Models: what are they and why are they useful

The Art of Modeling
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ABSTRACT

This paper discussed what the models are and why they are useful. The paper elaborated the characteristics, costs and different kinds of models. The art of modeling is also highlighted with respect to descriptive and decisional problems. The paper also discussed the mathematical modeling of Love dynamics, and analyzed the Robust & Fragile couples as well as High & Low quality relationships.

1. INTRODUCTION

Models are used to describe the behaviour and inter-relate the parameters of a physical process or system. It provides the information about the physical systems' response and behaviour. A model can be composed of a group of conjectures. The model can be consistent with the observed fact. The model should be precise, user friendly, flexible and low cost. There are wide variety of models such as verbal, physical & mathematical models such as differential equations, partial differential equations, ordinary differential equations, statistical model and dynamic models etc. The art of modelling is also useful to analyze the descriptive and decisional problems faced while developing models.

Love dynamics is the subject of social psychology, in which interpersonal relationships are mostly discussed. When two individuals, who are completely indifferent to each other, meet then the love story starts, develop and ends in a regime, that's why Love is a dynamic process. Love dynamics can be modelled using mathematical models. These mathematical equations give an insight of the behaviour of standard couples and their relationship quality.

2. Models

The term model can be referred to group of equations that relates the variables and parameters that describes a physical system. A model can be composed of a group of conjectures. The model can be consistent with the observed fact. Models are often nicer than reality but on the other hand there exists conflicts between the people who developed or use the model and those who observe or measure the real phenomenon. Most real systems can be modelled. Depending on the inputs and outputs the models changes. The models have different characteristics and kinds as discussed below:

2.1 Characteristics of Model

The important characteristic of a model is that it should be precise and accurate. It should describe the behaviour of the real system that is being modelled. On the other hand, it should be flexible, so that the model is able to work well in different situations and if necessary it is possible to add/subtract more variables inside it.

The model should not be so complex, which means, it should be user friendly. Another factor which effects model is the cost. The model should not be of very high cost, it must have low cost, so cost is an important characteristic of the model. So, the model should be precise, user friendly, flexible and low cost.

2.2 Types of Costs

There are different costs associated with the model. The first step is to develop the model and developing the model requires the cost. Every model need some data to process, so some cost is also require for collecting the data for the model. The cost vary depend on the way the data is collected. The model needs to be updated in order to meet the new requirements or needs and this updating of model have associated a cost with it. So, each model has an associated cost with it as discussed previously but a model should have a low cost.

2.3 Different kinds of Models

There are different kinds of models such as verbal, physical and mathematical models. In verbal model, words are used instead of numbers. The verbal model involves symbols to indicate operations or processes and it uses the words to highlight the important and necessary information.

The physical model is a physical copy of an object, regardless of the size of the object. The size of the object to be modelled ranges from small atomic particle to giant Solar system [1]. The physical model provides a visualization of information relating to the object being modelled. The two dimensional and three dimensional models are also being widely used.

The mathematical models involve the variables and parameters that describe the behaviour of the system to be modelled. The mathematical model can be of the form of differential equations, partial differential equations, ordinary differential equations, statistical model and dynamic models etc. The examples of the mathematical models can be air pollution, global warming, population growth, bacterial growth and birth-death rate etc. [2] [3].

There may be one system and it can be represented by many models e.g. total fish biomass in a river at a given time. On the other hand, there can be many systems represented by same model e.g. Love, War as depicted by the following set of equations:

$$x_1(t+1)= x_1(t)-f_1x_1(t)+r_1x_2(t) \quad \text{....................(2.3.1)}$$
$$x_2(t+1)= x_2(t)-f_2x_2(t)+r_2x_1(t) \quad \text{....................(2.3.2)}$$

The set of equations i.e., 2.3.1 and 2.3.2 can be used for modelling both Love and War. But in the case of Love, the f_1 and f_2 are the forgetting coefficients and r_1 and r_2 are the reaction of the individual 1 and 2 respectively. While in case of War, the f_1 and f_2 are the technological decay coefficients and r_1 and r_2 are the defence coefficients.

3. The Art of Modelling

Modelling is basically the derivation of equations that are solved for a set of system variables. The two kinds of problems involved in modelling are descriptive and decisional problems that will be discussed in the following sections.

3.1 Descriptive Problems

The descriptive problems are classified as long and short term problems. The long term problems involve simulations while the short term involves forecasting. The examples of long term problems include modelling human population, global warming and city structures etc. The short term problems include modelling about forecasting the weather, stock exchange and tourism etc.

3.2 Decisional Problems

The decisional problems are also classified as long and short term problems as the descriptive problems. The long term decisional problems refer to planning such as planning about train schedule, hospital or dam etc. The short term decisional problems refer to management such as managing the minimizations of delays or any real time operation etc.

In the following section of this paper, mathematical modelling of Love dynamics will be discussed.

4. Love Dynamics

Love dynamics is the subject of social psychology, in which interpersonal relationships are mostly discussed. Among the interpersonal relationships, the romantic relationships are said to be simple because just two individuals are involved in that relationship [4]. When two individuals, who are completely indifferent to each other, meet then the love story starts and it start developing & ends in a regime, that's why Love is a dynamic process. The dynamics of the feelings between the two persons can be modelled by differential equations [4].

In the following sections, the effects of appeal on the development of love will be discussed. The outcomes of positive and negative appeals will also be discussed. It's also important to mention that appeal is not an absolute quantity but rather an immanent quantity given by the love partner.

4.1 Mathematical Model of Love dynamics

Before discussing the mathematical model of love dynamics, it's necessary to define the standard couples. The secure couples are those respond positively if the love of the other partner increases [5]. The non-secure individuals decrease their response when involvement becomes very high. The non-synergic persons' response to love is independent of the degree of involvement [6].

The mathematical model of the love dynamics is composed of two variables i.e. x_1 and x_2 which represents the love of person 1 and 2 for the love partner. The mathematical equations representing the love dynamics are the following [7] [8] [9]:

$$\dot{x_1} = -\alpha_1 x_1 + R_1(x_2) + A_2 \text{ .... } (4.1.1)$$
$$\dot{x_2} = -\alpha_2 x_2 + R_2(x_1) + A_1 \text{ .... } (4.1.2)$$

where,
α_i is forgetting coefficient of individual i.
A_i is appeal of individual i.
x represents love
$\dot{x}$ represents evolution of feelings
$R_i(x_j)$ represents the reactions of individual i to love x_j.

Now, different standard couples will be discussed with respect to appeals A_1 and A_2. Initial conditions are $x_1(0) = x_2(0) = 0$, because the couple is completely indifferent to each other at their first meeting.

Case 1: Robust & Fragile couples

The set of mathematical equations representing love dynamics i.e. 4.1.1 and 4.1.2 have generically one or three equilibrium points [4]. The two fold bifurcation curves in state space of appeals A_1 and A_2 are computed using software package *AUTO* [10] [11] [12] and are shown in figure 4.1.1.

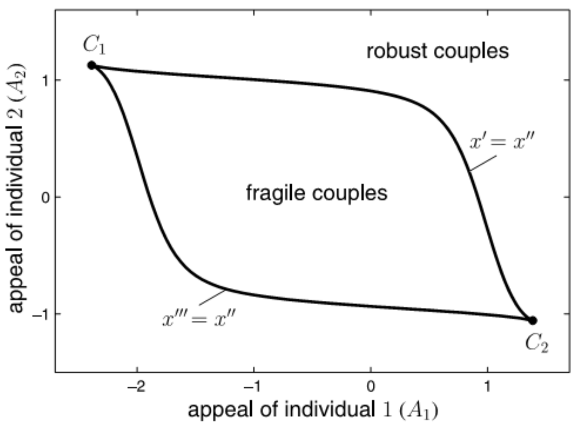

Figure 4.1.1 Two-fold bifurcation curves [10]

In figure 4.1.1, the couples corresponding to two stable states x' & x''' and saddle x'', are called fragile couples. The fragile couples have a romantic relationship that switch from high to low and vice versa. On the contrary, the robust couples correspond to points outside the closed region shown in figure 4.1.1. The robust couples have a single stable equilibrium point which means that if a disturbance occur within the robust couples, then their romantic relationship always return to same quality after the disturbance.

Case 2: High & Low quality Relationships

The relationships with states of positive feelings represent the high quality relationships. On the contrary, the relationships with states of negative feelings represent the low quality relationships [4].

The figure 4.1.2 represents the curves that give the information about high and low quality relationships [4].

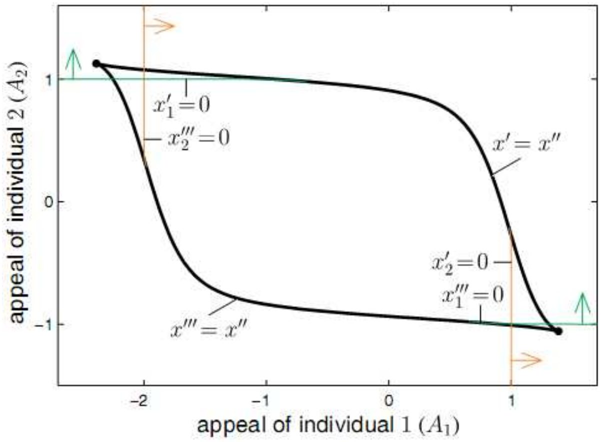

Figure 4.1.2 Curves representing signs of feelings [4]

The figure 4.1.2 reveals the fact that robust relationships have high quality relationships in long run despite of fact if one of the two partners has negative appeal. But if both the partners have negative appeal then robust couples experience low quality relationships. On the contrary, the fragile couples have been partitioned into seven classes as shown in figure 4.1.2. The x' & x''' are the two stable equilibrium points that are balanced and the signs of feelings at x' & x''' change from negative to positive crossing the green and orange curves in the directions of arrows as shown in figure 4.1.2.

5. CONCLUSION

The main aim was to discuss the models & their usefulness and their application to Love dynamics. The model should be precise, user friendly, flexible and low cost. The different kinds of models are verbal, physical and mathematical models.

Modelling is basically the derivation of equations that are solved for a set of system variables. It involved the descriptive and decisional problems.

Love dynamics is the subject of social psychology, in which interpersonal relationships are mostly discussed. The fragile couples have two stable states and a saddle point. The fragile couples have a romantic relationship that switch from high to low and vice versa. The robust couples have a single stable equilibrium point. The robust relationships have high quality relationships in long run despite of fact if one of the two partners has negative appeal. But if both the partners have negative appeal then robust couples experience low quality relationships

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