

Ergodic Capacity Analysis of MIMO Relay Network over Rayleigh-Rician Channels

*Original*

Ergodic Capacity Analysis of MIMO Relay Network over Rayleigh-Rician Channels / Zhou, Siyuan; Nordio, Alessandro; Alfano, Giuseppa; Chiasserini, Carla Fabiana. - In: IEEE COMMUNICATIONS LETTERS. - ISSN 1089-7798. - STAMPA. - 19:4(2015), pp. 601-604. [10.1109/LCOMM.2015.2396068]

*Availability:*

This version is available at: 11583/2585590 since:

*Publisher:*

IEEE - INST ELECTRICAL ELECTRONICS ENGINEERS INC

*Published*

DOI:10.1109/LCOMM.2015.2396068

*Terms of use:*

This article is made available under terms and conditions as specified in the corresponding bibliographic description in the repository

*Publisher copyright*

(Article begins on next page)

# Ergodic Capacity Analysis of MIMO Relay Network over Rayleigh-Rician Channels

Siyuan Zhou, *Student Member, IEEE*, Alessandro Nordio, *Member, IEEE*, Giuseppa Alfano, *Member, IEEE*, Carla-Fabiana Chiasserini, *Senior Member, IEEE*

**Abstract**—We present an analytical characterization of the ergodic capacity for an amplify-and-forward (AF) multiple-input multiple-output (MIMO) relay network over asymmetric channels. In the two-hop system that we consider, the source-relay and relay-destination channels undergo Rayleigh and Rician fading, respectively. Considering arbitrary-rank means for the relay-destination channel, we first investigate the marginal distribution of an unordered eigenvalue of the cascaded AF channel, and we provide an analytical expression for the ergodic capacity of the system. The closed-form expressions that we derive are computationally efficient and validated by numerical simulation. Our results also show the impact of the signal-to-noise ratio and of the Rician factor on this asymmetric relay network.

**Index Terms**—Ergodic capacity, MIMO, Amplify-and-Forward relay, Rayleigh fading, Rician fading.

## I. INTRODUCTION

Data transmission through relay channel has been proved to improve coverage, reliability and quality-of-service in wireless systems. Among several proposed relay schemes, amplify-and-forward (AF) has attracted significant attention since it can be easily analyzed and implemented. An AF two-hop system is a classic half-duplex model, where the source sends a signal to the relay in the first hop, and then the relay broadcasts the received signal to the destination after simple amplification. This model can then be enhanced by introducing multiple-input multiple-output (MIMO) technology, which can bring remarkable improvements in network performance over conventional single-input single-output systems.

The performance of AF MIMO relay networks has been widely analyzed by applying either asymptotic analysis [1], [2] (i.e., assuming an infinite number of antennas or nodes), or finite random matrix theory [3]. What these two approaches have in common is the assumption that both channels in the relay system are subject to Rayleigh fading. In real-world environments, however, the relay node may be deployed closer to the destination. In this case, a strong Line-of-Sight (LoS) path between the two close-by nodes may exist and the channel on the second hop is affected by Rician, rather than Rayleigh, fading. Note that such an asymmetric channel model can be seen as a generalization of the traditional two-hop Rayleigh

fading channel, since Rayleigh fading can be considered as a limiting case of Rician fading.

In the first of the above asymmetric scenarios (i.e., a relay close to the source), the analysis of the system performance is trivial. Much more challenging, instead, is the case where the channels of the two hops are modeled as Rayleigh and Rician, respectively. This second scenario has indeed attracted significant attention in the literature. In particular, [4] provides an exact expression of the moment generating function and the moments of the instantaneous signal-to-noise ratio (SNR), under the assumption that an orthogonal space-time block coding scheme is applied. In [5], the network performance is studied assuming that the relay is equipped with a single antenna. To the best of our knowledge, no analytical expression instead exists for the ergodic capacity in a AF MIMO relay network over asymmetric fading channels in the presence of a multiple-antenna relay node. In this letter, we therefore fill this gap. By using finite-dimensional random matrix theory, we provide a closed-form expression of the unordered eigenvalue distribution of the cascaded asymmetric relay channel when the two hops are characterized by Rayleigh and Rician fading, respectively. Through this expression, we also derive an analytical expression of the ergodic capacity of the system, with arbitrary-rank means of the Rician channel. Furthermore, by numerical simulation, we investigate the network performance as the Rician factor and the SNR vary.

## II. NOTATION AND DEFINITIONS

Throughout the paper, matrices are denoted by uppercase boldface letters, and vectors by lowercase boldface.  $\mathbb{E}[\cdot]$  denotes statistical expectation,  $(\cdot)^H$  is the conjugate transpose operator, and  $|\cdot|$  and  $\text{Tr}\{\cdot\}$  denote, respectively, the determinant and the trace of a square matrix. Also, we indicate with  $\{a_{ij}\}$  the matrix whose elements are  $a_{ij}$ .  $\Gamma_m(a)$  is the complex multivariate Gamma function defined in [6] as

$$\Gamma_m(a) = \pi_m \prod_{\ell=1}^m \Gamma(a - \ell + 1),$$

where  $\Gamma(\cdot)$  indicates the standard gamma function and  $\pi_m = \pi^{m(m-1)/2}$ . We also denote by  $\mathbf{I}_m$  the  $m \times m$  identity matrix.

Let  $\mathbf{A}$  be an  $m \times m$  Hermitian matrix with distinct non-negative eigenvalues  $a_1, \dots, a_m$ , sorted in descending order, and let  $\mathbf{F} = \{f_i(a_j)\}$ ,  $i, j = 1, \dots, m$ , be an  $m \times m$  matrix where the  $f_i(\cdot)$  are arbitrary differentiable functions. We denote by  $\mathcal{V}(\mathbf{A})$  the Vandermonde determinant of  $\mathbf{A}$ , i.e.,

$$\mathcal{V}(\mathbf{A}) = \prod_{1 \leq i < j \leq m} (a_i - a_j).$$

A. Nordio is with IEIIT-CNR (Institute of Electronics, Telecommunications and Information Engineering of the National Research Council of Italy), Italy. G. Alfano, S. Zhou, and C.-F. Chiasserini are with the Dipartimento di Elettronica and Telecomunicazioni, Politecnico di Torino, Italy. C.-F. Chiasserini is also a Research Associate at IEIIT-CNR.

This paper was made possible by NPRP grant #5-782-2-322 from the Qatar National Research Fund (a member of Qatar Foundation). The statements made herein are solely the responsibility of the authors.

When the eigenvalues of  $\mathbf{A}$  are not distinct,  $\mathcal{V}(\mathbf{A}) = 0$  and  $|\mathbf{F}| = 0$ . However, for any integer  $0 < n < m$  [7, Lemma 5],

$$\lim_{a_{n+1}, \dots, a_m \rightarrow a} \frac{|\mathbf{F}|}{\mathcal{V}(\mathbf{A})} = \frac{\pi_m \Gamma_n(m)}{\pi_n \Gamma_m(m)} \frac{|\tilde{\mathbf{F}}|}{\mathcal{V}(\tilde{\mathbf{A}})} |\tilde{\mathbf{A}} - a \mathbf{I}_n|^{n-m}, \quad (1)$$

where  $\tilde{\mathbf{A}}$  is of size  $n \times n$  and has eigenvalues  $a_1, \dots, a_n$ , whereas

$$(\tilde{\mathbf{F}})_{ij} = \begin{cases} f_i(a_j) & i = 1, \dots, m; j = 1, \dots, n \\ f_i^{(m-j)}(a) & i = 1, \dots, m; j = n+1, \dots, m \end{cases}$$

with  $f_i^{(k)}(\cdot)$  denoting the  $k$ -th derivative of  $f_i(\cdot)$ .

The generalized hypergeometric function is denoted by  ${}_pF_q(\mathbf{a}; \mathbf{b}; \mathcal{X})$ , where  $\mathbf{a} = [a_1, \dots, a_p]^T$ ,  $\mathbf{b} = [b_1, \dots, b_q]^T$ , and  $\mathcal{X}$  represents a set of arguments that can be either scalars or square matrices. In the case of a scalar argument  $\mathcal{X} = \{x\}$ , the hypergeometric function is defined as [6]

$${}_pF_q(\mathbf{a}; \mathbf{b}; x) = \sum_{k=0}^{\infty} \frac{(a_1)_k \cdots (a_p)_k}{(b_1)_k \cdots (b_q)_k} \frac{x^k}{k!},$$

where  $(a)_k = \Gamma(a+k)/\Gamma(a)$  denotes the Pochhammer symbol. When  $\mathcal{X} = \{\Phi, \Psi\}$ , with  $\Phi$  and  $\Psi$  both of size  $m \times m$ ,  ${}_pF_q(\mathbf{a}; \mathbf{b}; \mathcal{X})$  can be written through hypergeometric functions of scalar arguments as [8, eq. (2.34)]

$${}_pF_q(\mathbf{a}; \mathbf{b}; \Phi, \Psi) = c \frac{[{}_pF_q(\tilde{\mathbf{a}}; \tilde{\mathbf{b}}; \phi_h \psi_k)]}{\mathcal{V}(\Phi) \mathcal{V}(\Psi)} \quad h, k = 1, \dots, m. \quad (2)$$

In (2), the constant  $c$  is given by

$$c = \frac{\Gamma_m(m)}{\pi_m^{q-p+1}} \left[ \prod_{j=1}^q \frac{\Gamma_m(b_j)}{(b_j - m)!^m} \right] \left[ \prod_{i=1}^p \frac{(a_i - m)!^m}{\Gamma_m(a_i)} \right],$$

$\tilde{a}_i = a_i - m + 1$ ,  $i = 1, \dots, p$ ,  $\tilde{b}_j = b_j - m + 1$ ,  $j = 1, \dots, q$ , and the eigenvalues of  $\Phi$  and  $\Psi$  are denoted by  $\phi_1, \dots, \phi_m$  and  $\psi_1, \dots, \psi_m$ , respectively.

### III. SYSTEM MODEL

We consider a two-hop relay network where the source, the relay and the destination nodes are equipped with  $n$ ,  $r$  and  $m$  antennas, respectively. All nodes operate in half-duplex mode. We assume that no direct link exists between the source and the destination. The destination has perfect channel state information (CSI) on the source-relay and relay-destination channels, while the source and relay have no CSI.

Following [2] and [3], we assume that data transmission takes place in two phases, according to the following scheme. In the first phase, the source transmits signal  $\mathbf{x}$ , which is a vector with  $n$  components, towards the relay. The entries of  $\mathbf{x}$  are assumed to be i.i.d., zero-mean, circular symmetric, complex Gaussian random variables and the power irradiated by each antenna is assumed to be equal to  $\rho/n$ , i.e.,  $\mathbb{E}[\mathbf{x}\mathbf{x}^H] = \rho/n \mathbf{I}_n$ , where  $\rho$  is the signal-to-noise ratio. In the second phase, the relay simply forwards a scaled version of the signal it has received from the source. Let  $\mathbf{H}_1 \in \mathbb{C}^{r \times n}$  be the channel matrix between the source and the relay, and  $\mathbf{H}_2 \in \mathbb{C}^{m \times r}$

be the channel matrix between the relay and the destination. Then the signal received at the destination can be written as

$$\mathbf{y} = \mathbf{H}_2 \mathbf{A} \mathbf{H}_1 \mathbf{x} + \mathbf{H}_2 \mathbf{A} \mathbf{n}_r + \mathbf{n}_d, \quad (3)$$

where  $\mathbf{A} = \sqrt{a} \mathbf{I}_r$  is an  $r \times r$  linear transformation matrix representing the power amplification at the relay, and  $\mathbf{n}_r$  and  $\mathbf{n}_d$  are, respectively, the noise vectors at the relay and at the destination, whose entries are modeled as i.i.d. zero-mean, unit-variance, Gaussian random variables. Note that the above assumption on  $\mathbf{A}$  reflects the case where the relay has no CSI.

The source-relay channel is assumed to be affected by Rayleigh fading. Thus, the entries of  $\mathbf{H}_1$  are i.i.d. complex Gaussian random variables with zero mean and unit variance. Instead, the relay-destination channel is assumed to be affected by Rician fading so that the entries of  $\mathbf{H}_2$  can be written as

$$\mathbf{H}_2 = \sqrt{\frac{\kappa}{\kappa+1}} \bar{\mathbf{H}}_2 + \sqrt{\frac{1}{\kappa+1}} \tilde{\mathbf{H}}_2, \quad (4)$$

where  $\kappa$  is the Rician factor,  $\bar{\mathbf{H}}_2$  is deterministic and the entries of  $\tilde{\mathbf{H}}_2$  are i.i.d. complex Gaussian with zero mean and unit variance. For simplicity of notation, we define  $\tilde{\kappa} \triangleq 1 + \kappa$ .

Let  $q = \min(r, m)$ ,  $s = \min(n, q)$ ,  $\Lambda = \text{diag}(\lambda_1, \dots, \lambda_q)$  be the non-zero eigenvalues of  $\mathbf{H}_2 \mathbf{H}_2^H$ . Then the ergodic capacity<sup>1</sup> of the AF-MIMO relay channel described above is given by [3, eq. (13)]

$$C(\rho) = \frac{s}{2} \int_0^\infty \log_2 \left( 1 + \frac{\rho a}{n} z \right) p(z) dz, \quad (5)$$

where  $z$  denotes an unordered eigenvalue of the random matrix  $\mathbf{Z} = \mathbf{H}^H \mathbf{B} \mathbf{H}$  and  $p(z)$  denotes the probability density function (pdf) of  $z$ . In the expression for  $\mathbf{Z}$ ,  $\mathbf{H}$  is an  $n \times q$  random matrix with i.i.d. circularly symmetric, complex Gaussian entries with zero mean and unit variance, and  $\mathbf{B} = \Lambda(\mathbf{I} + a\Lambda)^{-1}$  is a  $q \times q$  diagonal matrix.

### IV. PERFORMANCE ANALYSIS

The ergodic capacity of the AF-MIMO channel described above can be obtained by deriving a closed-form expression for  $p(z)$  and plugging it into (5). In order to do so, we first assume the LoS component of the Rician channel (i.e.,  $\bar{\mathbf{H}}_2$ ) to be full-rank; this case is referred to as ‘‘non-i.i.d. Rician fading’’ in [9] and allows for relatively simpler analysis (Section IV-A). Then we will deal with the case where  $\bar{\mathbf{H}}_2$  is low-rank (Section IV-B).

#### A. Closed-form expression for the ergodic capacity

The pdf of the unordered eigenvalue of  $\mathbf{Z} = \mathbf{H}^H \mathbf{B} \mathbf{H}$ , with  $\mathbf{H}$  and  $\mathbf{B}$  as defined above, can be written as

$$p(z) = \int_{\mathbf{B}} p_{z|\mathbf{B}}(z|\mathbf{B}) p_{\mathbf{B}}(\mathbf{B}) d\mathbf{B}, \quad (6)$$

where  $p_{z|\mathbf{B}}(z|\mathbf{B})$  is the pdf of  $z$  conditioned on  $\mathbf{B}$  and can be written as [3, eq. (95)]:

$$p_{z|\mathbf{B}}(z|\mathbf{B}) = \frac{1}{s \mathcal{V}(\mathbf{B})} \sum_{k=q-s+1}^q \frac{z^{n+k-q-1}}{\Gamma(n-q+k)} |\mathbf{V}_k|. \quad (7)$$

<sup>1</sup>It can be seen that by applying a whitening filter, the output can be rewritten as  $\mathbf{y} = \hat{\mathbf{H}} \mathbf{x} + \mathbf{n}$ , for which the capacity is achieved with a Gaussian codebook.

In (7),  $\mathbf{V}$  is a  $q \times q$  matrix with entries given by:

$$(\mathbf{V}_k)_{i,j} = \begin{cases} b_i^{q-j}, & q-j+1 \neq k \\ e^{-z/b_i} b_i^{q-n-1}, & q-j+1 = k. \end{cases}$$

An expression for  $p_{\mathbf{B}}(\mathbf{B})$  is given by the proposition below.

**Proposition 1:** Consider a communication system and matrix  $\mathbf{B}$  as described above. Then the pdf of  $\mathbf{B}$  is given by

$$p_{\mathbf{B}}(\mathbf{B}) = \frac{\tilde{\kappa}^{pq} \mathcal{V}(\mathbf{B}) |\mathbf{B}|^{p-q} |\mathbf{I} - a\mathbf{B}|^{-p-1} |\mathbf{F}|}{(p-q)! q e^{\text{Tr}\{\tilde{\kappa}\mathbf{B}(\mathbf{I}-a\mathbf{B})^{-1} + \kappa\mathbf{M}\}} \mathcal{V}(\tilde{\kappa}\mathbf{M})}, \quad (8)$$

where  $\mathbf{F} = \{ {}_0F_1(; p-q+1; \kappa\tilde{\kappa}\mu_i b_j / (1-ab_j)) \}$  and  $\mathbf{M} = \bar{\mathbf{H}}_2 \bar{\mathbf{H}}_2^H$ .

*Proof:* Note that  $\mathbf{H}_2 = \hat{\mathbf{H}}/\sqrt{\tilde{\kappa}}$ , where  $\hat{\mathbf{H}} = \sqrt{\tilde{\kappa}}\bar{\mathbf{H}}_2 + \tilde{\mathbf{H}}_2$  is a standard noncentral Wishart matrix with mean  $\sqrt{\tilde{\kappa}}\bar{\mathbf{H}}_2$ . The joint pdf of the ordered eigenvalues,  $\hat{\Lambda}$ , of  $\hat{\mathbf{H}}^H \hat{\mathbf{H}}$  is given by [6, eq. (102)]; using (2), it can be written as:

$$p_{\hat{\Lambda}}(\hat{\Lambda}) = \frac{|\hat{\Lambda}|^{p-q} \mathcal{V}(\hat{\Lambda}) \{ {}_0F_1(; p-q+1; \kappa\tilde{\kappa}\mu_i \hat{\lambda}_j) \}}{(p-q)! q e^{\text{Tr}\{\hat{\Lambda} + \kappa\mathbf{M}\}} \mathcal{V}(\kappa\mathbf{M})}, \quad (9)$$

where  $\mu_i$ ,  $i = 1, \dots, q$ , are the eigenvalues of  $\mathbf{M} = \bar{\mathbf{H}}_2 \bar{\mathbf{H}}_2^H$  and  $\hat{\Lambda}$  is a diagonal matrix whose elements are  $\hat{\lambda}_j$ ,  $j = 1, \dots, q$ . The pdf of  $\Lambda$  can then be obtained as

$$\begin{aligned} p_{\Lambda}(\Lambda) &= \tilde{\kappa}^q p_{\hat{\Lambda}}(\tilde{\kappa}\Lambda) \\ &= \frac{\tilde{\kappa}^{pq} \mathcal{V}(\Lambda) \{ {}_0F_1(; p-q+1; \kappa\tilde{\kappa}\mu_i \lambda_j) \}}{(p-q)! q |\Lambda|^{q-p} e^{\text{Tr}\{\tilde{\kappa}\Lambda + \kappa\mathbf{M}\}} \mathcal{V}(\tilde{\kappa}\mathbf{M})}. \end{aligned} \quad (10)$$

Since  $\mathbf{B} = \Lambda(\mathbf{I} + a\Lambda)^{-1}$ , the pdf of  $\mathbf{B}$  can be written as a function of the pdf of  $\Lambda$ , i.e.,  $p_{\mathbf{B}}(\mathbf{B}) = |\mathbf{I} - a\mathbf{B}|^{-2} p_{\Lambda}(\mathbf{B}(\mathbf{I} - a\mathbf{B})^{-1})$ , which yields (8). In the derivation of (8), we have exploited the property  $\mathcal{V}(\mathbf{B}(\mathbf{I} - a\mathbf{B})^{-1}) = \mathcal{V}(\mathbf{B}) |\mathbf{I} - a\mathbf{B}|^{1-q}$ . ■

Then we replace  $p_{\mathbf{B}}(\mathbf{B})$  in (6) with the expression in (8) and obtain the result below.

**Proposition 2:** The pdf of an unordered eigenvalue  $z$  of  $\mathbf{Z} = \mathbf{H}^H \mathbf{B} \mathbf{H}$  is given by

$$p(z) = \frac{A}{\mathcal{V}(\mathbf{M})} \sum_{k=q-s+1}^q \frac{z^{c_k-1}}{\Gamma(c_k)} |\mathbf{W}_k|, \quad (11)$$

where  $c_k = n - q + k$ , the constant  $A$  is given by

$$A = \frac{\tilde{\kappa}^{pq} e^{-\kappa \text{Tr}\{\mathbf{M}\}}}{s(p-q)! q (\kappa\tilde{\kappa})^{q(q-1)/2}}, \quad (12)$$

and  $\mathbf{W}_k$  is a  $q \times q$  matrix whose entries are as follows:

$$(\mathbf{W}_k)_{i,j} = \sum_{h=0}^{j-1} \frac{\binom{j-1}{h} \Gamma(d_{j,h}) {}_1F_1(d_{j,h}; p-q+1; \kappa\mu_i)}{a^{-h} \tilde{\kappa}^{d_{j,h}}} \quad (13)$$

for  $i, j = 1, \dots, q$ ,  $j \neq q - k + 1$  and

$$(\mathbf{W}_k)_{i,j} = \frac{2e^{-za}}{\tilde{\kappa}^{p-n}} \sum_{\ell=0}^{\infty} \sum_{h=0}^n \frac{\binom{n}{h} (\kappa\mu_i)^{\ell} (z\tilde{\kappa})^{g_{\ell,h}/2}}{a^{-h} \tilde{\kappa}^h \ell! (p-q+1)_{\ell}} K_{g_{\ell,h}}(2\sqrt{z\tilde{\kappa}}) \quad (14)$$

for  $i = 1, \dots, q$ ,  $j = q - k + 1$ , where  $d_{j,h} = p + 1 - j + h$  and  $g_{\ell,h} = p - n + \ell + h$ . In (14),  $K_v(x)$  denotes the modified Bessel function of the second kind.

*Proof:* As mentioned above,  $p(z)$  can be computed by using (8) in (6). As for the integration domain, we observe that the  $i$ -th eigenvalue of  $\mathbf{H}_2 \bar{\mathbf{H}}_2^H$ ,  $\lambda_i$ , is such that  $0 \leq \lambda_i < +\infty$ .

Thus,  $b_i = \lambda_i / (1 + a\lambda_i)$  has support in  $[0, 1/a]$ . Moreover, the expression of  $p_{\mathbf{B}}(\mathbf{B})$  provided in (8) refers to the ordered eigenvalue distribution of  $\mathbf{B}$ , hence the integral in (6) should be taken under the constraint  $0 \leq b_q < \dots < b_1 \leq 1/a$ . By substituting (7) and (8) in (6), we obtain:

$$p(z) = \sum_{k=q-s+1}^q \frac{Az^{c_k-1}}{\mathcal{V}(\mathbf{M}) \Gamma(c_k)} \int \frac{|\mathbf{I} - a\mathbf{B}|^{-p-1} |\mathbf{F}| |\mathbf{V}_k| d\mathbf{B}}{|\mathbf{B}|^{q-p} e^{\text{Tr}\{\tilde{\kappa}\mathbf{B}(\mathbf{I}-a\mathbf{B})^{-1}\}}} \quad (15)$$

$$= \frac{A}{\mathcal{V}(\mathbf{M})} \sum_{k=q-s+1}^q \frac{z^{c_k-1}}{\Gamma(c_k)} |\mathbf{W}_k|, \quad (16)$$

where the constant  $A$  is given by (12) and  $c_k = n - q + k$ . The  $q \times q$  matrix  $\mathbf{W}_k$  in (16) derives from the application of [10, Corollary 2] to the integral in (15) and its entries are given by

$$(\mathbf{W}_k)_{i,j} = \begin{cases} \int_0^{\frac{1}{a}} \frac{e^{-\frac{\tilde{\kappa}x}{1-ax}} x^{p-j} f_{ij}}{(1-ax)^{p+1}} dx, & j \neq q - k + 1 \\ \int_0^{\frac{1}{a}} \frac{e^{-\frac{\tilde{\kappa}x}{1-ax} - z/x} f_{ij}}{x^{1-p+n} (1-ax)^{p+1}} dx, & j = q - k + 1 \end{cases} \quad (17)$$

where  $f_{ij} = {}_0F_1(; p-q+1; \kappa\tilde{\kappa}\mu_i x / (1-ax))$  has been defined below (8). The integrals in (17) can be solved by changing the integration variable into  $x = y / (\tilde{\kappa} + ay)$  and by expanding the powers of the binomial  $\tilde{\kappa} + ay$ . As a result, after some computations (omitted for lack of space), the matrix  $\mathbf{W}_k$  can be rewritten as in (13) and (14). ■

Eventually, the analytical expression for the ergodic capacity can be obtained by substituting (11) into (5).

### B. Low-rank LoS Rician fading component

We now consider the case where the LoS component of the Rician channel,  $\bar{\mathbf{H}}_2$ , does not have full rank, i.e., the terms  $|\mathbf{W}_k|$  and  $\mathcal{V}(\mathbf{M})$ , respectively, at the numerator and denominator of (11) vanish, thus leading to a 0/0 indeterminate form. In order to circumvent this problem, a limit must be taken, which can be evaluated using l'Hôpital's rule. In particular, in the following we assume that  $\bar{\mathbf{H}}_2^H \bar{\mathbf{H}}_2$  has  $0 < g < q$  non-zero eigenvalues, i.e.,  $\mu_{g+1} = \mu_{g+2} = \dots = \mu_q = 0$ . Then the pdf of an unordered eigenvalue  $z$  of  $\mathbf{Z} = \mathbf{H}^H \mathbf{B} \mathbf{H}$  can be derived by taking the following limit:

$$\begin{aligned} p(z)_{\text{low}} &= \lim_{\mu_{g+1}, \dots, \mu_q \rightarrow 0} p(z) \\ &= A \sum_{k=q-s+1}^q \frac{z^{c_k+1}}{\Gamma(c_k)} \lim_{\mu_{g+1}, \dots, \mu_q \rightarrow 0} \frac{|\mathbf{W}_k|}{\mathcal{V}(\mathbf{M})} \\ &= \frac{\pi_q \Gamma_g(q)}{\pi_g \Gamma_q(q)} \frac{|\widetilde{\mathbf{W}}_k|}{\mathcal{V}(\widetilde{\mathbf{M}})} |\widetilde{\mathbf{M}}|^{g-q}, \end{aligned} \quad (18)$$

where we have used the result reported in (1). In (18),  $\widetilde{\mathbf{M}}$  is a  $g \times g$  matrix with eigenvalues  $\mu_1, \dots, \mu_g$  and  $\widetilde{\mathbf{W}}_k$  is a  $q \times q$  matrix whose  $(i, j)$ -th entry is given by  $(\mathbf{W}_k)_{i,j}$ , for  $i = 1, \dots, g$  and  $j = 1, \dots, q$ ;

$$(\widetilde{\mathbf{W}}_k)_{i,j} = \sum_{h=0}^{j-1} \frac{\binom{j-1}{h} \Gamma(d_{j,h} + q - i) \kappa^{q-i}}{(p-q+1)_{q-i} a^{-h} \tilde{\kappa}^{d_{j,h}}}$$

for  $i = g + 1, \dots, q$ , and  $j \neq q - k + 1$ ; and

$$\left(\widetilde{\mathbf{W}}_k\right)_{i,j} = \frac{2e^{-za}}{\tilde{\kappa}^{p-n}} \sum_{h=0}^n \frac{\binom{n}{h} \kappa^{q-i} (z\tilde{\kappa})^{g_{q-i,h}/2}}{a^{-h} \tilde{\kappa}^h (p-q+1)_{q-i}} K_{g_{q-i,h}}(2\sqrt{z\tilde{\kappa}})$$

for  $i = g + 1, \dots, q$ ,  $j = q - k + 1$ . This result has been obtained by taking the derivative  $q - j$  times (with respect to  $\mu_j$ ) of the  $(i, j)$ -th entry of  $\mathbf{W}_k$  and by evaluating it in  $\mu_j = 0$ , for  $i = 1, \dots, q$  and  $j = g + 1, \dots, q$ . The other entries of  $\mathbf{W}_k$  have been left unchanged. Again, the expression for the ergodic capacity for the low-rank LoS Rician fading component can be obtained by substituting (18) into (5).

## V. NUMERICAL RESULTS

We now validate our analytical derivations through Monte Carlo simulation, and show the information theoretic performance of the channel of (3). We consider  $n = 2$ ,  $r = 3$ ,  $m = 4$ , and the Rician channel with with LoS matrix that satisfies  $\text{Tr}\{\bar{\mathbf{H}}_2^H \bar{\mathbf{H}}_2\} = rm$ . More specifically, in Figs. 1 and 2 we have employed both the full-rank LoS matrix  $\bar{\mathbf{H}}_2^f$  and the low-rank LoS matrix  $\bar{\mathbf{H}}_2^l$ , given by

$$\bar{\mathbf{H}}_2^f = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & \sqrt{2} & 2 \\ 1 & 1 & \sqrt{2} \\ 1 & \sqrt{3} & 1 \\ 1 & \sqrt{2} & \sqrt{5} \end{bmatrix}; \bar{\mathbf{H}}_2^l = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}.$$

While computing (5) and (6), we upper bound the integration variables  $z$  and  $\lambda$  so as to obtain an expression that can be efficiently computed. In numerical simulation, an accurate result can be obtained already for  $\lambda \leq 20$  and  $z \leq 100$ .

Figure 1 shows the excellent match between the pdf of  $z$ ,  $p(z)$ , computed through the analytical expression in (6) and the results obtained via Monte Carlo simulation. In particular, the top plot shows  $p(z)$  for the full-rank LoS component, as the Rician factor  $\kappa$  and the SNR ( $\rho$ ) vary. It also compares our result to that provided in [3] for the Rayleigh-Rayleigh channel. As expected, as  $\kappa$  decreases,  $p(z)$  converges to that obtained for Rayleigh-Rayleigh channel ( $\kappa = 0$ ). The bottom plot presents the same comparison between analytical and simulation results considering the low-rank LoS component. With respect to the full-rank LoS case, we note a higher concentration of  $p(z)$  for smaller  $z$ 's, which leads to a lower ergodic system capacity in the low-rank LoS case, as shown in Figure 2. The latter depicts the ergodic capacity of the relay system computed through (5) and Monte Carlo simulation, for different values of  $\kappa$  and rank of the LoS component. Again, analytical and numerical results are remarkably close. The fact that the lower the Rician factor is, the higher the ergodic capacity becomes, confirms the validity of our results.

## VI. CONCLUSION

We have investigated the ergodic capacity performance of AF MIMO relay networks over asymmetric Rayleigh-Rician channels. We derived a closed-form expression for the marginal pdf of an unordered eigenvalue of the cascaded AF channel for the two cases where the Rician channel has full-rank and low-rank means. Using these analytical expressions, we derived the ergodic capacity of the system. Our analysis

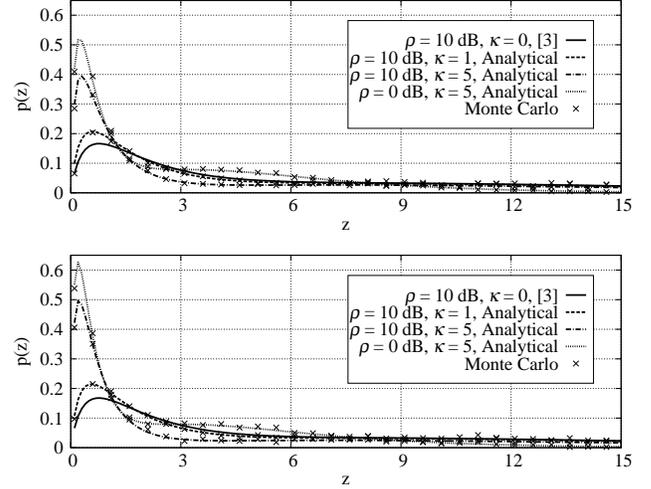


Fig. 1. Exact analysis vs. Monte Carlo simulation: pdf of an unordered eigenvalue  $z$ , for full-rank (top) and low-rank (bottom) LoS component.

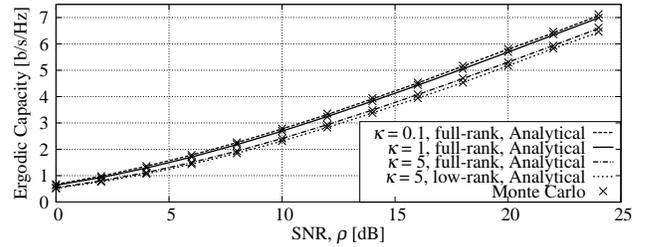


Fig. 2. Exact analysis vs. Monte Carlo simulation: Ergodic capacity vs. SNR for different values of the Rician factor and rank of LoS component.

was validated by showing the excellent match between the results obtained through our exact expressions and those obtained via Monte Carlo simulation.

## REFERENCES

- [1] J. Wagner, B. Rankov, and A. Wittneben, "On the asymptotic capacity of the Rayleigh fading amplify-and-forward MIMO relay channel," *Proc. IEEE Int. Symp. Inf. Theory (ISIT)*, Nice, France, June 2007.
- [2] V. I. Morgenshtern and H. Bölcskei, "Crystallization in large wireless networks," *IEEE Trans. Inf. Theory*, Vol. 53, pp. 3319–3349, Oct. 2007.
- [3] S. Jin, M.R. McKay, C. Zhong and K-K Wong, "Ergodic capacity analysis of amplify and forward MIMO dual-hop systems" *IEEE Trans. Inf. Theory*, Vol. 56, No. 5, pp. 2204–2224, May 2010.
- [4] L.K.S. Jayasinghe, N. Rajatheva, P. Dharmawansa, and M. L. Aho, "Noncoherent amplify-and-forward MIMO relaying with OSTBC over Rayleigh-Rician fading channels," *IEEE Trans. Veh. Technol.*, Vol. 62, No. 4, pp. 1610–1622, May 2013.
- [5] M. Lin, K. An, J. Ouyang, Y. Huang, and M. Li, "Effect of beamforming on multi-antenna two hop asymmetric fading channels with fixed gain relays," *Progress In Electromagnetics Research*, Vol. 133, pp. 367–390, 2013.
- [6] A. T. James, "Distribution of matrix variates and latent roots derived from normal samples," *The Annals of Mathematical Statistics*, Vol. 35, No. 2, pp. 474–501, June 1964.
- [7] A. Ghaderipour, C. Tellambura, and A. Paulraj, "On the application of character expansions for MIMO capacity analysis," *IEEE Trans. on Information Theory*, Vol. 58, No. 5, pp. 2950–2962, 2012.
- [8] M.R. McKay, "Random Matrix theory analysis of multiple antenna communication systems," Ph.D. dissertation, Oct. 2006.
- [9] M. Kang and M.-S. Alouini, "Capacity of MIMO rician channels," *IEEE Trans. Wireless Commun.*, Vol. 5, No. 1, pp. 112–122, Jan. 2006.
- [10] M. Chiani, M. Z. Win, and A. Zanella, "On the capacity of spatially correlated MIMO Rayleigh-fading channels," *IEEE Trans. Inf. Theory*, Vol. 49, No. 10, pp. 2363–2371, Oct. 2003.