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The $\mathcal{N} > 2$ supersymmetric AdS vacua in maximal supergravity

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Abstract: We perform a systematic search for anti-de Sitter vacua of maximal supergravity with $\mathcal{N} > 2$ residual supersymmetries. We find that maximal supergravity admits two 1-parameter classes of $\mathcal{N} = 3$ and $\mathcal{N} = 4$ vacua, respectively. They are embedded, for the different values of an angular parameter, in the ω-rotated SO(8) ($\mathcal{N} = 3$) and SO(1, 7) ($\mathcal{N} = 4$ and 3) gauged models. All vacua disappear in the $\omega \to 0$ limit. We determine the mass spectra and the AdS-supermultiplet structure. These appear to be the first and only $\mathcal{N} > 2$ supersymmetric AdS vacua in maximal supergravity, aside from the $\mathcal{N} = 8$ vacua of the SO(8)-gauged models. We also prove on general grounds that no such vacua can exist for $4 < \mathcal{N} < 8$.

Keywords: Supersymmetry Breaking, Extended Supersymmetry, Supergravity Models, Superstring Vacua

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Contents

1 Introduction

Since the seminal paper [1] in which the first gauged maximal supergravity was constructed with gauge group SO(8), much work has been done to study the vacua of this model and to construct new gauged maximal supergravities. Certain vacua of the original SO(8)-gauged model,[1] like the anti-de Sitter (AdS) vacuum with $\mathcal{N}=8$ residual supersymmetry, were put in correspondence with compactifications of $D=11$ supergravity on a seven-dimensional sphere or on warped/stretched versions of a seven-sphere, possibly with torsion. Non-compact and even non-semisimple gaugings, defined by groups of the form $\mathrm{CSO}(p,q,r)$,

[1] See [2] for early results in the search for vacua of the original theory and [3] for a recent study.

$p+q+r=8$, were first constructed in [4] and their de Sitter vacua put in correspondence with reductions on non-compact spaces with negative curvature [5]. Flat-gaugings in $D=4$ describing Scherk-Schwarz reductions of maximal $D=5$ supergravity and yielding no-scale models, were first constructed in [6].

A new formulation of gauged extended supergravities in terms of the so called *embedding tensor* [7–9], has opened the way for a more systematic analysis of the possible gaugings and their vacuum structure. All possible choices of gauge groups in the maximal supergravity are encoded in a single object $\Theta_M{}^\alpha$ (the embedding tensor), which defines the embedding of the gauge algebra inside the algebra $\mathfrak{e}_{7(7)}$ of the on-shell global symmetry group $\mathrm{E}_{7(7)}$ of the ungauged theory. This object is formally $\mathrm{E}_{7(7)}$-covariant and is constrained, by linear and quadratic conditions originating from the requirement of supersymmetry and gauge invariance, to belong to certain orbits of the **912** representation. An interesting feature of this formulation is that the field equations and Bianchi identities of the gauged model are formally $\mathrm{E}_{7(7)}$-covariant if the fields are transformed together with the embedding tensor. In other words there is a mapping (or duality) between gauged theories defined by embedding tensors that are related by $\mathrm{E}_{7(7)}$ transformations. Such mapping should encode the effect of string/M-theory dualities on flux compactifications. In particular the scalar potential $V(\Theta,\phi)$ is a quadratic function of $\Theta_M{}^\alpha$ and is invariant under the simultaneous action of $\mathrm{E}_{7(7)}$ on the 70 scalar fields $\phi=(\phi^{ijkl})$ of the model and the embedding tensor:

$$\forall g\in \mathrm{E}_{7(7)}\ ;\quad V(\Theta,\phi)=V(g\star\Theta,g\star\phi)\,, \tag{1.1}$$

where $g\star$ denotes the generic action of a group element g on the scalars (non-linear action) and on $\Theta_M{}^\alpha$ (linear action). The above property and the homogeneity of the scalar manifold has motivated what has been dubbed as the "going-to-the-origin" approach for the study of vacua of gauged supergravities [10–12]: any vacuum of a given gauged model can be mapped into the origin of the scalar manifold[2] by means of a suitable $\mathrm{E}_{7(7)}$-transformation, provided the embedding tensor is transformed accordingly. This means that the vacua of gauged maximal supergravity can be systematically studied by restricting to the origin of the manifold so that the extremization condition on V becomes another condition on $\Theta_M{}^\alpha$ only. In this way, one can search for vacua with particular properties without committing to a particular gauge group, i.e. while simultaneously scanning through all possible gaugings.

In [13] a new family of SO(8)-gauged maximal supergravities was constructed by exploiting the freedom in the original choice of the symplectic frame defining the electric and magnetic gauge fields. These models were obtained as a deformation of the original de Wit and Nicolai model, parametrized by an angle ω. They all exhibit an $\mathcal{N}=8$ vacuum at the origin. Their spectrum is identical while the ω parameter only affects the higher-order interactions. Similar generalizations of non-compact gaugings were studied in [14, 15]. Adopting the "going-to-the-origin" approach, the authors of [16–18] systematically searched for vacua with certain residual symmetries and found several vacua of the

[2]By origin we mean the point in the scalar coset manifold $\mathrm{E}_{7(7)}/\mathrm{SU}(8)$ at which all scalar fields ϕ^{ijkl} vanish and is thus manifestly SU(8)-invariant.

new ω-deformed models. An interesting feature observed in all the above works, is that the ω-deformed models in general exhibit a much richer vacuum structure that the original $\omega = 0$ models from [1, 4]. In other words, many vacua of these theories disappear in the limit $\omega \to 0$.

In the present paper we start a systematic analysis of vacua of maximal supergravity with a minimal amount of residual supersymmetry. We focus on AdS vacua preserving $\mathcal{N} > 2$ supersymmetries by implementing the supersymmetry conditions (Killing spinor equations) directly on the embedding tensor (with the scalar fields fixed at the origin). Our (computer aided) analysis is systematic and we find, aside from the known $\mathcal{N} = 8$ vacua, only two other classes of solutions with residual supersymmetry $\mathcal{N} = 4$ and $\mathcal{N} = 3$, respectively. *These are, to our knowledge, the first AdS vacua of maximal supergravity with residual $\mathcal{N} > 2$ supersymmetry, aside from the $\mathcal{N} = 8$ ones.* We can exclude, by general argument, solutions with $8 > \mathcal{N} > 4$. Each class of the newly found vacua is parametrized by an angle φ and, depending on its values, the corresponding vacua are embedded in different (ω-deformed) $\mathrm{CSO}(p, q, r)$ models. In particular the $\mathcal{N} = 4$ vacua, depending on φ, belong to gaugings of the form $\mathrm{SO}(1, 7)$ and $[\mathrm{SO}(1, 1) \times \mathrm{SO}(6)] \ltimes T^{12}$, while $\mathcal{N} = 3$ vacua to models with gauge group $\mathrm{SO}(8)$, $\mathrm{SO}(1, 7)$ and $\mathrm{ISO}(7)$.

We compute the mass spectra on these vacua, which turn out to be φ-independent, and determine the corresponding AdS-supermultiplet structure. Our analysis shows that, while there are several AdS $\mathcal{N} = 8 \longrightarrow \mathcal{N} = 3$ supersymmetry breaking patterns, only one, for each residual symmetry, seems to be dynamically realized in the full non-linear theory.

As a last comment, vacua with residual $\mathrm{SO}(4)$ symmetry were investigated in [18]. This analysis however missed the vacua discussed here since it restricted the $\mathrm{SO}(8)$ singlets to a sector which is invariant under a D_4 discrete subgroup of $\mathrm{SU}(8)$.[3]

The paper is organized as follows. In section 2 we formulate the problem of systematically studying the spontaneous $\mathcal{N} = 8$ supersymmetry breaking on an AdS vacuum with residual extended supersymmetry: after a first introduction of the embedding tensor formalism, we consider the spontaneous supersymmetry breaking to $\mathcal{N} > 2$ on AdS vacua and derive the corresponding system of quadratic equations on the non-vanishing components of the embedding tensor. We show that $\mathcal{N} > 2$ residual supersymmetry requires the massive gravitinos to transform non-trivially under the associated $\mathrm{SO}(\mathcal{N})$ R-symmetry group. In particular, we deduce the absence of solutions with $8 > \mathcal{N} > 4$ residual supersymmetry.

In section 3, we study the possible AdS $\mathcal{N} = 8 \longrightarrow \mathcal{N} = 3$ supersymmetry breaking patterns at the level of the corresponding supersymmetry multiplets. In section 4, we then describe the $\mathcal{N} = 4$ and $\mathcal{N} = 3$ classes of solutions to the quadratic equations. We identify, for the different values of the angular parameter φ, the corresponding gauge groups through the signature of the Cartan-Killing metric and by identifying the $\mathrm{E}_{7(7)}$-invariant quantities constructed out of the embedding tensor with the same quantities evaluated on ω-rotated $\mathrm{SO}(8)$ [13], $\mathrm{SO}(1, 7)$ groups and on $\mathrm{ISO}(7)$. We show that these vacua disappear in the $\omega \to 0$ limit. Finally we give the AdS-supermultiplet structure and bosonic mass

[3]As a consequence, the gravitino mass matrix which is consistent with these symmetry requirements is proportional to the identity matrix and thus is different from the one we obtain for $\mathcal{N} = 4$.

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spectrum for the two classes of solutions. Appendix A summarizes our conventions and normalizations for the mass matrices; appendix B collects some of the computational details for the results of the main text.

2 AdS vacua with extended supersymmetry

2.1 Gauged $\mathcal{N} = 8$ supergravity

Let us briefly review some key formulas of gauged $\mathcal{N} = 8$ supergravity, for details we refer to [7, 9, 19]. Gaugings of maximal $\mathcal{N} = 8$ supergravity are described by the gauge group generators $X_{MN}{}^K$, $(M, N = 1, \ldots, 56)$ which in turn are obtained by contracting the $\mathrm{E}_{7(7)}$ generators t_α $(\alpha = 1, \ldots, 133)$ with a given embedding tensor $\Theta_M{}^\alpha$

$$X_{MN}{}^K = \Theta_M{}^\alpha \, (t_\alpha)_N{}^K \,. \tag{2.1}$$

They satisfy the quadratic identity

$$[X_M, X_N] = -X_{MN}{}^K X_K \qquad \Longleftrightarrow \qquad \Omega^{MN} \Theta_M{}^\alpha \Theta_N{}^\beta = 0 \,, \tag{2.2}$$

which poses a quadratic constraint on the embedding tensor $\Theta_M{}^\alpha$, and exhibits the closure of the gauge algebra. The dressing of the generators (2.1) with the scalar dependent complex vielbein $\{\mathcal{V}_{M\,[ij]}, \mathcal{V}_M{}^{[ij]} \equiv (\mathcal{V}_{M\,[ij]})^*\}$, $i, j = 1, \ldots, 8$, defines the T-tensor

$$(T_{ij})^{klmn} \equiv \frac{1}{2} (\mathcal{V}^{-1})_{ij}{}^M (\mathcal{V}^{-1})^{klN} \, (X_M)_N{}^K \, \mathcal{V}_K{}^{mn} \,, \qquad \text{etc.}\,. \tag{2.3}$$

The various components of this tensor will show up in the field equations of the gauged theory and parametrize the couplings. The fact that the embedding tensor $\Theta_M{}^\alpha$ is restricted to the **912** representation of $\mathrm{E}_{7(7)}$ can be expressed by parametrizing the components of the T-tensor according to

$$\begin{aligned}
(T_{ij})_{kl}{}^{mn} &= \frac{1}{2} \left(\delta_{[k}{}^{[m} A^{n]}{}_{l]ij} + \delta_{i[k}{}^{mn} A_{l]j} - \delta_{j[k}{}^{mn} A_{l]i} \right) \,, \\
(T_{ij})^{rs}{}_{pq} &= -\frac{1}{2} \left(\delta_{[p}{}^{[r} A^{s]}{}_{q]ij} + \delta_{i[p}{}^{rs} A_{q]j} - \delta_{j[p}{}^{rs} A_{q]i} \right) \,, \\
(T_{ij})_{kl\,pq} &= \frac{1}{24} \, \varepsilon_{klpqrstu} \, \delta_{[i}{}^r A_{j]}{}^{stu} \,, \\
(T_{ij})^{rs\,mn} &= \delta_{[i}{}^{[r} A_{j]}{}^{smn]} \,,
\end{aligned} \tag{2.4}$$

in terms of the scalar tensors[4] A_{ij}, $A_i{}^{jkl}$, satisfying $A_{[ij]} = 0$, $A_i{}^{jkl} = A_i{}^{[jkl]}$, and $A_i{}^{jki} = 0$. These tensors represent the **36** and **420** representations of SU(8), respectively, and parametrize the Yukawa-type couplings in the Lagrangian as

$$\begin{aligned}
\mathcal{L}_{\text{Yuk}} = e \Big\{ &\frac{1}{2} \sqrt{2} \, A_{1\,ij} \, \bar{\psi}^i_\mu \gamma^{\mu\nu} \psi^j_\nu + \frac{1}{6} A_i{}^{jkl} \, \bar{\psi}^i_\mu \gamma^\mu \chi_{jkl} \\
&+ \frac{1}{144} \sqrt{2} \, \varepsilon^{ijkpqrlm} \, A^n{}_{pqr} \, \bar{\chi}_{ijk} \, \chi_{lmn} \; + \; \text{h.c.} \Big\} \,,
\end{aligned} \tag{2.5}$$

[4] Here and in the following the coupling constant g is absorbed in the definition of the tensors A_{ij}, $A_i{}^{jkl}$.

for the eight gravitini ${\psi_\mu}^i$ and the 56 fermions χ_{ijk}. The quadratic constraints (2.2) on the embedding tensor induce the following identities among the scalar dependent tensors A_{ij}, $A_i{}^{jkl}$

$$
\begin{aligned}
0 &= A^k{}_{lij}\, A_n{}^{mij} - A_l{}^{kij}\, A^m{}_{nij} - 4A^{(k}{}_{lni} A^{m)i} - 4A_{(n}{}^{mki} A_{l)i} \\
&\quad - 2\,\delta_l^m\, A_{ni} A^{ki} + 2\,\delta_n^k\, A_{li} A^{mi}\,, \\
0 &= A^i{}_{jk[m}\, A^k{}_{npq]} + A_{jk}\delta^i_{[m} A^k{}_{npq]} - A_{j[m} A^i{}_{npq]} \\
&\quad + \frac{1}{24}\,\varepsilon_{mnpqrstu}\,\left(A_j{}^{ikr}\, A_k{}^{stu} + A^{ik}\delta^r_j A_k{}^{stu} - A^{ir} A_j{}^{stu}\right)\,, \\
0 &= A^r{}_{ijk}\, A_r{}^{mnp} - 9\,A^{[m}{}_{r[ij}\, A_{k]}{}^{np]r} - 9\,\delta_{[i}{}^{[m}\, A^n{}_{|rs|j}\, A_{k]}{}^{p]rs} \\
&\quad - 9\,\delta_{[ij}{}^{[mn}\, A^{|u|}{}_{k]rs}\, A_u{}^{p]rs} + \delta_{ijk}{}^{mnp}\, A^u{}_{rst}\, A_u{}^{rst}\,.
\end{aligned}
\tag{2.6}
$$

Let us finally note that the scalar potential of the theory is given in terms of these tensors by

$$
V = -\frac{3}{4}\left(A_{kl}A^{kl} - \frac{1}{18}\, A_n{}^{jkl} A^n{}_{jkl}\right), \tag{2.7}
$$

and that its extremal points are given by those values for the scalar fields at which the tensor

$$
\mathcal{C}_{ijkl} = A^m{}_{[ijk} A_{l]m} + \frac{3}{4}\, A^m{}_{n[ij} A^n{}_{kl]m}\,, \tag{2.8}
$$

becomes anti-selfdual:

$$
\mathcal{C}_{ijkl} + \frac{1}{24}\,\varepsilon_{ijklmnpq}\,\mathcal{C}^{mnpq} = 0\,. \tag{2.9}
$$

At these extremal points, the couplings (2.5) give rise to the fermionic mass terms. For example, the gravitino masses are obtained as the eigenvalues of the properly normalized tensor A_{ij}. For vanishing gauge fields and constant scalars, the Killing spinor equations of the theory reduce to

$$
\begin{aligned}
0 \overset{!}{\equiv} \delta_\epsilon \psi^i_\mu &= 2\,\mathcal{D}_\mu \epsilon^i + \sqrt{2}\, A^{ij}\gamma_\mu \epsilon_j\,, \\
0 \overset{!}{\equiv} \delta_\epsilon \chi^{ijk} &= -2\, A_l{}^{ijk}\epsilon^l\,.
\end{aligned}
\tag{2.10}
$$

Let us give, for the sake of completeness, the mass matrices for the various fields [19]. The linearization of the scalar field equations yields, to lowest order,

$$
\Box\, \delta\phi_{ijkl} = \mathcal{M}_{ijkl}{}^{mnpq}\, \delta\phi_{mnpq} + \mathcal{O}(\delta\phi^2)\,, \tag{2.11}
$$

where $\delta\phi_{ijkl}$ are fluctuations of the self-dual scalar fields $\phi_{ijkl} = \frac{1}{24}\varepsilon_{ijklpqrs}\,\phi^{pqrs}$ around their vacuum value $\phi_0 = (\phi_0^{ijkl})$ and the scalar mass matrix $\mathcal{M}_{ijkl}{}^{mnpq}$ is given by

$$
\begin{aligned}
\mathcal{M}_{ijkl}{}^{mnpq}\, \delta\phi^{ijkl}\delta\phi_{mnpq} &= 6\,\left(A_m{}^{ijk} A^l_{ijn} - \tfrac{1}{4} A_i{}^{jkl} A^i_{jmn}\right)\delta\phi^{mnpq}\delta\phi_{klpq} \\
&\quad + \left(\tfrac{5}{24}\, A_i{}^{jkl} A^i{}_{jkl} - \tfrac{1}{2} A_{ij} A^{ij}\right)\delta\phi^{mnpq}\delta\phi_{mnpq} \\
&\quad - \tfrac{2}{3}\, A_i{}^{jkl} A^m{}_{npq}\, \delta\phi^{inpq}\delta\phi_{jklm} \\
&= 12\, V^{(2)}(\delta\phi)\,,
\end{aligned}
\tag{2.12}
$$

where we have denoted by $V^{(2)}(\delta\phi)$ the terms on the scalar potential (2.7) which are second order in $\delta\phi$ upon expansion around the vacuum ϕ_0:

$$V(\phi) \;=\; V_0 + V^{(2)}(\delta\phi) + \mathcal{O}(\delta\phi^3)\,. \tag{2.13}$$

The vector mass matrix reads

$$\mathcal{M}_{\rm vec} \;=\; \begin{pmatrix} \mathcal{M}_{ij}{}^{kl} & \mathcal{M}_{ijkl} \\ \mathcal{M}^{ijkl} & \mathcal{M}^{ij}{}_{kl} \end{pmatrix}\,, \tag{2.14}$$

with

$$\begin{aligned} \mathcal{M}_{ij}{}^{kl} \;&=\; -\tfrac{1}{6}\,A_{[i}{}^{npq}\delta^{[k}_{j]}A^{l]}{}_{npq} + \tfrac{1}{2}\,A_{[i}{}^{pq[k}A^{l]}{}_{j]pq}\,, \\ \mathcal{M}_{ijkl} \;&=\; \tfrac{1}{36}\,A_{[i}{}^{pqr}\epsilon_{j]pqrmns[k}A_{l]}{}^{mns}\,. \end{aligned} \tag{2.15}$$

We can also give this matrix a manifestly symplectic covariant form

$$\mathcal{M}_{\text{vec}\,M}{}^N = \frac{1}{6}\,\left[\text{Tr}(X_M\,X_P) + \text{Tr}(\mathbb{M}^{-1}\,X_M\,\mathbb{M}\,(X_P)^T)\right]\,\mathbb{M}^{PN}\,, \tag{2.16}$$

where $\mathbb{M}_{MN} = (\mathcal{V}\,\mathcal{V}^T)_{MN}$ is the symmetric, symplectic, positive definite matrix constructed from the coset representative $\mathcal{V}_M{}^N$ in the **56** of $\text{E}_{7(7)}$. By virtue of the quadratic constraint (2.2) on Θ, the matrix $\mathcal{M}_{\rm vec}$ always has 28 vanishing eigenvalues (corresponding to the magnetic vector fields), while the remaining eigenvalues define the masses of the (electric) vector fields.

Finally, the gravitino and fermion mass matrices are:

$$\mathcal{M}_{\psi}{}^{ij} \;=\; \sqrt{2}\,A^{ij}\,, \qquad \mathcal{M}_{\chi}{}^{ijk,lmn} \;=\; \tfrac{1}{12}\sqrt{2}\,\epsilon^{ijkpqr[lm}A^{n]}{}_{pqr}\,. \tag{2.17}$$

The first matrix $\mathcal{M}_\psi$ carries the information about the breaking of supersymmetry and the latter matrix has to be evaluated after projecting out the fermions that are eaten by the massive gravitinos. Explicitly, at an AdS vacuum and in a basis in which A_{ij} is diagonal, the effective fermion mass matrix is given by

$$\mathcal{M}_{\chi}{}^{ijk,lmn} \;=\; \frac{1}{12}\sqrt{2}\left(\epsilon^{ijkpqr[lm}A^{n]}{}_{pqr} \;+\; \frac{4}{3}\,{\sum_{p,q}}'A_p{}^{ijk}A_q{}^{lmn}\left(\frac{A}{A^2+\mathbf{1}\,V/6}\right)^{pq}\right), \tag{2.18}$$

with the sum running only over the massive gravitino directions.

2.2 $\mathcal{N}>2$ AdS vacua

We have reviewed, how a given embedding tensor defines the scalar potential (2.7) of gauged supergravity which in turn may carry extremal points (2.9) at which supersymmetry is (partially) broken. The embedding tensor formalism allows to nicely invert the problem and to search for vacua with given properties by simultaneously scanning the set of all possible gaugings. That strategy has e.g. been applied in [10, 11, 16–18] in order to identify and analyze vacua with a given residual symmetry group. Concretely, any joint solution to the

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quadratic equations (2.6) and the vacuum condition (2.9) defines a vacuum in some maximal gauged supergravity. The associated embedding tensor and gauge group generators can then be restored via (2.4), (2.3), and (2.1).

In this section, we will investigate AdS vacua in maximal supergravity that preserve more than 2 supersymmetries. Let us assume that the matrices A_{ij}, $A_i{}^{jkl}$ describe an AdS vacuum preserving $\mathcal{N}$ supersymmetries, i.e. assume the existence of $\mathcal{N}$ independent solutions of (2.10). Without loss of generality, we may then choose a basis $i = (\alpha, a)$ in which

$$\begin{aligned} &|A_{\alpha\beta}| = g\,\delta_{\alpha\beta}\,, \quad A_{\alpha a} = 0\,, \qquad A^{\alpha}{}_{ijk} = 0\,, \\ &\qquad\qquad \text{for} \quad \alpha,\beta = 1,\dots,\mathcal{N}, \quad a = \mathcal{N}+1,\dots,8\,, \end{aligned} \tag{2.19}$$

and try to solve the quadratic equations (2.6), (2.9) under these assumptions for the remaining components of the tensors A_{ij}, $A_i{}^{jkl}$. First, let us note that for all non-vanishing $\mathcal{N} > 0$, equations (2.9) follow directly as upon reduction of equations (2.6) by (2.19). This is nothing but a remnant of the fact that the existence of a Killing spinor in general implies part of the remaining bosonic equations of motion (in this case the scalar field equations for constant scalars). It thus remains to solve equations (2.6) with the ansatz (2.19). Since they are homogeneous, we may furthermore set $g = 1$. Some contraction of the first equation from (2.6) then allows to deduce the value of the potential as

$$V = -6\,. \tag{2.20}$$

On the other hand, the first equation of (2.6) with $k = \alpha, l = \beta, n = \gamma$ yields

$$0 = -2A^{m}{}_{\beta\gamma\delta}\,A^{\alpha\delta} \quad \Longrightarrow \quad A^{m}{}_{\alpha\beta\gamma} = 0\,, \tag{2.21}$$

thus imposes the absence of the components $A^{m}{}_{\alpha\beta\gamma}$. For later use, we also note that the second equation of (2.6) in particular implies that

$$0 = 3\,A^{a}{}_{\alpha e[b}A^{e}{}_{cd]\beta} + A^{a}{}_{e\alpha\beta}A^{e}{}_{bcd} + A_{\alpha\beta}\,A^{a}{}_{bcd} - \frac{1}{6}\,\epsilon_{\alpha\beta\,bcd\,ijk}A^{ae}A_{e}{}^{ijk}\,. \tag{2.22}$$

Let us now specialize to the case of $\mathcal{N} > 2$ preserved supersymmetries. In this case, the preserved supercharges transform in the vector representation of the AdS R-symmetry SO($\mathcal{N}$). We can then give a systematic discussion of these vacua according to the transformation of the broken supercharges (i.e. the massive gravitino fields) under that SO($\mathcal{N}$). In particular, all non-vanishing components of the tensors A_{ij}, $A_i{}^{jkl}$ must be singlets under SO($\mathcal{N}$). Let us consider as an example the case when all broken supercharges are singlet under SO($\mathcal{N}$). If $\mathcal{N} > 4$ this is the only option (in the absence of non-trivial SO($\mathcal{N}$) representations of sufficiently small size). The non-vanishing components of the tensors A_{ij}, $A_i{}^{jkl}$ are thus given by

$$\{A_{\alpha\beta},\, A_{ab},\, A^{a}{}_{bcd}\}\,, \tag{2.23}$$

with all other possible singlets under SO($\mathcal{N}$) vanishing, in view of (2.19) and (2.21). Now (2.22) immediately implies that also $A^{a}{}_{bcd} = 0$, i.e. the entire tensor $A_i{}^{jkl}$ vanishes.

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$\Delta \setminus s$	$\frac{3}{2}$	1	$\frac{1}{2}$	0
E_0+3				$[j]$
$E_0+\frac{5}{2}$			$[j+1]+[j]+[j-1]$	
E_0+2		$[j+1]+[j]+[j-1]$		$[j+2]+[j+1]+[j]$ $+[j]+[j-1]+[j-2]$
$E_0+\frac{3}{2}$	$[j]$		$[j+2]+[j+1]+[j+1]$ $+2[j]+2[j-1]+[j-2]$	
E_0+1		$[j+1]+[j]+[j-1]$		$[j+2]+[j+1]+2[j]$ $+[j-1]+[j-2]$
$E_0+\frac{1}{2}$			$[j+1]+[j]+[j-1]$	
E_0				$[j]$

Table 1. The long $\mathcal{N}=3$ gravitino multiplet $DS(3/2,E_0,j)_{\mathrm{L}}$, organized by energy Δ and spin s. When the energy saturates the unitarity bound $E_0=j+1$, the blue states in the table form a semi-short multiplet $DS(3/2,j+1,j)_{\mathrm{S}}$ and the other states decouple as a vector multiplet $DS(1,j+2)$.

Then however the first equation of (2.6) implies that

$$A_{ac}\,A^{cb}=\delta_a{}^b\,, \tag{2.24}$$

i.e. after diagonalisation the eigenvalues of A_{ab} are of absolute value 1 and all correspond to unbroken supersymmetries. The resulting vacuum thus is an $\mathcal{N}=8$ vacuum. We conclude that there are no $\mathcal{N}>2$ AdS vacua (other than the $\mathcal{N}=8$ ones) if the broken supercharges transform as singlets under the SO($\mathcal{N}$) R-symmetry. In particular, there are no AdS vacua in maximal supergravity preserving $4<\mathcal{N}<8$ supersymmetries. For $\mathcal{N}=6$ AdS vacua, this is consistent with the result of [20].

In the following, we will thus assume that the broken supercharges transform non-trivially under the SO($\mathcal{N}$) R-symmetry and determine the general solution of (2.6) for $\mathcal{N}>2$.

3 AdS $\mathcal{N}=8\longrightarrow\mathcal{N}=3$ supersymmetry breaking patterns

Before we start the analysis of the $\mathcal{N}=3$ solutions of the quadratic constraints (2.6), it is instructive to study the possible decompositions of the $\mathcal{N}=8$ supergravity multiplet into $\mathcal{N}=3$ multiplets, i.e. to identify the possible kinetic scenarios of supersymmetry breaking. The multiplet structure of the $\mathcal{N}=3$ AdS supergroup OSp(3|4) is well known, see [21–23], in the following we will adopt the notation from [23]. The relevant multiplets for our discussion are the massive gravitino multiplets which accommodate the five massive gravitinos after the supersymmetry breaking $\mathcal{N}=8\longrightarrow\mathcal{N}=3$. The structure of the generic long gravitino multiplet $DS(3/2,E_0,j)_{\mathrm{L}}$ is recollected in table 1. It is characterized by two numbers: the energy E_0 of its ground state, and the isospin j, characterising the

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representation of the gravitino under the R-symmetry group SO(3). Unitarity imposes the bound $E_0 \geq j+1$ for the ground state energy (the ground state having spin 0). As usual for such supergroups, multiplet shortening occurs when the unitarity bound is saturated. At this value of E_0, the multiplet splits into a short massive gravitino multiplet together with a vector multiplet according to

$$DS(3/2, E_0, j)_{\rm L}\Big|_{E_0=j+1} \longrightarrow DS(3/2, j+1, j)_{\rm S} + DS(1, j+2)\ , \tag{3.1}$$

with the structure of the vector multiplet $DS(1, j+2)$ given in table 2. At low values of j, the multiplet structure becomes non-generic, but the tables still capture the correct representation content upon formally extending the definition of SO(3) representations $[j]$ to negative j according to[5]

$$[-j] \equiv -[j-1]\ . \tag{3.2}$$

In particular, the lowest-lying short gravitino multiplet $DS(3/2, 1, 0)_{\rm S}$ carries a massless gravitino, three massless vectors, three fermions, and two scalars of energy 1 and 2. Due to the massless gauge fields, its presence in the spectrum implies an enhancement of supersymmetry and gauge symmetry. Similarly, the massless vector multiplet $DS(1, 1)$ carries six scalars together with a massless vector and four fermions.

With the multiplet structure given in tables 1, 2, the possible supersymmetry breaking patterns correspond to the different ways of splitting up the $\mathcal{N} = 8$ supergravity multiplet into $\mathcal{N} = 3$ multiplets. At this stage, we do not make any assumption about the energies of the various states (other than those implied by unitarity). The $\mathcal{N} = 8$ supergravity multiplet consists of the graviton, 8 gravitinos, 28 vectors, 56 fermions and 70 scalars. Upon subtracting the $\mathcal{N} = 3$ supergravity multiplet $DS(2, 3/2, 0)_{\rm S}$, given in table 3, we are left with 5 gravitinos, 25 vectors, 55 fermions and 70 scalars, to be packaged into $\mathcal{N} = 3$ multiplets. There are various options for the splitting of the five massive gravitinos into SO(3) R-symmetry representations:

$$\begin{aligned} \text{I)}\ 5\ &\longrightarrow 5\ , \\ \text{II)}\ 5\ &\longrightarrow 3+1+1\ , \\ \text{III)}\ 5\ &\longrightarrow 2+2+1\ , \end{aligned} \tag{3.3}$$

where we have taken into account the reality property of the gravitinos which rules out decompositions such as $4+1$, $3+2$, etc. Moreover, the general discussion of section 2.2 has ruled out the trivial decomposition $5 \longrightarrow 1+1+1+1+1$. Let us discuss the patterns (3.3) one by one.

Option I) in (3.3) leaves the five massive gravitinos in the irreducible spin-2 representation of SO(3). According to table 1, they can sit either in a long multiplet $DS(3/2, E_0, 2)_{\rm L}$ or in its shortened version $D(3/2, 3, 2)_{\rm S}$. Simple counting of states shows that the long multiplet carries 30 vector fields and thus does not fit into $\mathcal{N} = 8$ supergravity. The short

[5] By $-[j-1]$ we mean that the isospin multiplet structure is obtained by deleting the representations with negative isospin ($[-j]$) and, for each of them, a representation $[j-1]$.

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$\Delta \diagdown s$	1	$\frac{1}{2}$	0
j_0+2			$[j_0-2]$
$j_0+\frac{3}{2}$		$[j_0-1]+[j_0-2]$	
j_0+1	$[j_0-1]$		$[j_0]+[j_0-1]+[j_0-2]$
$j_0+\frac{1}{2}$		$[j_0]+[j_0-1]$	
j_0			$[j_0]$

Table 2. The $\mathcal{N}=3$ vector multiplet $DS(1,j_0)$.

$\Delta \diagdown s$	2^*	$\frac{3}{2}^*$	1^*	$\frac{1}{2}$
3	$[0]$			
$\frac{5}{2}$		$[1]$		
2			$[1]$	
$\frac{3}{2}$				$[0]$

Table 3. The $\mathcal{N}=3$ massless gravity multiplet $DS(2,3/2,0)_{\rm S}$.

multiplet $DS(3/2,E_0,2)$ on the other hand does fit into $\mathcal{N}=8$ supergravity with the remaining states filling precisely two vector multiplets. A first possible kinetic pattern thus is given by

$$\text{I)}:\mathcal{N}=8\longrightarrow DS(2,3/2,0)_{\rm S}+DS(3/2,3,2)_{\rm S}+2\cdot DS(1,1)\,. \tag{3.4}$$

The next option in (3.3) is the partition $3+1+1$, in which case there are various possibilities depending on the embedding of these gravitinos into the corresponding long or short gravitino multiplets. Naive counting allows for the following possibilities

$$\begin{array}{r|cccc} \text{II)} & 3 & 1 & 1 & \text{vectors} \\ \hline \text{a):} & \text{L} & \text{S} & \text{S} & 1 \\ \text{b):} & \text{S} & \text{L} & \text{L} & 0 \\ \text{c):} & \text{S} & \text{L} & \text{S} & 3 \\ \text{d):} & \text{S} & \text{S} & \text{S} & 6 \end{array}\quad. \tag{3.5}$$

The last column denotes the number of vector multiplets that describe the remaining matter spectrum, once the gravity and gravitino multiplets are subtracted from $\mathcal{N}=8$. Here, we note the following property of the vector multiplet: when ignoring the energy of the states, the field content of $DS(1,j_0)$ coincides with the tensor product of $DS(1,1)$ with the SO(3) representation $[j_0-1]$ of the vector fields. As a consequence, for instance the 3 vectors in the third row of (3.5) can either correspond to three multiplets $DS(1,1)$ or to a single multiplet $DS(1,2)$, the field content only differs in energies. Let us take a closer look at the decompositions of (3.5): the cases a) and d) both carry two gravitinos in the short massless $DS(3/2,1,0)_{\rm S}$, i.e. both cases in fact correspond to a supersymmetry enhancement to $\mathcal{N}=5$. Such vacua have been ruled out by the general discussion in section 2 and cannot be dynamically realized. We are thus left with the options IIb) and IIc), of which the latter corresponds to a supersymmetry enhancement to $\mathcal{N}=4$.

The third option in (3.3) is the partition $2+2+1$, for which we find two possibilities

$$\begin{array}{c|cccc} \text{III)} & 2 & 2 & 1 & \text{vectors} \\ \hline \text{a):} & \text{S} & \text{S} & \text{L} & 3 \\ \text{b):} & \text{S} & \text{S} & \text{S} & 6 \end{array} \quad . \tag{3.6}$$

In summary, the possible AdS $\mathcal{N}=8 \longrightarrow \mathcal{N}=3$ supersymmetry breaking patterns are given by the following decompositions of the $\mathcal{N}=8$ supergravity multiplet

$$\begin{aligned} \text{I)} &: DS(2,3/2,0)_{\text{S}} + DS(3/2,3,2)_{\text{S}} + 2\cdot DS(1,1)\,, \\ \text{IIb)} &: DS(2,3/2,0)_{\text{S}} + DS(3/2,2,1)_{\text{S}} + DS(3/2,E_1,0)_{\text{L}} + DS(3/2,E_2,0)_{\text{L}}\,, \\ \text{IIc)} &: DS(2,3/2,0)_{\text{S}} + DS(3/2,2,1)_{\text{S}} + DS(3/2,E_0,0)_{\text{L}} + DS(3/2,1,0)_{\text{S}} + DS(1,2)\,, \\ \text{IIIa)} &: DS(2,3/2,0)_{\text{S}} + 2\cdot DS(3/2,3/2,1/2)_{\text{S}} + DS(3/2,E_0,0)_{\text{L}} + DS(1,2)\,, \\ \text{IIIb)} &: DS(2,3/2,0)_{\text{S}} + 2\cdot DS(3/2,3/2,1/2)_{\text{S}} + DS(3/2,1,0)_{\text{S}} + 2\cdot DS(1,2)\,. \end{aligned} \tag{3.7}$$

In the following we will study which of these patterns can actually be dynamically realized in $\mathcal{N}=8$ supergravity and determine the specific gaugings which allow for the corresponding vacua.

4 $\mathcal{N}=3$ and $\mathcal{N}=4$ vacua

4.1 Solutions of the quadratic equations

The SO(8) subgroup of SU(8) naturally splits into SO(3) × SO(5). We require the vacuum at the origin (and thus the tensors A_{ij}, $A^i{}_{jkl}$) to be invariant under the diagonal group $\text{SO}(3)_{\text{d}}$ of the SO(3) group acting only on the Killing spinors and a second SO(3) embedded inside SO(5) according to the transformation of the massive gravitinos. We shall separately discuss the three cases corresponding to the allowed inequivalent embeddings (3.3) of SO(3) inside SO(5) and in each if them study the solutions to the system (2.6). In all cases we have reduced the system by implementing the most general ansatz in terms of singlets under $\text{SO}(3)_{\text{d}}$ and (with the help of mathematica) systematically scanned the remaining equations for their real solutions. Such solutions turn out to be extremely rare. Some computational details are relegated to appendix B.

4.1.1 Case $\mathbf{5} \to \mathbf{5}$

With this decomposition, there are six $\text{SO}(3)_{\text{d}}$ singlets in the tensors A_{ij}, $A_i{}^{jkl}$, three of which are killed by the general discussion of section 2.2. It is straightforward to verify that the remaining system of quadratic equations for three parameters does not possess any real solution (other than the known $\mathcal{N}=8$ solution), such that this possibility is ruled out by direct computation.

4.1.2 Case $\mathbf{5} \to \mathbf{2}+\mathbf{2}+\mathbf{1}$

Let us first split the $A, B, \ldots$ indices into $\Lambda, \Sigma, \ldots = 4,5,6,7$ labeling the fundamental of the SO(4) inside SO(5), and identify the singlet in the decomposition with the value $i=8$.

The index i thus splits in $i = \alpha, \Lambda, 8$. Next we embed SO(3) inside SO(4) by identifying its generators with the anti-self-dual matrices $(t^{(-)}_\alpha)^\Lambda{}_\Sigma$:

$$t^{(-)}_1 = \begin{pmatrix} 0 & \frac{1}{2} & 0 & 0 \\ -\frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 \end{pmatrix} ; \; t^{(-)}_2 = \begin{pmatrix} 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & -\frac{1}{2} \\ \frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 \end{pmatrix} ; \; t^{(-)}_3 = \begin{pmatrix} 0 & 0 & 0 & -\frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 \\ 0 & -\frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 \end{pmatrix} \tag{4.1}$$

We also have the complementary set of generators $(t^{(+)}_{\hat\alpha})^\Lambda{}_\Sigma$, commuting with t_α, obtained by changing the sign of the 4th row and columns of the latter. The following properties hold:

$$t^{(\pm)}_{\alpha\,\Lambda\Sigma} = \pm\frac{1}{2}\epsilon_{\Lambda\Sigma\Gamma\Delta}\, t^{(\pm)}_{\alpha\,\Gamma\Delta} \,. \tag{4.2}$$

The $\mathrm{SO}(3)_\mathrm{d}$ generators in the $\mathbf{8}$ of SO(8) read:

$$t_\alpha = \begin{pmatrix} \epsilon_{\beta\alpha\gamma} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & (t^{(-)}_\alpha)_\Lambda{}^\Sigma & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & 0 \end{pmatrix} , \tag{4.3}$$

and close the $\mathfrak{so}(3)_\mathrm{d}$ algebra:

$$[t_\alpha,\, t_\beta] = \epsilon_{\alpha\beta\gamma}\, t_\gamma \,. \tag{4.4}$$

With the general ansatz for A_{ij}, $A_i{}^{jkl}$ in terms of singlets under this $\mathrm{SO}(3)_\mathrm{d}$ (cf. appendix B), we find aside from the known $\mathcal{N} = 8$ solution $A_i{}^{jkl} = 0$, $A_{77} = \pm 1$, $A_{88} = e^{i\varphi}$, only the following $\mathcal{N} = 3$ solution:

$$\begin{aligned} &A_{\alpha\beta} = \delta_{\alpha\beta} \,, && A_{\Lambda\Sigma} = \frac{3}{2}\,\epsilon\,\delta_{\Lambda\Sigma} \,, && A_{88} = -\sqrt{3}\, e^{3i\varphi} \,, \\ &A^\Lambda{}_{\Sigma\alpha\beta} = \epsilon_{\alpha\beta\gamma}\,(t^{(-)}_\gamma)^\Lambda{}_\Sigma \,, && A^\Lambda{}_{\Sigma\alpha 8} = -\sqrt{3}\, e^{i\varphi}\,(t^{(-)}_\alpha)^\Lambda{}_\Sigma \,, \\ &A^\Lambda{}_{\Sigma\Gamma\Delta} = \epsilon\frac{\sqrt{3}}{2}\, e^{-i\varphi}\,\epsilon_{\Lambda\Sigma\Gamma\Delta} \,, && A^8{}_{\alpha\beta\gamma} = 0 \,, \quad A^8{}_{8\Lambda\Sigma} = 0 \,, && A^8{}_{\alpha\Lambda\Sigma} = -2\,\epsilon\, e^{-2i\varphi}\, t^{(-)}_{\alpha\,\Lambda\Sigma} \,, \end{aligned} \tag{4.5}$$

with real φ and $\epsilon = \pm 1$. It effectively depends only on the phase φ, since the sign $\epsilon = \pm 1$ can be absorbed by an SU(8) transformation.

4.1.3 Case $\mathbf{5} \to \mathbf{3} + \mathbf{1} + \mathbf{1}$

Let us split the index i into $i = \alpha, \alpha', a$, where $\alpha = 1, 2, 3$, $\alpha' = 4, 5, 6$ and $a = 7, 8$ is the index labeling the singlets. The $\mathrm{SO}(3)_\mathrm{d}$ generators in the $\mathbf{8}$ of SO(8) read:

$$t_\alpha = \begin{pmatrix} \epsilon_{\beta\alpha\gamma} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \epsilon_{\beta'\alpha\gamma'} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0}_2 \end{pmatrix} , \tag{4.6}$$

and satisfy the relations (4.4). With the general ansatz for A_{ij}, $A_i{}^{jkl}$ in terms of singlets under this $\mathrm{SO}(3)_\mathrm{d}$ (cf. appendix B), we find aside from the known $\mathcal{N} = 8$ solution only the

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following solution:

$$
\begin{aligned}
A_{\alpha\beta} &= \delta_{\alpha\beta}\,, \qquad A_{\alpha'\beta'} = 2\xi\,\delta_{\alpha'\beta'}\,, & A_{77} &= 2\,\eta\,, \qquad A_{88} = \xi\,\eta\,e^{i\varphi}\,,\\
A^{7}{}_{\alpha'\beta'\gamma} &= \sqrt{2}\,\xi\,\eta\,e^{-i\frac{\varphi}{4}}\,\epsilon_{\alpha'-3\,\beta'-3\,\gamma}\,, & A^{\alpha'}{}_{\beta'\gamma 7} &= -\sqrt{2}\,e^{-i\frac{\varphi}{4}}\,\epsilon_{\alpha'-3\,\beta'-3\,\gamma}\,,\\
A^{\alpha'}{}_{\beta'\gamma' 8} &= \sqrt{2}\,\xi\,e^{i\frac{\varphi}{4}}\,\epsilon_{\alpha'-3\,\beta'-3\,\gamma'-3}\,, & A^{\alpha'}{}_{\alpha a b} &= -\eta\,e^{i\frac{\varphi}{2}}\,\delta_{\alpha}^{\alpha'-3}\,\epsilon_{ab}\,,\\
A^{7}{}_{8\alpha'\alpha} &= -\xi\,e^{i\frac{\varphi}{2}}\,\delta_{\alpha'-3\,\alpha}\,, & A^{\alpha'}{}_{\beta'\alpha\beta} &= -2\,\delta_{\alpha\beta}^{\alpha'-3\,\beta'-3}\,,
\end{aligned} \tag{4.7}
$$

where η, $\xi = \pm 1$. The parameter ξ can be disposed of by means of a SU(8) transformation while the sign η can be changed by shifting $\varphi \to \varphi + 2\pi$. We can thus set $\xi = \eta = +1$. Notice that $A^{8}{}_{ijk} = 0$ which implies that this is actually an $\mathcal{N} = 4$ solution and that the residual symmetry group is enhanced to SO(4).

4.2 Gauge groups and $\mathrm{E}_{7(7)}$-invariants

We have identified two AdS vacua in maximal supergravity by solving the system of quadratic constraints (2.6) for the embedding tensor. As the next step, we will have to determine the associated gauge groups, i.e. identify in which gauged maximal supergravity these vacua live. We can compute the associated gauge group generators via (2.4), (2.3), and (2.1). Much of the structure of the gauge group can already be inferred from the $\mathrm{E}_{7(7)}$-invariant signature of the (generalized) Cartan-Killing metric

$$
\mathrm{sign}[\mathrm{Tr}(X_M \cdot X_N)]\,. \tag{4.8}
$$

The above matrix has 28 vanishing eigenvalues (due to the locality constraint (2.2)) while the other 28 eigenvalues define the Cartan-Killing metric of the gauge algebra.

4.2.1 The $\mathcal{N} = 4$ vacuum

We first compute the Cartan-Killing metric (4.8) for the $\mathcal{N} = 4$ vacuum (4.7) as a function of the angular parameter φ. This allows the following identification of the corresponding underlying gauge group:

parameter	signature of C.-K. metric	gauge group
$\varphi = 2\pi$	$(1_+, 15_-, 12_0)$	$[\mathrm{SO}(1,1) \times \mathrm{SO}(6)] \ltimes T^{12}$
$0 \leq \varphi < 2\pi$	$(7_+, 21_-)$	$\mathrm{SO}(1,7)$

(4.9)

where T^{12} denotes a subgroup generated by twelve nilpotent operators. Notice that AdS vacua in theories with gauged $\mathrm{SO}(1,7)$ and $[\mathrm{SO}(1,1) \times \mathrm{SO}(6)] \ltimes T^{12}$ groups were found in [11]. The residual supersymmetry, symmetry group (SO(4)) and spectrum of our vacuum distinguishes it from those found in the same reference.

The gauge group $\mathrm{SO}(1,7)$ alone is not sufficient to determine the gauged supergravity, since there is a one-parameter class of such theories [13, 15]. Rather, we expect to find a mapping between the angular parameter φ which defines our vacua, and the ω-angle that labels the one-parameter class of $\mathrm{SO}(1,7)$ theories. To this end, we compute other

$E_{7(7)}$-invariants on the $\mathcal{N}=4$ vacuum, to compare with the same quantities evaluated for the ω-rotated SO(1,7) gauge group. In particular, we consider the 1540×1540 matrix:

$$\mathbb{K}_{MN}{}^{PQ} \equiv \frac{1}{4}\, d^{R_1R_2R_3R_4}\, X_{R_1M}{}^{K}\, X_{R_2NK}\, X_{R_3}{}^{PL}\, X_{R_4L}{}^{Q}\,, \tag{4.10}$$

quartic in the gauge group generators (2.1), which is antisymmetric in $[MN]$ and $[PQ]$, by virtue of the total symmetry of the $E_{7(7)}$-invariant tensor $d^{R_1R_2R_3R_4}$. This tensor $\mathbb{K}$ is related to one computed in [13] for the ω-deformed SO(8) gauging. Instead of evaluating the eigenvalues of this matrix, as was done for the corresponding tensor in [13] for a specific gauging, we evaluate the traces of its powers. We compute them for our $\mathcal{N}=4$ solution and for the (ω-deformed) SO(1,7) and SO(8) gauging.

For the invariant d-tensor we use the following form in the SU(8)-basis:

$$\begin{aligned} d^{MNPQ}\,\Lambda_M\Lambda_N\Lambda_P\Lambda_Q &= \Lambda^{i_1i_2}\Lambda^{i_3i_4}\Lambda^{i_5i_6}\Lambda^{i_7i_8}\,\epsilon_{i_1\dots i_8} + \Lambda_{i_1i_2}\Lambda_{i_3i_4}\Lambda_{i_5i_6}\Lambda_{i_7i_8}\,\epsilon^{i_1\dots i_8} + \\ &\quad + 96\,\mathrm{Tr}(\Lambda\bar{\Lambda}\Lambda\bar{\Lambda}) - 24\,\mathrm{Tr}(\Lambda\bar{\Lambda})^2\,, \\ \Lambda_M &\equiv (\Lambda,\bar{\Lambda}) \;\equiv\; (\Lambda_{ij},\Lambda^{ij})\,. \end{aligned} \tag{4.11}$$

For the SO(8) and SO(1,7) gaugings the X-tensor (2.1) is computed via (2.3), (2.4), starting from fermion shift tensors of the form [11]:

$$A_{ij} = e^{i\,\omega}\,\mathrm{Tr}(\theta)\,\delta_{ij}\quad,\quad A_i{}^{jkl} = e^{-i\,\omega}\,(\Gamma_i{}^{jkl})_{IJ}\theta^{IJ}\,, \tag{4.12}$$

where

$$\theta^{IJ} = \mathrm{diag}(1,1,1,1,1,1,1,\kappa)\,, \tag{4.13}$$

with $\kappa=+1$ for SO(8) and -1 for SO(1,7). We find for the traces of the various powers of (4.10)

$$\begin{aligned} &\mathrm{Tr}(\mathbb{K}) = 0\,, \\ &\mathrm{Tr}(\mathbb{K}^2) = 2^{23}\times 3^4\times 5\times 7\times (7(5\kappa+3)+28(\kappa-1)\cos(4\omega)+(\kappa+7)\cos(8\omega))\;, \\ &\mathrm{Tr}(\mathbb{K}^3) = 2^{36}\times 3^7\times 5\times 7\times (35\kappa+4(7\kappa+1)\cos(4\omega)+(\kappa-1)\cos(8\omega)-3)\sin^2(2\omega)\,, \\ &\mathrm{Tr}(\mathbb{K}^4) \propto \mathrm{Tr}(\mathbb{K}^2)^2\,. \end{aligned} \tag{4.14}$$

Notice that in the SO(8) case these invariants have half-period $\pi/8$, namely they assume all possible values in the interval $\omega\in(0,\pi/8)$, while for the SO(1,7) gauging the half-period is $\pi/4$. In the former case we have independent gaugings only for $\omega\in(0,\pi/8)$, while in the latter case for $\omega\in(0,\pi/4)$, consistently with the results of [13, 14]. Eqs. (4.14) do not hold for $\kappa=0$, corresponding to ISO(7), in which case all traces are zero.

On our $\mathcal{N}=4$ solution (4.7), (denoting the corresponding tensor by $\mathbb{K}_s$) these traces become

$$\begin{aligned} &\mathrm{Tr}(\mathbb{K}_s) = 0\,, \\ &\mathrm{Tr}(\mathbb{K}_s^2) = 2^6\times 3^4\times 5\times 7\times \left(3\cos\left(\frac{\varphi}{2}\right)+\cos(\varphi)+2\right)\;, \\ &\mathrm{Tr}(\mathbb{K}_s^3) = -2^{10}\times 3^7\times 5\times 7\times \cos^4\left(\frac{\varphi}{4}\right)\;, \\ &\mathrm{Tr}(\mathbb{K}_s^4) \propto \mathrm{Tr}(\mathbb{K}_s^2)^2\,. \end{aligned} \tag{4.15}$$

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These expressions are symmetric under $\varphi \to -\varphi$ and $\varphi \to \varphi + 4\pi$, so that they assume all possible values in the interval $(0, 2\pi)$. Following the same reasoning as for the SO(1,7) and SO(8) cases, we can then argue that X tensors in this class with generic φ can be SU(8)-rotated to one with $\varphi \in (0, 2\pi)$. In comparing the traces for the $\mathcal{N} = 4$ vacuum (4.15) to those of the SO(1,7) gauging (4.14), we assume that

$$X^{(s)}_{MN}{}^{P} = \lambda(\varphi)\, \mathrm{E}_{7(7)} \star (X_{MN}{}^{P}) \,, \tag{4.16}$$

where $X^{(s)}_{MN}{}^{P}$ is the X-tensor on the $\mathcal{N} = 4$ vacuum, $\mathrm{E}_{7(7)} \star (X_{MN}{}^{P})$ is the $\mathrm{E}_{7(7)}$-rotated X tensor of the SO(1,7) gauging, and we allowed for a proportionality factor $\lambda(\varphi)$ depending on the parameter ϕ. Clearly the traces of $\mathbb{K}$ do not depend on the $\mathrm{E}_{7(7)}$-rotation, so that eq. (4.16) implies:

$$\mathrm{Tr}(\mathbb{K}_s^2) = \lambda(\varphi)^8\, \mathrm{Tr}(\mathbb{K}^2) \quad , \quad \mathrm{Tr}(\mathbb{K}_s^3) = \lambda(\varphi)^{12}\, \mathrm{Tr}(\mathbb{K}^3) \,. \tag{4.17}$$

We immediately realize that for $\omega = 0$ the above system has no solution since $\mathrm{Tr}(\mathbb{K}_s^2) = 0$ while $\mathrm{Tr}(\mathbb{K}^2) \neq 0$, thus implying $\lambda = 0$. Similarly for $\varphi = 2\pi$, the second of (4.17) implies $\omega = 0$ while in the first $\mathrm{Tr}(\mathbb{K}_s^2) = 0$ while $\mathrm{Tr}(\mathbb{K}^2) \neq 0$, againg implying $\lambda = 0$. This is compatible with our finding (4.9) that the gauge group at $\varphi = 2\pi$ degenerates to $[\mathrm{SO}(1,1) \times \mathrm{SO}(6)] \ltimes T^{12}$.

For generic values of ω and φ the relation between the two parameters can then be deduced from the (λ-independent) equation

$$\frac{\mathrm{Tr}(\mathbb{K}_s^3)^2}{\mathrm{Tr}(\mathbb{K}_s^2)^3} = \frac{\mathrm{Tr}(\mathbb{K}^3)^2}{\mathrm{Tr}(\mathbb{K}^2)^3} \iff \frac{\cos^2\left(\frac{\varphi}{4}\right)}{\left(2\cos\left(\frac{\varphi}{2}\right)+1\right)^3} = \frac{32\,(\cos(4\omega)+3)^4 \sin^4(2\omega)}{(-28\cos(4\omega)+3\cos(8\omega)-7)^3} \,, \tag{4.18}$$

whose solution $\omega(\varphi)$ is plotted in figure 1. We conclude that the $\mathcal{N} = 4$ vacuum can be found (for generic values of φ), only in the ω-rotated SO(1,7) gauging, while in the limit $\omega \to 0$ it disappears. At the corresponding point $\varphi = 2\pi$ in the parameter space of solutions it turns into a vacuum of the gauging with non-semisimple gauge group $[\mathrm{SO}(1,1) \times \mathrm{SO}(6)] \ltimes T^{12}$.

4.2.2 The $\mathcal{N} = 3$ vacuum

The same analysis can be repeated for the $\mathcal{N} = 3$ vacuum (4.5). In this case the computation of the Cartan-Killing metric (4.8) for the gauge group indicates the following correspondence between the values of φ and the gauge group

parameter	signature of C.-K. metric	gauge group
$0 \leq \varphi < \frac{\pi}{6}$	$(0_+, 28_-)$	SO(8)
$\frac{\pi}{6} < \varphi \leq \pi$	$(7_+, 21_-)$	SO(1,7)
$\varphi = \frac{\pi}{6}$	$(0_+, 21_-, 7_0)$	ISO(7)

(4.19)

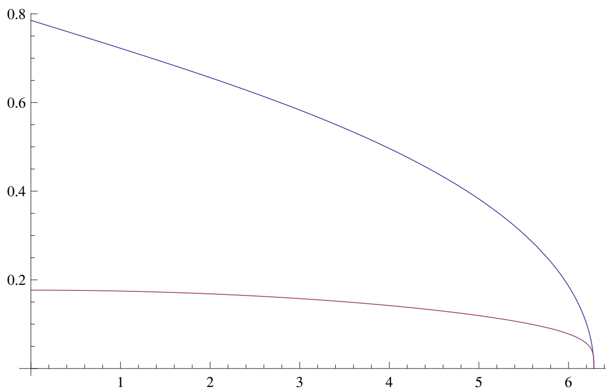

Figure 1. The parameters ω (blue) and λ (red) as function of φ for the $\mathcal{N}=4$ vacuum. The gauge group is $\mathrm{SO}(1,7)$ except for the point $\varphi=2\pi$ where λ vanishes and the gauge group degenerates to $[\mathrm{SO}(1,1)\times\mathrm{SO}(6)]\ltimes T^{12}$.

Computing the traces of the tensor (4.10) on our $\mathcal{N}=3$ solution we find in this case:

$$
\begin{aligned}
&\mathrm{Tr}(\mathbb{K}_s)=0\,,\\
&\mathrm{Tr}(\mathbb{K}_s^2)=-2^{-1}\times 3^8\times 5\times 7\times\cos(\varphi)\left(\sqrt{3}(\cos(2\varphi)+3)-7\cos(\varphi)\right)\,,\\
&\mathrm{Tr}(\mathbb{K}_s^3)=2^{-2}\times 3^{11}\times 5\times 7\times(24\sqrt{3}\cos(\varphi)-18\cos(2\varphi)+2\sqrt{3}\cos(3\varphi)-27)\,,\\
&\mathrm{Tr}(\mathbb{K}_s^4)\propto\mathrm{Tr}(\mathbb{K}_s^2)^2\,,
\end{aligned}
\tag{4.20}
$$

which should now be compared to (4.14) for $\kappa=\pm1$ in the different intervals of (4.19). The expressions of (4.20) are symmetric under $\varphi\to-\varphi$ and $\varphi\to\varphi+2\pi$, so that they assume all possible values in the interval $(0,\pi)$. We then argue that an X-tensor in this class with a generic φ can be SU(8)-rotated to one within $\varphi\in(0,\pi)$. The correspondence between φ and ω for the ω-rotated $\mathrm{SO}(1,7)$ and $\mathrm{SO}(8)$ groups is obtained from an equation analogous to (4.18) and plotted in figure 2. This illustrates that λ vanishes, as $\omega\to0$ ($\varphi\to\pi/6$), so that the vacuum disappears from the corresponding gauged theories. At this point in the φ parameter space it becomes a vacuum of an ISO(7) gauged theory. Indeed, for $\varphi=\pi/6$, all traces of (4.20) vanish.

The existence of the proportionality parameter $\lambda(\varphi)$ depending on φ (or, equivalently on ω) is due to the fact that we have fixed the value the potential in our vacua to a given value (-6) by choosing the coupling constant. This function therefore encodes the dependence, in the corresponding ω-rotated theories, of the cosmological constant of these vacua on ω, which is a generic feature of all extrema of the potential aside from the $\mathcal{N}=8$ one [13, 17].

4.3 Mass spectra

We can eventually compute the spectra around the new vacua by evaluating the mass formulas (2.12), (2.14), and (2.17) for our solutions A_{ij}, $A_i{}^{jkl}$ and compare the result to the general multiplet structure discussed in section 3. We find that in all cases the spectra are independent of the parameter φ.

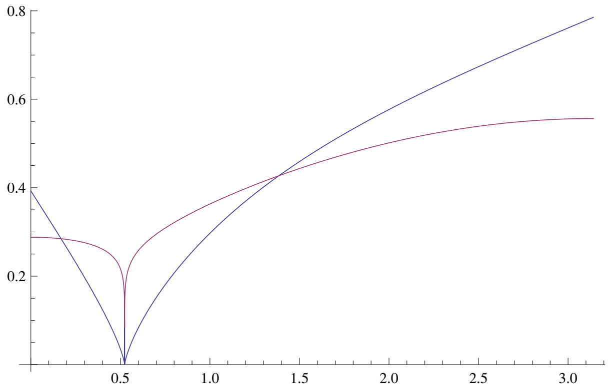

Figure 2. The parameters ω (blue) and λ (red) as function of φ for the $\mathcal{N}=3$ vacuum. The gauge group is SO(8) for $\varphi < \pi/6$ and SO(1,7) for $\varphi > \pi/6$. A the point $\varphi = \pi/6$ where λ vanishes, the gauge group degenerates to ISO(7).

4.3.1 The $\mathcal{N} = 3$ vacuum

The scalar mass spectrum on the $\mathcal{N} = 3$ vacuum is:

$$\begin{aligned} m^2\,L_0^2 \qquad &: 1\times\Big(3(1+\sqrt{3})\Big)\ ;\ 6\times\Big(1+\sqrt{3}\Big)\ ;\ 1\times\Big(3(1-\sqrt{3})\Big)\ ;\ 6\times\Big(1-\sqrt{3}\Big)\ ; \\ & 4\times\left(-\frac{9}{4}\right)\ ;\ 18\times(-2)\ ;\ 12\times\left(-\frac{5}{4}\right)\ ;\ 22\times(0)\ , \end{aligned} \tag{4.21}$$

in units of the inverse anti- de Sitter radius $1/L_0$ from (A.7). The Breitenlohner-Freedman bound $m^2\,L_0^2 \geq -\frac{9}{4}$ [24] is satisfied by virtue of supersymmetry. The normalized vector masses are given by:

$$m^2\,L_0^2 \quad : 3\times\Big(3+\sqrt{3}\Big)\ ;\ 3\times\Big(3-\sqrt{3}\Big)\ ;\ 4\times\left(\frac{15}{4}\right)\ ;\ 12\times\left(\frac{3}{4}\right)\ ;\ 6\times(0)\ , \tag{4.22}$$

The 22 massless scalar fields are the Goldstone bosons for the massive vector fields. Together, we conclude that the $\mathcal{N} = 3$ vacuum realizes option IIIa) from (3.7) with one long spin 3/2 multiplet of energy $E_0 = \sqrt{3}$

$$DS(2,3/2,0)_{\mathrm{S}} + 2\cdot DS(3/2,3/2,1/2)_{\mathrm{S}} + DS(3/2,\sqrt{3},0)_{\mathrm{L}} + 3\cdot DS(1,1) \tag{4.23}$$

The three extra massless vectors describe an extra SO(3) symmetry. Explicit computation of the fermionic mass matrices (2.17) also confirms the multiplet structure.

4.3.2 The $\mathcal{N} = 4$ vacuum

The scalar mass spectrum on the $\mathcal{N} = 4$ vacuum is:

$$m^2\,L_0^2 : 1\times(10)\ ;\ 10\times(4)\ ;\ 11\times(-2)\ ;\ 48\times(0)\ . \tag{4.24}$$

The vector masses are

$$m^2\,L_0^2 : 7\times(6)\ ;\ 15\times(2)\ ;\ 6\times(0)\ , \tag{4.25}$$

22 of the massless scalar fields are the Goldstone bosons for the massive vector fields, while the six massless vectors gauge the residual SO(4) group. This solution thus realizes option IIc) from (3.7) with a long spin 3/2 multiplet of energy $E_0 = 2$

$$DS(2,3/2,0)_{\rm S} + DS(3/2,1,0)_{\rm S} + DS(3/2,2,1)_{\rm S} + DS(3/2,2,0)_{\rm L} + DS(1,2)\ , \quad (4.26)$$

and supersymmetry enhancement to $\mathcal{N} = 4$, under which the first two multiplets combine into the $\mathcal{N} = 4$ massless supergravity multiplet and the remaining three multiplets combine into a single $\mathcal{N} = 4$ massive spin 3/2 multiplet. Again, an explicit computation of the fermionic mass matrices (2.17) confirms this multiplet structure.

5 Conclusions

In this paper we have studied AdS vacua of maximal supergravity in four dimensions with residual $\mathcal{N} > 2$ supersymmetry. We exclude on general grounds $8 > \mathcal{N} > 4$ vacua and find two 1-parameter classes of $\mathcal{N} = 3$ and 4 vacua, which can be embedded only in the ω-rotated gauged models. Of particular importance are the models with SO(8) gauging since they exhibit in addition an $\mathcal{N} = 8$ vacuum. The eleven dimensional origin of the latter is still debated and in [13] it was conjectured to corrrespond to certain to ABJ theories [25], through the AdS/CFT duality [26]. Understanding the higher dimensional origin of the new $\mathcal{N} = 3$ and 4 vacua is an important problem which deserves investigation.

Still in the light of the AdS/CFT correspondence, these new vacua should describe conformal fixed points of some dual (three-dimensional) field theory. It would be also interesting, in this respect, to study RG flows between the conformal critical points dual to the two kinds of vacua in the ω-deformed SO(1,7) models, or interpolating between the $\mathcal{N} = 8$ and $\mathcal{N} = 3$ vacua in the ω-deformed SO(8) theories, thus generalizing the analysis of [27, 28].

An other issue which deserves investigation is the study of black holes asymptoting the new $\mathcal{N} = 3$ and 4 vacua, along the lines of [29]. It would also be interesting to understand to which extend the methods developed in this paper can be extended to a systematic analysis of the AdS (and Minkowski) vacuum in maximal supergravity with $\mathcal{N} = 2$ and $\mathcal{N} = 1$ supersymmetry.

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A Normalizations and conventions

Let us recall the relevant notations for the scalar masses. The bosonic Lagrangian of $\mathcal{N} = 8$ supergravity reads (setting $\kappa^2 = 8\pi G = 1$):[6]

$$\mathcal{L} = e\left[-\frac{R}{2} + \frac{1}{12}\,P_\mu^{ijkl}\,P^\mu_{ijkl} + \frac{1}{4}\,I_{\Lambda\Sigma}(\phi)\,F^\Lambda_{\mu\nu}F^{\Sigma\,\mu\nu} + \frac{1}{8\,e}\,\epsilon^{\mu\nu\rho\sigma}\,R_{\Lambda\Sigma}(\phi)\,F^\Lambda_{\mu\nu}F^\Sigma_{\rho\sigma} - V(\phi)\right]. \quad (A.1)$$

[6]We use the notations of [9], though in the 'mostly minus' notation.

We choose the vacuum at the origin $\phi_0 = 0$ in which the scalar potential is negative $V_0 = V(0) < 0$ (AdS vacuum), and we expand about it:

$$\phi^{ijkl} = \phi_0^{ijkl} + \delta\phi^{ijkl} = \delta\phi^{ijkl} \,. \tag{A.2}$$

Being interested in the scalar kinetic and mass terms, we set $A_\mu^\Lambda = 0$ so that vielbein P_μ^{ijkl} on the vacuum reads:

$$P_\mu^{ijkl} = \partial_\mu \delta\phi^{ijkl} \,. \tag{A.3}$$

Let us denote by ϕ^α, $\alpha = 1, \ldots, 70$ the real and imaginary parts of ϕ^{ijkl} subject to the self-duality condition. On the vacuum the kinetic and mass terms of the scalar fluctuations read:

$$\mathcal{L}_s^{(2)} = 4 \sum_\alpha \partial_\mu \delta\phi^\alpha \partial^\mu \delta\phi^\alpha - \frac{1}{2} \left. \frac{\partial^2 V}{\partial\phi^\alpha \partial\phi^\beta} \right|_{\phi=0} \delta\phi^\alpha \delta\phi^\beta \,, \tag{A.4}$$

from which we deduce the mass matrix:

$$(m^2)_{\alpha\beta} = \frac{1}{8} \left. \frac{\partial^2 V}{\partial\phi^\alpha \partial\phi^\beta} \right|_{\phi=0} = \frac{1}{8} \frac{\partial^2 V^{(2)}}{\partial\delta\phi^\alpha \partial\delta\phi^\beta} \,, \tag{A.5}$$

where $V^{(2)}(\delta\phi)$ is given in (2.12).

Four-dimensional anti-de Sitter space can be defined as the connected hyperboloid in $\mathbb{R}^5$ described by the equation:

$$\eta_{AB} y^A y^B = L_0^2 \;;\;\; \eta = \mathrm{diag}(+ - - - +) \,, \tag{A.6}$$

L_0 being the "radius" of the AdS space-time. The Ricci tensor reads:

$$R_{\mu\nu} = -\Lambda \, g_{\mu\nu} \;, \quad \Lambda = -\frac{3}{L_0^2} < 0 \,, \tag{A.7}$$

where Λ is the cosmological constant. From (A.1) we can identify:

$$\Lambda = V_0 = -\frac{3}{L_0^2} \,. \tag{A.8}$$

If m^2 is a generic eigenvalue of $(m^2)_{\alpha\beta}$, stability of the vacuum implies the following condition:

$$m^2 \, L_0^2 = \frac{3}{|V_0|} \, m^2 \geq -\frac{9}{4} \,, \tag{A.9}$$

which is the Breitenlohner-Freedman bound [24].

For the reader's convenience we give the relations between the AdS energy E_0 and the masses of the various fields [30]:

$$\begin{aligned} \text{scalars}: \;\; E_0 &= \frac{1}{2} \left(3 \pm \sqrt{9 + 4 \, m^2 \, L_0^2}\right) \,, \\ \text{vectors}: \;\; E_0 &= \frac{1}{2} \left(3 + \sqrt{1 + 4 \, m^2 \, L_0^2}\right) \,, \\ \text{spinors, gravitino}: \;\; E_0 &= \frac{1}{2} \left(3 + 2 \, |m \, L_0|\right) \,. \end{aligned} \tag{A.10}$$

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B $\mathcal{N} = 3$ vacua: computational details

B.1 Case $\mathbf{5} \to \mathbf{2} + \mathbf{2} + \mathbf{1}$

The $\mathrm{SO}(3)_{\mathrm{d}}$-invariant tensors A_{ij}, $A^i{}_{jkl}$ have, in general, the following non-vanishing components:

$$
\begin{aligned}
A_{\alpha\beta} &= \delta_{\alpha\beta}\;; \qquad\qquad A_{\Lambda\Sigma} = A_{77}\,\delta_{\Lambda\Sigma}\;; \qquad\qquad A_{88}\,,\\
A^\Lambda{}_{\Sigma\alpha\beta} &= A^{(0)}\,\epsilon_{\alpha\beta\gamma}\,(t^{(-)}_\gamma)^\Lambda{}_\Sigma + A^{(\hat\alpha)}\,\epsilon_{\alpha\beta\gamma}\,(t^{(-)}_\gamma t^{(+)}_{\hat\alpha})^\Lambda{}_\Sigma\,,\\
A^\Lambda{}_{\Sigma\alpha 8} &= C^{(0)}\,(t^{(-)}_\alpha)^\Lambda{}_\Sigma + C^{(\hat\alpha)}\,(t^{(-)}_\alpha t^{(+)}_{\hat\alpha})^\Lambda{}_\Sigma\,,\\
A^\Lambda{}_{\Sigma\Gamma\Delta} &= D^{(0)}\,\epsilon_{\Lambda\Sigma\Gamma\Delta} + D^{(\hat\alpha)}\,\delta^\Lambda_{[\Sigma}(t^{(+)}_{\hat\alpha})_{\Gamma\Delta]}\,,\\
A^8{}_{\alpha\beta\gamma} &= B^{(0)}\,\epsilon_{\alpha\beta\gamma}\;;\quad A^8{}_{8\Lambda\Sigma} = B^{(\hat\alpha)}\,(t^{(+)}_{\hat\alpha})_{\Lambda\Sigma}\;;\quad A^8{}_{\alpha\Lambda\Sigma} = E^{(0)}\,t^{(-)}_{\alpha\,\Lambda\Sigma}\,.
\end{aligned}
\tag{B.1}
$$

The traceless condition on $A^i{}_{jkl}$ sets $B^{(\hat\alpha)} = -2\,D^{(\hat\alpha)}/3$, so that we end up with the 16 independent parameters:

$$
A_{77},\ A_{88},\ A^{(0)},\ A^{(\hat\alpha)},\ B^{(0)},\ C^{(0)},\ C^{(\hat\alpha)},\ D^{(0)},\ D^{(\hat\alpha)},\ E^{(0)}\,. \tag{B.2}
$$

Using the residual SU(8) symmetry we can set A_{77} to be real. It is useful to identify the last 14 parameters with entries of $A_i{}^{jkl}$:

$$
\begin{aligned}
&A^{(0)} = 2\,A^7{}_{236}; && A^{(1)} = 4\,A^7{}_{237}; && A^{(2)} = 4\,A^7{}_{234}; && A^{(3)} = -4\,A^7{}_{235}\,,\\
&C^{(0)} = -2\,A^7{}_{348}; && C^{(1)} = -4\,A^7{}_{358}; && C^{(2)} = 4\,A^7{}_{368}; && C^{(3)} = -4\,A^7{}_{378}\,,\\
&D^{(0)} = -A^7{}_{456}; && D^{(1)} = -3\,A^8{}_{678}; && D^{(2)} = -3\,A^8{}_{578}; && D^{(3)} = -3\,A^8{}_{568}\,,\\
&B^{(0)} = A^8{}_{123}\,, && E^{(0)} = 2\,A^8{}_{356}\,.
\end{aligned}
\tag{B.3}
$$

Aside from the known $\mathcal{N} = 8$ solution $A_i{}^{jkl} = 0$, $A_{77} = \pm 1$, $A_{88} = e^{i\varphi}$, we only find the the following $\mathcal{N} = 3$ solution (4.5), corresponding to $A^{(\hat\alpha)} = D^{(\hat\alpha)} = C^{(\hat\alpha)} = 0$, $B^{(0)} = 0$ and

$$
\begin{aligned}
A_{77} &= \frac{3}{2}\,\epsilon; && A_{88} = -\sqrt{3}\,e^{2i\varphi}; \quad A^{(0)} = 1; \quad C^{(0)} = -\sqrt{3}\,e^{i\varphi}\,,\\
D^{(0)} &= \epsilon\frac{\sqrt{3}}{2}\,e^{-i\varphi}; && E^{(0)} = -2\,\epsilon\,e^{-2i\varphi}\,.
\end{aligned}
\tag{B.4}
$$

B.2 Case $\mathbf{5} \to \mathbf{3} + \mathbf{1} + \mathbf{1}$

Let us split the index i into $i = \alpha, \alpha', a$, where $\alpha = 1,2,3$, $\alpha' = 4,5,6$ and $a = 7,8$ is the index labeling the singlets. The $\mathrm{SO}(3)_{\mathrm{d}}$ generators in the $\mathbf{8}$ of SO(8) read:

$$
t_\alpha = \begin{pmatrix} \epsilon_{\beta\alpha\gamma} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \epsilon_{\beta'\alpha\gamma'} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0}_2 \end{pmatrix}\,, \tag{B.5}
$$

and satisfy the relations (4.4).

The $\mathrm{SO}(3)_{\mathrm{d}}$-invariant tensors A_{ij}, $A^i{}_{jkl}$ have the following non-vanishing components:

$$\begin{aligned}
A_{\alpha\beta} &= \delta_{\alpha\beta}; & A_{\alpha'\beta'} &= A_{66}\,\delta_{\alpha'\beta'}; \quad A_{ab}\,, \\
A^a{}_{\alpha'\beta'\gamma'} &= A^a\,\epsilon_{\alpha'-3\,\beta'-3\,\gamma'-3}; & A^a{}_{\alpha'\beta'\gamma} &= B^a\,\epsilon_{\alpha'-3\,\beta'-3\,\gamma}\,, \\
A^a{}_{\alpha'\beta\gamma} &= D^a\,\epsilon_{\alpha'-3\,\beta\gamma}; & A^a{}_{\alpha\beta\gamma} &= C^a\,\epsilon_{\alpha\beta\gamma}\,, \\
A^a{}_{b'\beta'\gamma} &= A^a{}_b\,\delta_{\beta'-3\,\gamma} \\
A^{\alpha'}{}_{\beta'\gamma'a} &= \tilde{A}_a\,\epsilon_{\alpha'-3\,\beta'-3\,\gamma'-3}; & A^{\alpha'}{}_{\beta'\gamma a} &= \tilde{B}_a\,\epsilon_{\alpha'-3\,\beta'-3\,\gamma}; \quad A^{\alpha'}{}_{\beta\gamma a} = \tilde{D}_a\,\epsilon_{\alpha'-3\,\beta\,\gamma}\,, \\
A^{\alpha'}{}_{\alpha ab} &= B\,\delta^{\alpha'-3}_{\alpha}\,\epsilon_{ab}; & A^{\alpha'}{}_{\beta'\alpha\beta} &= E\,\delta^{\alpha'-3\,\beta'-3}_{\alpha\beta}; \quad A^{\alpha'}{}_{\beta\alpha\gamma} = C\,\delta^{\alpha'-3\,\beta}_{\alpha\gamma}\,.
\end{aligned} \tag{B.6}$$

We can always set $A_{78}=0$ and A_{66}, A_{77} to be real and the 21 complex parameters entering $A^i{}_{ijk}$:

$$A^a{}_b,\ A^a,\ B^a,\ C^a,\ D^a,\ \tilde{A}^a,\ \tilde{B}^a\ \tilde{D}^a,\ B,\ C,\ E\,, \tag{B.7}$$

are subject to the tracelessness condition:

$$C = A^a{}_a\,, \tag{B.8}$$

which leaves us with a total of 23 complex parameters

$$A_{66},\ A_{aa},\ A^a{}_b,\ A^a,\ B^a,\ C^a,\ D^a,\ \tilde{A}^a,\ \tilde{B}^a\ \tilde{D}^a,\ B,\ C,\ E\,,$$

two of which (A_{66}, A_{77}), as previously mentioned, can be made real. The relation of the parameters (B.7) to the entries of $A^i{}_{ijk}$ is:

$$\begin{aligned}
A^a{}_b &= -A^a{}_{b36}; & A^a &= A^a{}_{456}; & B^a &= A^a{}_{345}; & C^a &= A^a{}_{123}\,, \\
D^a &= A^a{}_{234}; & \tilde{A}_a &= A^6{}_{45a}; & \tilde{B}_a &= -A^6{}_{24a}; & \tilde{D}_a &= A^6{}_{12a}\,, \\
B &= A^6{}_{378}; & E &= -2\,A^6{}_{235}\,.
\end{aligned} \tag{B.9}$$

Aside from the known $\mathcal{N}=8$ solution , we find the following solution

$$\begin{aligned}
A_{66} &= 2\,\xi\,; & A_{77} &= 2\,\eta; & A_{88} &= \xi\,\eta\,e^{i\varphi}\,, \\
A^a &= C^a = D^a = 0\,; & B^7 &= \sqrt{2}\,\xi\,\eta\,e^{-i\frac{\varphi}{4}}; & B^8 &= 0\,, \\
\tilde{A}_7 &= 0\,; & \tilde{A}_8 &= \sqrt{2}\,\xi\,e^{i\frac{\varphi}{4}}\,; & \tilde{B}_7 &= -\sqrt{2}\,e^{-i\frac{\varphi}{4}}\,; \quad \tilde{B}_8 = 0\,;\ \tilde{D}_a \quad = 0\,, \\
C &= 0; & B &= -\eta\,e^{i\frac{\varphi}{2}}; & E &= -2\,, \\
A^7{}_7 &= A^8{}_8 = A^8{}_7 = 0; & A^7{}_8 &= -\xi\,e^{i\frac{\varphi}{2}}\,,
\end{aligned} \tag{B.10}$$

where η, $\xi = \pm 1$. Using the above identifications we can write the non-vanishing components of the tensors as in (4.7).

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