tau(\omega)$, the set of available bins of type τ at the second stage; $\mathcal{K}(\omega) = \cup_\tau \mathcal{K}^\tau$, the set of available bins at the second stage; $g^\tau(\omega)$, $k \in \mathcal{K}^\tau(\omega)$, the associated fixed costs; $\mathcal{I}(\omega)$, the set of items to be packed; and v_i , $i \in \mathcal{I}(\omega)$, the item volumes. The second-stage variables are defined as follows: $z_k^\tau = 1$ if bin $k \in \mathcal{K}^\tau(\omega)$ is selected, 0 otherwise; $x_{ij} = 1$ if item $i \in \mathcal{I}(\omega)$ is packed in bin $j \in \mathcal{J}(\omega)$, 0 otherwise; and $x_{ik} = 1$ if item $i \in \mathcal{I}(\omega)$ is packed in bin $k \in \mathcal{K}(\omega)$, 0 otherwise.

We now define the function $Q(y, \xi(\omega))$ as

$$Q(y, \xi(\omega)) = \min_{y, z, x} \sum_{\tau \in \mathcal{T}} \sum_{k \in \mathcal{K}^\tau(\omega)} g^\tau(\omega) z_k^\tau \quad (4.3)$$

$$\text{s.t. } \sum_{j \in \mathcal{J}} x_{ij} + \sum_{k \in \mathcal{K}(\omega)} x_{ik} = 1, \quad \forall i \in \mathcal{I}(\omega) \quad (4.4)$$

$$\sum_{i \in \mathcal{I}(\omega)} v_i(\omega) x_{ij} \leq V^\tau y_j^\tau, \quad \forall \tau \in \mathcal{T}, j \in \mathcal{J}^\tau \quad (4.5)$$

$$\sum_{i \in \mathcal{I}(\omega)} v_i(\omega) x_{ik} \leq V^\tau z_k^\tau, \quad \forall \tau \in \mathcal{T}, k \in \mathcal{K}^\tau(\omega) \quad (4.6)$$

$$x_{ij} \in \{0,1\}, \quad \forall i \in \mathcal{I}(\omega), j \in \mathcal{J} \quad (4.7)$$

$$x_{ik} \in \{0,1\}, \quad \forall i \in \mathcal{I}(\omega), k \in \mathcal{K}^\tau(\omega) \quad (4.8)$$

$$z_k^\tau \in \{0,1\}, \quad \forall \tau \in \mathcal{T}, k \in \mathcal{K}^\tau(\omega). \quad (4.9)$$

The objective function (4.3) minimizes the cost associated with the extra bins selected at the second stage. Constraints (4.4) ensure that each item is packed in a

single bin. Constraints (4.5) and (4.6) ensure that the total volume of items packed in each bin does not exceed the bin volume. Finally, constraints (4.7) to (4.9) impose the integrality requirements on all second-stage variables.

4.3 A lower bound for the SVCSBPP

We present in this section a *lower bound* (LB) for the **SVCSBPP** (4.1)–(4.9), which provides a way to measure the quality of the heuristic proposed in Section 4.4.

LB is obtained by removing the item-to-bin assignment constraints (4.4) and aggregating the individual bin feasibility constraints (4.5) and (4.6). The resulting formulation (4.10)–(4.13) is a two-stage stochastic model with fixed recourse, which yields an optimal set of bins, involving both the capacity plan and extra bins, with a total capacity sufficient for the items considered (see constraints (4.11)). Thus, the LB does not guarantee feasibility for individual bins.

$$\min_{y,z} \sum_{\tau \in \mathcal{T}} \sum_{j \in \mathcal{J}^\tau} f_j^\tau y_j^\tau + \mathbb{E}_\xi \left[\sum_{\tau \in \mathcal{T}} \sum_{k \in \mathcal{K}^\tau(\omega)} g_k^\tau(\omega) z_k^\tau \right] \quad (4.10)$$

$$\text{s.t.} \quad \sum_{\tau \in \mathcal{T}} \sum_{j \in \mathcal{J}^\tau} V_j^\tau y_j^\tau + \sum_{\tau \in \mathcal{T}} \sum_{k \in \mathcal{K}^\tau(\omega)} V_k^\tau z_k^\tau \geq \sum_{i \in \mathcal{I}(\omega)} v_i(\omega), \quad (4.11)$$

$$y_j^\tau \in \{0,1\} \quad \forall \tau \in \mathcal{T}, j \in \mathcal{J}^\tau, \quad (4.12)$$

$$z_k^\tau \in \{0,1\} \quad \forall \tau \in \mathcal{T}, k \in \mathcal{K}^\tau(\omega). \quad (4.13)$$

Note that the LB formulation is independent of the number of items. This reduces the number of variables in the model and makes it possible to find an optimal solution in a reasonable time, (e.g. using commercial MIP solvers).

4.4 Heuristic based on progressive hedging

Algorithm 1 describes the proposed heuristic for the **SVCSBPP**, which is inspired by the PH algorithm.

The method first applies a scenario decomposition (SD) technique based on the

augmented Lagrangian relaxation, which separates the stochastic problem by scenario. Here, the Lagrangian multipliers are used to penalize a lack of implementability due to differences in the first-stage variable values among scenario subproblems. Section 4.4.1 describes how the **SVCSBPP** can be decomposed into deterministic VCSBPP subproblems with modified fixed costs. Then, the method proceeds in two phases. Phase 1 aims to obtain consensus among the subproblems. At each iteration, the subproblems are first solved separately. Their solutions are then aggregated into a temporary overall solution, as defined in Section 4.4.2. The search process is gradually guided toward scenario consensus by adjusting the Lagrangian multipliers and the subproblem penalties (Section 4.4.2), based on the deviations of the scenario solutions from the overall solution, and by a variable bundle-fixing strategy (Section 4.4.2). The search process continues until the consensus is achieved or one of the termination criteria is met (see Section 4.4.3). When consensus is not achieved in the first phase, Phase 2 (see Section 4.4.4) solves the restricted **SVCSBPP** obtained by fixing the first-stage variables for which consensus has been reached, e.g., the bins used in all the scenario subproblems. Section 4.4.5 finally describes the parallel implementation of the algorithm solving the subproblems concurrently on multiple processors.

4.4.1 Scenario decomposition of the **SVCSBPP**

We first reformulate the **SVCSBPP** stochastic model (4.1)–(4.9) using scenario decomposition. Sampling is applied to obtain a set of representative scenarios, namely the set \mathcal{S} , and these are used to approximate the expected cost associated with the second stage.

For the first stage, let $y_j^{\tau s} = 1$ if bin $j \in \mathcal{J}^\tau$ of type $\tau \in \mathcal{T}$ is selected under scenario $s \in \mathcal{S}$ and 0 otherwise. For the second stage, define $\mathcal{K}^s = \cup_{\tau} \mathcal{K}^{\tau s}$, where $\mathcal{K}^{\tau s}$ is the set of extra bins of type $\tau \in \mathcal{T}$ in scenario $s \in \mathcal{S}$, and let \mathcal{I}^s be the set of items to pack under scenario $s \in \mathcal{S}$. Then, variable $z_k^{\tau s}$ is one if and only if extra bin $k \in \mathcal{K}^{\tau s}$ of type $\tau \in \mathcal{T}$ is selected in scenario $s \in \mathcal{S}$, and x_{ij}^s and x_{ik}^s are item-to-bin assignment variables for scenario $s \in \mathcal{S}$. Given the probability p_s of each scenario $s \in \mathcal{S}$, the **SVCSBPP** problem (4.1)–(4.9) can be approximated by the following

Algorithm 1 PH-based meta-heuristic for the **SVCSBPP**

Scenario decomposition

Generate a set of scenarios \mathcal{S} ;

Decompose the resulting deterministic model (4.14)–(4.22) by scenario using augmented Lagrangian relaxation;

Phase 1

$\nu \leftarrow 0$; $\lambda_j^{\tau s \nu} \leftarrow 0$; $\rho_j^{\tau \nu} \leftarrow f^\tau / 10$;

while Termination criteria not met **do**

For all $s \in \mathcal{S}$, solve the corresponding Variable Cost and Size Bin Packing subproblem $\rightarrow y_j^{\tau s \nu}$;

Compute temporary global solution

$$\bar{y}_j^{\tau \nu} \leftarrow \sum_{s \in \mathcal{S}} p_s y_j^{\tau s \nu}$$

$$\bar{\delta}^{\tau \nu} \leftarrow \sum_{s \in \mathcal{S}} p_s \delta^{\tau s \nu}$$

Penalty adjustment

$$\lambda_j^{\tau s \nu} = \lambda_j^{\tau s \nu - 1} + \rho_j^{\tau(\nu-1)} (y_j^{\tau s \nu} - \bar{y}_j^{\tau \nu})$$

$$\rho_j^{\tau \nu} \leftarrow \alpha \rho_j^{\tau(\nu-1)}$$

if consensus is at least $\sigma\%$ **then**

 Adjust the fixed costs $f^{\tau s \nu}$ according to (4.46);

Bundle fixing

$$\bar{\delta}_m^{\tau \nu} \leftarrow \min_{s \in \mathcal{S}} \delta^{\tau s \nu}$$

$$\bar{\delta}_M^{\tau \nu} \leftarrow \max_{s \in \mathcal{S}} \delta^{\tau s \nu}$$

 Apply variable fixing;

$\nu \leftarrow \nu + 1$

Phase 2

if consensus not met for a single bin type τ' ($\bar{\delta}_m^{\tau'} < \bar{\delta}_M^{\tau'}$) **then**

 Identify the consensus number of bins δ of type τ' by enumerating $\delta \in [\bar{\delta}_m^{\tau'}, \bar{\delta}_M^{\tau'}]$ (and variable fixing)

else

 Fix consensus variables in model (4.14)–(4.22);

 Solve restricted (4.14)–(4.22) model using a MIP solver.

equivalent deterministic model:

$$\min_{y,z,x} \sum_{s \in \mathcal{S}} p_s \left[\sum_{\tau \in \mathcal{T}} \sum_{j \in \mathcal{J}^\tau} f^\tau y_j^{\tau s} + \sum_{\tau \in \mathcal{T}} \sum_{k \in \mathcal{K}^{\tau s}} g^{\tau s} z_k^{\tau s} \right] \quad (4.14)$$

$$\text{s.t.} \quad \sum_{j \in \mathcal{J}} x_{ij}^s + \sum_{k \in \mathcal{K}^s} x_{ik}^s = 1 \quad \forall i \in \mathcal{I}^s, s \in \mathcal{S}, \quad (4.15)$$

$$\sum_{i \in \mathcal{I}^s} v_i^s x_{ij}^s \leq V^\tau y_j^{\tau s} \quad \forall \tau \in \mathcal{T}, j \in \mathcal{J}^\tau, s \in \mathcal{S}, \quad (4.16)$$

$$\sum_{i \in \mathcal{I}^s} v_i^s x_{ik}^s \leq V^\tau z_k^{\tau s} \quad \forall \tau \in \mathcal{T}, k \in \mathcal{K}^{\tau s}, s \in \mathcal{S}, \quad (4.17)$$

$$y_j^{\tau s} = y_j^{\tau t} \quad \forall \tau \in \mathcal{T}, j \in \mathcal{J}^\tau, s, t \in \mathcal{S}, \quad (4.18)$$

$$y_j^{\tau s} \in \{0,1\} \quad \forall \tau \in \mathcal{T}, j \in \mathcal{J}^\tau, s \in \mathcal{S}, \quad (4.19)$$

$$z_k^{\tau s} \in \{0,1\} \quad \forall \tau \in \mathcal{T}, k \in \mathcal{K}^{\tau s}, s \in \mathcal{S}, \quad (4.20)$$

$$x_{ij}^s \in \{0,1\} \quad \forall i \in \mathcal{I}^s, j \in \mathcal{J}, s \in \mathcal{S}, \quad (4.21)$$

$$x_{ik}^s \in \{0,1\} \quad \forall i \in \mathcal{I}^s, k \in \mathcal{K}^s, s \in \mathcal{S}. \quad (4.22)$$

Constraints (4.18) are referred to as the non-anticipativity constraints. They ensure that the first-stage decisions are not tailored according to the scenarios considered in S . Indeed, all the scenario solutions must be equal to produce a single implementable plan. Thus, the non-anticipativity constraints link the first-stage variables to the second-stage variables, and so the model is not separable.

To apply Lagrangean relaxation and make the model separable, we need a different expression of the non-anticipativity constraints. Let $\bar{y}_j^\tau \in \{0,1\}$, $\forall \tau \in \mathcal{T}, j \in \mathcal{J}^\tau$, be the *global capacity plan* (i.e., the set of bins selected at the first stage). The following constraints are equivalent to (4.18):

$$\bar{y}_j^\tau = y_j^{\tau s} \quad \tau \in \mathcal{T}, \quad j \in \mathcal{J}^\tau, \quad s \in \mathcal{S} \quad (4.23)$$

$$\bar{y}_j^\tau \in \{0,1\} \quad \tau \in \mathcal{T}, \quad j \in \mathcal{J}^\tau. \quad (4.24)$$

Constraints (4.23) force the first-stage solution of each scenario to be equal to the global capacity plan. Constraints (4.24) are simply the integrality conditions on the selection of the bins. With this formulation of the non-anticipativity constraints, when we apply Lagrangean relaxation to (4.23), we can penalize individually the

difference between the scenario solution and the global solution for each bin in the plan.

Following the decomposition scheme proposed by [Rockafellar and Wets \(1991\)](#), we relax constraints (4.23) using an augmented Lagrangean strategy. We thus obtain the following objective function for the overall problem:

$$\min_{y,z} \sum_{s \in S} p_s \left[\sum_{\tau \in \mathcal{T}} \sum_{j \in \mathcal{J}^\tau} f^\tau y_j^{\tau s} + \sum_{\tau \in \mathcal{T}} \sum_{k \in \mathcal{K}^{\tau s}} g^{\tau s} z_k^{\tau s} + \sum_{\tau \in \mathcal{T}} \sum_{j \in \mathcal{J}^\tau} \lambda_j^s (y_j^{\tau s} - \bar{y}_j^\tau) + \frac{1}{2} \sum_{\tau \in \mathcal{T}} \sum_{j \in \mathcal{J}^\tau} \rho_j^\tau (y_j^{\tau s} - \bar{y}_j^\tau)^2 \right] \quad (4.25)$$

where $\lambda_j^s, \forall j \in \mathcal{J}$ and $\forall s \in S$, define the Lagrangean multipliers for the relaxed constraints and ρ_j^τ is a penalty ratio associated with bin $j \in \mathcal{J}^\tau$ of type $\tau \in \mathcal{T}$. Within function (4.25), let us consider the quadratic term. Given the binary requirements of $y_j^{\tau s}$ and \bar{y}_j^τ , this term becomes:

$$\begin{aligned} \sum_{\tau \in \mathcal{T}} \sum_{j \in \mathcal{J}^\tau} \rho_j^\tau (y_j^{\tau s} - \bar{y}_j^\tau)^2 &= \sum_{\tau \in \mathcal{T}} \sum_{j \in \mathcal{J}^\tau} (\rho_j^\tau (y_j^{\tau s})^2 - 2\rho_j^\tau y_j^{\tau s} \bar{y}_j^\tau + \rho_j^\tau (\bar{y}_j^\tau)^2) = \\ &= \sum_{\tau \in \mathcal{T}} \sum_{j \in \mathcal{J}^\tau} (\rho_j^\tau y_j^{\tau s} - 2\rho_j^\tau y_j^{\tau s} \bar{y}_j^\tau + \rho_j^\tau \bar{y}_j^\tau). \end{aligned} \quad (4.26)$$

Therefore, the objective function can be formulated as follows:

$$\begin{aligned} \min_{y,z} \sum_{s \in S} p_s \left[\sum_{\tau \in \mathcal{T}} \sum_{j \in \mathcal{J}^\tau} \left(f^\tau + \lambda_j^s - \rho_j^\tau \bar{y}_j^\tau + \frac{\rho_j^\tau}{2} \right) y_j^{\tau s} + \sum_{\tau \in \mathcal{T}} \sum_{k \in \mathcal{K}^{\tau s}} g^{\tau s} z_k^{\tau s} \right] \\ - \sum_{\tau \in \mathcal{T}} \sum_{j \in \mathcal{J}^\tau} \lambda_j^s \bar{y}_j^\tau + \frac{\rho_j^\tau}{2} \sum_{\tau \in \mathcal{T}} \sum_{j \in \mathcal{J}^\tau} \bar{y}_j^\tau. \end{aligned} \quad (4.27)$$

Given constraints (4.15)–(4.22) and the objective function (4.27), the relaxed problem is not separable by scenario. However, if the overall plan $\bar{y}_j^\tau, \forall \tau \in \mathcal{T}$ and $\forall j \in \mathcal{J}^\tau$, is fixed to a given value vector (i.e., the expected value of the scenario solutions), then the model decomposes according to the scenarios in S and the

scenario subproblems can be expressed as follows:

$$\min_{y,z,x} \sum_{\tau \in \mathcal{T}} \sum_{j \in \mathcal{J}^\tau} \left(f^\tau + \lambda_j^s - \rho_j^\tau \bar{y}_j^\tau + \frac{\rho_j^\tau}{2} \right) y_j^{\tau s} + \sum_{\tau \in \mathcal{T}} \sum_{k \in \mathcal{K}^{\tau s}} g^{\tau s} z_k^{\tau s} \quad (4.28)$$

$$\text{s.t.} \quad \sum_{j \in \mathcal{J}} x_{ij}^s + \sum_{k \in \mathcal{K}^s} x_{ik}^s = 1 \quad \forall i \in \mathcal{I}^s, s \in \mathcal{S}, \quad (4.29)$$

$$\sum_{i \in \mathcal{I}^s} v_i^s x_{ij}^s \leq V^\tau y_j^{\tau s} \quad \forall \tau \in \mathcal{T}, j \in \mathcal{J}^\tau, s \in \mathcal{S}, \quad (4.30)$$

$$\sum_{i \in \mathcal{I}^s} v_i^s x_{ik}^s \leq V^\tau z_k^{\tau s} \quad \forall \tau \in \mathcal{T}, k \in \mathcal{K}^{\tau s}, s \in \mathcal{S}, \quad (4.31)$$

$$y_j^{\tau s} \in \{0,1\} \quad \forall \tau \in \mathcal{T}, j \in \mathcal{J}^\tau, s \in \mathcal{S}, \quad (4.32)$$

$$z_k^{\tau s} \in \{0,1\} \quad \forall \tau \in \mathcal{T}, k \in \mathcal{K}^{\tau s}, s \in \mathcal{S}, \quad (4.33)$$

$$x_{ij}^s \in \{0,1\} \quad \forall i \in \mathcal{I}^s, j \in \mathcal{J}^\tau, s \in \mathcal{S}, \quad (4.34)$$

$$x_{ik}^s \in \{0,1\} \quad \forall i \in \mathcal{I}^s, k \in \mathcal{K}^s, s \in \mathcal{S}. \quad (4.35)$$

Furthermore, by noting that λ_j^s and ρ_j^τ are exogenous constants for the model (4.28)–(4.35), we can reformulate each scenario subproblem as follows. For scenario s , let $\mathcal{B}^{\tau s} = \mathcal{J}^\tau \cup \mathcal{K}^{\tau s}$ be the set of available bins of type τ in the subproblem. For $b \in \mathcal{B}^{\tau s}$, let f_b^τ define the fixed cost associated with bin $b \in \mathcal{B}^{\tau s}$. The value of f_b is given by

$$f_b^\tau = \begin{cases} f^\tau + \lambda_j^s - \rho_j^\tau \bar{y}_j^\tau + \frac{\rho_j^\tau}{2} & \tau \in \mathcal{T}, b, j \in \mathcal{J}^\tau \\ g^{\tau s} & \tau \in \mathcal{T}, b \in \mathcal{K}^{\tau s}. \end{cases} \quad (4.36)$$

Thus, each scenario subproblem can be reduced to a deterministic VCSBPP with modified fixed costs:

$$\min_{y,x} \sum_{\tau \in \mathcal{T}} \sum_{b \in \mathcal{B}^{\tau s}} f_b^\tau y_b^\tau \quad (4.37)$$

$$\text{s.t.} \quad \sum_{\tau \in \mathcal{T}} \sum_{b \in \mathcal{B}^{\tau s}} x_{ib}^s = 1 \quad \forall i \in \mathcal{I}^s, s \in \mathcal{S}, \quad (4.38)$$

$$\sum_{i \in \mathcal{I}^s} v_i^s x_{ib}^s \leq V^\tau y_b^\tau \quad \forall \tau \in \mathcal{T}, \forall b \in \mathcal{B}^{\tau s}, s \in \mathcal{S}, \quad (4.39)$$

$$y_b^\tau \in \{0,1\} \quad \forall \tau \in \mathcal{T}, \forall b \in \mathcal{B}^{\tau s}, s \in \mathcal{S}, \quad (4.40)$$

$$x_{ib}^s \in \{0,1\} \quad \forall \tau \in \mathcal{T}, \forall b \in \mathcal{B}^{\tau s}, \forall i \in \mathcal{I}^s, s \in \mathcal{S} \quad (4.41)$$

where $y_b^\tau = 1$ if bin $b \in \mathcal{B}^{\tau s}$ of type $\tau \in \mathcal{T}$ is selected, 0 otherwise.

It is time-consuming to solve a large VCSBPP to optimality using a commercial MIP solver (Correia et al., 2008). Moreover, in the PH algorithm VCSBPP subproblems must be solved multiple times. Thus, we need to use one of the effective algorithms developed for the VCSBPP (Crainic et al., 2007; Baldi et al., 2011; Crainic et al., 2011c). We choose the algorithm of Crainic et al. (2011c), because of its efficiency on instances with up to 1000 items. The algorithm implements an adapted best-first decreasing strategy that sorts items and bins by nonincreasing order of volume and unit cost, respectively. The heuristic then sequentially assigns each item to the best bin, which is the bin with the maximum free space once the item is assigned.

4.4.2 Obtaining consensus among subproblems

At each iteration of the meta-heuristic, the solutions of the scenario subproblems are used to build a temporary global solution (the overall capacity plan). "Consensus" is then defined as scenario solutions being similar with regard to the first-stage decisions with the overall capacity plan and, thus, being similar among themselves. Section 4.4.2 describes how the overall plan is computed given the symmetry challenge of the bin packing formulation. Moreover, we introduce strategies for the penalty adjustment when nonconsensus is observed and techniques to guide the search process by bounding the number of bins that can be selected at the first stage.

Defining the overall capacity plan

Let ν be the iteration counter in the PH algorithm. At each iteration, the algorithm solves subproblems (4.37)–(4.41), obtaining local solutions $y_j^{\tau s \nu}$, $\forall s \in \mathcal{S}, \forall \tau \in \mathcal{T}$, and $\forall j \in \mathcal{J}^\tau$. The subproblem solutions are then combined in the overall capacity plan $\bar{y}_j^{\tau \nu}$ by using the expected value operator, as shown in Equation (4.42). The weight used for each component is the probability p_s associated with the corresponding

scenario.

$$\bar{y}_j^{\tau\nu} = \sum_{s \in \mathcal{S}} p_s y_j^{\tau s\nu}, \quad \forall \tau \in \mathcal{T}, \forall j \in \mathcal{J}^\tau. \quad (4.42)$$

However, this definition does not take into account the presence of a large number of equivalent solutions that is typical of packing problems (Baldi et al., 2012a). In fact, packing problems present a strong symmetry in the solution space. Two solutions are considered symmetric (and equivalent) if they involve the same set of bins in different orders. Equation (4.42) concerns the use of the specific bin $j \in \mathcal{J}^\tau$ and is therefore dependent on the order of the bins in the solution. Thus, it provides misleading information on the consensus. For this reason, (4.42) cannot be used to measure the convergence of the overall solution.

To deal with the symmetry of the solution space, we define an overall solution based on the number of bins in the capacity plan. Let $\delta^{\tau s\nu} = \sum_{j \in \mathcal{J}^\tau} y_j^{\tau s\nu}$ be the total number of bins of type $\tau \in \mathcal{T}$ in the capacity plan for scenario subproblem $s \in \mathcal{S}$ at iteration ν . Equivalently to (4.42), using the expected value operator on $\delta^{\tau s\nu} \forall s \in \mathcal{S}$, we can define the overall capacity plan for each bin type $\tau \in \mathcal{T}$ as

$$\bar{\delta}^{\tau\nu} = \sum_{s \in \mathcal{S}} p_s \delta^{\tau s\nu} = \sum_{s \in \mathcal{S}} p_s \sum_{j \in \mathcal{J}^\tau} y_j^{\tau s\nu} = \sum_{j \in \mathcal{J}^\tau} \sum_{s \in \mathcal{S}} p_s y_j^{\tau s\nu} = \sum_{j \in \mathcal{J}^\tau} \bar{y}_j^{\tau\nu}. \quad (4.43)$$

Equation (4.43) breaks the symmetry of the solutions (the order of the bins in the solution does not change the value of $\delta^{\tau s\nu}$) and can be used to define the stopping criterion. Thus, we consider consensus to be achieved when the values of $\delta^{\tau s\nu}$, $\forall s \in \mathcal{S}$, are equal to $\bar{\delta}^{\tau\nu}$.

It is important to note that (4.42) and (4.43) do not necessarily produce a feasible capacity plan. When consensus is not achieved the overall solution may not satisfy the integrality constraints on the first-stage decision variables. For nonconvex problems such as the **SVCSBPP** using the expected value as an aggregation operator does not guarantee that the algorithm converges to the optimal solution. Moreover, it cannot ensure that a good (feasible) solution will be obtained for the stochastic problem. Therefore, (4.42) and (4.43) are used as reference solutions with the goal of helping the search process of the PH algorithm to identify bins for which consensus is possible. Both are used in the penalty adjustment, while (4.43) is also used in the bounding strategy.

Penalty adjustment strategies

To induce consensus among the scenario subproblems, we adjust the penalties in the objective function at each iteration to penalize a lack of implementability and dissimilarity between local solutions and the overall solution. We propose two different strategies for these adjustments, both working at the local level in the sense that they affect every scenario subproblem separately.

The first strategy was originally proposed by [Rockafellar and Wets \(1991\)](#). Using information on the bin selection (e.g., variable $y_j^{\tau s \nu}$), it operates on the fixed costs by changing the Lagrangean multipliers. For a given iteration ν , let $\lambda_j^{s \nu}$ be the Lagrangean multiplier associated with bin $j \in \mathcal{J}^\tau$ for scenario $s \in \mathcal{S}$, and let $\rho_j^{\tau \nu}$ be the penalty deriving from the quadratic term. Note that the value of $\rho_j^{\tau \nu}$ is variable-specific. This approach outperforms scalar ρ strategies and guarantees faster convergence of the algorithm ([Watson and Woodruff, 2011](#)). At each iteration, we update the values $\lambda_j^{s \nu}$ and $\rho_j^{\tau \nu}$, $\forall j \in \mathcal{J}$ and $\forall s \in \mathcal{S}$, as follows:

$$\lambda_j^{s \nu} = \lambda_j^{s(\nu-1)} + \rho_j^{\tau(\nu-1)}(y_j^{\tau s \nu} - \bar{y}_j^{\tau \nu}) \quad (4.44)$$

$$\rho_j^{\tau \nu} \leftarrow \alpha \rho_j^{\tau(\nu-1)}, \quad (4.45)$$

where $\alpha > 1$ is a given constant and $\rho_j^{\tau 0}$ is fixed to a positive value to ensure that $\rho_j^{\tau \nu} \rightarrow \infty$ as the number of iterations ν increases.

We initialize $\lambda_j^{s 0} = 0$ for each scenario $s \in \mathcal{S}$. Equation (4.44) can then reduce, increase, or maintain this contribution according to the difference between the value of the bin-selection variables in the subproblem solutions and the overall capacity plan. The initial choice of $\rho_j^{\tau \nu}$ is important. An inaccurate choice may cause premature convergence to a solution that is far from optimal or cause slow convergence of the search process. To avoid this, we set $\rho_j^{\tau 0}$ proportional to the fixed cost associated with the bin-selection variable: $\rho_j^{\tau 0} = \max(1, f^\tau/10)$, $\forall j \in \mathcal{J}^\tau$ and $\forall \tau \in \mathcal{T}$. The value of $\rho_j^{\tau \nu}$ increases according to (4.45) as the number of iteration grows.

The second penalty adjustment is a heuristic strategy, which directly tunes the fixed costs of bins of the same type. The goal of this strategy is to accelerate the search process when the overall solution is close to consensus. When consensus is close, the difference between the subproblem solution and the overall solution may

be small, and adjustments (4.44) and (4.45) lose their effectiveness, requiring several iterations to reach consensus.

Let $f^{\tau s \nu}$ be the fixed cost of bin $j \in \mathcal{J}^\tau$ of type $\tau \in \mathcal{T}$ for scenario $s \in \mathcal{S}$ at iteration ν . At the beginning of the algorithm ($\nu = 0$), we impose $f^{\tau s 0} = f^\tau$. Then, when at least $\sigma\%$ of the variables have reached consensus, we perturb every subproblem by changing $f^{\tau s \nu}$ as follows:

$$f^{\tau s \nu} = \begin{cases} f^{\tau s(\nu-1)} \cdot M & \text{if } \delta^{\tau s(\nu-1)} > \bar{\delta}^{\tau(\nu-1)} \\ f^{\tau s(\nu-1)} \cdot \frac{1}{M} & \text{if } \delta^{\tau s(\nu-1)} < \bar{\delta}^{\tau(\nu-1)} \\ f^{\tau s(\nu-1)} & \text{otherwise.} \end{cases} \quad (4.46)$$

Here M is a constant greater than 1, while $\sigma\%$ is a constant such that $0.5 \leq \sigma\% \leq 1$. The current implementation of this heuristic strategy uses $\sigma\% = 0.75$ and $M = 1.1$. The rationale for (4.46) is the following: if $\delta^{\tau s(\nu-1)} > \bar{\delta}^{\tau(\nu-1)}$, this means that in the previous iteration the number of bins of a given bin type τ in scenario s was larger than the number of bins in the reference solution $\bar{\delta}^{\tau(\nu-1)}$. Thus, the use of bins of type τ is penalized by increasing the fixed cost by M . On the other hand, if $\delta^{\tau s(\nu-1)} < \bar{\delta}^{\tau(\nu-1)}$, we promote bins of type τ by reducing the fixed cost by $1/M$.

Bundle fixing

To guide the search process, we introduce a variable-fixing strategy. Because there are multiple equivalent solutions, it might not be efficient to fix a single bin-selection variable $\bar{y}_j^{\tau \nu}$. We instead restrict the number of bins of each type that can be used, specifying lower and upper bounds. We call this strategy *bundle fixing*.

Let $\bar{\delta}_m^{\tau \nu}$ and $\bar{\delta}_M^{\tau \nu}$ be the minimum and maximum number of bins of type τ involved in the overall solution at iteration ν :

$$\bar{\delta}_m^{\tau \nu} \leftarrow \min_{s \in \mathcal{S}} \delta^{\tau s \nu}, \quad (4.47)$$

$$\bar{\delta}_M^{\tau \nu} \leftarrow \max_{s \in \mathcal{S}} \delta^{\tau s \nu}. \quad (4.48)$$

At each iteration, the bundle strategy applies two bounds as follows. The lower bound $\bar{\delta}_m^{\tau \nu}$ determines a set of compulsory bins that must be used in each subproblem;

to implement this we set the decision variables $y_j^{\tau s(\nu+1)}$ to one for $j = 1, \dots, \bar{\delta}_m^{\tau\nu}$. The upper bound $\bar{\delta}_M^{\tau\nu}$ is an estimate of the maximum number of bins of type τ available in the next iteration; this reduces the number of decision variables in the subproblems. To implement this we remove decision variables $y_j^{\tau s(\nu+1)}$ for $j = \bar{\delta}_M^{\tau\nu} + 1, \dots, \|\mathcal{J}^\tau\|$.

4.4.3 Termination criteria

There are to date no theoretical results on the convergence of the PH algorithm for integer problems. Thus, we implement three stopping criteria for the search phase of the proposed meta-heuristic, based on the level of consensus reached and the number of iterations.

The level of consensus is measured through equations (4.47) and (4.48), as consensus is reached when $\bar{\delta}_m^{\tau\nu} = \bar{\delta}_M^{\tau\nu}, \forall \tau \in \mathcal{T}$. To speed up the algorithm, we actually stop the search, and proceed to Phase 2, as soon as consensus has been reached for all the bin types except one, type τ' , for which $\bar{\delta}_m^{\tau'} < \bar{\delta}_M^{\tau'}$.

When neither of the preceding conditions has been reached within a maximum number of iterations (200 in our experiments), the search is stopped and the meta-heuristic proceeds to the Phase 2.

4.4.4 Phase 2 of the meta-heuristic

Phase 2 is thus invoked either when consensus is not achieved within a given maximum number of iterations, or the search was stopped when all but one bin type were in consensus.

In the first case, there is only one bin type τ' with $\bar{\delta}_m^{\tau'} < \bar{\delta}_M^{\tau'}$, that is, not in consensus. Given the efficiency of the item-to-bin heuristic, Phase 2 computes the final solution by iteratively examining the possible number of bins for τ' (a consensus solution is always possible because $\bar{\delta}_M^{\tau'}$ is feasible in all scenarios):

For all $\delta \in [\bar{\delta}_m^{\tau'}, \bar{\delta}_M^{\tau'}]$ **do**

Set the number of bins of type τ' to δ ;

Solve all the scenario subproblems with the VCSBPP heuristic;

Check the feasibility of the solutions;

Update the overall solution if a better solution has been found;

Produce the consensus solution.

When the maximum number of iterations is reached, consensus is less close. Phase 2 of the meta-heuristic then builds a restricted version of the formulation (4.14)–(4.22) by fixing the bin-selection first-stage variables for which consensus has been achieved, together with the associated item-to-bin assignment variables. The range of the bin types not in consensus is reduced through bundle fixing, and the resulting MIP is solved exactly.

4.4.5 Parallel implementation

Given the straightforward parallelization of the PH algorithm, we developed a synchronous master–slave implementation. This implementation extends our heuristic to a multiprocessor environment. The subproblems are assigned to a number of slaves. The master collects the solutions from the slaves and waits until all the scenario subproblems have been solved. The master then has all the solutions and can proceed to calculate the overall solution. If consensus is not reached, the master updates the penalties of each subproblem and starts a new iteration. The parallelization reduces the computational time for each iteration and thus speeds up the convergence to a consensual solution. The quality of the solutions is not affected by the parallel execution. In fact, the search process follows the same dynamics as in the sequential case.

It should be noted that when the computational time for the subproblems is unbalanced, the parallel algorithm is less efficient. To mitigate this effect, the master checks the current load of each slave (the number of subproblems remaining) and, if necessary, it can reassign subproblems among the slaves.

The complete list of duties of the master and slaves is reported below.

- **Master**

1. Creates the pool of scenario problems;
2. Assigns each slave an equal number of problems;
3. Checks the load of the slaves and adjusts the assignments;

4. Computes the global solutions, computes the bounds, and updates the penalties.

- **Slave**

1. Solves the assigned subproblems;
2. Saves the solutions in a pool accessible by the master.

4.5 Computational results

We performed an extensive set of experiments. The goals of the experimental campaign were to

1. analyze the performance of the proposed PH-based meta-heuristic by comparing it to a state-of-the-art commercial MIP solver;
2. measure the impact of uncertainty and determine whether building a stochastic programming model is really useful;
3. explore the potential of the proposed model and algorithm by performing a number of analyzes of the structure, sensitivity and robustness of the logistics capacity plan under various problem settings.

We start by introducing the test instances generated for the numerical experiments (Section 4.5.1). We then proceed with the following experimental plan organized as follows:

PH algorithm validation (Section 4.5.2): We analyze the performance of the proposed meta-heuristic by comparing its results (objective values and computational times), those of the direct solution of the multi-scenario deterministic problem (called *RP* in the following) (4.14)–(4.22), and the lower bound (LB) (4.10)–(4.13). We also analyze the efficiency of the parallel implementation by studying the scalability to 16 thread.

Impact of the uncertainty (Section 4.5.3): These tests show the benefits of using the two-stage model with recourse compared to the perfect information problem (the so-called *wait and see* approach, WS) and the *expected value problem*

(EV). An important point when comparing RP and EV is to determine the difference in the first-stage decisions, i.e., the bins booked in advance. We do this via the well-known *EVPI* and *VSS* measures (Birge, 1982; Maggioni and Wallace, 2012).

Solution analysis (Section 4.5.4): We study the solutions to determine whether basic structure for the capacity planning exists and the dependence of the plan on attributes of the problem setting (e.g., number of bin types, spread of items to be loaded).

Sensitivity analysis (Section 4.5.5): We explore the sensitivity of the capacity planning when policies on the bins available in advance are considered in the contract. In particular, we consider the effect of lower and upper bounds on the number of each type of bin.

Effect of the capacity planning as actual policy (Section 4.5.6): We implemented a simulation approach to evaluate the robustness and reliability of the capacity planning decisions for one year under various demand scenarios. These scenarios represent realistic economic situations such as higher and lower demands. We focus on the use of planned bins, the extra capacity required to cover higher demand, and the corresponding cost.

We now list the approaches that we consider.

Exact approach. The RP and the LB are solved using CPLEX versione 12.5 (ILOG Inc., 2012), one of the most widely used commercial MIP solvers. We impose a maximum running time of 24 h. The solutions obtained by RP are the reference solutions for the validation of the PH algorithm.

Heuristic approach. As already noted, in the case of nonconvex problems, the PH algorithm may not converge to an optimal solution and it can thus be considered a heuristic method. We do not impose a maximum running time. We terminate the algorithm when consensus is achieved or the number of iterations reaches 200.

Expected-value approach. Since SMIPs are difficult to solve, it is common to solve a simpler deterministic problem in which the random parameters are replaced by their expected values. This approach does not guarantee optimality or feasibility.

Monte Carlo simulation. We use Monte Carlo simulation to evaluate the robustness of the solutions.

The tests are performed on a machine at the high-performance computing cluster of Politecnico di Torino (DAUIN, 2014) with 16 AMD Bulldozer cores at 2.3 GHz and 64 GB of RAM.

4.5.1 Instance set

We have developed two new sets of instances, denoted T and R, based on existing instances for different BPP problem variants (Monaci, 2002; Crainic et al., 2007, 2011c, 2012c, 2014b) and real applications of parcel delivery. Set T represents test cases with a limited number of items, and set R reproduces realistic cases of parcel delivery applications in urban and inter-city contexts. These instances are characterized by the number of bin types, the availability and the cost of the bins, and the number and volume of the items. The characteristics of the two sets are as follows:

Set T allows us to explore the structure of the capacity planning solutions for different configurations of bin types and items to be loaded. Moreover, this set aims to measure the effect of different levels of uncertainty in the demand and the extra capacity. The instances have the following characteristics:

- **Number of types.** We consider instances with 3 (T3), 5 (T5), and 10 (T10) bin types. The volumes are:
 - 50, 100, 150 for T3;
 - 50, 80, 100, 120, 150 for T5;
 - 50, 60, 70, 80, 100, 110, 120, 130, 140, 150 for T10.
- **Availability of bins.** The number of bins of type $\tau \in \mathcal{T}$ available in advance, $\|\mathcal{J}^\tau\|$, is the minimum number of bins of volume V^τ needed

to pack all items in the worst-case scenario (the scenario with the most items). This number is

$$\|\mathcal{J}^\tau\| = \left\lceil \frac{1}{V^\tau} \max_{s \in \mathcal{S}} \sum_{i \in \mathcal{I}^s} v_i^s \right\rceil. \quad (4.49)$$

At the second stage, we consider a large variability. The number of bins of type $\tau \in \mathcal{T}$ at the second stage, $\|\mathcal{K}^{\tau s}\|$, is uniformly distributed in the range $[0, \|\mathcal{J}^\tau\|]$. This means that the worst-case scenario may involve a limited number of extra bins.

- **Fixed cost of bins.** For the set of bins available in advance the fixed cost is

$$f^\tau = V^\tau(1 + \gamma), \quad (4.50)$$

where γ is uniformly distributed in the range $[-0.3, 0.3]$. According to [Correia et al. \(2008\)](#) this range replicates realistic situations. The fixed cost for extra bins $g^{\tau s}$ at the second stage is the original fixed cost f^τ multiplied by a factor $(1 + \alpha_\tau^s)$, inversely proportional to the availability of extra bins of type τ in scenario $s \in \mathcal{S}$, where

$$\alpha_\tau^s = 1 - \frac{\|\mathcal{K}^{\tau s}\|}{\sum_{\tau \in \mathcal{T}} \|\mathcal{K}^{\tau s}\|} \cdot \beta, \quad (4.51)$$

and $\beta \in U[0, 0.5]$. Thus, the maximum increase in the fixed cost is 50%.

- **Number of items.** The number of items at the second stage is uniformly distributed in the range
 - $[25, 100]$ for T3 and T5;
 - $[100, 500]$ for T10.
- **Volume of items.** The items are organized as follows:
 - Small item (S): volume in the range $[5, 10]$;
 - Medium item (M): volume in the range $[15, 25]$;
 - Big item (B): volume in the range $[20, 40]$.

These categories are then combined into four volume-spread classes reflecting different realistic settings:

- SP1 has a high percentage of small items $S = 60\%$, $M = 20\%$, $B = 20\%$;
 - SP2 has a high percentage of medium items $S = 20\%$, $M = 60\%$, $B = 20\%$;
 - SP3 has a high percentage of big items $S = 20\%$, $M = 20\%$, $B = 60\%$;
 - SP4 has no restrictions on the maximum number of items in each category.
- **Number of instances.** For each combination of the parameters mentioned above we define 10 instances with different scenarios. This gives a total of 120 instances.

Set R represents instances involved in real applications of parcel delivery companies (DHL, TNT, UPS, etc.), which need sufficient vehicles to deliver the unknown demand to the customers. We consider two applications: (R2) inter-city parcel delivery and (R3) parcel delivery in urban areas. The differences between the two settings are the volumes of the vehicles (and thus of the bins) and the cost of the bins. In these applications, the bins represent the vehicles used for the delivery of the items to the customers. There are usually two or three vehicle types, and the number of items per vehicle is of the order of 100 for urban delivery and 1000 for inter-city delivery. The different loads are a result of different logistics (e.g., maximum driver hours per day) and different vehicle types (inter-city services use large vehicles).

The instances have the following characteristics:

- **Number of types.** We consider up to 3 bin types with the volumes
 - 1000, 1200, 1500 for R3;
 - 10000, 15000 for R2.
- **Availability of bins.** The number of bins of type $\tau \in \mathcal{T}$ available in advance, $\|\mathcal{J}^\tau\|$, is defined by Equation (4.49). At the second stage, the

number of bins of type $\tau \in \mathcal{T}$, $\|\mathcal{K}^{\tau s}\|$, is uniformly distributed in the range $[0, \|\mathcal{J}^{\tau}\|]$.

- **Fixed cost of bins.** Equation (4.50) defines the fixed cost of the set of bins available in advance, and the fixed cost for extra capacity is the original cost multiplied by $(1 + \alpha_{\tau}^s)$ where α_{τ}^s is defined in Equation (4.51).
- **Number of items.** The number of items at the second stage is uniformly distributed in the range
 - $[3000, 4000]$ for R3;
 - $[9000, 11000]$ for R2.
- **Volume of items.** The volumes are uniformly distributed in the range
 - $[5, 20]$ for R3;
 - $[10, 15]$ for R2.
- **Number of instances.** For each combination of the parameters mentioned above we define 10 instances with different scenarios S . This gives a total of 20 instances.

Regarding the scenario tree, we assume that the random parameters have a finite number of possible outcomes at the end of the period considered. The discrete values that the random variable can assume are represented by a finite set of scenarios and are assumed to be exogenous to the problem. Consequently, the probability distribution is not influenced by the decisions. With these assumptions we can represent the stochastic parameters using a scenario tree that contains a root and a finite set of leaves. A total of $s = 1, \dots, 100$ scenarios are generated. This number of scenarios guarantees that the model has in-sample stability (Kaut et al., 2007), which means that the value of the optimal decisions of the first-stage variables does not change when a different set of scenarios is considered.

4.5.2 PH algorithm validation

Before studying the performance of the PH algorithm, we analyze the computational effort of CPLEX. Commercial MIP solvers such as CPLEX struggle to compute a good lower bound for models that include integer variables, and thus to prove the

optimality of the best incumbent solution in a reasonable computational time. The difficulty is even greater if there is a high degree of symmetry in the solutions, which is unfortunately typical in packing problems.

The quality of the solutions is measured by the optimality gap. This gap is defined as the difference between the best known solution UB^{CP} (i.e., the incumbent solution) and the best bound LB^{CP} . We present the relative optimality gap (Equation (4.52)), which is the optimality gap expressed as a percentage:

$$\Delta^{CP} = \frac{UB^{CP} - LB^{CP}}{LB^{CB}} \cdot 100. \quad (4.52)$$

The initial experiments on T3 and T5 showed that Δ^{CP} was greater than 10% after 24 hours of execution. We then considered using CPLEX in parallel mode. Figure 4.1 shows the trend of the average relative optimality gap for T3 and T5 with respect to the number of parallel threads. The gap decreases rapidly as the number of threads increases. With 16 parallel threads, the CPLEX has an average optimality gap below 6%. We therefore decided to execute CPLEX in parallel with 16 threads to obtain relevant computational results.

| Set | Spread | CPLEX | | PH | |
|-----|--------|---------------|--------------|--------------------|--------------|
| | | Δ^{CP} | t^{CP} [s] | Δ_{UB}^{PH} | t^{PH} [s] |
| T3 | SP1 | 5.38 | 86400 | -1.19 | 1.21 |
| | SP2 | 5.74 | 86400 | -1.39 | 2.38 |
| | SP3 | 6.38 | 86400 | -1.06 | 0.40 |
| | SP4 | 5.77 | 86400 | -0.86 | 0.48 |
| T5 | SP1 | 4.99 | 86400 | -1.17 | 2.08 |
| | SP2 | 5.23 | 86400 | -1.38 | 0.75 |
| | SP3 | 6.44 | 86400 | -1.98 | 0.62 |
| | SP4 | 5.20 | 86400 | -1.36 | 2.70 |

Table 4.1: Comparison of RP solutions from CPLEX and PH

Table 4.1 shows a detailed comparison of CPLEX and PH on T3 and T5; CPLEX is unable to solve T10, R2, and R3 with a reasonable optimality gap. We report, for each combination of instance set (Column 1) and item spread class (Column 2), the average optimality gaps Δ^{CP} of CPLEX (Column 3), the average running times

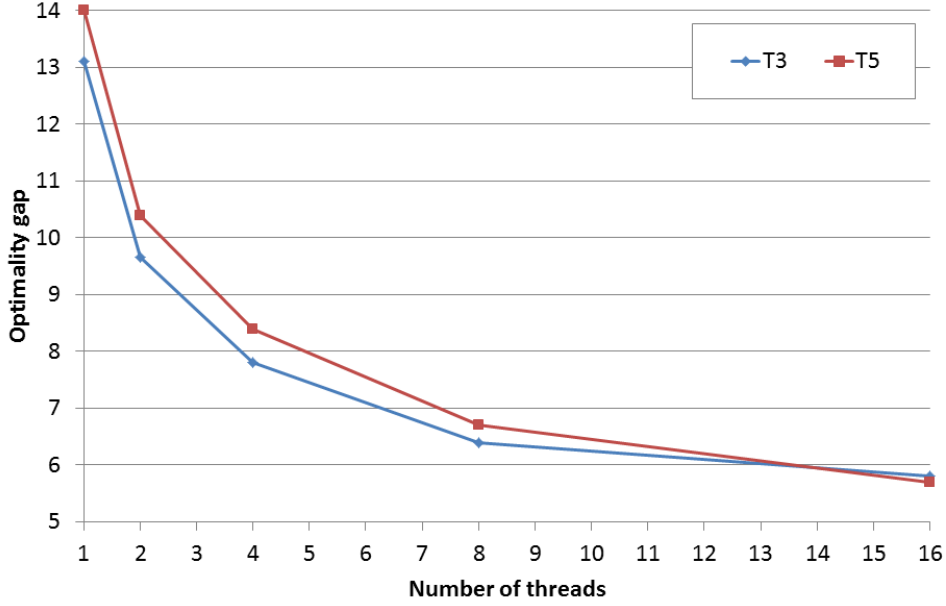


Figure 4.1: Average relative optimality gap (%) with respect to the number of parallel threads used by CPLEX

t^{CP} of CPLEX (Column 4), the average running times t^{PH} of PH (Column 6), and, in Column 5, the average gaps Δ_{UB}^{PH} of PH computed as

$$\Delta_{UB}^{PH} = \frac{UB^{PH} - UB^{CP}}{UB^{CP}} \quad (4.53)$$

where UB^{PH} is the PH objective value. Note that the value of Δ_{UB}^{PH} may be positive or negative. When Δ_{UB}^{PH} is negative, the PH solution is better than the CPLEX solution.

As stated before, CPLEX cannot solve **SVCSBPP** to optimality with a reasonable time limit for any instances considered. The average optimality gap after 24 hours is always greater than 5%, with a maximum value of 6.44%. This is mainly due to the number of variables in the two-stage models and the presence of equivalent solutions. Reducing the number of scenarios and thus the number of variables does not guarantee in-sample stability, so the capacity plan cannot be used in practice. The PH algorithm is accurate and effective on all T3 and T5 instances. It always converges quickly to better solutions than those obtained by CPLEX. The

gap Δ_{UB}^{PH} is negative for all instances and, on average, the improvement in the solutions is between 0.86% and 1.98%. PH always converges to a consensual solution in less than 3 s. This performance is directly related to the efficiency of the heuristic solver for the VCSBPP, which is able to solve deterministic subproblems in negligible computational times.

Despite the use of concurrent computation, CPLEX is not able to compute a good feasible solution for T10, R2, and R3 in a reasonable computing time (e.g., 24 h). These instances can easily involve more than 10 million variables. They require a huge amount of memory for the branch and bound (e.g., more than 64 GB of memory occupied just after 2 h of computation). For these instances, the memory becomes the bottleneck, and CPLEX cannot run for the assigned time. Thus, to validate the PH algorithm on the remaining instances, we compare the PH solutions with those obtained by solving the LB (4.10)–(4.13). We recall that the LB does not consider item-to-bin assignments, which drastically reduces the number of variables in the model. The reduced model can be solved to optimality by CPLEX with a limited computational effort. The time depends on the number of bins involved, so CPLEX requires only a few seconds to solve LB for sets T3, T5, R2, and R3, and it needs at most 120 s for set T10.

Table 4.2 reports, for each instance set (Column 1) and item spread class (Column 2), the percentage gap Δ_{LB}^{PH} between the objective functions obtained by PH and LB (Column 3), and the average and maximum percentage increase in the objective function (Columns 4 and 5) resulting from the use of the LB solutions for the capacity planning. Except for set R2, for which the gap is greater than 4%, on average, the gaps are smaller than 2%. These results demonstrate the accuracy of the PH algorithm. We can further investigate its accuracy by evaluating the quality of the LB solutions. We do this by fixing the capacity-planning decisions defined by LB in the original problem and computing the recourse cost. For T3, T5, and T10, the average increase in the objective function is smaller than 0.25%, and in some cases there is no increase. When C^{LB} is zero, the capacity plans defined by LB and PH match exactly. However, for R2 and R3, C^{LB} becomes significant (e.g., 2.77% and 1.23%, respectively), showing that LB struggles to correctly identify a tight bound when the number of bin types is limited. This is further underlined by the maximum increase, C_{max}^{LB} . The capacity plan may cost 25% more than expected.

In fact, when the choice is limited to a few types of bins (e.g., R2 includes only two types), an error in the selection can involve a high recourse cost. For this reason, although on average the LB decisions are similar to or the same as those of PH, LB cannot be used for real applications such as those represented by R2 and R3.

| Set | Spread | Δ_{LB}^{PH} | C^{LB} | C_{max}^{LB} |
|-----|--------|--------------------|----------|----------------|
| T3 | SP1 | 0.88 | 0.21 | 1.45 |
| | SP2 | 0.72 | 0.22 | 1.79 |
| | SP3 | 0.68 | 0.05 | 0.45 |
| | SP4 | 1.21 | 0.00 | 0.00 |
| T5 | SP1 | 1.57 | 0.09 | 0.63 |
| | SP2 | 1.45 | 0.00 | 0.00 |
| | SP3 | 1.03 | 0.06 | 0.47 |
| | SP4 | 2.03 | 0.10 | 0.77 |
| T10 | SP1 | 0.81 | 0.04 | 0.31 |
| | SP2 | 0.79 | 0.03 | 0.18 |
| | SP3 | 0.76 | 0.02 | 0.19 |
| | SP4 | 0.86 | 0.01 | 0.06 |
| R2 | - | 4.3 | 2.77 | 24.9 |
| R3 | - | 0.71 | 1.23 | 8.5 |

Table 4.2: Comparison of LB and PH solutions

We conclude our validation of the PH algorithm by analyzing the scalability of the parallel implementation up to 16 threads. The computational times t^{PH} for each instance set (Column 1) and item spread class (Column 2) are reported in Table 4.3, and the speed-up, defined as the ratio of the sequential running time to the parallel running time, is shown in Figure 4.2.

The computational times are negligible for T3 and T5 and become significant for the larger sets. However, the maximum sequential time is of the order of 600 s, several orders smaller than the time required by CPLEX. The computational effort increases with the number of items and especially with the number of bin types. Indeed, despite the large number of items involved in the realistic instances of R2 and R3, the computational times for these instances are shorter than those for T10. The latter has 10 bin types and at most 500 items, whereas R2 and R3 have only 2 and 3 bin types, and at most 11000 and 4000 items.

| Set | Spread | t_1^{PH} | t_2^{PH} | t_4^{PH} | t_8^{PH} | t_{16}^{PH} |
|-----|--------|------------|------------|------------|------------|---------------|
| T3 | SP1 | 1.21 | 0.87 | 0.73 | 0.50 | 0.30 |
| | SP2 | 2.36 | 1.73 | 1.41 | 0.97 | 0.59 |
| | SP3 | 0.41 | 0.29 | 0.25 | 0.17 | 0.10 |
| | SP4 | 0.49 | 0.35 | 0.29 | 0.20 | 0.12 |
| T5 | SP1 | 2.08 | 1.48 | 1.15 | 0.83 | 0.50 |
| | SP2 | 0.75 | 0.53 | 0.42 | 0.30 | 0.18 |
| | SP3 | 0.62 | 0.44 | 0.35 | 0.25 | 0.15 |
| | SP4 | 2.70 | 1.93 | 1.50 | 1.08 | 0.65 |
| T10 | SP1 | 625.17 | 350.10 | 187.08 | 89.02 | 45.62 |
| | SP2 | 580.59 | 325.52 | 174.53 | 85.05 | 42.56 |
| | SP3 | 432.48 | 241.24 | 122.87 | 74.25 | 31.52 |
| | SP4 | 524.87 | 293.35 | 155.24 | 75.85 | 38.47 |
| R2 | - | 366.76 | 201.98 | 99.70 | 52.20 | 26.38 |
| R3 | - | 521.63 | 280.98 | 137.79 | 72.91 | 36.99 |

Table 4.3: Average computational times for PH algorithm with respect to parallel threads

The scaling of the parallel implementation is quite linear. This is guaranteed by the performance of the heuristic, which solves every problem in a similar computational time. These times ensure that the slaves have balanced loads and lead to a linear speedup. For T3 and T5, the best speedup is about four, obtained with 16 parallel threads. This is due to the negligible computational times for the sequential execution of these instances. For the larger instances, the average speedup is below the ideal value: it is 80% of the latter in the worst case. The loss in performance is related to the functions that are inherently sequential in the algorithm. The master calculates the overall solution, checks the termination criteria, and updates the penalties, while the slaves wait for the assignment of new subproblems.

4.5.3 Impact of uncertainty

In this section, we show the benefit of using the two-stage model with recourse for the **SVCSBPP**. We do this by considering the most important measures in stochastic programming: the expected value of perfect information (*EVPI*) and the value of the stochastic solution (*VSS*).

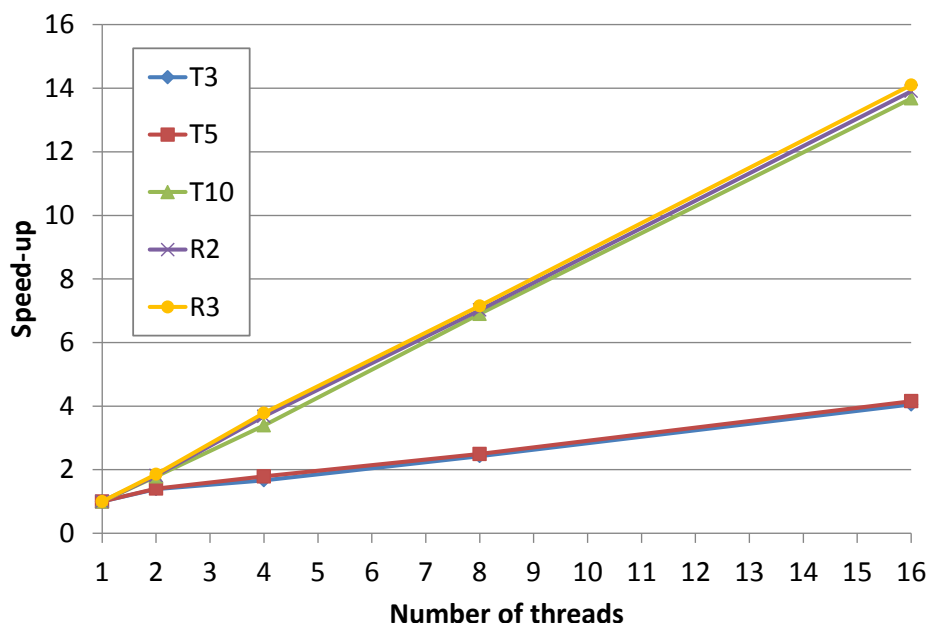


Figure 4.2: Speed-up of PH algorithm for different numbers of parallel threads

The $EVPI$ is defined by the difference between the objective values of the stochastic solutions and the wait-and-see solutions when the realizations of all the random parameters are known at the first stage:

$$EVPI := RP - WS. \quad (4.54)$$

The VSS indicates the expected gain from solving the stochastic model rather than its deterministic counterpart in which the random parameters are replaced with their expected values:

$$VSS := EEV - RP \quad (4.55)$$

where EEV denotes the solution value of the RP model, where the first-stage decision variables are fixed at the optimal values obtained by using the expected values of the coefficients.

We present the results in an aggregated form in Table 4.4. The results are grouped for instances in the same set and spread class. Table 4.4 reports, for each combination of instance set (Column 1) and item spread class (Column 2),

the average and maximum percentage $EVPI$ (Columns 3 and 4), computed as $EVPI/RP \cdot 100$, and the average and maximum percentage VSS (Columns 5 and 6), computed as $VSS/RP \cdot 100$.

| Set | Spread | $EVPI$ [%] | $EVPI_{max}$ [%] | VSS [%] | VSS_{max} [%] |
|-----|--------|------------|------------------|-----------|-----------------|
| T3 | SP1 | 5.66 | 6.97 | 5.63 | 6.89 |
| | SP2 | 4.35 | 7.08 | 5.28 | 7.66 |
| | SP3 | 5.42 | 7.10 | 4.21 | 6.94 |
| | SP4 | 6.18 | 6.67 | 8.10 | 11.25 |
| T5 | SP1 | 5.72 | 7.01 | 5.98 | 6.92 |
| | SP2 | 6.28 | 7.29 | 4.18 | 6.07 |
| | SP3 | 5.51 | 7.38 | 4.47 | 5.76 |
| | SP4 | 5.35 | 8.10 | 12.34 | 14.17 |
| T10 | SP1 | 11.73 | 14.52 | 8.09 | 15.95 |
| | SP2 | 11.52 | 12.62 | 6.66 | 9.37 |
| | SP3 | 10.87 | 12.51 | 7.25 | 12.00 |
| | SP4 | 11.17 | 15.20 | 9.67 | 17.70 |
| R2 | - | 2.85 | 4.96 | 2.75 | 5.92 |
| R3 | - | 2.82 | 4.60 | 2.69 | 5.06 |

Table 4.4: $EVPI$ and VSS comparison

We first analyze T3, T5, and T10. The $EVPI$ percentage is around 10%, showing the benefit of having information about the future in advance. It increases with the problem dimension to a maximum value above 15%. The average and maximum values of the VSS increase as the size of the instance increases. The gap between the expected-value solution and the stochastic solution is significant for all sets considered. Even for small instances, the maximum VSS reaches 11%, and it shows the losses incurred by following the capacity plan of the deterministic solution. The most critical item spread class is SP4; this is the most representative class for the shipping of freight over long distances. SP4 represents the situation in which we have no information about the category distribution of the items to be shipped. Its maximum $EVPI$ is 15.20% and its maximum VSS is always greater than 7% and reaches 17.70%.

We now discuss the realistic instances of R2 and R3. These applications are usually characterized by a limited number of vehicle (bin) types and limited variability

in the items, which are quite small. For these reasons, both $EVPI$ and VSS are smaller than the corresponding values for the testing instances. The average values are below 3%, and the maximum values reach 5.92%. However, the maximum values are sufficiently large to justify the use of the stochastic approach.

We now consider to what extent the first-stage decisions differ. On average, the EV problem overestimates the demand to be loaded (the total volume of the items is larger than the actual volume) and the availability of extra bins (a larger set of bins is available for the recourse action). This can lead to two situations. First, EV may plan to use a set of bins that is not actually required for the set of scenarios considered. Although the capacity plan is more expensive (i.e., the cost increases by 17.70%), the solution is feasible and thus implementable. Second, EV may plan an insufficient capacity for a subset of scenarios in which the actual availability of bins is very limited. The capacity plan is infeasible for these scenarios. This is rare, occurring for only 2% of the instances. However, the results clearly show the need to explicitly consider uncertainty in capacity-planning applications.

4.5.4 Solution analysis

This analysis focuses on the structure of the capacity-planning solutions and in particular on the use of bins in the plan. Table 4.5 reports, for each combination of instance set (Column 1) and item spread class (Column 2), the average number of bin types $N_{\mathcal{T}}$ used in the capacity plan (Column 3), the percentage of the objective function value achieved in the first stage Obj_{FS} (Column 4), and the average percentage fill level of the bins at the two stages, f_{FS} and f_{SS} respectively (Columns 5 and 6).

Only a few bin types are used in the plan. The number is close to 1 for T3, R2, and R3, and it is between 2 and 3 for T5 and T10, which have five and ten bin types, respectively. Furthermore, the number of bins is not evenly distributed among the types. Almost all of the bins included in the capacity plan are the same type; only one or two bins are of different types. This is, in our opinion, a result compatible with the rules used in practice, i.e., the larger the number of bin types used, the higher the loading/unloading costs because it becomes impossible to use standardized loading schemes. Concerning the spread classes, it is interesting to

| Set | Spread | $N_{\mathcal{T}}$ | Obj_{FS} [%] | f_{FS} [%] | f_{SS} [%] |
|-----|--------|-------------------|----------------|--------------|--------------|
| T3 | SP1 | 1.4 | 80.59 | 91.94 | 86.28 |
| | SP2 | 1.2 | 78.10 | 93.25 | 87.27 |
| | SP3 | 1 | 81.91 | 94.47 | 79.47 |
| | SP4 | 1.3 | 86.02 | 95.68 | 85.78 |
| T5 | SP1 | 2 | 77.10 | 95.38 | 87.29 |
| | SP2 | 1.8 | 80.57 | 97.34 | 85.35 |
| | SP3 | 1.5 | 82.00 | 95.64 | 81.46 |
| | SP4 | 2.1 | 81.60 | 94.66 | 85.68 |
| T10 | SP1 | 2.5 | 80.99 | 94.76 | 83.32 |
| | SP2 | 2.1 | 79.88 | 94.23 | 84.35 |
| | SP3 | 2 | 78.90 | 95.58 | 84.62 |
| | SP4 | 2.3 | 81.63 | 95.17 | 82.49 |
| R2 | - | 1.1 | 94.59 | 96.23 | 71.57 |
| R3 | - | 1.5 | 88.12 | 99.26 | 89.45 |

Table 4.5: Structure of solutions

note that when the stochastic problem considers a demand with a high percentage of small items (classes SP1 and SP4), $N_{\mathcal{T}}$ is maximum. In fact, small items may be loaded in any bins (large items cannot be placed in bins with limited capacities), and they are usually used to fill near-empty bins. Thus, it becomes attractive for the model to mix bin types.

The percentage of the objective function achieved in advance is about 80%, reaching 90% for the realistic instances. This means that the tactical decisions must be conservative: the company should book sufficient capacity in advance to limit the adjustment necessary when the actual demand becomes known. Moreover, this indicates that 10%-20% of recourse is a good compromise between cost reduction and uncertainty management.

Finally, the fill level of the bins selected at the first (second) stage is greater than 90% (70%). This indicates the effectiveness of the capacity plan, which requires only targeted adjustments at the second stage.

4.5.5 Sensitivity analysis

We now analyze the changes necessary when contractual policies are imposed by the 3PL. We define two policies. The first policy (P1) imposes a minimum number of bins for each bin type τ included in the capacity plan. The second policy (P2) reduces the number of bins of each type, which requires us to combine different types of bins in the most efficient way.

Let $u^\tau = \|\mathcal{J}^\tau\|$ be the number of bins of type τ that can be selected for the capacity plan. We define the minimum number of bins to be $m \cdot u^\tau$, with $m = \{10\%, 20\%, 30\%\}$. The maximum availability is reduced to $m \cdot u^\tau$ with $m = \{0.50, 0.75, 0.80\}$. To impose these policies, we introduce additional constraints (4.56) for the first policy and (4.57) for the second policy:

$$\sum_{j \in \mathcal{J}^\tau} y_j^{\tau s} \geq m \cdot u^\tau \quad \forall \tau \in \mathcal{T}, \quad s \in \mathcal{S} \quad (4.56)$$

$$\sum_{j \in \mathcal{J}^\tau} y_j^{\tau s} \leq m \cdot u^\tau \quad \forall \tau \in \mathcal{T}, \quad s \in \mathcal{S}. \quad (4.57)$$

We use the PH algorithm and compare the new solutions with the original solutions. Tables 4.6 and 4.7 report the average results of the sensitivity analysis for each set of instances (Column 1), the original number of bin types used in plan $N_\mathcal{T}$ (Column 2), the policy type (Column 3), and the associated value of factor m (Column 4). For each policy, we show the changes in the solutions with respect to the number of bin types used ($N'_\mathcal{T}$, Column 5) and the extra cost for the policy (Δ'_{Obj} , Column 6).

We can immediately note that the instances with a limited availability of bin types (T3, R2, and R3) are more sensitive to policy P1. For these sets P1 penalizes combinations of bin types (e.g., for $m = 30\%$ all the problems in R2 and R3 use a single type), causing an increase in the cost of the capacity plan or the recourse action, which purchases more bins. This cost increases as the bin types available decrease. It is approximately 2% for instances with 3 types and reaches 3.1% for R2. Similarly, P1 tends to penalize the use of bin types for T5 and T10, but the impact on the objective function is limited. The wide choice of bins that characterizes these instances allows them to efficiently implement the policy, reducing the cost

| Set | $N_{\mathcal{T}}$ | Policy | m (%) | $N'_{\mathcal{T}}$ | Δ'_{Obj} |
|-----|-------------------|--------|---------|--------------------|-----------------|
| T3 | 1.2 | P1 | 10 | 1.2 | 0.90 |
| | | P1 | 20 | 1.2 | 1.46 |
| | | P1 | 30 | 1.2 | 2.35 |
| | | P2 | 50 | 1.2 | 0.83 |
| | | P2 | 75 | 1.2 | 0.00 |
| | | P2 | 90 | 1.2 | 0.00 |
| T5 | 1.8 | P1 | 10 | 1.8 | 0.18 |
| | | P1 | 20 | 1.8 | 0.21 |
| | | P1 | 30 | 1.8 | 0.34 |
| | | P2 | 50 | 2.0 | 0.86 |
| | | P2 | 75 | 1.8 | 0.00 |
| | | P2 | 90 | 1.8 | 0.00 |
| T10 | 2.2 | P1 | 10 | 2.0 | 0.14 |
| | | P1 | 20 | 1.8 | 0.25 |
| | | P1 | 30 | 1.5 | 0.37 |
| | | P2 | 50 | 2.2 | 0.00 |
| | | P2 | 75 | 2.2 | 0.00 |
| | | P2 | 90 | 2.2 | 0.00 |

Table 4.6: Sensitivity of capacity-planning solutions

| Set | $N_{\mathcal{T}}$ | Policy | m | $N'_{\mathcal{T}}$ | Δ'_{Obj} |
|-----|-------------------|--------|-----|--------------------|-----------------|
| R2 | 1.1 | P1 | 10 | 1.0 | 3.1 |
| | | P1 | 20 | 1.0 | 3.1 |
| | | P1 | 30 | 1.0 | 3.1 |
| | | P2 | 50 | 2.0 | 9.67 |
| | | P2 | 75 | 1.1 | 0.00 |
| | | P2 | 90 | 1.1 | 0.00 |
| R3 | 1.5 | P1 | 10 | 1.3 | 0.69 |
| | | P1 | 20 | 1.2 | 1.01 |
| | | P1 | 30 | 1.0 | 1.38 |
| | | P2 | 50 | 2.0 | 5.25 |
| | | P2 | 75 | 1.6 | 0.33 |
| | | P2 | 90 | 1.5 | 0.00 |

Table 4.7: Sensitivity of capacity-planning solutions

associated with the second stage.

P2 reduces the set of bins available in advance. Thus, it is necessary to combine the available bins for each type (replacing the large bins that cannot be selected with more small bins). The policy becomes significant only in extreme cases in which the original availability of the bins is halved. Because of the high number of items to be loaded in R2 and R3, the increase in the cost may be higher than 9% and 5%, respectively. The solutions will include more bins, resulting in a higher fixed cost. In the worst case, where the plan does not meet the demand, the recourse selects extra capacity at a premium cost.

4.5.6 Effect of the capacity planning

This section focuses on capacity-planning decisions over a long period (e.g., 1 year). The goal is to evaluate the robustness and reliability of the decisions when the demand differs from the estimation (e.g., average demand larger than demand considered in the set of scenarios \mathcal{S}).

We use a Monte Carlo simulation on a subset (10%) of the testing and realistic instances. The simulation considers four demand scenarios:

1. **High demand** (Figure 4.3a): The average demand is higher than the demand of the scenarios in \mathcal{S} . This case study measures the cost of overusing the spot market to obtain the extra capacity.
2. **Low demand** (Figure 4.3b): The average demand is lower than the demand of the scenarios in \mathcal{S} . This case study measures the cost of not using part of the planned capacity.
3. **Economic crisis** (Figure 4.3c): The demand initially decreases rapidly to a value below the estimated demand. It then stabilizes below the average demand of the scenarios in \mathcal{S} .
4. **Economic recovery** (Figure 4.3d): The demand is initially below the average estimated demand, but it increases rapidly to a peak above the maximum estimation. It then stabilizes.

The overall process of the simulation can be described as follows:

- Given an instance and an associated set of scenarios \mathcal{S} , find the capacity-planning solution using the PH algorithm.
- Create a new set of scenarios \mathcal{S}' as follows. Extract from \mathcal{S} the maximum and minimum demand and compute the difference Δ between them. Select the demand scenario and find $K\Delta$, where $K = \{10, 20, 30, 40, 50\}\%$ is an offset factor that defines how far the demand distribution is from that estimated in set \mathcal{S} (see Figure 4.3). Finally, define 365 scenarios respecting the trends of the demand and the characteristics of the instance (number of bin types and spread class of items).
- For each scenario $s \in \mathcal{S}'$, build a VCSBPP with the bin set formed by the bins included in the capacity-planning solution and the bins at premium cost defined in the scenario, and the demand in terms of the items to be loaded. The resulting VCSBPP is then solved by the heuristic solver.
- Given the optimal solution, compute the expected value of the total cost and statistics related to the use of the capacity plan and the extra capacity.

To obtain the most reliable results, we repeat this process ten times and average the results.

Figures 4.4 and 4.5 show the results of the Monte Carlo simulation for a testing instance from T10 and a realistic instance from R3. The figures report, for each demand scenario, the percentage usage of the capacity plan defined as the ratio of the number of bins used in the solution to the total number of bins available in the plan (Figures 4.4a and 4.5a), the percentage of extra bins in the solution defined as the ratio of the number of extra bins to the total number of bins in the solution (Figures 4.4b and 4.5b), and the percentage of the objective function associated with the capacity plan (Figures 4.4c and 4.5c) for the offset factor K .

Despite the great difference between T10 and R3 in terms of the number of items and thus of the total capacity to be loaded, the trends are similar. This indicates that capacity-planning decisions are valid for widely varying problem settings. We now analyze the individual results of the simulation. Increasing the offset factor K increases or decreases the percentage of the capacity actually used according to the demand trend in different demand scenarios. In fact, for the LOW and CRISIS

scenarios, the use of the planned bins decreases, in the worst case, to 77% for T10 and 96% for R3. In contrast, for the HIGH and GROWTH scenarios, the number of bins actually used in the solution is always greater than 90% and close to 100% for realistic instances.

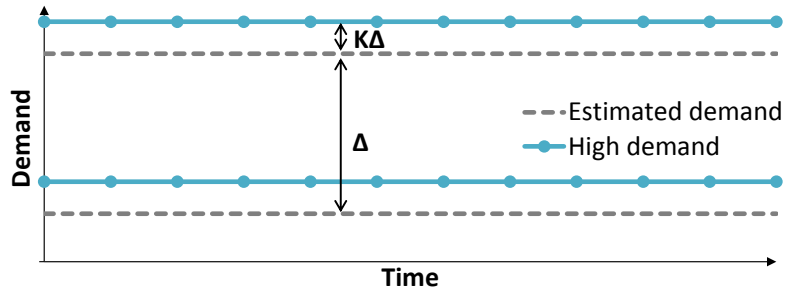
The results for the extra capacity are similar. The reduction in the capacity used corresponds to a decrease in the need for extra capacity. This is particularly evident for T10, for which no extra capacity is needed when the demand is minimum (LOW scenario and $K = 50\%$). Similarly, R3 needs only 5% of extra capacity, which in this case corresponds to less than 2 bins. Conversely, when the demand is underestimated, it may be necessary to purchase additional capacity: more than 20% for the HIGH scenario (about 10 bins for R3 and 7 bins for T10) and between 10% and 15% for the GROWTH scenario (about 7 bins for R3 and 4 for T10). These results show the limited use of the spot market and further demonstrate the validity of the solutions found.

The limited use of the spot market is also indicated by the objective function for the capacity planning. It corresponds to the total fixed cost of the bins that are selected in advance. This cost increases as demand decreases in the demand scenarios. Indeed, when the demand is overestimated (scenarios LOW and CRISIS) the capacity plan is generally adequate, and only a few adjustments are required for a small set of scenarios. This means that most of the cost is due to the capacity plan. For the instances considered, the range is 75% to 100%, values that indicate the robustness of the solution even when the estimation errors are considerable.

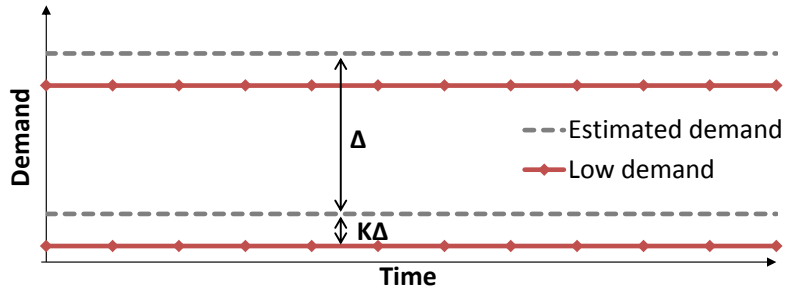
The fill level of the planned bins and the extra bins is always between 99% and 80% even for the LOW and CRISIS scenarios. This result demonstrates the robustness of the capacity-planning decisions that need targeted adjustments in the spot market to meet the demand. Other instances give similar results, and thus the analysis is still valid.

Regarding the sensitivity to the various demand scenarios considered, we note that the GROWTH scenario has a limited impact on the results with variations of a few percentage points, while the most significant scenarios are LOW and HIGH. Moreover, it is interesting that the CRISIS and LOW scenarios exchange roles for values of K greater than 30%. For small values of K , CRISIS is the worst-case scenario because of the decrease in the demand. For high values of K , LOW has

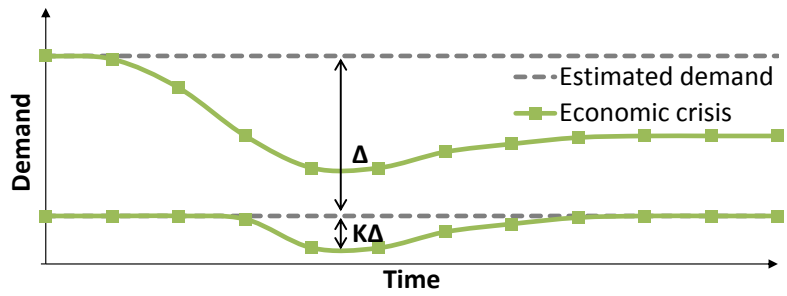
the most significant impact because of the error in the estimation of the demand, which is constant over time (for each scenario in \mathcal{S}').



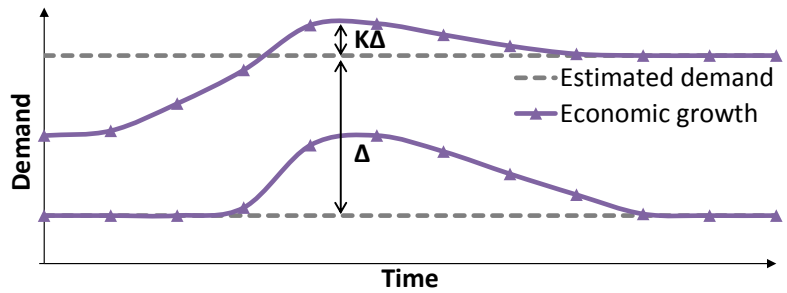
(a) Distribution of high-demand scenario



(b) Distribution of low-demand scenario

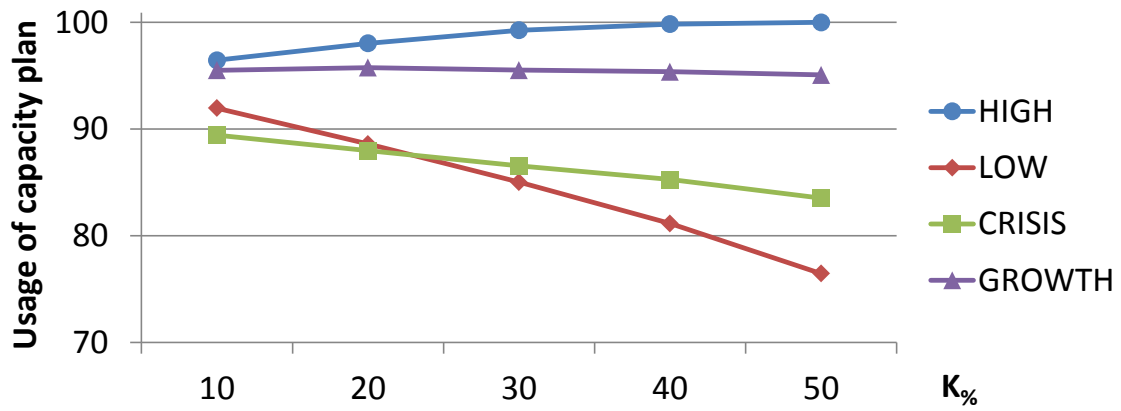


(c) Distribution of economic-crisis scenario

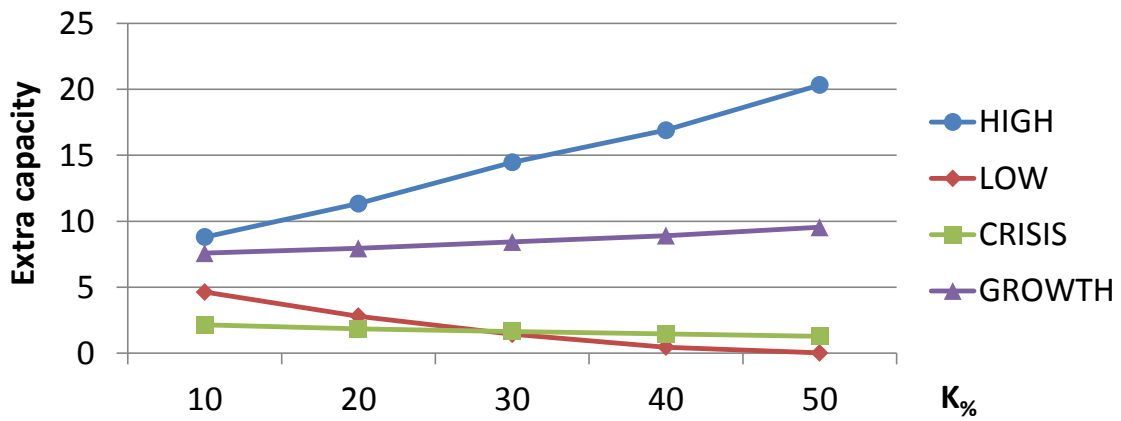


(d) Distribution of economic-growth scenario

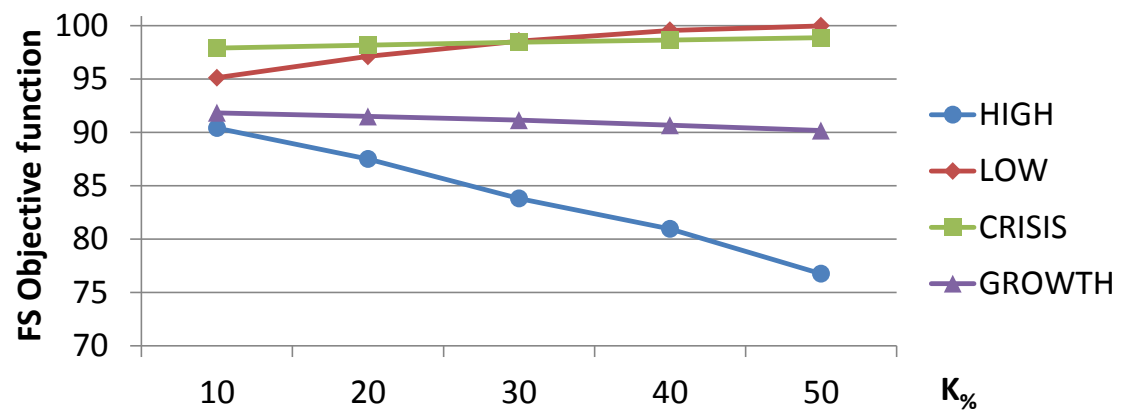
Figure 4.3: Demand trends for the Monte Carlo simulation



(a) Percentage usage of bins in capacity plan

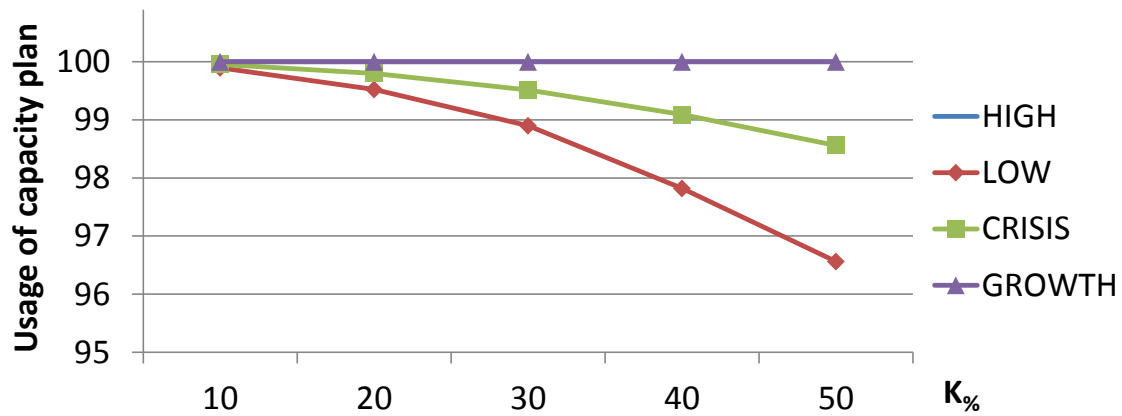


(b) Percentage of extra bins in the solution

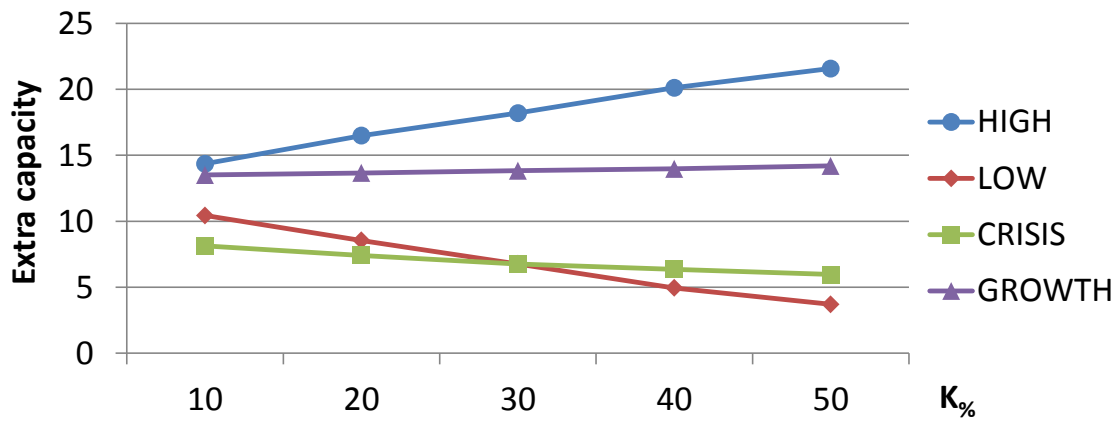


(c) Objective function associated with the capacity plan

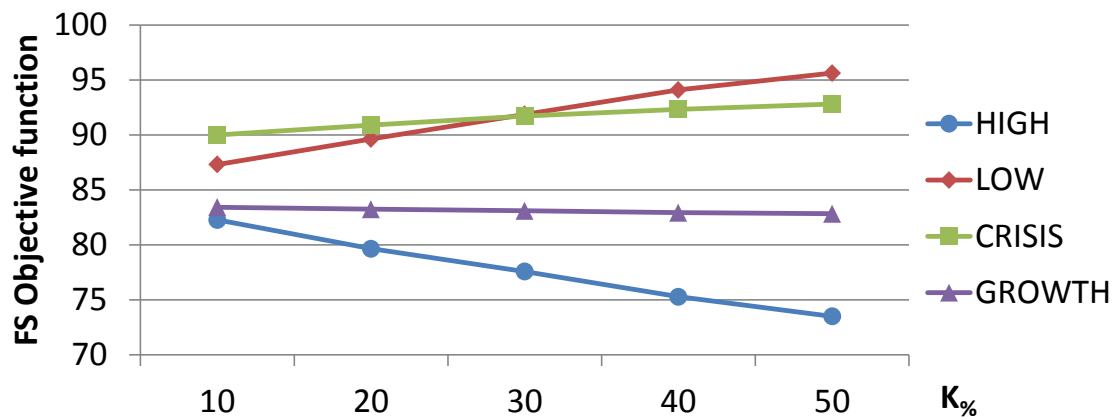
Figure 4.4: Monte Carlo simulation for a testing instance



(a) Percentage usage of bins in capacity plan



(b) Percentage of extra bins in the solution



(c) Objective function associated with the capacity plan

Figure 4.5: Monte Carlo simulation for a realistic instance

Chapter 5

The Multi-Handler Knapsack Problem under Uncertainty

This chapter introduces the $\text{MHKP}_{\mathbf{u}}$ together with the stochastic formulation. From this formulation, a deterministic approximation and a two-stage stochastic model are derived. We show that, under a mild hypothesis on the unknown probability distribution, the deterministic approximation becomes a knapsack problem where the total expected profit of the loaded items is proportional to the logarithm of the total accessibility of those items to the set of handlers. The accuracy of such a deterministic approximation is tested against the two-stage model. Very promising results are obtained on a large set of instances in terms of both accuracy of solutions and computational effort.

This chapter is organized as follows. In Section 5.1 the description of the problem context and the literature review are reported. The stochastic model of the $\text{MHKP}_{\mathbf{u}}$ is described in Section 5.2 Section 5.3 is devoted to presenting the deterministic approximation of the $\text{MHKP}_{\mathbf{u}}$, while in Section 5.4 its two-stage program with fixed recourse is given. Finally, in Section 5.5 the deterministic approximation and the two-stage program with fixed recourse are tested and compared on a set of newly introduced instances.

5.1 Problem description and literature review

A large number of real-life situations can be satisfactorily modeled as a $\text{MHKP}_{\mathbf{u}}$, e.g. in financial and resource allocation. The general idea is to think of the capacity of the knapsack as the available amount of a resource (i.e. budget) and the items as activities to which this resource can be allocated (i.e. shares). Moreover, these items present profits which are random variables. The $\text{MHKP}_{\mathbf{u}}$ may also appear as a subproblem of larger optimization problems. A specific application of the $\text{MHKP}_{\mathbf{u}}$ can be found in the automotive sector (Tadei et al., 2002). There the delivery of cars from manufacturers to dealers is not managed by the manufacturers themselves, but is delegated to specialized companies. These companies manage both the finishing operations on the cars (e.g. removal of the protective wax, installation of specific accessories, etc.) and the logistics operations linked to delivery to the dealers. In order to have a more flexible structure, the fleet of auto-carriers used to deliver the cars is only partially owned by each company, while a substantial part of the deliveries is sub-contracted to micro-companies with highly variable random costs. Moreover, the auto-carriers have different capacities due to the presence of specific technical features. From the point of view of the cars that must be delivered, the net profit for the company is affected by different factors, including delays in the finishing operations, additional costs due to violations of the negotiated deadlines or additional transportation costs.

Another example of real-world applications of the $\text{MHKP}_{\mathbf{u}}$ comes from trans-continental naval shipping operations, where freight transportation from eastern ports to Europe and North America is managed by specialized companies. The competition between the transportation companies, as well as the possibility of managing the port cranes by different operators, force the companies to consider both the profit given by the shipped items and the additional costs due to the logistics operations.

The problem can be also seen as a relaxation of container loading problems where the capacity of the given containers are collapsed into one single container. This leads to an approximation of the real problem suitable in strategic and tactical planning, where the stochastic nature of the profits is more relevant than the actual loading of the items. Moreover, it is required to obtain accurate solutions within a

limited computational effort in order to explore multiple scenarios of the underlying business model.

Other applications of the $\text{MHKP}_{\mathbf{u}}$ come from the domain of Smart City and City Logistics, and particularly in the last mile delivery. In Chapter 3, we described the managements of consolidation operations in the satellites. A similar problem is present in yard management, where the profit oscillations are given by the operations done by workers working for yard management companies, different in skills and reliability.

In general, the $\text{MHKP}_{\mathbf{u}}$ arises in logistics and production scheduling applications, where a single item can be managed by several handlers (third-party logistics providers or sub-contractors), whose costs affect the net profit of the item itself. The large number of possible handler cost scenarios and the difficulty to measure the associated handler costs suggest the representation of these net profits as stochastic variables with unknown probability distribution.

5.1.1 Literature review

While different variants of the stochastic knapsack problem are present in the literature, the $\text{MHKP}_{\mathbf{u}}$ is absent. For this reason, we will consider some relevant literature on similar problems, highlighting the main differences with the problem faced in this paper.

A first group of studies consider deterministic profits and random volumes, with the goal of maximizing the total expected value of selected items, while ensuring that the probability to satisfy the knapsack capacity is limited by some upper bounds. Usually, heavy assumptions on the distribution of the random volumes are considered (e.g. (Kleinberg et al., 2000; Goel and Indyk, 1999) where item volumes have a Bernoulli distribution, and (Merzifonluoglu et al., 2012; Cohn and Barnhart, 1998) where the distribution is a Normal one). These assumptions on the distributions heavily limit the possibility to extend the results to other variants of the problem.

A second group of studies deals with random profits and the goal to assign a set of items to the knapsack in order to maximize the probability of achieving some target total value. They are usually more related to financial and economic issues than to the impact of the operations on the final revenue (Lisser and Lopez, 2010;

Henig, 1990; Steinberg and Parks, 1979). Unfortunately, these problems differ from $\text{MHKP}_{\mathbf{u}}$ because they consider the random profit associated only to the item, while in $\text{MHKP}_{\mathbf{u}}$ the randomness is given by the interaction between the item and the handler managing the loading/unloading operations.

Finally, from a methodological point of view, the study most similar to the present paper is (Perboli et al., 2012), where the authors consider the stochastic version of the Generalized Bin Packing Problem, a recently introduced packing problem where, given a set of bins characterized by volume and cost and a set of items characterized by volume and profit (which also depends on bins), a subset of items is selected for loading into a subset of bins which maximizes the total net profit, while satisfying the volume and bin availability constraints (Baldi et al., 2012a). Similarly to $\text{MHKP}_{\mathbf{u}}$, the item profits are random variables and the probability distribution of these random variables is assumed to be unknown.

5.2 Problem formulation

In the $\text{MHKP}_{\mathbf{u}}$ the item profits are random variables. They are composed by a deterministic profit plus a random term, which represents the profit oscillation due to the handling costs occurred by the different handlers for preparing items for loading. In practice, such profit oscillations randomly depend on the handling scenarios adopted by the handlers for preparing items for loading and are actually very difficult to be measured. This implies that the probability distribution of these random terms must be assumed as unknown.

Let it be

- I : set of items
- J : set of handlers
- L : set of handling scenarios for loading items into the knapsack
- p_i : non-negative deterministic profit of item i
- p_{ij} : non-negative deterministic profit of item i when loaded by handler j

- $\tilde{\theta}^{jl}$: random profit oscillation of any item when it is loaded by handler j under scenario $l \in L$
- $\tilde{p}_{ij}(\tilde{\theta}^{jl}) = p_{ij} + \tilde{\theta}^{jl}$: random profit of item i when loaded by handler j under scenario l
- y_i : boolean variable equal to 1 if item i is loaded, 0 otherwise
- x_{ij} : percentage of item i handled by handler j
- w_i : volume of item i
- W : knapsack capacity.

The **MHKP_u** is formulated as follows

$$\max_{\{y,x\}} \sum_{i \in I} p_i y_i + \mathbb{E}_{\{\tilde{\theta}^{jl}\}} \left[\sum_{i \in I} \sum_{j \in J} \sum_{l \in L} \tilde{p}_{ij}(\tilde{\theta}^{jl}) x_{ij} \right] \quad (5.1)$$

subject to

$$\sum_{i \in I} w_i y_i \leq W \quad (5.2)$$

$$\sum_{j \in J} x_{ij} = y_i \quad i \in I \quad (5.3)$$

$$y_i \in \{0, 1\} \quad i \in I \quad (5.4)$$

$$x_{ij} \geq 0 \quad i \in I, \quad j \in J. \quad (5.5)$$

The objective function (5.1) expresses the maximization of the profit of the items loaded into the knapsack plus the expected value of the handling profit; constraint (5.2) ensures that the capacity of the knapsack is not exceeded; constraints (5.3) guarantee that any item is completely processed by some handlers only if it is loaded. Finally, (5.4)-(5.5) are the integrality and non-negativity constraints, respectively.

5.3 The deterministic approximation

Let us assume that $\tilde{\theta}^{jl}$ are independent and identically distributed (i.i.d.) random variables with a common and unknown probability distribution

$$F(x) = Pr\{\tilde{\theta}^{jl} \leq x\}. \quad (5.6)$$

Let us define with $\tilde{\theta}^j$ the maximum of the random profit oscillations $\tilde{\theta}^{jl}$ for handler j among the alternative scenarios $l \in L$

$$\tilde{\theta}^j = \max_{l \in L} \tilde{\theta}^{jl} \quad j \in J. \quad (5.7)$$

Because $F(x)$ is unknown, $\tilde{\theta}^j$ is still of course a random variable with unknown probability distribution given by

$$B_j(x) = Pr\{\tilde{\theta}^j \leq x\} \quad j \in J. \quad (5.8)$$

As, for any handler j , $\tilde{\theta}^j \leq x \iff \tilde{\theta}^{jl} \leq x$, $l \in L$ and $\tilde{\theta}^{jl}$ are independent, using (5.6) one gets

$$B_j(x) = \prod_{l \in L} Pr\{\tilde{\theta}^{jl} \leq x\} = \prod_{l \in L} F(x) = [F(x)]^{|L|} \quad j \in J. \quad (5.9)$$

We assume that the knapsack loading is efficiency-based so that, for any item i and any handler j , among the alternative scenarios $l \in L$ the one which maximizes the random profit $\tilde{p}_{ij}(\tilde{\theta}^{jl})$ will be selected. This does not mean that, in the stochastic problem, we select the model scenario which maximizes the profit, but that, when the actual profits become known (e.g. in day-by-day operations, the profits of a given day), the choice among the different alternatives is done by taking the most profitable one.

Then, the random profit of item i when it is loaded (i.e. $y_i = 1$) by handler j becomes

$$\tilde{p}_{ij}(\tilde{\theta}^j) = \max_{l \in L} \tilde{p}_{ij}(\tilde{\theta}^{jl}) = p_{ij} + \max_{l \in L} \tilde{\theta}^{jl} = p_{ij} + \tilde{\theta}^j \quad i \in I : y_i = 1, j \in J. \quad (5.10)$$

The maximum profit oscillation $\tilde{\theta}^j$ can be either positive or negative, but, in practice, its absolute value does not overcome the profit p_{ij} , so that $\tilde{p}_{ij}(\tilde{\theta}^j)$ is always non negative.

The expected maximum total profit of the loaded items is obtained by solving the following problem

$$\mathbb{E}_{\{\tilde{\theta}\}} \left[\max_{\{x\}} \sum_{i \in I: y_i=1} \sum_{j \in J} \tilde{p}_{ij}(\tilde{\theta}^j) x_{ij} \right] \quad (5.11)$$

$$\sum_{j \in J} x_{ij} = 1 \quad i \in I : y_i = 1 \quad (5.12)$$

$$x_{ij} \geq 0 \quad i \in I : y_i = 1, \quad j \in J. \quad (5.13)$$

The objective function (5.11) maximizes the expected total profit for the loaded items. Constraints (5.12) guarantee that each loaded item is completely processed by some handlers, while (5.13) are the non-negativity constraints.

For each item i , let us consider the handler $j = i^*$ (for the sake of simplicity, we assume it is unique), which gives the maximum random profit for loading the item.

The maximum random profit for loading item i then becomes

$$\tilde{p}_i(\tilde{\theta}^{i^*}) = \max_{j \in J} \tilde{p}_{ij}(\tilde{\theta}^j) \quad i \in I : y_i = 1 \quad (5.14)$$

and the optimal variables $\{x_{ij}\}$ are

$$x_{ij} = \begin{cases} 1, & \text{if } j = i^* \\ 0, & \text{otherwise} \end{cases}, \quad (5.15)$$

which satisfy (5.12) and (5.13).

Using (5.14), (5.15), and the linearity of the expected value operator \mathbb{E} , the objective function (5.11) becomes

$$\mathbb{E}_{\{\tilde{\theta}^*\}} \left[\sum_{i \in I: y_i=1} \tilde{p}_i(\tilde{\theta}^{i^*}) \right] = \sum_{i \in I: y_i=1} \mathbb{E}_{\{\tilde{\theta}^*\}} [\tilde{p}_i(\tilde{\theta}^{i^*})] = \sum_{i \in I: y_i=1} \hat{p}_i \quad (5.16)$$

where

$$\hat{p}_i = \mathbb{E}_{\{\tilde{\theta}^*\}} [\tilde{p}_i(\tilde{\theta}^{i*})] \quad i \in I : y_i = 1. \quad (5.17)$$

The **MHKP_u** (5.1)-(5.5) then becomes

$$\max_{\{y\}} \sum_{i \in I} (p_i + \hat{p}_i) y_i \quad (5.18)$$

$$\sum_{i \in I} w_i y_i \leq W \quad (5.19)$$

$$y_i \in \{0,1\} \quad i \in I. \quad (5.20)$$

However, the calculation of \hat{p}_i in (5.18) requires to know the probability distribution of the maximum random profit for loading item i , i.e. $\tilde{p}_i(\tilde{\theta}^{i*})$ in (5.17), which will be derived in the next section.

By (5.10) and (5.14), let

$$G_i(x) = Pr \left\{ \tilde{p}_i(\tilde{\theta}^{i*}) \leq x \right\} = Pr \left\{ \max_{j \in J} [p_{ij} + \tilde{\theta}^j] \leq x \right\} \quad i \in I \quad (5.21)$$

be the probability distribution of the maximum random profit for loading item i .

As, for any item i , $\max_{j \in J} [p_{ij} + \tilde{\theta}^j] \leq x \iff [p_{ij} + \tilde{\theta}^j] \leq x, \quad j \in J$, and the random variables $\tilde{\theta}^j$ are independent (because $\tilde{\theta}^{jl}$ are independent), due to (5.8) and (5.9), $G_i\{x\}$ in (5.21) becomes a function of the total number $|L|$ of handling scenarios for loading as follows

$$\begin{aligned} G_i(x, |L|) &= Pr \left\{ \max_{j \in J} [p_{ij} + \tilde{\theta}^j] \leq x \right\} = \prod_{j \in J} Pr \left\{ [p_{ij} + \tilde{\theta}^j] \leq x \right\} \\ &= \prod_{j \in J} Pr \left\{ \tilde{\theta}^j \leq x - p_{ij} \right\} = \prod_{j \in J} B_j(x - p_{ij}) \\ &= \prod_{j \in J} [F(x - p_{ij})]^{|L|} \quad i \in I. \end{aligned} \quad (5.22)$$

First, let us consider the following aspect: the optimal solution of problem (5.11)-(5.13) does not change if any arbitrary constant is added or subtracted to the random variables $\tilde{\theta}^j$.

Let us choose this constant as the root a of the equation

$$1 - F(a) = 1/|L|. \quad (5.23)$$

Let us assume that $|L|$ is large enough to use the asymptotic approximation $\lim_{|L| \rightarrow +\infty} G_i(x, |L|)$ as a good approximation of $G_i(x)$, i.e.

$$G_i(x) = \lim_{|L| \rightarrow +\infty} G_i(x, |L|) \quad i \in I. \quad (5.24)$$

The calculation of the limit in (5.24) would require to know the probability distribution $F(\cdot)$ in (5.6), which is still unknown. From [Perboli et al. \(2012\)](#), we know that under a mild assumption on the shape of the unknown probability distribution $F(\cdot)$ (i.e. it is asymptotically exponential in its right tail), the limit in (5.24) tends towards the following Gumbel ([Gumbel, 1958](#)) probability distribution

$$G_i(x) = \lim_{|L| \rightarrow +\infty} G_i(x, |L|) = \exp(-A_i e^{-\beta x}) \quad i \in I \quad (5.25)$$

where $\beta > 0$ is a parameter to be calibrated and

$$A_i = \sum_{j \in J} e^{\beta p_{ij}} \quad i \in I \quad (5.26)$$

is the *accessibility*, in the sense of [Hansen \(1959\)](#), of item i to the set of handlers.

The accessibility in the sense of Hansen is defined as the potential of opportunities for interaction and is a measure of the intensity of the possibility of interaction. Here the interaction is between item and handlers. (5.26) shows that the accessibility of an item to the set of handlers is proportional to a function of profits associated to the different handlers.

Using the probability distribution $G_i(x)$ given by (5.25), after some manipulations, \hat{p}_i in (5.17) becomes

$$\hat{p}_i = \int_{-\infty}^{+\infty} x dG_i(x) = \int_{-\infty}^{+\infty} x \exp(-A_i e^{-\beta x}) A_i e^{-\beta x} \beta dx = 1/\beta (\ln A_i + \gamma) \quad i \in I \quad (5.27)$$

where $\gamma \simeq 0.5772$ is the Euler constant.

By (5.27), the **MHKP_u** (5.18)-(5.20) becomes

$$\max_{\{y\}} \sum_{i \in I} p_i y_i + \frac{1}{\beta} \sum_{i \in I} y_i \ln A_i + \frac{\gamma}{\beta} \sum_{i \in I} y_i =$$

$$\begin{aligned}
 &= \max_{\{y\}} \sum_{i \in I} p_i y_i + \frac{1}{\beta} \ln \prod_{i \in I} A_i^{y_i} + \frac{\gamma}{\beta} \sum_{i \in I} y_i = \\
 &= \max_{\{y\}} \sum_{i \in I} p_i y_i + \frac{1}{\beta} \ln \Phi + \frac{\gamma}{\beta} \sum_{i \in I} y_i
 \end{aligned} \tag{5.28}$$

subject to (5.19)-(5.20), where $\Phi = \prod_{i \in I} A_i^{y_i}$ is the *total accessibility* of the loaded items to the set of handlers.

It is interesting to observe that the total expected profit of the loaded items is proportional to the logarithm of the total accessibility of those items to the set of handlers.

In the following, we will refer to the deterministic approximation of the **MHKP_u** as **DA-MHKP_u**.

The following theorem holds

Theorem 1. *At optimality, the percentage of each item i handled by handler j , x_{ij} , is given by*

$$x_{ij} = \frac{e^{\beta p_{ij}}}{\sum_{j' \in J} e^{\beta p_{ij'}}}, \quad i \in I, j \in J. \tag{5.29}$$

Proof. At optimality, the probability that item i is handled by handler j is equal to the probability that handler j is that one of maximum profit. Then, from the Total Probability Theorem (DeGroot and Schervish, 2002), one obtains

$$\begin{aligned}
 x_{ij} &= \int_{-\infty}^{+\infty} \prod_{v \neq j} \exp \left[-e^{-\beta(x-p_{iv})} \right] d \left[\exp \left(-e^{-\beta(x-p_{ij})} \right) \right] = \\
 &= e^{\beta p_{ij}} \int_{-\infty}^{+\infty} \beta e^{-\beta x} \exp(-A_i e^{-\beta x}) dx = \\
 &= e^{\beta p_{ij}} \int_0^{+\infty} e^{-A_i t} dt = \frac{e^{\beta p_{ij}}}{A_i} = \\
 &= \frac{e^{\beta p_{ij}}}{\sum_{j' \in J} e^{\beta p_{ij'}}} \quad i \in I, j \in J
 \end{aligned} \tag{5.30}$$

where $t = e^{-\beta x}$. □

It is trivial to check for x_{ij} the satisfaction of constraints (5.12) and (5.13).

Expression (5.29) represents a multinomial Logit model, which is widely used in choice theory (Domencich and McFadden, 1975). In our case, it describes how the

optimal handling of item i is split among different handlers j , due to the stochastic handling profit of item i .

5.4 A two-stage stochastic model with fixed recourse

Approximating the profit stochasticity by discretizing the probability distributions and generating a set of scenarios $S \subseteq L$, the $\text{MHKP}_{\mathbf{u}}$ (5.1)-(5.5) may be interpreted as a two-stage program with fixed recourse.

Let be the variables

- y_i : first stage decision variable equals to 1 if item i is loaded, 0 otherwise
- x_{ij} : first stage decision variable which represents the percentage of item i handled by handler j
- y_i^{+s} : second stage decision variable equals to 1 if item i is loaded under scenario s , 0 otherwise
- y_i^{-s} : second stage decision variable equals to 1 if item i is unloaded under scenario s , 0 otherwise
- x_{ij}^{+s} : second stage decision variable which represents the percentage of loaded item i handled by handler j under scenario s
- x_{ij}^{-s} : second stage decision variable which represents the percentage of unloaded item i handled by handler j under scenario s .

Moreover, we define by

$$\pi_{ij}^{+s} = p_i + p_{ij} + \tilde{\theta}^{js} \quad (5.31)$$

and

$$\pi_{ij}^{-s} = -\pi_{ij}^{+s} - \pi'_{ij} \quad (5.32)$$

the stochastic profits related to loading and unloading operations in the second stage, respectively, where $-\pi'_{ij}$ represents an extra cost to be paid for unloading item i by handler j in the second stage.

Finally, given the probability ρ_s of each second-stage scenario s , the two-stage program with fixed recourse, named **2S-MHKP_u**, is formulated as follows

$$\max_{\{y,x\}} \quad \sum_{i \in I} p_i y_i + \sum_{i \in I} \sum_{j \in J} p_{ij} x_{ij} + \sum_{s \in S} \rho_s \left[\sum_{i \in I} \sum_{j \in J} \pi_{ij}^{+s} x_{ij}^{+s} + \sum_{i \in I} \sum_{j \in J} \pi_{ij}^{-s} x_{ij}^{-s} \right] \quad (5.33)$$

$$\sum_{i \in I} w_i y_i \leq W \quad (5.34)$$

$$\sum_{j \in J} x_{ij} = y_i \quad i \in I \quad (5.35)$$

$$\sum_{i \in I} w_i y_i + \sum_{i \in I} \sum_{s \in S} w_i y_i^{+s} - \sum_{i \in I} \sum_{s \in S} w_i y_i^{-s} \leq W \quad (5.36)$$

$$\sum_{j \in J} x_{ij}^{+s} = y_i^{+s} \quad i \in I, \quad s \in S \quad (5.37)$$

$$\sum_{j \in J} x_{ij}^{-s} = y_i^{-s} \quad i \in I, \quad s \in S \quad (5.38)$$

$$y_i^{+s} \leq 1 - y_i \quad i \in I, \quad s \in S \quad (5.39)$$

$$y_i^{-s} \leq y_i \quad i \in I, \quad s \in S \quad (5.40)$$

$$x_{ij}^{+s} = x_{ij}^{+s'} \quad i \in I, \quad j \in J, \quad s, s' \in S \quad (5.41)$$

$$x_{ij}^{-s} = x_{ij}^{-s'} \quad i \in I, \quad j \in J, \quad s, s' \in S \quad (5.42)$$

$$y_i \in \{0, 1\} \quad i \in I \quad (5.43)$$

$$y_i^{+s} \in \{0, 1\} \quad i \in I, \quad s \in S \quad (5.44)$$

$$y_i^{-s} \in \{0, 1\} \quad i \in I, \quad s \in S \quad (5.45)$$

$$x_{ij} \geq 0 \quad i \in I, \quad j \in J \quad (5.46)$$

$$x_{ij}^{+s} \geq 0 \quad i \in I, \quad j \in J, \quad s \in S \quad (5.47)$$

$$x_{ij}^{-s} \geq 0 \quad i \in I, \quad j \in J, \quad s \in S. \quad (5.48)$$

The objective function (5.33) expresses the maximization of the total profit, given by the sum of the first stage profit plus the expected profit of the items handled in the second stage. Note that constraints (5.34) and (5.35) are the first stage constraints, while constraints (5.36)-(5.42) are the second stage ones. In particular, constraints (5.34) and (5.36) ensure that the capacity of the knapsack is not exceeded in first and second stages, respectively. Constraints (5.35) guarantee that any item is completely processed by some handlers only if it is loaded. Constraints

(5.37) and (5.38) guarantee that if an item is loaded or unloaded in the second stage it is completely processed by some handlers. Constraints (5.39) establish that no item can be handled for loading in the second stage if it has already been loaded in the first stage. Similarly, constraints (5.40) establish that no item can be handled for unloading in the second stage if it has not been loaded in the first stage. Constraints (5.41) and (5.42) are the non-anticipativity constraints. Finally, constraints (5.43)-(5.45) and (5.46)-(5.48) are the integrality and the non-negativity constraints, respectively.

The optimal solutions of the two-stage model $\mathbf{2S-MHKP}_{\mathbf{u}}$ and the deterministic approximation $\mathbf{DA-MHKP}_{\mathbf{u}}$ are strictly related. Let us suppose to have an optimal solution of model $\mathbf{DA-MHKP}_{\mathbf{u}}$. This model gives us a feasible approximation of the first-stage variables y_i , while it gives, by (5.29), a continuous relaxation of the assignment variables x_{ij} . Notice that, given the values of the variables x_{ij} in (5.29), one can derive a feasible first-stage solution of $\mathbf{2S-MHKP}_{\mathbf{u}}$ by fixing to one, for each item with $y_i = 1$, any x_{ij} variable associated to it (for example, the one with the greatest value). This means that the information given by model $\mathbf{DA-MHKP}_{\mathbf{u}}$ is related to the first level only, while to obtain the possible recursion we need to force the first-level solution in the two-stage model.

5.5 Experimental plan

The first goal is to assess the behavior of the $\mathbf{2S-MHKP}_{\mathbf{u}}$, the two-stage program with fixed recourse for the $\mathbf{MHKP}_{\mathbf{u}}$ proposed. In particular, we aim to study the structure of the solutions and impacts of uncertainty on them. The second is to evaluate the effectiveness of the deterministic approximation of the $\mathbf{MHKP}_{\mathbf{u}}$ we derived. Moreover, we want to calculate and evaluate the handling costs obtained by using our approximated results as first-stage decisions of the $\mathbf{2S-MHKP}_{\mathbf{u}}$.

The two-stage program with fixed recourse was implemented with CPLEX 12.4. Experiments were performed on a Intel i7 2 Ghz workstation with 6 GB of RAM.

5.5.1 Instance set

No instances are present in the literature for this stochastic version of the knapsack problem. We then generated instances, partially based on those available for the deterministic knapsack problem (Pisinger, 2005).

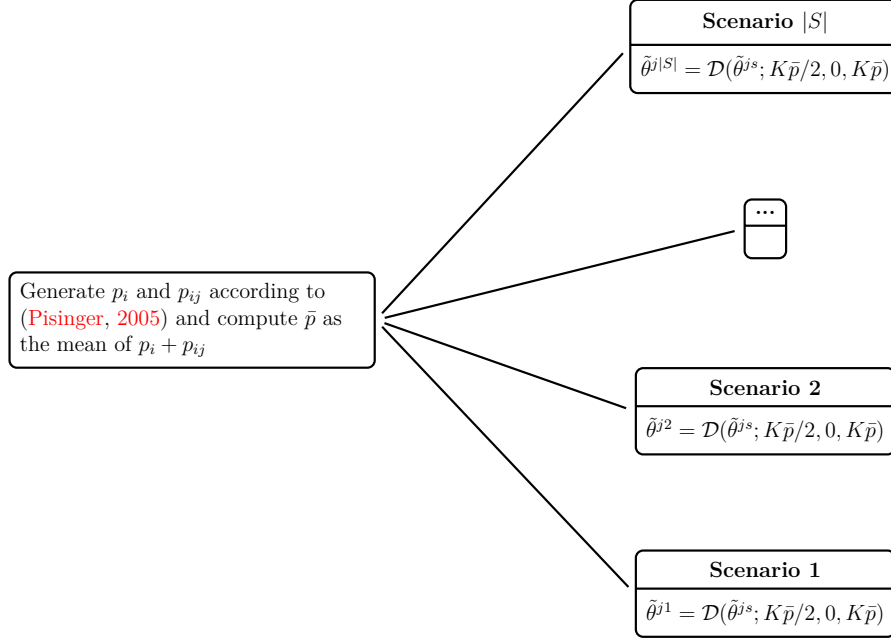


Figure 5.1: Scenario tree generation

Instances were created with the goal of providing the means to explore the impact of both the correlation between volume and profit of the items and the different probability distributions of the profit oscillations. Thus, the instances are characterized by various correlation strengths, as well as different probability distributions. Ten instances were randomly generated for each combination of the parameters.

- Number of items in the interval $[100, 1000]$.
- Number of handlers in the interval $[3, 5]$.
- Item volume uniformly distributed in the interval $[1, R]$, where $R = 1000$.
- Deterministic item profits generated according to the following three rules

UC: the deterministic item profits are uncorrelated to item volumes. They are uniformly generated in the interval $[1, R]$ (Martello and Toth, 1979; Pisinger, 2005).

SC: the deterministic item profits are strongly correlated with the item volumes. The profit is defined as $w_i + R/10$, where w_i is the item volume (Martello and Toth, 1979; Pisinger, 2005).

PC: the deterministic item profits are proportionally correlated with the item volumes. The profit is defined as αw_i , where α is uniformly drawn from the interval $[1, 5]$.

- Capacity of the knapsack was computed according to $\frac{h}{H+1} \sum_{i \in I} w_i$, where H is the number of instances for a set of parameters and $h \in [1, H]$ is the identification of an instance in that subset. This approach covers a large number of cases, diversifying the correlation between the parameters and the maximum capacity of the knapsack.
- Scenario generation. For each combination of the parameters described above, we first generate, for all the scenarios, the deterministic the item profits according to the above three rules. Given the average value of the deterministic profits, let \bar{p} , let $P_K = K\bar{p}$ be the maximum profit oscillation, where K belongs to the set $\{0.1, 0.3, 0.5\}$. The item profit oscillations were generated as $\tilde{\theta}^{j|S|} = \mathcal{D}(\tilde{\theta}^{js}; K\bar{p}/2, 0, K\bar{p})$, where $\mathcal{D}(\tilde{\theta}^{js}; \mu, min, max)$ is the distribution \mathcal{D} with mean μ and truncated between the values min and max (see Figure 5.1). In our tests we used the Uniform and the Gumbel distributions.

Having solved the instances 10 times each and computed the standard deviation and the mean of the optima over the runs, we derived that the appropriate number of scenarios is 50. For each instance, this value ensures a maximum ratio between the standard deviation and the mean for the optima which is less than 1%.

The parameters were also chosen to reflect realistic cases of supply chain applications. In details, the different levels of correlation between item volumes and profits have the double effect to explore more challenging instances from the computation point of view (Pisinger, 2005) and explore price policies quite common in transportation Crainic et al. (2011c). The interval of item volumes and their link

to the knapsack capacity is derived from [Pisinger \(2005\)](#). Finally, the bound of the stochastic oscillations has been set as in [Tadei et al. \(2012\)](#) and reflects typical boundaries for profit oscillations in logistics.

5.6 Computational results

In this section, we report the computational results for the instances presented. After the calibration of the β parameter (Section 5.6.1), we first study the effects of the uncertainty on the **2S-MHKP_u** in Section 5.6.2. Then, we compare the solutions obtained by means the two-stage program and the approximation in Section 5.6.3, while the use of solutions of the approximation as first-stage decision of the **2S-MHKP_u** is analyzed in Section 5.6.4.

5.6.1 Calibration of the model

The deterministic approximation of the **MHKP_u** given by (5.28) requires an appropriate value of the positive parameter β . This parameter describes the propensity of the model to choose among the set of the handlers characterized by different handling profits.

β is obtained by calibration as follows. Let us consider the standard Gumbel distribution $G(x) = \exp(e^{-x})$. If an approximation error of 2‰ is accepted, then $G(x) = 1 \Leftrightarrow x = 6.08$ and $G(x) = 0 \Leftrightarrow x = -1.76$. Let us consider the range $[m, M]$ ($[0, P_K]$ in our case) where the stochastic profit oscillations are drawn from. The following equations hold

$$\beta(m - \zeta) = -1.76 \tag{5.49}$$

$$\beta(M - \zeta) = 6.08 \tag{5.50}$$

where ζ is the mode of the Gumbel distribution $G(x) = \exp(e^{-\beta(x-\zeta)})$. From (5.49) and (5.50) one gets the corresponding value of the parameter β

$$\beta = \frac{7.84}{M - m} = \frac{7.84}{P_K} = \frac{7.84}{K\bar{p}}. \tag{5.51}$$

More sophisticated methods to calibrate β can be found in Galambos et al. (1994).

5.6.2 Impact of uncertainty

In this section, we show the benefit of using the two-stage model with recourse for the **2S-MHKP_u**. We do this by considering *EVPI* and *VSS*.

For each combination of the number of item (Column 1) and item profit generation rule (Column 2), Table 5.1 reports the average and maximum percentage *EVPI* (Columns 3 and 4 for the gumbel distribution, and Columns 5 and 6 for the uniform distribution). Similarly, Table 5.2 reports the average and maximum percentage *VSS*.

| IT | PROFIT | GUMBEL | | UNIFORM | |
|------|--------|-----------------|-------------------------------|-----------------|-------------------------------|
| | | <i>EVPI</i> [%] | <i>EVPI_{max}</i> [%] | <i>EVPI</i> [%] | <i>EVPI_{max}</i> [%] |
| 100 | UC | 3.99 | 7.79 | 4.57 | 8.71 |
| | SC | 4.15 | 7.90 | 5.11 | 8.96 |
| | PC | 4.25 | 8.49 | 4.88 | 9.54 |
| 1000 | UC | 4.14 | 8.11 | 4.79 | 9.19 |
| | SC | 4.31 | 8.42 | 5.23 | 9.33 |
| | PC | 4.68 | 9.47 | 5.39 | 10.24 |

Table 5.1: EVPI comparison

| IT | PROFIT | GUMBEL | | UNIFORM | |
|-----|--------|----------------|------------------------------|----------------|------------------------------|
| | | <i>VSS</i> [%] | <i>VSS_{max}</i> [%] | <i>VSS</i> [%] | <i>VSS_{max}</i> [%] |
| 100 | UC | 3.08 | 6.24 | 3.54 | 6.67 |
| | SC | 3.24 | 6.37 | 3.69 | 6.94 |
| | PC | 3.37 | 6.71 | 3.78 | 6.93 |
| 100 | UC | 3.34 | 6.35 | 3.99 | 6.71 |
| | SC | 3.52 | 6.52 | 4.03 | 7.14 |
| | PC | 3.98 | 6.93 | 4.15 | 7.19 |

Table 5.2: VSS comparison

The *EVPI* percentage is, in average, larger than 4%. The maximum varies between 7.79% and 10.24%. It increases with the problem dimension and it is maximum when the uniform distribution is considered. The average and maximum

values of the VSS increase as the size of the instance increases. The gap between the expected-value solution and the stochastic solution is always larger than 3% for all sets considered. Even for smaller instances, the maximum VSS reaches 6%, showing the losses incurred by following the the expected solution. It is interesting to note that the most critical item profit generation rule is PC. PC, in fact, presents the higher variability of the stochastic item profit. The results clearly show the benefit of SP approach for the $\mathbf{2S-MHKP}_u$.

5.6.3 Comparison of the two-stage program and the approximation

The two-stage program solutions showed a common trend: the $\mathbf{2S-MHKP}_u$ reserves half of the total knapsack capacity in the second stage. For this reason, no items are unloaded in 99% of the instances. This means that the $\mathbf{2S-MHKP}_u$ uses the knowledge given by the scenarios in order to forecast what items can be immediately loaded, while preserving a proper loading space for items to be arranged in the recourse. This leads to a drastic reduction of the unloading and rearranging operations. Thus, the $\mathbf{2S-MHKP}_u$ policy is quite far from the usual supply chain approach of almost fully loading the knapsack in advance, while the unloading/rearranging operations are made at a later time and the percentage gap with the optimal solution can easily overcome 10%.

Here we summarize the results for all instances and different combinations of the parameters. The performance, in terms of optimality gap, is defined by the relative percentage error of the approximated solution when compared to the optimum. Moreover, we estimate the solution likelihood as the percentage of items loaded by the approximated solution which are also present in the optimal solution.

Note that the comparison results do not consider the number of handlers, which does not seem to affect the average performance of the deterministic approximation.

Table 5.3 reports the percentage optimality gap and the solution likelihood of the deterministic approximation for all combinations of the parameters, while varying the probability distribution (either Gumbel or Uniform). The first column displays the number of items, while Columns 2-3 and 5-6 report the mean and variance of the optimality gap, and Columns 4 and 7 show the mean percentage solution likelihood.

The best mean values are obtained for the Gumbel distribution, that is very close to the optimum (0.12% for larger instances) and characterized by a negligible variance. Moreover, increasing the number of items gives better results for the deterministic approximation. Similarly, in terms of likelihood, this method guarantees results close to the optimum for both distributions.

| IT | GUMBEL | | | UNIFORM | | |
|-------------|--------|------|------------|---------|------|------------|
| | mean | var | likelihood | mean | var | likelihood |
| 100 | 0.25 | 0.05 | 97.12 | 1.33 | 0.74 | 96.26 |
| 1000 | 0.12 | 0.01 | 98.54 | 1.33 | 0.52 | 97.31 |

Table 5.3: Optimality gap and solution likelihood of the deterministic approximation

Table 5.4 reports the percentage optimality gap and the solution likelihood performance of the deterministic approximation while varying the correlation between profits and volumes of items (Column 2) and the probability distribution (Columns 3-8). The results indicate that the strongly correlated instances (SC) yield the worst gaps, as well as the worst solution likelihood, with an average optimality gap of about 0.27% and 1.54% for the Gumbel and the Uniform distributions, respectively. For uncorrelated instances (UC) and the Gumbel distribution, some solutions of the deterministic approximation exactly match the two-stage program solutions.

| IT | PROFIT | GUMBEL | | | UNIFORM | | |
|-------------|-----------|--------|------|------------|---------|------|------------|
| | | mean | var | likelihood | mean | var | likelihood |
| 100 | UC | 0.23 | 0.05 | 98.78 | 1.20 | 0.57 | 98.21 |
| | SC | 0.27 | 0.06 | 95.28 | 1.54 | 0.93 | 95.19 |
| | PC | 0.25 | 0.05 | 97.31 | 1.26 | 0.68 | 95.19 |
| 1000 | UC | 0.10 | 0.01 | 99.31 | 1.14 | 0.38 | 98.92 |
| | SC | 0.15 | 0.02 | 98.06 | 1.56 | 0.55 | 96.40 |
| | PC | 0.11 | 0.01 | 98.23 | 1.29 | 0.55 | 96.62 |

Table 5.4: Optimality gap and solution likelihood for the profit correlation rules

The analysis of the impact of the maximum profit oscillations on the results accuracy is proposed in Table 5.5, considering different probability distributions (Columns 3-8). Recalling the definition of the maximum profit oscillation $P_K = K\bar{p}$, column 2 represents the percentage K of the mean profit \bar{p} of the instances. The

gap and the solution likelihood are clearly inversely proportional to the range of the oscillations. Indeed, the best mean values are obtained for $K = 0.1$.

| IT | K | GUMBEL | | | UNIFORM | | |
|------|-----|--------|------|------------|---------|------|------------|
| | | mean | var | likelihood | mean | var | likelihood |
| 100 | 0.1 | 0.09 | 0.01 | 97.56 | 0.53 | 0.04 | 96.67 |
| | 0.3 | 0.30 | 0.06 | 97.02 | 1.33 | 0.27 | 96.54 |
| | 0.5 | 0.36 | 0.06 | 96.80 | 2.13 | 0.64 | 95.58 |
| 1000 | 0.1 | 0.04 | 0.00 | 98.81 | 0.55 | 0.04 | 97.88 |
| | 0.3 | 0.12 | 0.01 | 98.56 | 1.39 | 0.13 | 97.29 |
| | 0.5 | 0.30 | 0.01 | 98.23 | 2.09 | 0.26 | 96.76 |

Table 5.5: Optimality gap and solution likelihood for the maximum profit oscillations

In conclusion, the results are very promising. The procedure performs very well for all types of instances and distributions and guarantees a high accuracy. The best performance is obtained if the random profits have a Gumbel distribution, that is usually the case for real market oscillations. Moreover, the variance of the results is tight and in some cases close to zero. With respect to the solution likelihood, the mean values are all greater than 95% and increase according to the number of items.

As we expected, the mean optimality gap slightly increases for instances with Uniform distributed profit oscillations, but results are stable for each combination of the parameters and improve with respect to the number of items. Furthermore, it is interesting to note that the hardest subset of instances are the strongly correlated ones. In fact, this kind of instance is characterized by a peculiar profit-volume correlation, as shown in [Pisinger \(2005\)](#).

From a computational point of view, the average CPU-times to solve to optimality the $\mathbf{2S-MHKP}_u$ and to compute the deterministic approximation are about 120 seconds and less than one second, respectively.

5.6.4 Usage of the approximated model as a decision tool

In the last part of this computational analysis, we analyze the losses, in terms of optimality gap, obtained by plugging the solution of the deterministic approximation of the \mathbf{MHKP}_u into the first-stage decision of the $\mathbf{2S-MHKP}_u$ ([Maggioni and Wallace, 2012](#)). In this way we can measure the accuracy of the approximated model

when used not only as a method to calculate the optimum, but also as a decision tool to actually choose the items to be loaded. This means that the only degrees of freedom to maximize the objective function are the item-to-handler assignments and the handling operations in the second stage. Indeed, the effect of this strategy is the increase of the unloading operations, which does not exceed 6% of total operations, however.

Next, we present the comparison results organized as in Section 5.3. Tables 5.6, 5.7, and 5.8 summarize the average gap for all combinations of the parameters, for the profit correlations and for the maximum profit oscillations, respectively.

| IT | GUMBEL | | UNIFORM | |
|-------------|--------|------|---------|------|
| | mean | var | mean | var |
| 100 | 1.18 | 0.44 | 1.99 | 1.40 |
| 1000 | 1.22 | 0.48 | 1.99 | 1.15 |

Table 5.6: Optimality gap with fixed first stage decision

| IT | PROFIT | GUMBEL | | UNIFORM | |
|-------------|-----------|--------|------|---------|------|
| | | mean | var | mean | var |
| 100 | UC | 1.05 | 0.31 | 1.77 | 1.06 |
| | SC | 1.36 | 0.59 | 2.31 | 1.74 |
| | PC | 1.12 | 0.38 | 1.88 | 1.28 |
| 1000 | UC | 1.01 | 0.28 | 1.75 | 0.88 |
| | SC | 1.49 | 0.64 | 2.32 | 1.25 |
| | PC | 1.15 | 0.42 | 1.91 | 1.18 |

Table 5.7: Optimality gap with fixed first stage decision for the profit correlation rules

As expected, the best performance is obtained by instances with the Gumbel distribution (Table 5.6). With respect to the profit correlations, the observed gap of SC instances is worse than other rules (Table 5.7). Finally, the gap increases according to the maximum range of the random profits (Table 5.8).

| IT | K | GUMBEL | | UNIFORM | |
|------|----|--------|------|---------|------|
| | | mean | var | mean | var |
| 100 | 10 | 0.47 | 0.03 | 0.81 | 0.09 |
| | 30 | 1.24 | 0.10 | 2.01 | 0.48 |
| | 50 | 1.82 | 0.26 | 3.13 | 0.93 |
| 1000 | 10 | 0.47 | 0.02 | 0.82 | 0.08 |
| | 30 | 1.26 | 0.10 | 2.08 | 0.29 |
| | 50 | 1.92 | 0.26 | 3.07 | 0.53 |

Table 5.8: Optimality gap with fixed first stage decision for the maximum profit oscillations

Chapter 6

The Multi-Path Travelling Salesman Problem with stochastic travel times

In this chapter we consider the **mpTSP_s**, a recently introduced variant of the standard TSP related to Smart City and City Logistics applications (Tadei et al., 2014). After defining the problem and the source of the uncertainty, we propose two two-stage stochastic formulation of **mpTSP_s**: one based on the sub-tour elimination constraints and one based on network flow constraints. We then present an heuristic strategy based on the PH algorithm for the first formulation, which decomposes the **mpTSP_s** in deterministic TSP problems and guides the search process towards the consensus among them. Whilst, the second formulation is solve by means of CPLEX proving a reference solution to validate the heuristic. The accuracy and the efficiency of the latter is demonstrated through extensive experimental campaign. A set of realistic instance sets based on the traffic sensor network of the city of Turin is introduced. The instance set is designed to analyze the effectiveness of the heuristic proposed and to measure the impacts of uncertain on the problem.

The paper is organized as follows. In Section 6.1 the description of the problem and a detailed literature review on related works are given. In Section 6.2 the two-stage models with recourse are presented, while the PH algorithm is introduced in Section 6.3. The experimental plan and the computational results are then given in

Section 6.4.

6.1 Problem description and literature review

The **mpTSP_s** arises mainly in City Logistics applications. In fact, while at the operational level it is possible to know with a good approximation, for each path, the actual travel time, this is not the case at the planning level, where the tour must be built in order to cope with different working days. In the case of delivery express companies, for example, new experimental delivery policies concerns some small storage boxes (BentoBox) in malls ([CITYLOG Consortium, 2010](#)). This means that their drivers have a fixed tour to reach the malls, valid for a period of several days (usually one month). Moreover, for operational reasons due to the usual door-to-door deliveries, the tour can start in different working hours. Thus, at this level, even knowing the order of the nodes to visit, the exact paths between the nodes and their travel times are random variable. A similar behavior can be seen in the management of fleets of hybrid vehicles in urban freight delivery. In this case the different paths represent the different operational modes of the hybrid vehicle, e.g. pure electric, traditional with battery recharge or traditional without battery recharge. The actual battery consumption/recharge is affected by several factors, including time distribution of the vehicle speed, street type, road congestion, number of Stop&Go. As this information is only partially known at the moment of the trip planning, it is surrogated in a unknown oscillation of the mean usage of the battery itself.

6.1.1 Literature review

While different stochastic and/or dynamic variants of TSP (and more in general of vehicle routing problem) are present in the literature ([Gendreau et al., 1996](#); [Golden et al., 2008](#); [Pillac et al., 2013](#)), the **mpTSP_s** has been recently defined. For this reason, we also consider some relevant literature on similar problems, highlighting the main differences with the problem faced in this paper.

In [Tadei et al. \(2014\)](#) the authors introduce the problem and derive a deterministic approximation. The quality of the deterministic approximation is then evaluated

by comparing it with the Perfect Information results obtained by means of a Monte Carlo method. The comparison shows a good accuracy of the deterministic approximation, with a reduction of the computational times of two orders of magnitude.

In the literature several stochastic variants of the TSP problems can be found. In these problems a known distribution affecting some problem parameters is given and the theoretical results are strongly connected with the hypotheses on such distribution. The main sources of uncertainty are related to the arc costs (Leipala, 1978; Toriello et al., 2012) and the subset of cities to be visited with their location (Jaillet, 1988; Goemans and Bertsimas, 1991).

If we consider general routing problems, different types of uncertainty and dynamics can be considered. The most studied variants are related to the online arrival of customers, with the requests being both goods (Hvattum et al., 2006, 2007; Ichoua et al., 2006; Mitrović-Minić and Laporte, 2004) and services (Beaudry et al., 2010; Bertsimas and Van Ryzin, 1991; Gendreau et al., 1999; Larsen et al., 2004). Only in recent years the dynamics related to travel times has been considered in the literature (Chen et al., 2006; Fleischmann et al., 2004; Güner et al., 2012; Kenyon and Morton, 2003; Tagmouti et al., 2011; Taniguchi and Shimamoto, 2004), while, to the best of our knowledge, service time has not been explicitly studied. The last variants of vehicle routing problems are related to the dynamically revealed demands of a known set of customers (Novoa and Storer, 2009; Secomandi and Margot, 2009) and the vehicle availability (Li et al., 2009a,b; Mu et al., 2011). For a recent review, the reader can refer to (Pillac et al., 2013).

All the papers presented in this survey deal with uncertainty and/or dynamic aspects of the routing problems where the magnitude of the uncertainty is limited and the parameter values are revealed in a time interval compatible with the operations optimization. Then the multi-path aspects can be ignored, being possible an a priori choice of the path connecting the two nodes. In our case, the **mpTSP_s** is thought to be used for planning a service. Thus, the enlarged time horizon as well as strong dynamic changes in travel times due to traffic congestion and other nuisances, typical of the urban transportation, induce the presence of multiple paths connecting every pair of nodes, each one with its stochastic cost. This is, to our knowledge, an aspect of the transportation literature considered only in transshipment problems, where the routing aspect is heavily relaxed (Baldi et al., 2012b; Maggioni et al.,

2009; Tadei et al., 2012).

6.2 Problem formulation

To include the random nature of the travel time process in a urban context, we consider a two-stage stochastic linear program with recourse. The travel time oscillation Δ_{ij}^k by using path k between nodes i and j , is then described by a stochastic process represented using a discrete random variable. All possible discrete values that the random variable can assume, is represented by a finite set of vectors, named scenarios. We represent each realization (scenario) of random travel time oscillation process by Δ_{ij}^{ks} . We denote with S the set of time scenarios, and the probability of each scenario $s \in S$ by p_s . In two-stage stochastic programming, we explicitly classify the decision variables according to whether they are implemented before or after an outcome of the random travel time variable of each path is observed. In the multi-path traveling salesman problem, in the *first stage* the decision maker does not have any information about the travel time oscillation. However, the routing among the nodes should be determined before the complete information is available. Thus, the first-stage decision variable y_{ij} is represented by the nodes i and j to be visited in a tour. In the second stage, travel time oscillations are available and the paths k between each pair of nodes i and j , x_{ij}^{ks} (recourse actions) can be calculated. The objective of the two-stage stochastic model with recourse for the **mpTSP_s** is the minimization of the total cost due to paths congestion. In the next subsections we consider two different model for this problem:

1. A sub-tour elimination based two-stage stochastic model with recourse, which is an extension of the classical deterministic sub-tour elimination model (Cook, 2012);
2. A flow-based two-stage stochastic model with recourse derived from the MIP model of the Two-Echelon VRP problem by Perboli et al. (2011).

6.2.1 A sub-tour elimination based two-stage model with recourse for the mpTSPs

First we consider a sub-tour elimination based two-stage stochastic model with recourse. The notation adopted is the following:

Sets:

- N : set of nodes
- U : subset of N
- S : set of time scenarios
- K_{ij} : set of paths between nodes i and j

Parameters:

- $\bar{c}_{ij} \geq 0$: estimation of the mean unit travel time cost between nodes i and j
- $c_{ij}^{ks} \geq 0$: unit random travel time cost of the path $k \in K_{ij}$ under the time scenario $s \in S$
- $\Delta_{ij}^{ks} = c_{ij}^{ks} - \bar{c}_{ij}$: error on the travel time cost estimated for the path $k \in K_{ij}$ under time scenario $s \in S$
- p_s : probability of time scenario $s \in S$

Variables:

- y_{ij} : boolean variable equal to 1 if node j is visited just after node i , 0 otherwise
- x_{ij}^{ks} : boolean variable equal to 1 if path $k \in K_{ij}$ is selected at stage 2, 0 otherwise.

The deterministic equivalent formulation of the sub-tour elimination based two-stage stochastic model with recourse is as follows:

$$\min_{\{y,x\}} \left[\sum_{i \in N} \sum_{j \in N} \bar{c}_{ij} y_{ij} + \sum_{s \in S} p_s \sum_{i \in N} \sum_{j \in N} \sum_{k \in K_{ij}} \Delta_{ij}^{ks} x_{ij}^{ks} \right] \quad (6.1)$$

subject to

$$\sum_{j \in N: j \neq i} y_{ij} = 1 \quad i \in N \quad (6.2)$$

$$\sum_{i \in N: i \neq j} y_{ij} = 1 \quad j \in N \quad (6.3)$$

$$\sum_{i \in U} \sum_{j \notin U} y_{ij} \geq 1 \quad \forall U \subset N \quad (6.4)$$

$$\sum_{k \in K_{ij}} x_{ij}^{ks} = y_{ij} \quad i \in N, \quad j \in N, \quad s \in S \quad (6.5)$$

$$x_{ij}^{ks} \in \{0,1\} \quad k \in K_{ij}, \quad i \in N, \quad j \in N, \quad s \in S \quad (6.6)$$

$$y_{ij} \in \{0,1\} \quad i \in N, \quad j \in N \quad (6.7)$$

Problem (6.1)-(6.7) is a large-scale binary problem. The first sum in the objective function (6.1) represents the first-stage travel cost, while the second sum represents the recourse action, consisting in choosing the best path $k \in K_{ij}$ under time scenario realization $s \in S$. Constraints (6.2) and (6.3) are the standard first-stage assignment constraints and (6.4) represent the first-stage sub-tour elimination constraints. Constraints (6.5) guarantee that path k between nodes i and j can be chosen at stage 2 only if nodes i and j were part of the tour fixed at stage 1. Finally, the integrality constraints (6.6)-(6.7) define the first-stage and second-stage decision variables of the problem. By solving problem (6.1)-(6.7), one finds a single tour y_{ij} , $\forall i, j \in N$, with minimum travel time cost overall scenarios included in S .

The main problem when dealing with this formulation are the constraints (6.4), which are exponential in their number and they require a dynamic cut generation method only (Applegate et al., 2007). This makes difficult to incorporate such a model in a MIP solver. On the other hand, as we will show in Section 6.3, this model makes quite easy to define an efficient PH-based algorithm.

6.2.2 A flow-based two-stage model with recourse for the mpTSPs

We consider now a flow-based two-stage stochastic model with recourse for the mpTSP_s. The notation adopted is the same as in the previous subsection. A first

stage real variable ϕ_{ij} associated to the flow on the arc (i, j) is also introduced.

The deterministic equivalent formulation of the flow-based two-stage stochastic model with recourse is as follows:

$$\min_{\{y,x\}} \left[\sum_{i \in N} \sum_{j \in N} \bar{c}_{ij} y_{ij} + \sum_{s \in S} p_s \sum_{i \in N} \sum_{j \in N} \sum_{k \in K_{ij}} \Delta_{ij}^{ks} x_{ij}^{ks} \right] \quad (6.8)$$

subject to

$$\sum_{j \in N: j \neq i} y_{ij} = 1 \quad i \in N \quad (6.9)$$

$$\sum_{i \in N: i \neq j} y_{ij} = 1 \quad j \in N \quad (6.10)$$

$$\sum_{i \in N: i \neq j} \phi_{ij} - \sum_{k \in N: k \neq j} \phi_{jk} = 1 \quad \forall j \in N \setminus \{1\} \quad (6.11)$$

$$\sum_{i \in N: i \neq 1} \phi_{i1} - \sum_{k \in N: k \neq 1} \phi_{1k} = 1 - |N| \quad (6.12)$$

$$\sum_{k \in N: k \neq 1} \phi_{1k} = |N| \quad (6.13)$$

$$\phi_{ij} \leq |N| y_{ij} \quad i \in N, \quad j \in N \quad (6.14)$$

$$\sum_{k \in K_{ij}} x_{ij}^{ks} = y_{ij} \quad i \in N, \quad j \in N, \quad s \in S \quad (6.15)$$

$$x_{ij}^{ks} \in \{0,1\} \quad k \in K_{ij}, \quad i \in N, \quad j \in N, \quad s \in S \quad (6.16)$$

$$y_{ij} \in \{0,1\} \quad i \in N, \quad j \in N \quad (6.17)$$

While the meaning of the objective function (6.8) and the constraints (6.9) and (6.10) is the same of (6.1), (6.2), and (6.3) in the previous model, the sub-tour elimination constraints (6.4) are rewritten by means of the constraints (6.11), (6.12), (6.13), and (6.14). Without losing in generality, let us consider node 1 as the starting point of our tour. Constraints (6.12) and (6.13) force the node 1 to have an outbound flow equal to to the number of nodes $|N|$ and an inbound with value 1. Thus, the subtours are forbidden by constraint (6.11), which obliges every node to reduce by 1 the outbound flow if compared to its inbound one. If a subtour exists, this constraint is violated by at least one of the nodes in the subtour (see [Perboli et al. \(2011\)](#) for further details). Finally, constraint (6.14) links the binary variables y_{ij}

to the existence of a flow in an arc. This constraint, with the constraints (6.9) and (6.10), force to have exactly one arc with a non zero flow both as inbound and outbound.

This flow-based model is more suitable to be solved by means of MIP solvers, involving $O(N)$ constraints instead of the exponential number of constraints (6.4).

6.3 Heuristic based on progressive hedging

This section introduces an heuristic method based on the PH algorithm for the \mathbf{mpTSP}_s . The steps of the method are summarized in the Algorithm 2.

As stated in the Chapter 4 for the $\mathbf{SVCSBPP}$, the method applies a SD technique based on the augmented Lagrangean relaxation, which separates the stochastic problem following the possible scenarios of the random event. Section 6.3.1 shows how the \mathbf{mpTSP}_s can be decomposed into deterministic TSP subproblems with modified fixed costs. Then, the method proceeds in two phases. Phase 1 aims to obtain consensus among the subproblems formed by the SD (see Section 6.3.2), iteratively solving the individual penalized subproblems at each iteration. The individual solutions are then aggregated in order to obtain a reference solution. Section 6.3.3 proposes adjustment strategy of the penalties based on the deviation of the scenario solutions from the reference solutions that gradually guides the search process toward scenario consensus. The search process continues until the consensus or termination criteria are met (see Section 6.3.4). When consensus is not achieved in the first phase, Phase 2 solves the restricted \mathbf{mpTSP}_s obtained by fixing the first-stage variables for which consensus has been reached.

6.3.1 Scenario decomposition of the \mathbf{mpTSP}_s

We refer to the sub-tour formulation (6.1) - (6.7), which presents a structure, in terms of variables and constraints, closes to the classical formulation of the TSP problem. This enables the use of a specialized method in the PH algorithm.

To apply the SD scheme proposed by [Rockafellar and Wets \(1991\)](#), we define the following vectors: $y_{ij}^s \in \{0,1\}$, $\forall i, j \in N$ and $\forall s \in S$. In doing so, a copy of the first stage variables is created for each scenario $s \in S$. Model (6.1)-(6.7) can now

Algorithm 2 PH-based meta-heuristic for the **mpTSP_s**

Scenario decomposition

Generate a set of scenarios \mathcal{S} ;

Decompose the resulting deterministic model ((6.1)–(6.7)) by scenario using augmented Lagrangian relaxation;

Phase 1

$\nu \leftarrow 0, \lambda_{ij}^{s\nu} \leftarrow 0, \rho_{ij}^\nu \leftarrow \bar{c}_{ij}/2$

while Termination criteria not met **do**

For all $s \in \mathcal{S}$,

 solve the corresponding Traveling Salesman Problem subproblem $\rightarrow y_{ij}^{s\nu}$;

Compute temporary global solution

$$\bar{y}_{ij}^\nu \leftarrow \sum_{s \in \mathcal{S}} p_s y_{ij}^{s\nu}$$

Penalty adjustment

$$\lambda_{ij}^{s\nu} = \lambda_{ij}^{s\nu-1} + \rho_{ij}^{(\nu-1)} (y_{ij}^{s\nu} - \bar{y}_{ij}^{s\nu})$$

$$\rho_{ij}^\nu \leftarrow \alpha \rho_{ij}^{(\nu-1)}$$

Building a feasible tour

 Apply algorithm 3 to the reference solution $\bar{y}_{ij}^\nu \rightarrow \hat{y}_{ij}^\nu$;

 Evaluate \hat{y}_{ij}^ν and update the best solution if necessary;

if consensus is at least $\sigma\%$ **then**

$$\bar{y}_{ij}^\nu = \hat{y}_{ij}^\nu$$

Phase 2

if consensus is not met **then**

 Fix consensus variables in model ((6.1)–(6.7));

 Solve restricted ((6.1)–(6.7)) model using a MIP solver.

be rewritten as follows:

$$\min_{\{y,x\}} \sum_{s \in S} p_s \left[\sum_{i \in N} \sum_{j \in N} \bar{c}_{ij} y_{ij}^s + \sum_{i \in N} \sum_{j \in N} \sum_{k \in K_{ij}} \Delta_{ij}^{ks} x_{ij}^{ks} \right] \quad (6.18)$$

subject to

$$\sum_{j \in N: j \neq i} y_{ij}^s = 1 \quad i \in N, \quad s \in S \quad (6.19)$$

$$\sum_{i \in N: i \neq j} y_{ij}^s = 1 \quad j \in N, \quad s \in S \quad (6.20)$$

$$\sum_{i \in U} \sum_{j \notin U} y_{ij}^s \geq 1 \quad \forall U \subset N, \quad s \in S \quad (6.21)$$

$$\sum_k x_{ij}^{ks} = y_{ij}^s \quad i \in N, \quad j \in N, \quad s \in S \quad (6.22)$$

$$y_{ij}^s = y_{ij}^t \quad i \in N, \quad j \in N, \quad s, t \in S \quad (6.23)$$

$$x_{ij}^{ks} \in \{0,1\} \quad k \in K_{ij}, \quad i \in N, \quad j \in N, \quad s \in S \quad (6.24)$$

$$y_{ij}^s \in \{0,1\} \quad i \in N, \quad j \in N, \quad s \in S \quad (6.25)$$

Equations (6.23) are referred to as the non-anticipativity constraints. These constraints are used to make sure that the decisions on the tour are not tailored according to the scenarios considered in S . All the scenario tours must be equal to each other to produce a single implementable tour. At this point, an important observation may be made: relaxing constraints (6.23) the problem (6.18)-(6.25) becomes scenario separable. However, it should be noted that the number of constraints defined in (6.23) may become large given the size of S . Therefore, a different expression of the non-anticipativity constraints is required.

If $\bar{y}_{ij} \in \{0,1\}$, $\forall i, j \in N$ is defined as the *overall tour vector* (i.e. the tour for all scenarios considered), then the following constraints are equivalent to (6.23):

$$\bar{y}_{ij} = y_{ij}^s \quad i \in N, \quad j \in N, \quad s \in S \quad (6.26)$$

$$\bar{y}_{ij} \in \{0,1\} \quad i \in N, \quad j \in N, \quad (6.27)$$

Constraints (6.26) force each scenario tour to be equal to the overall tour. As for (6.27), they are simply the required integrality conditions on the overall design.

By using this particular formulation for the non-anticipativity constraints, when Lagrangean relaxation is applied on (6.26), one can penalize individually the difference between the scenario solution and the overall solution for each node within the tour.

Constraints (6.26) are relaxed using an augmented Lagrangean strategy. We thus obtain the following objective for the overall problem:

$$\begin{aligned} \min_{\{y,x\}} \sum_{s \in S} p_s \left[\sum_{i \in N} \sum_{j \in N} \bar{c}_{ij} y_{ij}^s + \sum_{i \in N} \sum_{j \in N} \sum_{k \in K_{ij}} \Delta_{ij}^{ks} x_{ij}^{ks} \right. \\ \left. + \sum_{i \in N} \sum_{j \in N} \lambda_{ij}^s (y_{ij}^s - \bar{y}_{ij}) + \frac{1}{2} \sum_{i \in N} \sum_{j \in N} \rho (y_{ij}^s - \bar{y}_{ij})^2 \right], \end{aligned} \quad (6.28)$$

where $\lambda_{ij}^s, \forall i, j \in N$ and $\forall s \in S$, define the Lagrangean multipliers for the relaxed constraints and ρ is a penalty ratio. Within function (6.28), let us consider the quadratic term. Given the binary requirements for the scenario tour variables and for the overall tour vector, this term becomes:

$$\begin{aligned} \sum_{i \in N} \sum_{j \in N} \rho (y_{ij}^s - \bar{y}_{ij})^2 &= \sum_{i \in N} \sum_{j \in N} (\rho (y_{ij}^s)^2 - 2\rho y_{ij}^s \bar{y}_{ij} + \rho (\bar{y}_{ij})^2) = \\ &= \sum_{i \in N} \sum_{j \in N} (\rho y_{ij}^s - 2\rho y_{ij}^s \bar{y}_{ij} + \rho \bar{y}_{ij}). \end{aligned} \quad (6.29)$$

Therefore, the objective of the relaxed problem can be formulated as follows:

$$\begin{aligned} \min_{\{y,x\}} \sum_{s \in S} p_s \left[\sum_{i \in N} \sum_{j \in N} \left(\bar{c}_{ij} + \lambda_{ij}^s - \rho \bar{y}_{ij} + \frac{\rho}{2} \right) y_{ij}^s + \sum_{i \in N} \sum_{j \in N} \sum_{k \in K_{ij}} \Delta_{ij}^{ks} x_{ij}^{ks} \right] \\ - \sum_{i \in N} \sum_{j \in N} \lambda_{ij}^s \bar{y}_{ij} + \frac{\rho}{2} \sum_{i \in N} \sum_{j \in N} \bar{y}_{ij}. \end{aligned} \quad (6.30)$$

Given the constraints of the model and considering the objective function (6.30), the relaxed problem is not scenario separable. However, if the overall design $\bar{y}_{ij}, \forall i, j \in N$, is fixed to a given value vector, then the model decomposes according to the time scenarios included in set S . All scenarios subproblems can then be expressed as follow:

$$\min_{\{y,x\}} \left[\sum_{i \in N} \sum_{j \in N} \left(\bar{c}_{ij} + \lambda_{ij}^s - \rho \bar{y}_{ij} + \frac{\rho}{2} \right) y_{ij}^s + \sum_{i \in N} \sum_{j \in N} \sum_{k \in K_{ij}} \Delta_{ij}^{ks} x_{ij}^{ks} \right] \quad (6.31)$$

subject to

$$\sum_{j \in N: j \neq i} y_{ij}^s = 1 \quad i \in N, \quad s \in S \quad (6.32)$$

$$\sum_{i \in N: i \neq j} y_{ij}^s = 1 \quad j \in N, \quad s \in S \quad (6.33)$$

$$\sum_{i \in U} \sum_{j \notin U} y_{ij}^s \geq 1 \quad \forall U \subset N, \quad s \in S \quad (6.34)$$

$$\sum_{k \in K_{ij}} x_{ij}^{ks} = y_{ij}^s \quad i \in N, \quad j \in N, \quad s \in S \quad (6.35)$$

$$x_{ij}^{ks} \in \{0,1\} \quad k \in K_{ij}, \quad i \in N, \quad j \in N, \quad s \in S \quad (6.36)$$

$$y_{ij}^s \in \{0,1\} \quad i \in N, \quad j \in N, \quad s \in S. \quad (6.37)$$

At optimality, the paths of the problem (6.31)-(6.37) within the tour are given by

$$\begin{cases} x_{ij}^{k^*s} = y_{ij}^s & i \in N, \quad j \in N, \quad k = k^* \\ x_{ij}^{ks} = 0 & i \in N, \quad j \in N, \quad k \neq k^*, \end{cases} \quad (6.38)$$

where $k^* \in K_{ij}$ is the path between nodes i and j with minimum travel time cost. Thus, each scenario subproblem can be reduced to a TSP as follows:

$$\min_{\{y\}} \sum_{i \in N} \sum_{j \in N} \left(\bar{c}_{ij} + \Delta_{ij}^{k^*s} + \lambda_{ij}^s - \rho \bar{y}_{ij} + \frac{\rho}{2} \right) y_{ij}^s \quad (6.39)$$

subject to

$$\sum_{j \in N: j \neq i} y_{ij}^s = 1 \quad i \in N, \quad s \in S \quad (6.40)$$

$$\sum_{i \in N: i \neq j} y_{ij}^s = 1 \quad j \in N, \quad s \in S \quad (6.41)$$

$$\sum_{i \in U} \sum_{j \notin U} y_{ij}^s \geq 1 \quad \forall U \subset N, \quad s \in S \quad (6.42)$$

$$y_{ij}^s \in \{0,1\} \quad i \in N, \quad j \in N, \quad s \in S. \quad (6.43)$$

An important observation can be made. The subproblems (6.39)-(6.43) are deterministic TSP with modified travel time costs. From a methodological perspective,

this turns out to be very interesting since one is able to use some of the more efficient algorithms that have been developed for the deterministic TSP. Concorde (Applegate et al., 2007; Cook, 2012) is used to address these problems, which, embedding a cutting-plane algorithm within a branch-and-bound search, is currently one of the best procedures for the TSP.

For all scenarios $s \in S$ within the subproblems (6.39)-(6.43), the Lagrangian multipliers λ_{ij}^s , $\forall s \in S$, and the value ρ , are used to penalize the differences that may exist between the scenario tour and the fixed overall tour, which serves as a reference point. Therefore, these penalties can be adjusted in order to drive all scenario subproblems to converge to a single design that is defined by \bar{y}_{ij} , $\forall i, j \in N$. Next sections describe how the search process guides the consensus among the scenarios.

6.3.2 Defining the overall capacity plan

Let ν define the index of the current iteration of the PH algorithm that sequentially solves subproblems (6.39)-(6.43) $\forall s \in S$ and then, produces an overall tour \bar{y}_{ij}^ν , $\forall i, j \in N$ using an aggregation operator on first-stage decision variables of each scenario subproblem $y_{ij}^{s\nu}$ $\forall s \in S$ and $\forall i, j \in N$. For the **mpTSP_s**, we simply use an average function to combine scenario solutions into a single solution where the weights are the probabilities associated with the scenarios:

$$\bar{y}_{ij}^\nu = \sum_{s \in S} p_s y_{ij}^{s\nu}, \quad \forall i, j \in N. \quad (6.44)$$

It is very important to note that (6.44) does not necessarily produce an overall feasible tour. Considering all scenario decision variable for a given pair of nodes $i, j \in N$ (e.g. $y_{ij}^{s\nu}$ $\forall s \in S$), if it has consensus for all scenarios, then $\bar{y}_{ij}^\nu \in \{0,1\}$. Otherwise, there is non-consensus and $0 < \bar{y}_{ij}^\nu < 1$, which is infeasible given the integrality constraints on the variables. In the case of non-convex problems, like the **mpTSP_s**, the aggregation operator defined above does not guarantee that the algorithm converge to an optimal solution. Moreover, it cannot ensure that a good (feasible) solution will be obtained for the present stochastic problem.

However, (6.44) is used as a reference solution with the objective of guiding the search process of the PH algorithm to identify arcs for which a consensus is possible.

In fact, when there is no consensus for a given pair of nodes $i, j \in N$, the value \bar{y}_{ij}^ν provides information concerning the convergence of the overall solution. More in detail, if \bar{y}_{ij}^ν is close to zero, the PH changes the Lagrangean multipliers in order to forbid the arc between nodes i and j in the overall tour. Otherwise, if the value of \bar{y}_{ij}^ν is close to one, the algorithm prompts the use of the associated arc.

To produce a feasible solution $\hat{y}_{ij}^\nu, \forall i, j \in N$ for the problem (6.39)-(6.43) using values $\bar{y}_{ij}^\nu, \forall i, j \in N$, a simple constructive heuristic is applied. The heuristics, summarized in the Algorithm 3, simply adds one arc after the other ordering them by non-increasing order of consensus. It should be noted that although the obtained feasible solution can be used as the reference point within the PH algorithm, whenever the consensus is low, \hat{y}_{ij}^ν it may wrongly bias the search process of penalization of the non-consensus scenario solutions with respect to the referencer point. Therefore, feasible solutions are only used to accelerate the convergence in advanced iterations of the algorithm. More in detail, when at least $\sigma\%$ of the first stage variables have reached consensus, we use the feasible solution as reference solution for the next iteration. The current implementation of this heuristic strategy uses $\sigma\% = 75\%$;

Algorithm 3 Constructive method for a feasible solution for **mpTSP_s**

Initialization

$$\{i', j'\} \leftarrow \{i, j\} : \max_{i, j \in N} \bar{y}_{ij}^\nu$$

$$N' \leftarrow \{i', j'\}$$

$$\hat{y}_{i'j'}^\nu \leftarrow 1$$

while An Hamiltonian cycle is not built **do**

$$i' \leftarrow j'$$

$$j' \leftarrow j : \max_{j \in N/\{N'\}} \bar{y}_{i'j}^\nu$$

$$N' \leftarrow N' \cup \{j'\}$$

$$\hat{y}_{i'j'}^\nu \leftarrow 1$$

6.3.3 Strategies for penalty adjustments

Given the scenario solutions obtained, i.e., the Hamiltonian cycle designed in each scenario, and using the aggregation operator (6.44), the methodology proposed generates a reference solution that serves as the overall tour. To induce consensus

among the scenario subproblems, the travel times are then adjusted to penalize non-consensus between scenario solutions and reference point. This section describes the strategy used to perform these adjustments for the **mpTSP_s**.

Recalling that PH uses a scenario decomposition based on an augmented Lagrangean relaxation, for a given iteration ν , let $\lambda_{ij}^{s\nu}$ define the value of Lagrangean multiplier associated with the relaxed non-anticipativity constraint for the decision variable on the arc between nodes i and j for the scenario s and let ρ_{ij}^ν define the value of the ratio for the quadratic penalty. Following the innovations for the PH algorithm proposed by [Watson and Woodruff \(2011\)](#), our strategy uses a variable-specific and per-iteration quadratic penalty ρ_{ij}^ν . Then, at each iteration ν , the values $\lambda_{ij}^{s\nu}$ and ρ_{ij}^ν are updated as follows, $\forall i, j \in N$ and $\forall s \in S$:

$$\lambda_{ij}^{s\nu} = \lambda_{ij}^{s\nu-1} + \rho_{ij}^{\nu-1}(y_{ij}^{s\nu} - \bar{y}_{ij}^\nu) \quad (6.45)$$

$$\rho_{ij}^\nu \leftarrow \alpha \rho_{ij}^{\nu-1}, \quad (6.46)$$

where $\alpha > 1$ is a given constant, ρ_{ij}^0 is fixed to a positive value to ensure that $\rho_{ij}^\nu \rightarrow \infty$ as the number of iterations ν increases. It is important to note that an inaccurate choice of ρ^0 may cause a premature convergence of the search process to a solution that may be quite far from an optimal one. To avoid this situation, we select ρ_{ij}^0 proportional to the unit-cost in the objective function of the associated decision variable: $\rho_{ij}^0 = \bar{c}_{ij}/2$.

For a particular scenario s , the equation (6.45) can reduce, increase or maintain unchanged the contribution in the objective function of the scenario problem. On the contrary, the value of ρ_{ij}^ν simply increases as the number of iteration grows according to (6.46).

6.3.4 Implementation of the methodology

We conclude the description of the algorithm by introducing the stopping criteria and the parallel implementation.

Although there are not, yet, theoretical results on the convergence of the PH algorithm in integer cases, the proposed methodology for **mpTSP_s** converges to a consensual solution in a finite number of iterations. However, stopping criteria

based on maximum limit of CPU time and number of iterations are implemented in the algorithm. Precisely, we defined the following criteria: one hour of CPU time and 200 iterations. However, letting the method stop on such criteria may entail solutions that have not been converged. Thus the algorithm proceeds in two phases. Firstly, the algorithm described in previous sections is executed until it stops if consensus is achieved for all scenarios or once it satisfies one the stopping criteria. Secondly, if necessary, the restricted RP (6.1)-(6.7), obtained by fixing all arcs for which the consensus has been achieved, is solved to optimality by means of branch-and-bound using CPLEX.

Concerning the parallel implementation, the proposed method implements the master-slave synchronous strategy presented in the Section 4.4.5 for the **SVCS-BPP**. The master controls the search, computes the global design, and performs the parameters updates, while the slave processors modify the costs associated to the arcs of the graph and solve the resulting scenario subproblems. Synchronization is performed at the end of each iteration.

6.4 Computational results

In this section, we report the experimental plan, the set of instances considered for the tests and the computational results.

PH algorithm validation (Section 6.4.2): **mpTSP_s** includes many binary variables in both the stages. This makes the model hard to solve when exact methods such as a commercial MIP solver. We discuss this issue and, then, we present extensive results showing that, compared with a commercial MIP solver, PH finds a good solutions in less time. More in detail, we compare the results (objective values and computational times) from the PH algorithm and the direct solution of the RP (6.8)-(6.17). Being the two models equivalent and following the computational results presented by [Perboli et al. \(2011\)](#), we consider the flow-based two-stage stochastic model, using CPLEX 12.5 as MIP solver.

Impact of the uncertainty (Section 6.4.3): These tests show the benefits of using the two-stage model with recourse compared to the WS and the EV. Here, the

goal is to measure the impact that uncertainty on the travel time has on the planning of the tour. An important point when comparing RP and EV is to determine the difference in the first-stage decisions, i.e., the arcs of the tour planned in advance. This analysis is done by comparing the solution obtained from PH and those obtained by solving a simpler deterministic problem in which the random parameters are replaced by their expected values. The latter does not guarantee optimality or feasibility.

All the tests were performed on an Intel I7 2 GHz workstation with 8 GB of RAM. Concerning the parallel computation, CPLEX and PH are executed with a limit of 8 parallel threads.

6.4.1 Instance set

No real-life instances are present in the literature for this stochastic version of the TSP problem. Then, with the purpose to analyze the effectiveness of the methodology proposed for the **mpTSP_s** with respect to the *RP* approach and to measure the impacts of uncertain travel time cost in TSP problems, we generate an instance set based on the real traffic sensor network of the medium sized city, Turin in Italy, which allows to better reflect real cases of City Logistics applications.

According to the guidelines presented in (Kenyon and Morton, 2003), instance are characteristics by the following inputs:

- Instance size. We considered instances with a number of nodes up to 200. This number is of the same order of magnitude of a day trip of the main parcel and courier delivery services as TNT and DHL. In particular, we split those instances into three sets: instances with up to 50 nodes (N50), up to 100 nodes (N100) and up to 200 nodes (N200).
- Nodes. Given a square of 14 km edge, which is equivalent to a medium sized city like Turin, nodes are mapped in such portion of plane and then partitioned into two subsets:
 - Central nodes: the nodes belonging to city center, which are the nodes in the circle with the center coincident with the geometric center of the 14 km square and a radius equals to 7 km;

- Suburban nodes: the nodes which are not central.
- Nodes distribution strategies. In order to define the spatial distribution of nodes, we divided the portion of plane in 12 neighborhoods (Q1 to Q12), 8 in the city center and 4 in the suburban area. For each neighborhood, nodes are randomly generated according with the following distribution strategies:
 - D1: the nodes are distributed only in the city center;
 - D2: the nodes are distributed only in the suburban area of the city;
 - D3: the nodes are distributed in all neighborhoods both central and suburban in ratio 3:1 respectively;
 - D4: the nodes are distributed in all neighborhoods both central and suburban in ratio 1:1 respectively.

Figure 6.1 shows the subdivision of the 14 km square in neighborhoods and, for each distribution strategy, the neighborhoods involved.

- Multiple paths. The number of paths between any pair of nodes is set to 3. The choice of this number is related to hybrid vehicles applications, where the typical number of power train modes is 3 (Tadei et al., 2014).
- Pair of nodes types: the pairs of nodes can be homogeneous or heterogeneous.
 - Homogeneous: they are pairs of nodes where the starting node i and the destination node j are both central or suburban. In this case all the multiple paths between the nodes present the empirical speed profile of a central or suburban speed sensor, respectively.
 - Heterogeneous: they are pairs of nodes where the starting node i and the destination node j belongs to a different subset. In this case the multiple paths between the nodes present the empirical speed profile of a central speed sensor for 1/3 of the paths and a suburban one for the 2/3 of them.
- Speed data. We build central and suburban speed profiles from real data on the traffic of Turin available at the website <http://www.5t.torino.it/5t/>. The data of the mean vehicle speed, expressed in kilometers per hour (km/h), are

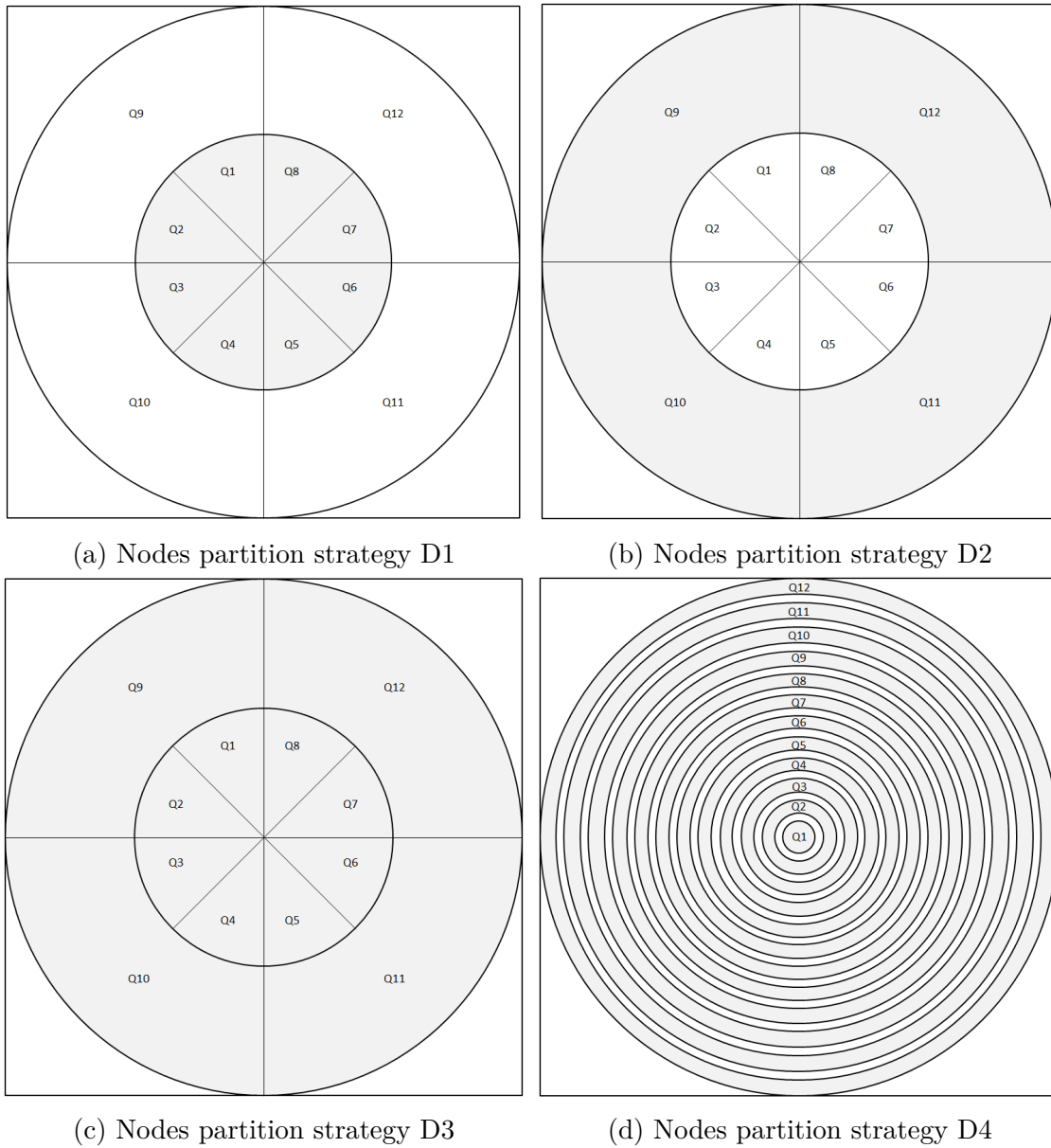


Figure 6.1: Subdivision in neighborhoods and admissible areas (grey) for each distribution strategy

accessible with an accuracy of 5 minutes. We aggregated them into blocks of 30 minutes, for a total of 48 observations per day. The instances refer to 50 central speed sensors locations and 100 suburban ones in the period since 13 to 17 February 2013 (see the two circles in Figure 6.2, giving the distribution of the actual sensors).

- Scenario tree generation. We assume that the random variable travel time oscillation has a finite number of possible outcomes at the end of the period considered. All possible discrete values that the random variable can assume, are represented by a finite set of scenarios and are assumed to be exogenous to the problem. Consequently, the probability distribution is not influenced as well by decisions. Making these assumptions we can represent the stochastic process travel time oscillation using a scenario tree which contains a root and finite set of leaves. Empirical velocity profile distributions v_{ij}^{ks} associated to the path k between i and j under scenario s are then generated as inverse of the Kaplan-Meier estimate of the cumulative distribution function (also known as the empirical cdf) of the speed real data on the traffic of Turin. From this distribution, a total of $s = 1, \dots, 100$ scenarios were generated both for the central and the suburban areas.
- Time blocks. Given the real observations of speed profiles and in order to represent different traffic flows cases, we use data corresponding to 8.00, 12.00 and 16.00, which represent the hours of maximum oscillation of the travel times due to traffic congestions.
- Path travel times. The travel time c_{ij}^{ks} is a function of the Euclidean distance between nodes i and $j \in N$, EC_{ij} , the type of pair of nodes, k , and the empirical velocity profile distributions v_{ij}^{ks} associated to the path k between i and j under scenario s . In details, this travel time has been computed as

$$c_{ij}^{ks} = \frac{EC_{ij}}{v_{ij}^{ks}} \quad (6.47)$$

and

$$\bar{c}_{ij} = \mathbb{E}_{s \in S} \frac{EC_{ij}}{\mathbb{E}_{k \in K_{ij}} [v_{ij}^{ks}]}. \quad (6.48)$$

is the average travel time over all scenarios $s \in S$ when an average empirical velocity is considered for all path $k \in K_{ij}$ between nodes i and j , where $\mathbb{E}_{s \in S}$ and $\mathbb{E}_{k \in K_{ij}}$ are the expectation operators. The random travel time oscillations

are then computed as

$$\Delta_{ij}^{ks} = c_{ij}^{ks} - \bar{c}_{ij} = \frac{EC_{ij}}{v_{ij}^{ks}} - \mathbb{E}_{s \in S} \frac{EC_{ij}}{\mathbb{E}_{k \in K_{ij}} [v_{ij}^{ks}]} \quad (6.49)$$

For each combination of the parameters mentioned above (i.e., 3 graph's sizes, 4 nodes distribution strategies, and 3 time blocks) we define 5 graphs. Moreover, for each graph, 10 different sets of scenarios S are generated according with the scenario tree generation method. This gives us a final set of 1800 instances.

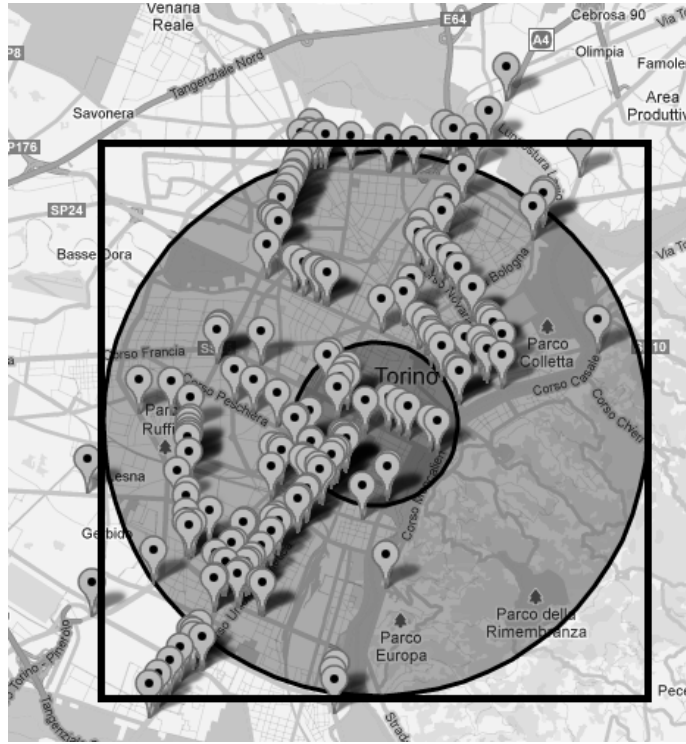


Figure 6.2: Distribution of central (dark gray circle) and suburban (light gray circle) speed sensors in the city of Turin in Italy.

6.4.2 Progressive Hedging validation

This first set of tests aims to qualify the usage of the PH method as a solution method for the \mathbf{mpTSP}_s . To make the comparison we consider the PH and we compare its results with respect to the two-stage models. In order to qualify the

results both of the PH and the two-stage models, we first perform a tuning of the number of scenarios needed to obtain stable results. The first outcome is that the two-stage models, from a pure computational effort perspective, cannot reach the optimality with a reasonable time limit (5 hours) for instances with 100 customers when the number of scenarios is more than 50 (e.g. the average optimality gap after 5 hours is greater than 3%). Moreover, even with such a small number of scenarios, they become impracticable with more than 100 customers. This is mainly due to the size of the models in terms of number of variables. This issue will be discussed in a more detailed way in the following part of this section. Thus, the two-stage models cannot be used to properly tune the number of scenarios. For this reasons, we performed this tuning by means of the PH. In order to perform this task we used a subset of 180 instances defined by selecting a single realization (i.e. a set of scenarios) for each combination of the parameters mentioned in Section 6.4.1. For every instance we considered both the first and the second stage objective functions. The results are presented in Figure 6.3, where in the horizontal axis we report the number of scenarios and in the vertical one the mean of the first and the total objective functions over the 180 instances. In particular, the continuous line represents the contribution of the first stage objective function, while the dotted line total objective function, respectively. Notice that, being the values in the first-stage objective functions the reference values of the costs for each arch (i, j) , the complete total objective function has, at convergence, a value at most equal to the first-stage one. The difference measures the improvement due to the recourse actions that selects the right path between two nodes. From the graph in Figure 6.3 it is evident how the convergence is substantially reached with 100 scenarios, while, for a full stability, one should fix the number of scenarios to 300. Unfortunately, fixing the scenarios to 300 means to have computational times with more than 1 hour with the PH and several hours with the two-stage with recourse models even with the smallest instances ($N50$). Thus, we fixed both for the PH and the two-stage with recourse models the number of scenarios to 100.

Table 6.1 reports the detailed comparison between CPLEX and the PH. This is performed on $N50$ instance set only, being the only one where CPLEX is able to solve and to prove optimality of all the instances in the time limit. We also tried to increase the time limit for CPLEX, but, while the number of nodes increases, the

optimality gap is not closed. This is mostly due to the large number of variables involved in the two-stage formulation. The first two columns reports the instance set and the node distribution strategy. Column 3 gives the percentage gap between the optimal solution given by CPLEX and the PH one. Finally columns 5 and 6 provide the computational time in seconds of the two solution methods, respectively.

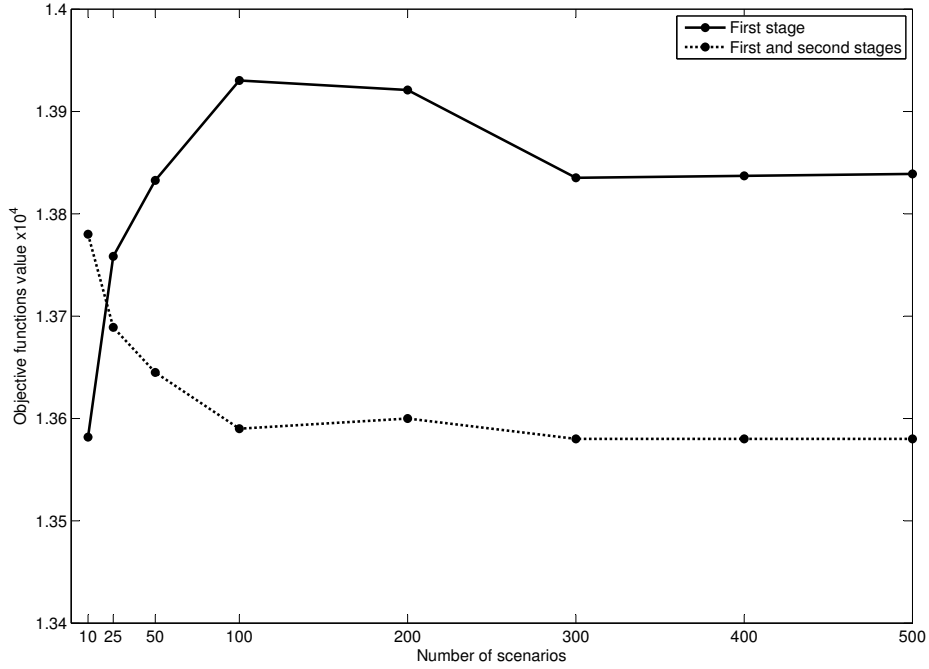


Figure 6.3: Sensitivity of the first stage objective function (solid line) and the total objective function (dotted line) with respect to the number of scenarios

| N | D | Gap% | T_{CPLEX} [s] | T_{PH} [s] |
|-----|----|------|-----------------|--------------|
| N50 | D1 | 0.04 | 60.6 | 170.8 |
| | D2 | 0.03 | 55.9 | 179.3 |
| | D3 | 0.04 | 138.8 | 171.9 |
| | D4 | 0.05 | 162.4 | 181.9 |

Table 6.1: Comparison of the results between CPLEX and the PH algorithm

The computational results show how the PH is accurate, with a gap from CPLEX

of less than 0.1% with all the node distribution strategy. By considering the computational effort, CPLEX is more efficient in distribution strategies $D1$ and $D2$, while the results become comparable in $D3$ and $D4$.

When we increase the number of nodes, the computational time of the PH increases almost linearly with the number of nodes, while CPLEX is unable to solve to optimality the instances in set $N100$ in the given time limit. Moreover, in these instances the solution found by CPLEX is or of the same quality or worst than the one found by the PH.

Up to $N100$ the PH never reaches neither the time limit, not the iteration one, while the iteration limit is reached in some of the $N200$ instances. From a pure computer architecture point of view, the PH uses far less resources than CPLEX. In fact, CPLEX requires up to 2GB for $N50$ and more than 4GB for $N100$ instances, thus imposing the use of 64 bit machine. On the contrary, the PH algorithm never uses more than 300 MB even when it addresses $N200$ instances. Due to the accuracy and the efficiency at dealing with larger size instances, we use for the results presented in the next sections the PH as the solver for the **mpTSP_s**.

Regarding the benefit of parallel implementation, the scaling of the parallel implementation is quite linear. This is guaranteed by the performance of the heuristic, which solves every problem in a similar computational time. These times ensure that the slaves have balanced loads and lead to a linear speedup. For $N50$, the best speedup is in average 5.6, obtained with 8 parallel threads. For the larger instances (e.g. $N100$ and $N200$), the average speedup is below the ideal value: it is 75% of the latter in the worst case.

6.4.3 Impact of uncertainty

This section is devoted to qualify the **mpTSP_s** by showing the benefits of using the two-stage with recourse models when compared to the Perfect Information case and the EV . This is done by considering the well known $EVPI$ and the VSS measures defined in equations (4.54) and (4.55), respectively.

As shown in subsection 6.4.2, using directly the two-stage models with recourse is not possible for instances with 100 customers and more. On the contrary, the PH is both accurate and efficient from the computational point of view. Thus, the

results presented in this section are all computed by means of the PH. Due to the large number of instances involved, the results are presented in an aggregated form in Table 6.2. The meaning of the columns reported is the following:

- Column 1: instance size (N);
- Column 2: time block ($Hour$);
- Column 3: node distribution strategy (D);
- Column 4: computational time in seconds needed by the PH. The value is a mean over the instances with the same values of the parameters N , $Hour$, and D ;
- Column 5: percentage $EVPI$ corresponding to the **mpTSP_s** stochastic solution RP , computed as $EVPI/RP \cdot 100$. The value is a mean of the instances with the same values of the parameters N , $Hour$, and D ;
- Column 6: maximum percentage $EVPI$ of the instances with the same values of the parameters N , $Hour$, and D ;
- Column 7: percentage VSS corresponding to the **mpTSP_s** stochastic solution RP , computed as $VSS/RP \cdot 100$. The value is a mean of the instances with the same values of the parameters N , $Hour$, and D ;
- Column 8: maximum percentage VSS of the instances with the same values of the parameters N , $Hour$, and D .

First of all we can highlight how the computational effort is stable with respect to the time block and the node distribution strategy and it increases almost linearly with the instance size. The overall computational effort is limited, being of the order of magnitude of 10 minutes for the 200 customers instances. This makes the PH a strategic tool that can be incorporated in larger Decision Support Systems.

The percentage $EVPI$ present values of about 30%, showing the relevance, for a decision maker, to have the information about the future in advance. Notice that the $EVPI$ is stable regardless the instance parameters N , $Hour$, and D .

When considering the *VSS*, we can see how it is increasing both in mean and in maximum values while the size of the instance increases. In particular, the gap between the expected value solution and the stochastic solution becomes relevant when the number of nodes is between 100 and 200, which is the typical size of the day tour of a single vehicle in the parcel delivery and courier services in a medium and large city. Even when considering small instances (50 nodes) the *VSS* is relevant, with values of the maximum gap up to 6% and it shows the losses obtained by following the tour suggested by the deterministic solution. When analyzing the results with respect to the node distribution strategy, the most critical ones are *D4* and *D3*. This finding is relevant, being the latter the most representative of the distribution of customers in a city (Perboli et al., 2013).

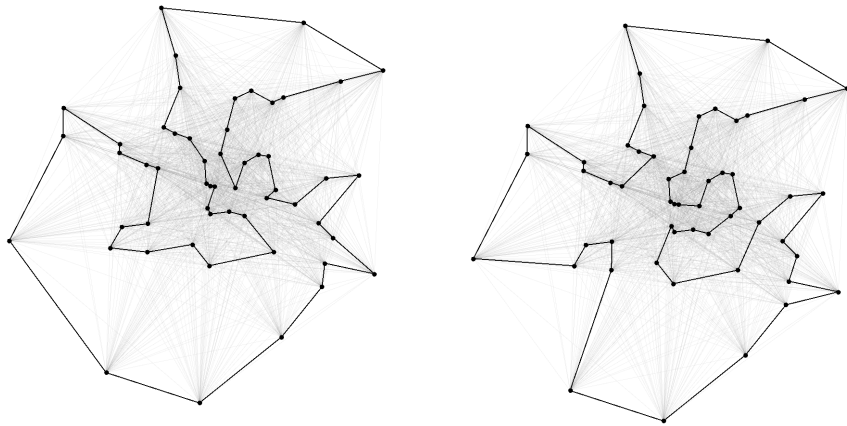
Notice that, due to the combinatorial nature of the problem, measures of the quality of deterministic solution (Maggioni and Wallace, 2012) like *loss using the skeleton solution LUSS*, obtained fixing at zero all first stage variables which are at zero in the expected value solution and then solving the stochastic program, correspond to *VSS*. The same for the *loss of upgrading the deterministic solution LUDS*, obtained by considering the expected value solution variables as a starting point to the stochastic model; the reason can be explained as follows: if an arc has been opened in the *EV* solution, then it must be used also in the stochastic setting, on the contrary if an arc is closed in the *EV* solution it can be opened in the stochastic one. But since the *EV* solution is a cycle and the stochastic solution cannot add or remove any arc because of the subtour elimination constraints and of the *LUDS* condition, then $LUDS = VSS$.

An important point when comparing the solutions of the Recourse and the Expected Value problems is to determine how much the first stage decisions, i.e., the sequence of the nodes to visit, differ in the two problem solutions. We analyzed this issue, seeing how the two decisions differ of more than 15%. An example of this issue is given in Figure 6.4, where the first stage decisions are highlighted for an instance with 50 customers. In particular, Figure 6.4c overlaps the two solutions, presenting the arcs differing in the two solutions with a long-dotted line for the Expected Value Problem solution and a short-dotted line for the Recourse Problem one, respectively. From the pictures we can see how the gap in terms of objective functions (and recourse actions in particular) is determined by a change of a relevant

part of the central area tour, with 11 arcs involved, corresponding to about 20% of the first stage decisions. A specific point is the suburban arc in the South-West portion of the Expected Value Problem solution, which is inserted in this solution for its mean value, which is unfortunately misleading in terms of actual cost for the variance that the cost oscillation has in the different scenarios. On the contrary, the Recourse Problem solution considers this issue, removing the suburban arc and consequently updating the overall tour, causing a reduction of the overall cost of 3.89%.

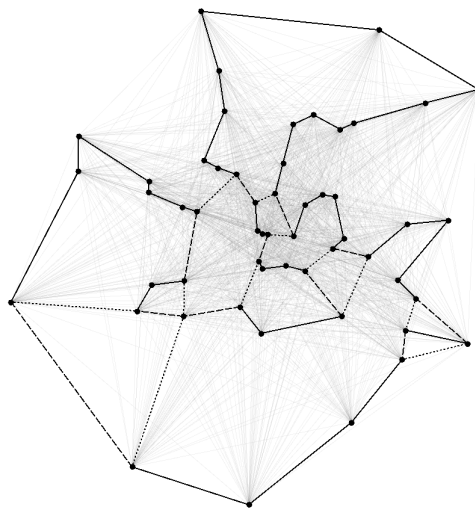
| N | Hour | D | T_{PH} [s] | EVPI% | max EVPI% | VSS% | max VSS% |
|------|-------|----|--------------|-------|-----------|------|----------|
| N50 | 8.00 | D1 | 174.71 | 32.67 | 38.81 | 2.50 | 4.92 |
| | | D2 | 174.06 | 30.62 | 35.48 | 3.09 | 5.66 |
| | | D3 | 174.49 | 27.41 | 32.07 | 2.94 | 6.43 |
| | | D4 | 178.49 | 28.04 | 31.70 | 2.76 | 8.52 |
| | 12.00 | D1 | 159.31 | 31.58 | 36.92 | 2.81 | 4.82 |
| | | D2 | 188.16 | 30.18 | 34.57 | 3.73 | 6.43 |
| | | D3 | 168.3 | 26.58 | 30.17 | 2.60 | 5.11 |
| | | D4 | 189.60 | 26.60 | 30.98 | 2.32 | 3.81 |
| | 16.00 | D1 | 178.42 | 32.32 | 38.45 | 3.21 | 6.28 |
| | | D2 | 175.68 | 29.97 | 33.29 | 3.70 | 5.83 |
| | | D3 | 172.97 | 29.40 | 30.89 | 2.69 | 5.60 |
| | | D4 | 171.40 | 26.65 | 30.77 | 2.56 | 4.71 |
| N100 | 8.00 | D1 | 335.91 | 29.40 | 34.44 | 6.82 | 11.77 |
| | | D2 | 336.09 | 30.09 | 34.68 | 6.80 | 10.13 |
| | | D3 | 338.38 | 29.27 | 33.30 | 6.70 | 9.77 |
| | | D4 | 368.45 | 29.00 | 33.71 | 7.20 | 14.07 |
| | 12.00 | D1 | 357.84 | 29.15 | 32.96 | 5.67 | 8.80 |
| | | D2 | 361.18 | 30.42 | 33.47 | 6.16 | 8.57 |
| | | D3 | 393.69 | 29.57 | 32.52 | 6.07 | 10.20 |
| | | D4 | 382.42 | 29.06 | 31.84 | 6.28 | 12.33 |
| | 16.00 | D1 | 332.12 | 28.71 | 33.23 | 6.06 | 8.46 |
| | | D2 | 367.57 | 29.99 | 34.45 | 7.53 | 13.84 |
| | | D3 | 336.48 | 28.62 | 32.81 | 6.47 | 10.41 |
| | | D4 | 387.26 | 28.85 | 32.92 | 6.63 | 14.51 |
| N200 | 8.00 | D1 | 704,13 | 31,76 | 38,19 | 7,66 | 10,16 |
| | | D2 | 694,85 | 31,87 | 36,53 | 8,12 | 12,87 |
| | | D3 | 727,09 | 28,85 | 34,59 | 7,61 | 11,51 |
| | | D4 | 702,16 | 28,89 | 34,17 | 7,88 | 11,67 |
| | 12.00 | D1 | 635,44 | 30,63 | 37,39 | 7,80 | 11,55 |
| | | D2 | 654,46 | 30,61 | 35,21 | 7,69 | 11,87 |
| | | D3 | 640,05 | 26,70 | 30,77 | 7,50 | 11,16 |
| | | D4 | 629,21 | 26,93 | 31,69 | 7,60 | 10,69 |
| | 16.00 | D1 | 667.12 | 31,70 | 38,76 | 7,87 | 12,99 |
| | | D2 | 633.10 | 30,55 | 35,16 | 7,76 | 10,63 |
| | | D3 | 646,27 | 26,46 | 31,94 | 7,65 | 11,36 |
| | | D4 | 695,66 | 27,00 | 31,75 | 7,22 | 10,50 |

Table 6.2: Full instance set results: *EVPI* and *VSS* values comparison



(a) Expected Value Problem solution

(b) Recourse Problem solution



(c) Solutions comparison

Figure 6.4: Comparison of the first-stage Expected Value Problem solution (a) and the Recourse Problem solution (b). Figure (c) shows the common arcs (solid line), arcs used only in the Expected Value Problem solution (long-dotted line) and arcs used only in Recourse Problem one (short-dotted line).

Chapter 7

Conclusions

In this thesis, we have introduced and studied three new planning problems arising in City Logistic applications, where the design and the optimization of the freight transportation process is essential. These problems are the stochastic variable cost and size bin packing problem (**SVCSBPP**), the multi-handler knapsack problem under uncertainty (**MHKP_u**) and the multi-path traveling salesman problem with stochastic travel times (**mpTSP_s**), which explicitly take into account the presence of uncertainty on parameters, constraints and activities that characterize the system of urban areas. We have addressed these new problems in order to overcome a noteworthy portion of a gap in the literature concerning the development of models for City Logistics solutions in terms of comprehensive study on the sources of uncertainty related to and methodologies in order to solve them in efficient and accurate way.

Our main results concerned the development of problems and methodologies of these new problems characterized by the presence of different sources of stochasticity that strongly affect the long and medium term decisions. The problems proposed play crucial part in the supply chain proposed in City Logistics solutions. The trend is to substitute traditional single-echelon routing structures with two-echelon ones and to introduce satellites and the use of environmental friendly vehicles. **SVCSBPP** may be used to plan the capacity of the fleet and of the transshipment satellites. In the satellites different sequences of consolidation operations are done by different handler. The selection of handler with different skill levels may be planned with

the \mathbf{MHKP}_u . Finally, the \mathbf{mpTSP}_s plans the tour of environmental friendly vehicles to cope with different working days and to synchronize the transshipment operations between urban truck and city freighters that occur in satellites. To mitigate the computational difficulty associated with these stochastic problems, we have proposed heuristic strategies based on the Progressive Hedging algorithm (with the exception of the \mathbf{MHKP}_u , for which the deterministic approximation derived is effective and guarantees an high level of accuracy of solutions). The heuristics use specialized solving strategies to solve the deterministic subproblems obtained from the scenario decomposition of the stochastic model. Moreover, we implemented advanced strategies that directly operate on the data of the problem (e.g. cost of the bins) in order to accelerate the convergence, and that deal with the symmetry of the solution space, a typical property of bin packing problems. Extensive computational tests on a large set of instances, including realistic applications, show that heuristics outperform the use of commercial solvers. The computational time is reduced of several orders, assuring an high accuracy of the solutions quality.

Two future directions need to be further investigated.

First, the problems proposed in this thesis only partially cover realistic applications in the urban context. The $\mathbf{SVCSBPP}$ is based on the hypothesis that the capacity is completely available at the shipping day. This hypothesis may be not always true. Considering the parcel delivery services, some deliveries have not been carried the previous day (e.g. missing customer), which have to be delivered with the next shipment, reducing the available capacity. The rental of extra capacity at premium cost is required to compensate this effect. One can generalize the $\mathbf{SVCSBPP}$ explicitly introducing this unknown loss of capacity. Similarly, the \mathbf{mpTSP}_s considers arcs with a stochastic time-independent travel time, which means that the travel time does not varies during the tour, or more in general, during the day. To represent more realistic applications with time-dependent travel times, a multi-stage stochastic model is needed and more sophisticated methodologies are required to deal with the complexity of the model. Moreover, additional technology constraints related to the state of charge level of the battery of hybrid vehicles should be considered. This issue becomes of particular interest, being both the stochastic oscillation of the costs and the state-of-charge level of the battery originated by the same sources of uncertainty (e.g. travel times, traffic congestion, number of

Stop&Go).

Second, two practical issues can drastically affect the efficiency of solving methods based on Progressive Hedging ideas and need to be investigated: the communication overhead and the solution time-variability. The first relates to the relative balance of communication and computation involved in the solution of scenario subproblems. When subproblems solve quickly, as in the case of the **SVCSBPP** and the **mpTSP_s**, communication may degrade the parallel efficiency. The second issue relates to the presence of the barrier synchronization in the master algorithm. If solving effort of subproblem presents high variability (e.g. using a commercial MIP solver), parallel efficiency will degrade dramatically as the number of scenarios increases. To mitigate this effects, alternative approaches have to be considered for parallel implementations. [Somervell \(1998\)](#) describes several asynchronous implementations of the PH algorithm, including asynchronism within each iteration and not solving all of the scenarios within each iteration. He showed that updating penalties immediately after a scenario subproblem has been solved fails to converge even on very small problems, while waiting for at least half of solutions leads to a good or the optimal solution. Instead, no formal proposals have been explored for cooperative parallel schemes ([Crainic and Toulouse, 2010](#)) applied to PH. The research on asynchronous and cooperative schemes for heuristics based on the Progressive Hedging ideas is still an open problem.

Currently, we are developing an innovative PH-based heuristic method that makes use of multiple solutions of scenario subproblems. We named these solutions as *candidate solutions*. In order to provide an overall solution, the PH proposed in literature aggregates one solution (e.g. the optimal solution) for each scenario subproblem weighted by the probability of the scenario. On the contrary, the new method involves an aggregation operator that assigns to each candidate solution a weight that reflects its relative importance. More in detail, the weight considers, on one hand, the probability of the scenario subproblem and, on the other hand, the quality of the candidate solution (e.g. difference of the objective function from the best candidate solution) and the similarity of the candidate solution with respect to the current overall solution. While the factor related to the probability of scenarios remains constant during the entire search process, the second factor may change according to the state of the search. At the beginning, the aggregation of candidate

solutions is guided by the quality of solutions, then, when the consensus is close, the similarity of solutions will be promoted.

The algorithm has been applied to the **SVCSBPP**. First experiments show its advantages: the computational time is always reduced by at least two times and the iterations needed to meet the consensus are less than 5 for all instances considered. Further experimentations are required on different stochastic problems characterized by a higher computational effort such as the network design ([Crainic et al., 2011b](#)). These tests allow the calibration of the algorithm before the definition of asynchronous and cooperative implementations.

Bibliography

- Aissaoui, N., Haouari, M., and Hassini, E. (2007). Supplier selection and order lot sizing modeling: A review. *Computers & Operations Research*, 34(12):3516 – 3540. Operations Research and Outsourcing.
- Applegate, D. L., Bixby, R. E., Chvátal, V., and Cook, W. J. (2007). *The Traveling Salesman Problem: A Computational Study*. Princeton University Press, Princeton, NJ, USA.
- Baldi, M. M., Crainic, T. G., Perboli, G., and Tadei, R. (2011). The generalized bin packing problem. Technical report, CIRRELT, CIRRELT-2011-39.
- Baldi, M. M., Crainic, T. G., Perboli, G., and Tadei, R. (2012a). The generalized bin packing problem. *Transportation Research Part E*, 48(6):1205–1220.
- Baldi, M. M., Ghirardi, M., Perboli, G., and Tadei, R. (2012b). The capacitated transshipment location problem under uncertainty: A computational study. *PRO-CEDIA - Social and Behavioral Sciences*, 39:424–436.
- Beaudry, A., Laporte, G., Melo, T., and Nickel, S. (2010). Dynamic transportation of patients in hospitals. *OR Spectrum*, 32:77–107.
- Benjelloun, A. and Crainic, T.G. (2008). Trends, Challenges, and Perspectives in City Logistics. In *Transportation and Land Use Interaction, Proceedings TRANSLU'08*, pages 269–284. Editura Politecnica Press, Bucharest, Romania.
- Bertazzi, L., Chan, L. M. A., and Speranza, M. G. (2007). Analysis of practical policies for a single link distribution system. *Naval Research Logistics (NRL)*, 54(5):497–509.

- Bertazzi, L. and Speranza, M. (2012). Inventory routing problems: an introduction. *EURO Journal on Transportation and Logistics*, 1(4):307–326.
- Bertazzi, L. and Speranza, M. G. (2005). Worst-case analysis of the full load policy in the single link problem. *International Journal of Production Economics*, 93â€“94(0):217 – 224. Proceedings of the Twelfth International Symposium on Inventories.
- Bertsimas, D. and Van Ryzin, G. (1991). A stochastic and dynamic vehicle-routing problem in the euclidean plane. *Operations Research*, 39:601–615.
- BESTUFS Consortium (2008). Best urban freight solutions. european union project.
- Birge, J. (1982). The value of the stochastic solution in stochastic linear programs with fixed recourse. *Mathematical Programming*, 24(1):314–325.
- Birge, J. and Louveaux, F. (1997). *Introduction to Stochastic Programming*. Springer Series in Operations Research and Financial Engineering. Springer.
- Boerkamps, J. H. K., van Binsbergen, A. J., and Bovy, P. H. L. (2000). Modeling Behavioral Aspects of Urban Freight Movement in Supply Chains. *Transportation Research Record*, 1725:17–25.
- Browne, M., Allen, J., and Leonardi, J. (2011). Evaluating the use of an urban consolidation centre and electric vehicles in central london. *{IATSS} Research*, 35(1):1 – 6.
- Chen, F., Federgruen, A., and Zheng, Y.-S. (2001). Coordination mechanisms for a distribution system with one supplier and multiple retailers. *Management Science*, 47(5):693–708.
- Chen, H.-K., Hsueh, C.-F., and Chang, M.-S. (2006). The real-time time-dependent vehicle routing problem. *Transportation Research Part E: Logistics and Transportation Review*, 42:383–408.
- CITY FREIGHT Consortium (2005). Inter- and intra- urban freight distribution networks, european union project.

- CITY PORTS Consortium (2005). A network of cities following a co-ordinated approach to develop feasible and sustainable city logistics solutions.
- CITYBOX Consortium (2002). City box - small loading unit for urban distribution.
- CITYLOG Consortium (2010). CITYLOG European project.
- CIVITAS I Consortium (2006). City-vitality-sustainability.
- Coffman Jr., E., So, K., Hofri, M., and Yao, A. (1980). A stochastic model of bin-packing. *Information and Control*, 44(2):105–115.
- Cohn, A. M. and Barnhart, C. (1998). The stochastic knapsack problem with random weights: A heuristic approach to robust transportation planning. In *Proceedings of the Triennial Symposium on Transportation Analysis*.
- Cook, W. J. (2012). *In Pursuit of the Traveling Salesman: Mathematics at the Limits of Computation*. Princeton University Press, Princeton, NJ, USA.
- Correia, I., Gouveia, L., and Saldanha-da-Gama, F. (2008). Solving the variable size bin packing problem with discretized formulations. *Computers & Operations Research*, 35:2103–2113.
- Crainic, T., Gobbato, L., Perboli, G., and Rei, W. (2014a). Logistics capacity planning: A stochastic bin packing formulation and a progressive hedging meta-heuristic. Technical report, CIRRELT, CIRRELT-2014-66.
- Crainic, T., Mancini, S., Perboli, G., and Tadei, R. (2013a). Grasp with path relinking for the two-echelon vehicle routing problem. In Di Gaspero, L., Schaerf, A., and Stützle, T., editors, *Advances in Metaheuristics*, volume 53 of *Operations Research/Computer Science Interfaces Series*, pages 113–125. Springer New York.
- Crainic, T., Ricciardi, N., and Storchi, G. (2009a). Models for evaluating and planning city logistics systems. *Transportation Science*, 43:432–454.
- Crainic, T. and Toulouse, M. (2010). Parallel meta-heuristics. In Gendreau, M. and Potvin, J.-Y., editors, *Handbook of Metaheuristics*, volume 146 of *International Series in Operations Research & Management Science*, pages 497–541. Springer US.

- Crainic, T. G., Errico, F., Rei, W., and Ricciardi, N. (2012a). Modelling demand uncertainty in two-tier city logistics tactical plan. Technical report, CIRRELT, CIRRELT-2012-65.
- Crainic, T. G., Fu, X., Gendreau, M., Rei, W., and Wallace, S. W. (2011a). Progressive hedging-based metaheuristics for stochastic network design. *Networks*, (58):114–124.
- Crainic, T. G., Fu, X., Gendreau, M., Rei, W., and Wallace, S. W. (2011b). Progressive hedging-based metaheuristics for stochastic network design. *Networks*, 58(2):114–124.
- Crainic, T. G., Gobbato, L., Perboli, G., Rei, W., Watson, J. P., and Woodruff, D. L. (2013b). Bin packing problems with uncertainty on item characteristics: An application to capacity planning in logistics. Technical Report CIRRELT-2013-31, CIRRELT.
- Crainic, T. G., Gobbato, L., Perboli, G., Rei, W., Watson, J. P., and Woodruff, D. L. (2014b). Bin packing problems with uncertainty on item characteristics: An application to capacity planning in logistics. *Procedia - Social and Behavioral Sciences*, 111:654–662.
- Crainic, T. G. and Kim, K. H. (2007). Intermodal Transportation. In Barnhart, C. and Laporte, G., editors, *Transportation*, volume 14 of *Handbooks in Operations Research and Management Science*, chapter 8, pages 467–537. North-Holland, Amsterdam.
- Crainic, T. G., Mancini, S., Perboli, G., and Tadei, R. (2010). Two-echelon vehicle routing problem: A satellite location analysis. *PROCEDIA - Social and Behavioral Sciences*, 2:5944–5955.
- Crainic, T. G., Perboli, G., Pezzuto, M., and Tadei, R. (2007). New bin packing fast lower bounds. *Computers & Operations Research*, 34:3439–3457.
- Crainic, T. G., Perboli, G., Rei, W., and Tadei, R. (2011c). Efficient lower bounds and heuristics for the variable cost and size bin packing problem. *Computers & Operations Research*, 38:1474–1482.

- Crainic, T. G., Perboli, G., Watson, J.-P., Woodruff, D. L., and Rei, W. (2012b). A stochastic bin packing model for logistical capacity planning. In *Proceedings of Odysseus 2012, Mykonos, Greece*, pages 351 – 355.
- Crainic, T. G., Tadei, R., Perboli, G., and Baldi, M. M. (2012c). The generalized bin packing problem: Models and bounds. In *Odysseus 2012, the 5th International Workshop on Freight Transportation and Logistics, Mykonos (Greece), May 21–25, 2012*.
- Crainic, T.G., Gendreau, M., and Potvin, J.-Y. (2009b). Intelligent Freight-Transportation Systems: Assessment and the Contribution of Operations Research. *Transportation Res. C*, 17:541–557. doi:10.1016/j.trc.2008.07.002.
- Crainic, T.G., Ricciardi, N., and Storchi, G. (2004). Advanced Freight Transportation Systems for Congested Urban Areas. *Transportation Res. C*, 12(2):119–137.
- DAUIN (2014). Hpc cluster at politecnico di torino, <http://www.dauin-hpc.polito.it>.
- DeGroot, M. and Schervish, M. (2002). *Probability and Statistics (Third Edition)*. Addison Wesley, Reading.
- Diabat, A. and Simchi-Levi, D. (2009). A carbon-capped supply chain network problem. In *Industrial Engineering and Engineering Management, 2009. IEEM 2009. IEEE International Conference on*, pages 523–527.
- Dimitrakopoulos, G. and Demestichas, P. (2010). Intelligent transportation systems. *Vehicular Technology Magazine, IEEE*, 5(1):77–84.
- Domencich, T. and McFadden, D. (1975). *Urban travel dynamics: a behavioral analysis*. North Holland, Amsterdam.
- eDRUL Consortium (2004). ecommerce enabled, demand responsive urban logistics.
- Epstein, L., Favrholt, L. M., and Levin, A. (2011). Online variable-sized bin packing with conflicts. *Discrete Optimization*, 8(2):333 – 343.
- Fazi, S., van Woensel, T., and Fransoo, J. C. (2012). A stochastic variable size bin packing problem with time constraints. Technical report, Technische Universiteit Eindhoven.

- Fleischmann, B., Gnutzmann, S., and Sandvoss, E. (2004). Dynamic vehicle routing based on online traffic information. *Transportation Science*, 38:420–433.
- Ford, D. J. (2001). Inbound freight consolidation: a simulation model to evaluate consolidation rules. Master’s thesis, Massachusetts Institute of Technology.
- FV-2000 Consortium (1999). Quality of freight villages structure and operations.
- Galambos, J., Lechner, J., and Simiu, E. (1994). *Extreme Value Theory and Applications*. Kluwer.
- Gendreau, M., Guertin, F., Potvin, J.-Y., and Taillard, E. (1999). Parallel tabu search for real-time vehicle routing and dispatching. *Transportation Science*, 33:381–390.
- Gendreau, M., Laporte, G., and Séguin, R. (1996). A Tabu Search Algorithm for the Vehicle Routing Problem with Stochastic Demands and Customers. *Operations Research*, 44(3):469–477.
- Gobbato, L., Maggioni, F., and Perboli, G. (2014). New models and methods for the multi-path travelling salesman problem with stochastic travel costs. In *Proceedings of the International Conference on Applied Mathematical Optimization and Modelling*.
- Goel, A. and Indyk, P. (1999). Stochastic load balancing and related problems. In *40th Annual Symposium on Foundations of Computer Science*, pages 579–586.
- Goemans, M. X. and Bertsimas, D. (1991). Probabilistic analysis of the Held and Karp lower bound for the Euclidean traveling salesman problem. *Math. Oper. Res.*, 16:72–89.
- Golden, B. L., Raghavan, S., and Wasil, E. A. (2008). *The Vehicle Routing Problem: Latest Advances and New Challenges*. Springer.
- Gonzalez-Feliu, J., Morana, J., et al. (2010). Are city logistics solutions sustainable? the cityporto case. *TeMA-Trimestrale del Laboratorio Territorio Mobilità Ambiente*, 3(2):55–64.

- Gumbel, E. J. (1958). *Statistics of Extremes*. Columbia University Press.
- Güner, A. R., Murat, A., and Chinnam, R. B. (2012). Dynamic routing under recurrent and non-recurrent congestion using real-time ITS information. *Computers & Operations Research*, 39:358–373.
- Han, X., Peng, C., Ye, D., Zhang, D., and Lan, Y. (2010). Dynamic bin packing with unit fraction items revisited. *Inf. Process. Lett.*, 110(23):1049–1054.
- Hansen, W. (1959). How accessibility shapes land use. *Journal of the American Institute of Planners*, 25:73–76.
- Henig, M. (1990). Risk criteria in a stochastic knapsack problem. *Operations Research*, 38:820–825.
- Hvattum, L. M., Lokketangen, A., and Laporte, G. (2006). Solving a dynamic and stochastic vehicle routing problem with a sample scenario hedging heuristic. *Transportation Science*, 40:421–438.
- Hvattum, L. M., Lokketangen, A., and Laporte, G. (2007). A branch-and-regret heuristic for stochastic and dynamic vehicle routing problems. *Networks*, 49:330–340.
- Ichoua, S., Gendreau, M., and Potvin, J.-Y. (2006). Exploiting knowledge about future demands for real-time vehicle dispatching. *Transportation Science*, 40:211–225.
- IDIOMA Consortium (2001). Innovative distribution with intermodal freight operation in metropolitan areas.
- ILOG Inc. (2012). *IBM ILOG CPLEX v12.1 User's Manual*.
- Institute of City Logistics (1999). Institute of city logistics web site.
- Iwama, K. and Taketomi, S. (2002). Removable online knapsack problems. In Widmayer, P., Eidenbenz, S., Triguero, F., Morales, R., Conejo, R., and Hennessy, M., editors, *Automata, Languages and Programming*, volume 2380 of *Lecture Notes in Computer Science*, pages 773 – 773. Springer Berlin / Heidelberg.

- Jaillet, P. (1988). A priori solution of a traveling salesman problem in which a random subset of the customers are visited. *Operations Research*, 36:929–936.
- Kall, P. and Wallace, S. W. (1994). *Stochastic Programming*. John Wiley.
- Kaut, M., Vladimirou, H., Wallace, S., and Zenios, S. (2007). Stability analysis of portfolio management with conditional value-at-risk. *Quantitative Finance*, 7(4):397–409.
- Kenyon, A. S. and Morton, D. P. (2003). Stochastic vehicle routing with random travel times. *Transportation Science*, 37:69–82.
- Kleinberg, J., Rabani, Y., and Tardos, E. (2000). Allocating bandwidth for bursty connections. *SIAM Journal on Computing*, 30:191–217.
- Klibi, W., Martel, A., and Guitouni, A. (2010). The design of robust value-creating supply chain networks: A critical review. *European Journal of Operational Research*, 203(2):283 – 293.
- Larsen, A., Madsen, O. B. G., and Solomon, M. M. (2004). The a priori dynamic traveling salesman problem with time windows. *Transportation Science*, 38:459–472.
- Leipala, T. (1978). On the solutions of stochastic traveling salesman problems. *European Journal of Operational Research*, 2(4):291–297.
- Li, F., Liu, T., Zhang, H., Cao, R., Ding, W., and Fasano, J. (2008). Distribution center location for green supply chain. In *Service Operations and Logistics, and Informatics, 2008. IEEE/SOLI 2008. IEEE International Conference on*, volume 2, pages 2951–2956.
- Li, J.-Q., Mirchandani, P. B., and Borenstein, D. (2009a). A Lagrangian heuristic for the real-time vehicle rescheduling problem. *Transportation Res. E*, 45:419–433.
- Li, J.-Q., Mirchandani, P. B., and Borenstein, D. (2009b). Real-time vehicle rerouting problems with time windows. *European Journal of Operational Research*, 194:711–727.

- Lisser, A. and Lopez, R. (2010). Stochastic quadratic knapsack with recourse. *Electronic Notes in Discrete Mathematics*, 36:97–104.
- Lueker, G. S. (1983). Bin packing with items uniformly distributed over intervals [a,b]. In *Proceedings of 24th Annual Symposium on Foundations of Computer Science*, pages 289–297. IEEE Comput. Soc. Press.
- Maggioni, F., Kaut, M., and Bertazzi, L. (2009). Stochastic optimization models for a single-sink transportation problem. *Computational Management Science*, 6:251–267.
- Maggioni, F. and Wallace, S. (2012). Analyzing the quality of the expected value solution in stochastic programming. *Annals of Operations Research*, 200:37–54.
- Mallidis, I., Dekker, R., and Vlachos, D. (2012). The impact of greening on supply chain design and cost: a case for a developing region. *Journal of Transport Geography*, 22(0):118 – 128.
- Marasco, A. (2008). Third-party logistics: A literature review. *International Journal of Production Economics*, 113(1):127 – 147. Research and Applications in E-Commerce and Third-Party Logistics Management Special Section on Meta-standards in Operations Management: Cross-disciplinary perspectives.
- Martello, S. and Toth, P. (1979). The 0-1 knapsack problem. In Christofides, N., Mingozzi, A., Toth, P., and Sandi, C., editors, *Combinatorial Optimization*, pages 237–279. John Wiley & Sons.
- Mayer, J. (1998). *Stochastic Linear Programming Algorithms: A Comparison Based on a Model Management System*. Optimization theory and applications. Taylor & Francis.
- McKinnon, P., Cullinane, S., Whiteing, A., and Browne, P. (2010). *Green Logistics: Improving the Environmental Sustainability of Logistics*. Kogan Page.
- Merzifonluoglu, Y., Geunes, J., and Romeijn, H. E. (2012). The static stochastic knapsack problem with normally distributed item sizes. *Mathematical Programming*, 134:459–489.

- Mitrović-Minić, S. and Laporte, G. (2004). Waiting strategies for the dynamic pickup and delivery problem with time windows. *Transportation Res. B*, 38:635–655.
- Monaci, M. (2002). *Algorithms for packing and scheduling problems*. PhD thesis, Università di Bologna, Bologna, Italy.
- Monczka, R., Handfield, R., Giunipero, L., and Patterson, J. (2008). *Purchasing and Supply Chain Management*. Cengage Learning.
- Mu, Q., Fu, Z., Lysgaard, J., and Eglese, R. (2011). Disruption management of the vehicle routing problem with vehicle breakdown. *Journal of the Operational Research Society*, 62:742–749.
- Novoa, C. and Storer, R. (2009). An approximate dynamic programming approach for the vehicle routing problem with stochastic demands. *European Journal of Operational Research*, 196:509–515.
- OECD (2012). Outlook to 2050: The consequences of inaction. Technical report, OECD Publishing.
- Peng, J. and Zhang, B. (2012). Bin packing problem with uncertain volumes and capacities. <http://or.sc.edu.cn/online/120601.pdf>.
- Perboli, G., Maggioni, F., and Tadei, R. (2013). The multi-path traveling salesman problem with stochastic travel costs: A city logistics computational study. In *Proceedings of the XIII International Conference on Stochastic Programming*.
- Perboli, G., Tadei, R., and Baldi, M. (2012). The stochastic generalized bin packing problem. *Discrete Applied Mathematics*, 160:1291–1297.
- Perboli, G., Tadei, R., and Gobbato, L. (2014). The multi-handler knapsack problem under uncertainty. *European Journal of Operational Research*, 236(3):1000–1007.
- Perboli, G., Tadei, R., and Vigo, D. (2011). The two-echelon capacitated vehicle routing problem: Models and math-based heuristics. *Transportation Science*, 45:364–380.

- PIEVERDE Consortium (2014). Piattaforma ibridi elettrici veicoli e reti di distribuzione ecosostenibile.
- Pillac, V., Gendreau, M., Guillet, C., and Medaglia, A. L. (2013). A review of dynamic vehicle routing problems. *European Journal of Operational Research*, 225(1):1–11.
- Pisinger, D. (2005). Where are the hard knapsack problems? *Computers & Operations Research*, 32:2271–2284.
- PROMIT Consortium (2009). Promote innovative intermodal freight transport.
- Rhee, W. T. and Talagrand, M. (1993a). On-line bin packing of items of random size, part i. *SIAM Journal on Computing*, 18:438–445.
- Rhee, W. T. and Talagrand, M. (1993b). On-line bin packing of items of random size, part ii. *SIAM Journal on Computing*, 18:473–486.
- Ricciardi, N., Tadei, R., and Grosso, A. (2002). Optimal facility location with random throughput costs. *Computers & Operations Research*, 29(6):593–607.
- Rizk, N., Martel, A., and D’Amours, S. (2008). Synchronized production-distribution planning in a single-plant multi-destination network. *Journal of the Operational Research Society*, 59(1):90 – 104.
- Rockafellar, R. T. and Wets, R. J.-B. (1991). Scenarios and policy aggregation in optimization under uncertainty. *Math. Oper. Res.*, 16(1):119–147.
- Russo, F. and Comi, A. (2004). A state of the art on urban freight distribution at european scale. In *Proceedings of the European Conference on Mobility Management*.
- Ruszczynski, A. (1997). Decomposition methods in stochastic programming. *Mathematical Programming*, 79(1-3):333–353.
- Ryan, S., Wets, R.-B., Woodruff, D., Silva-Monroy, C., and Watson, J.-P. (2013). Toward scalable, parallel progressive hedging for stochastic unit commitment. In *Power and Energy Society General Meeting (PES), 2013 IEEE*, pages 1–5.

- Secomandi, N. and Margot, F. (2009). Reoptimization approaches for the vehicle routing problem with stochastic demands. *Operations Research*, 57:214–230.
- Seiden, S. (2000). An optimal online algorithm for bounded space variable-sized bin packing. In Montanari, U., Rolim, J., and Welzl, E., editors, *Automata, Languages and Programming*, volume 1853 of *Lecture Notes in Computer Science*, pages 283–295. Springer.
- Seiden, S. S., Van Stee, R., and Epstein, L. (2003). New bounds for variable-sized online bin packing. *SIAM Journal on Computing*, 32(2):455–469.
- Seuring, S. and Muller, M. (2008). From a literature review to a conceptual framework for sustainable supply chain management. *Journal of Cleaner Production*, 16(15):1699 – 1710.
- Silva, A. D. and Abramson, D. (1994). Computational experience with the parallel progressive hedging algorithm for stochastic linear programs.
- SMARTFREIGHT Consortium (2008). Smart freight transportation in urban area.
- Snyder, L. V., Daskin, M. S., and Teo, C. (2007). The stochastic location model with risk pooling. *European Journal of Operational Research*, 179:1221–1238.
- Somervell, M. (1998). Progressive hedging in parallel. In *Proceedings of the 33rd ORSNZ Conference*.
- START Consortium (2009). Short term actions to reorganize transport of goods.
- Steinberg, E. and Parks, M. (1979). A preference order dynamic program for a knapsack problem with stochastic rewards. *Journal of the Operational Research Society*, 30:141–147.
- Tadei, R., Perboli, G., and Baldi, M. M. (2012). The capacitated transshipment location problem with stochastic handling costs at the facilities. *International Transactions in Operational Research*, 19(6):789–807.
- Tadei, R., Perboli, G., and Della Croce, F. (2002). A heuristic algorithm for the auto-carrier transportation problem. *Transportation Science*, 36(1):55–62.

- Tadei, R., Perboli, G., and Perfetti, F. (2014). The multi-path traveling salesman problem with stochastic travel costs. *EURO Journal on Transportation and Logistics*, pages 1–21.
- Tadei, R., Ricciardi, N., and Perboli, G. (2009). The stochastic p-median problem with unknown cost probability distribution. *Operations Research Letters*, 37:135–141.
- Tagmouti, M., Gendreau, M., and Potvin, J.-Y. (2011). A dynamic capacitated arc routing problem with time-dependent service costs. *Transportation Res. C*, 19:20–28.
- Taniguchi, E., Noritake, M., Yamada, T., and Izumitani, T. (1999). Optimal size and location planning of public logistics terminals. *Transportation Research Part E: Logistics and Transportation Review*, 35(3):207 – 222.
- Taniguchi, E. and Shimamoto, H. (2004). Intelligent transportation system based dynamic vehicle routing and scheduling with variable travel times. *Transportation Res. C*, 12:235–250.
- Taniguchi, E., Thompson, R., Yamada, T., and van Duin, J. (2001). *City Logistics: Network Modeling and Intelligent Transport Systems*. Pergamon, Amsterdam.
- Toriello, A., Haskell, W. B., and Poremba, M. (2012). A dynamic traveling salesman problem with stochastic arc costs. Technical report, Optimization Online.
- Van Mierlo, J., Vereecken, L., Maggetto, G., Favrel, V., Meyer, S., and Hecq, W. (2003). Comparison of the environmental damage caused by vehicles with different alternative fuels and drivetrains in a brussels context. *Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering*, 217(7):583–593.
- Van Rooijen, T. and Quak, H. (2009). Binnenstadservice. nl—a new type of urban consolidation centre. *European Transport and Contribution*.
- Wang, F., Lai, X., and Shi, N. (2011). A multi-objective optimization for green supply chain network design. *Decision Support Systems*, 51(2):262 – 269.

- Watson, J.-P., Strip, D. R., and Woodruff, D. L. (2014). Deployment of enterprise-scale systems sustainability optimization on commodity computing clusters. In *Logistics, optimization and transportation (LOT), 2014*.
- Watson, J.-P. and Woodruff, D. (2011). Progressive hedging innovations for a class of stochastic mixed-integer resource allocation problems. *Computational Management Science*, 8(4):355–370.
- Zhang, G. (1997). A new version of on-line variable-sized bin packing. *Discrete Applied Mathematics*, 72(3):193 – 197.