A Holistic View of ITS-Enhanced Charging Markets

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Francesco Malandrino, *Member, IEEE,* Claudio Casetti, *Member, IEEE,* Carla-Fabiana Chiasserini, *Senior Member, IEEE*

***Abstract*—We consider a network of electric vehicles (EVs) and its components: vehicles, charging stations, and coalitions of stations. For such a setting, we propose a model in which individual stations, coalitions of stations and vehicles interact in a market revolving around the energy for battery recharge. We start by separately studying (i) how autonomously-operated charging stations form coalitions; (ii) the price policy enacted by such coalitions; (iii) how vehicles select the charging station to use, pursuing a time/price tradeoff. Our main goal is to investigate how equilibrium in such a market can be reached. We also address the issue of computational complexity, showing that, through our model, equilibria can be found in polynomial time.**

We evaluate our model in a realistic scenario, focusing on its ability to capture the advantages of the availability of an Intelligent Transportation System (ITS) supporting the EV drivers. The model also mimics the anticompetitive behavior that charging stations are likely to follow, and it highlights the effect of possible countermeasures to such a behavior.

***Index Terms*—ITS, charging station selection, game theory.**

I. Introduction

It is now an established tenet of transportation technology that Electric Vehicles (EVs) will, at some point in the future, replace vehicles propelled by fossil fuel. Environmentally-friendly by definition, EVs enjoy favorable attention by industry and governments alike. Indeed, the mass production and widespread adoption of EVs seem around the corner if some concerns are overcome, such as short driving range, lack of recharging infrastructure and long charging time. The latter two issues will likely determine the gradual phasing-out of old-fashioned gas pumps in favor of public charging stations. It is thus to be expected that electric outlets will start cropping up at the curbside, in parking lots as well as in cab stands. However, the fact that someone will deploy and operate such charging stations is often taken for granted. In this work, we take a closer look at this issue, arguing that charging stations will be deployed and operated only if their owners find it profitable for themselves. Similarly, the drivers of electric vehicles will select the charging station to use pursuing their own benefit: a shorter trip time, a cheaper price, or both. Note that the trip time includes the detour time from the vehicle original route to a charging station and back, the wait time there, and the service time.

We present a model that captures the behavior of the two main actors involved in such a dynamic energy market, namely:

- the electric vehicles, i.e., their drivers;
- the charging stations, i.e., their owners.

Additionally, our model accounts for *coalitions* that charging stations may form, and their commercial strategies. Each of these actors pursues different, potentially conflicting, objectives. And, each of them is likely to change its behavior as a consequence of the actions the others take – or are expected to take. By exploiting game theory, we then describe a polynomial-complex algorithm to find an operational point for such a model, which turns out to be a Nash equilibrium [1].

Our model represents a novel contribution in two ways. For starters, it is the first to *jointly* address the behavior of charging stations and vehicles, as well as the interaction between buy and sell energy prices. Furthermore, we specifically address the issue of the computational cost of finding an equilibrium, and attain a complexity that is polynomial in the number of charging stations (note that finding a Nash equilibrium is, in general [1], NP-hard). Such an issue is traditionally disregarded by economists, who tend to focus on proving that an equilibrium exists rather than designing a way to compute it. In engineering applications, instead, it is crucial that large-scale scenarios are analyzed quickly, if not in real-time. Furthermore, it is important to stress that the main focus of our model is to study the steady-state equilibrium, as opposed to the dynamics of the transient. This is justified by the features of the scenario we deal with, e.g., the slow pace at which prices change.

The rest of the paper is organized as follows. We review previous work in Section II, while we describe the system model in Section III. In Section IV, we discuss the players, moves and payoffs of the related game, whose computational issues are dealt with in Section V. We test our model in the scenario described in Section VI, obtaining the results described in Section VII. Finally, Section VIII concludes the paper.

II. Related Work

Recently, both the academic and industrial communities have devoted a great deal of interest to EVs and charging station deployment.

As an example, in [2] Ferreira et al. consider the case where the behavior of EV drivers, i.e., whether they drive to the closest or the cheapest charging station, depends on their profile (age or gender). Similarly, [3] describes an intelligent transportation system (ITS) to support drivers in the selection of the charging station, accounting for the fact that they will act selfishly, and providing strategy-compatible suggestions. The work in [4], instead, accounts for real-time charging prices and envisions a centralized control of the EV's charging schedule

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so as to minimize user costs. An analytic model for the study of the EVs trip time is presented in [5]. The road topology is modeled as a graph whose edges are associated with a fixed waiting time, and a lower bound to the charging time is derived. With respect to our work, however, the study in [5] does not account for the strategic behavior of vehicles.

Such an issue is addressed in [6], which presents a set of decentralized policies assigning vehicles to charging stations and yielding socially optimal equilibria. A multi-objective decision-making model is also presented in [7], where the gas station selection depends on the driver personalized requirements and the gasoline price, and it aims at minimizing the travel distance and the refueling cost. In these works, however, individual EV routing and charging are optimized through standard techniques, and the effect of such decisions on each other is not taken into account.

Several works, e.g., [8]–[16], are mostly concerned with the impact of EVs on the power grid. In particular, the study in [10] assumes the presence of a central controller that predicts the EVs mobility and advises each EV about which charging station to use and when, so as to even the power demand over time. The work in [10], however, assumes that vehicles always follow the central controller's suggestions. Similarly, the goal of [12] is to ensure that EVs can obtain the energy they need to recharge their batteries without impairing the stability of the power grid. The study in [12] accounts for the behavior of EV drivers and aims at influencing it by means of monetary incentives. The work in [13] envisions that smart grids shall distinguish regular and EV-induced loads, and optimize the way such loads are served. In a similar setting, the authors of [14] propose a queue-based model for smart grids serving EVs, and use it to find the optimal size of local energy stores at charging stations. In [15], a centralized optimization problem and a low-complexity heuristic are presented with the aim to adjust EV charging to real-time prices and the power grid load.

The study in [16] is closer to our approach, indeed it accounts for both charging time and energy costs. With an emphasis on charging stations (as opposed to vehicle drivers), the authors present an optimal way to handle the EV load on the power grid, as well as a greedy heuristics.

As mentioned, to our knowledge, our work is the first that jointly investigates the behavior of charging stations and vehicles, as well as the interaction between buy and sell energy prices while taking into account its complexity.

III. System model

We aim at modeling a realistic scenario featuring *electric vehicles* travelling on an urban road topology where several *charging stations* are available for battery refill. Our model captures both vehicle and station viewpoints in a dynamic energy market setting. When its battery is depleted, a vehicle stops at one of the charging stations. The choice of the station must weigh the monetary service cost and the expected incurred delay (detour, waiting and service time). All stations can sell energy at a price of their own choice, accounting for demand and expected revenue, but they have to buy energy at market prices. Stations may also have the option of forming coalitions, aiming at lowering the buying price and at driving selling prices up to increase their revenues.

Our model hinges upon two kinds of agents: vehicles $v \in \mathcal{V}$ and charging stations $c \in \mathcal{C}$. Their behavior is detailed in III-A and III-B, respectively. Notice that set $\mathcal{V}$ only contains those vehicles that are interested in changing their battery during their trip. The other vehicles do not take part in the market and simply contribute to the road traffic, just like ordinary, fossil-fueled vehicles do. Such an effect is accounted for in the computation of the trip time, as detailed later in the paper.

We assume that batteries are *replaced*, not charged. This is due to the exceedingly long charging times of current and (likely) future technologies, and is consistent with early deployments [17]. As a consequence, charging stations can express buy and sell energy prices, b_c and s_c, respectively, in dollars per battery replacement instead of, for example, dollars per kilojoule. We focus on a given time period of the day characterized by uniform battery replacement demand and vehicular traffic conditions, and we consider that prices are kept constant during such period.

A. Vehicles

Vehicles are associated to an origin and a destination point on the road topology. During their trip, they stop at exactly one charging station among the possible ones, in order to replace their battery.

Let α_v, ω_v and η_v be, respectively, the points on the road topology marking the origin of trip of vehicle v, its destination and the point where its driver becomes aware that a battery replacement is needed. Also, let λ_c be the location of charging station c. Then, each vehicle v will select a charging station pursuing a tradeoff between cost and expected incurred delay. Specifically, EVs will try to optimize the following objective:

$$\min_{c \in \mathcal{C}} \{s_c + K \cdot [t(\alpha_v, \eta_v) + t(\eta_v, \lambda_c) + w_c + t(\lambda_c, \omega_v)]\} \quad (1)$$

where:

- s_c is the battery replacement cost at station c;
- $t(x_1, x_2)$ is the travel time between locations x_1 and x_2, with $x_1 = \alpha_v, \eta_v, \lambda_c$ and $x_2 = \eta_v, \lambda_c, \omega_v$;
- w_c is the waiting plus service time at station c;
- K is a coefficient used to convert time into cost. Indeed, while s_c is a cost, all other terms represent time periods.

In particular, note that the sum of terms multiplied by K represents the total time it will take to the vehicle to go from its origin to its destination, including the detour to and from the charging stating, the waiting time there and the service time. Thus, K can be read as the *value of time* as perceived by the drivers: if $K = 10$ EUR$/h$, drivers will be willing to spend one more hour reaching a farther charging station and/or waiting at a more crowded one if they can save at least 10 EUR on their replacement price. In other words, very high values of K mean that drivers are willing to pay any price to shorten their trip. Low values mean that they prefer the cheapest station, no matter how far away or crowded it may be. Also, we stress that time w_c depends on the number of service stalls at the station, the service time, and the number of customers that a vehicle finds upon arriving at the station. A closed-form expression of w_c is provided in the Appendix.

Vehicles with no ITS support: Most of the parameters in (1) are static or only change occasionally (e.g., the energy prices for the tagged time period may change once a day). To correctly account for such values while optimizing their objectives, vehicles do not need information from ITS. As an example, the up-to-date energy prices could be downloaded from the Internet upon starting the trip.

Conversely, the waiting times w_c's cannot be estimated a priori, as they depend on the decisions of all the other players. Therefore, vehicles with no ITS support will have to make their decisions, i.e., optimizing objective (1), without taking the waiting times into account. Without loss of generality, in the following we consider that such vehicles assume[1] $w_c = 0, \forall c$.

B. Charging stations

Each charging station c buys energy at a price b_c per replacement, and sells it at a price s_c. Additionally, stations have to pay a fixed cost f_c, i.e., a lump sum accounting for taxes, maintenance and labor costs. Unless coalitions are formed, as explained below, each station is free to decide its charging price. A station can thus increase the price in order to have a higher revenue, or decrease it in order to attract more costumers. Demand, i.e., the number of vehicles on the road topology that need to replace their battery, is a major factor determining the prices. As we focus on a given time period of the day, our model does not explicitly account for time-varying prices. However, as shown in Section VII, the model can be used to study different demand conditions.

Stations may form *coalitions*, for the purpose of obtaining a bulk rate from energy suppliers on their buy price b_c, and to enforce a common pricing strategy. We denote by $\mathcal{K}$ the set of coalitions that are formed and by k_c the coalition to which station c belongs. Also, A_c and A_{k_c} indicate the attendance (i.e., the number of vehicles headed to it) of station c and of its coalition, respectively. At the outset, $k_c \equiv c$ for all stations, i.e., each station belongs to a coalition formed by itself only (and, clearly, $A_{k_c} = A_c$). Stations cannot belong to more than one coalition at the same time.

Joining a coalition has two effects: on the one hand, the price b_c that station c is charged when buying energy may be lowered, as explained in III-C. On the other hand, station c forfeits its freedom to decide the selling price s_c. Indeed, the price policy within each coalition κ is determined so as to maximize the coalition revenue, i.e., to optimize the following objective:

$$\max \sum_{c \in \mathcal{C}: k_c = \kappa} s_c A_c. \tag{2}$$

We indicate with k_0 the virtual coalition formed by those stations that do not participate in the market, i.e., that are "not operating". Stations in k_0 have attendance $A_c = 0$, and do not pay the fixed cost f_c.

[1]Note that assuming any other constant value for w_c would be equivalent.

C. Energy price

We do not make any specific assumption on the presence of one or more energy suppliers. However, we do assume that the buy price b_c charged to a station c depends on the amount of energy bought by its coalition, i.e., on the coalition-wise attendance A_{k_c}, through a buy price function $p(\cdot)$, such that $b_c = p(A_{k_c})$. The exact dependence, i.e., the shape of $p(A_{k_c})$, is crucial in determining whether joining a coalition is a sensible move or not. As an example, $p(A_{k_c}) = b_0$ means that there is no incentive at all to form coalitions. On the other hand, functions such as $p(A_{k_c}) = b_0 - \log A_{k_c}$, $p(A_{k_c}) = b_0 - A_{k_c}^2$, $p(x) = b_0 - \exp A_{k_c}$ provide increasingly strong incentives.

Recall that a charging station joining a coalition forfeits its freedom to decide the sell price s_c. Hence, stations will not join a coalition if the incentive represented by the reduction in buy price b_c is not high enough.

We also assume that selling prices are chosen from a finite-sized set $\mathcal{P}$. This assumption simplifies our discussion but, as we will see in Sec. V, has no impact on the overall level of realism of the model.

IV. The Market Game

Game theory [1] studies the interaction among rational agents, called *players*. Players can choose among a set of *moves* and aim at maximizing their *payoff*. The payoff obtained by each player depends not only on its move, but also upon the other players' moves. This makes game theory a particularly powerful and convenient tool to study cooperation and conflict mechanisms, such as the EV charging model we have outlined above. We thus define the charging market game as follows.

Players: We have two categories of players:

- the vehicles in $\mathcal{V}$;
- the charging stations in $\mathcal{C}$.

Payoffs: Vehicles aim at optimizing the time-price trade-off in (1), which represents the cost they incur[2]. As for charging stations, their payoff is represented by the following monetary gain:

$$\mathbb{1}_{[c \notin k_0]} \left[A_c (s_c - b_c) - f_c \right]. \tag{3}$$

Notice that, from (3), it follows that equilibrium payoffs for charging stations are never negative.

Moves: Vehicles have to select a charging station, thus the set of their possible moves corresponds to the set $\mathcal{C}$ of charging stations. Charging stations, on the other hand, can decide to:

- form a new coalition formed by themselves alone;
- exit the market, i.e., joining coalition k_0;
- leave their current coalition and join one of the existing ones.

Recall that the game players are vehicles and charging stations, not coalitions, which therefore do not make any move. Indeed, the coalition prices are completely determined by the composition of the existing coalitions and by the behavior of vehicles.

[2]Recall that vehicles with no ITS support will still optimize (1), but accounting for an incorrect value of w_c.

Equilibrium: A strategy profile is a mapping of vehicles onto charging stations and of charging stations onto coalitions. In this work, we use the standard definition of Nash equilibrium: a strategy profile from which no player has interest to unilaterally deviate, i.e., when no vehicle selects a different station and no station changes coalition.

V. Complexity issues

The model we have described so far is characterized by several variables, accounting for both topological and economic aspects. Among the former, our model lists:

- a set of charging stations $c \in \mathcal{C}$, with their locations λ_c;
- a set of vehicles $v \in \mathcal{V}$, with their origin, destination, and points at which they become aware of their low battery, i.e., α_v, ω_v, and η_v);
- the distances $t(x_1, x_2)$ between any two points (x_1, x_2) in the road topology.

As for the economic aspects, our model comprises:

- the fixed costs to be paid by charging stations f_c, $c \in \mathcal{C}$;
- the buy price b_c, $c \in \mathcal{C}$, through function $p(\cdot)$;
- the set $\mathcal{P}$ of possible sell prices.

In order to find an equilibrium, our task is to identify:

- a mapping of vehicles in $\mathcal{V}$ onto stations in $\mathcal{C}$;
- a mapping of stations in $\mathcal{C}$ onto coalitions in $\mathcal{K}$;
- a mapping of stations in $\mathcal{C}$ onto sell prices in $\mathcal{P}$;

such that no vehicle or station would deviate from them. In other words, each step of the game requires the identification of (i) all possible coalitions formed by the $|\mathcal{C}|$ charging stations; (ii) for each coalition, the $|\mathcal{P}|$ prices a station $c \in \mathcal{C}$ can choose from; (iii) for each coalition-price combination, (up to) $|\mathcal{C}|$ possible charging stations, among which the $|\mathcal{V}|$ vehicles can choose.

The possible number of ways the stations in $\mathcal{C}$ can form coalitions corresponds to the number of combinations of $|\mathcal{C}|$ elements, i.e., $2^{|C|-1}$. It follows that the number of solutions to examine is:

$$2^{|C|-1} \cdot (|\mathcal{P}|+1)^{|\mathcal{C}|} \cdot |\mathcal{C}|^{|\mathcal{V}|} = O\left(|\mathcal{C}|^{|\mathcal{V}|+|\mathcal{C}|}\right). \qquad (4)$$

The number in (4) is clearly overwhelming, even for unrealistically small instances of the scenario.

We therefore try to reduce the overwhelming complexity of finding an equilibrium by means of some simplifying assumptions, and check that such assumptions do not jeopardize the realism of our model.

A. Vehicles book their battery replacement

A first assumption we can make is that vehicles can *book*, e.g., via an existing ITS, their battery replacement, as soon as they realize that their charge level is low. Such an approach is used, among others, by the Pod Point project [18]. This does not mean that they will not wait in line if they arrive at the station earlier than the allotted time. However, it has a very important consequence:

Proposition 5.1: The trip time of each vehicle depends solely on the moves of the vehicles booking before it.

This, in turn, yields the following important result.

Theorem 5.2: The strategy obtained under the assumption that each vehicle books a station when it needs to replace its battery is a Nash equilibrium.

Proof: The vehicle booking first will not deviate from the strategy. The payoff of the second one will only be affected by the decision of the first vehicle, so the second vehicle will not deviate from the strategy either. By induction, no other vehicle will deviate. ■

Theorem 5.2 yields a huge reduction of the complexity, indeed we can replace the third term in (4) with 1, obtaining:

$$2^{|\mathcal{C}|-1} \cdot (|\mathcal{P}|+1)^{|\mathcal{C}|} = O\left(|\mathcal{P}|^{|\mathcal{C}|}\right). \qquad (5)$$

Importantly, in (5) we have no dependence upon the number $|\mathcal{V}|$ of vehicles. Therefore, we can easily take into account realistic scenarios with heavy traffic levels and/or high penetration rates.

B. Deciding the prices

Let us assume that the coalitions are given and, hence, the buying prices b_c are known. Recall that $\mathcal{P}$ is the finite set of price levels, and that we impose $s_c \in \mathcal{P}, \forall c \in \mathcal{C}$.

The prices of all charging stations start at the minimum level, i.e., $s_c = \min \mathcal{P}, \forall c \in \mathcal{C}$. Then, coalitions take turns incrementing the price of at most one of their stations, by at most one level, until no coalition wants to increase any price anymore. Such a situation is not necessarily an equilibrium, as some coalitions may want to decrease their prices. Therefore, coalitions take turns reducing the price of at most one of their stations, by at most one level, until no coalition wants to decrease any price anymore.

Let us now consider the maximum number of solutions evaluated in this way. In the worst case, the price of each station goes from $\min \mathcal{P}$ to $\max \mathcal{P}$, and then goes back to $\min \mathcal{P}$. Therefore, the complexity of an "up-and-down" round is bounded by:

$$2 \cdot |\mathcal{C}| \cdot |\mathcal{P}| = O(|\mathcal{C}| \cdot |\mathcal{P}|).$$

Clearly, we are not guaranteed that after one "up-and-down" round we reach an equilibrium, i.e., that now no coalition would like to increase their prices again. The process may go on indefinitely. This is a consequence of the so-called "perfect rationality" assumption which is commonly made in game theory: each player is perfectly able to predict the opponents' moves, and to change her own accordingly. However, such an assumption is unrealistic in most practical cases, and is often dropped in favor of the notion of *bounded* rationality: each player will be able to reconstruct in her mind only the next $r \in \mathbb{N}$ moves of her opponents. Practical experiments [19], [20] show that players significantly underestimate their opponents' rationality, and suggest a value of r below 5.

Assuming bounded rationality, the complexity of the price decision is bounded by $2r \cdot |\mathcal{C}| \cdot |\mathcal{P}|$, and the total complexity (5) becomes:

$$2^{|\mathcal{C}|-1} \cdot 2r|\mathcal{C}||\mathcal{P}| = O\left(2^{|\mathcal{C}|}\right). \qquad (6)$$

Adding parameter r allows us to study how the coalition revenue depends on the level of rationality employed in defining their price policy, i.e., if "more clever" coalitions have a competitive advantage.

C. Forming the coalitions

We follow a similar approach in determining how the coalitions are formed.

Each charging station starts by forming its own coalition, i.e., $k_c = c, \forall c \in \mathcal{C}$, and $k_0 = \emptyset$. Then, charging stations take turns in deciding which coalitions to join. Each charging station can choose among at most $|\mathcal{C}|+1$ coalitions, including the current one and k_0. Thus, a "coalition forming" round includes exactly $|\mathcal{C}|$ turns, during which at most $|\mathcal{C}| + 1$ alternatives are examined. It follows that the complexity of each round is bounded by $|\mathcal{C}|^2$.

Again, we assume that players have bounded rationality r, i.e., at most r "coalition forming" rounds are performed. We can thus update the overall complexity (6) to:

$$r|\mathcal{C}|^2 \cdot 2r|\mathcal{C}||\mathcal{P}| = 2r^2|\mathcal{C}|^2 \cdot |\mathcal{C}||\mathcal{P}| = O(|\mathcal{C}|^3). \qquad (7)$$

D. Summary and discussion

We have been able to move from the worse-than-exponential complexity in (4) to the cubic complexity in (7), by subsequently addressing the following three stages:

- vehicles choosing the charging station to use;
- coalitions deciding their price policy;
- charging stations joining a coalition.

If dealt with naivety, each of these stages would have exponential complexity. However, we have shown that if vehicles can book their charging station, the complexity of the first stage reduces to a constant. Furthermore, the complexity of the latter two stages becomes polynomial as a consequence of the fact that players have bounded rationality.

Interestingly, none of the assumptions we made impairs the realism of our model. Specifically, vehicles *do* book their replacement [18] and players, including businesses, *do* have bounded rationality [19], [20].

We can therefore be quite satisfied with the final complexity reached in (7), for a variety of reasons.

First and foremost, cubic is better than exponential. Roughly speaking, it means that we can tackle realistically-sized scenarios with a complexity comparable to that of linear programming problems. Furthermore, the dominant term in the expression of the complexity only depends on the number of charging stations we have. In other words, the number of price levels in $\mathcal{P}$ and the rationality r do have an impact on the computation time, however as the values of the involved parameters grow very large, it is $|\mathcal{C}|$ that dominates. This means that we will be able to increase r and $|\mathcal{P}|$ as much as needed, without incurring in an exceedingly high complexity.

VI. Scenario and settings

We now apply our model to a realistic scenario, so as to get some insight in the way the decisions of individual charging stations, coalitions of stations and vehicles interact.

We consider the road topology in Figure 1, depicting a 10×10 km^2 section of the urban area of Ingolstadt, Germany.

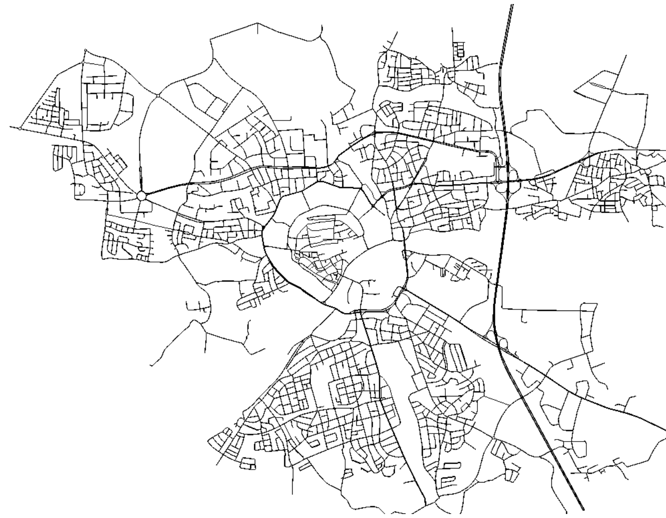


Fig. 1. Road topology used for our experiments.

There are $|\mathcal{V}| = 300$ vehicles traveling on the topology, each needing to stop at one of $|\mathcal{C}| = 14$ charging stations. Charging stations are located at major intersections throughout the topology. For each station, we set the number of service stalls to 6 and the service time, i.e., the time to replace a vehicle battery, to 60 s. Travel times within the topology are obtained through SUMO [21] simulations, tabulated and input to our model. As detailed in the Appendix, the waiting time at a charging station, i.e., the time a vehicle waits in line there, varies depending on the number of vehicles it finds already waiting at the charging station.

Unless otherwise specified, the fixed cost is $f_c = 50$ for each station, the possible sell prices are $\mathcal{P} = \{9, 10, \dots, 17, 18\}$ and the buy prices are determined as follows:

$$b_c = 3 - L \log_{10} A_{k_c}. \qquad (8)$$

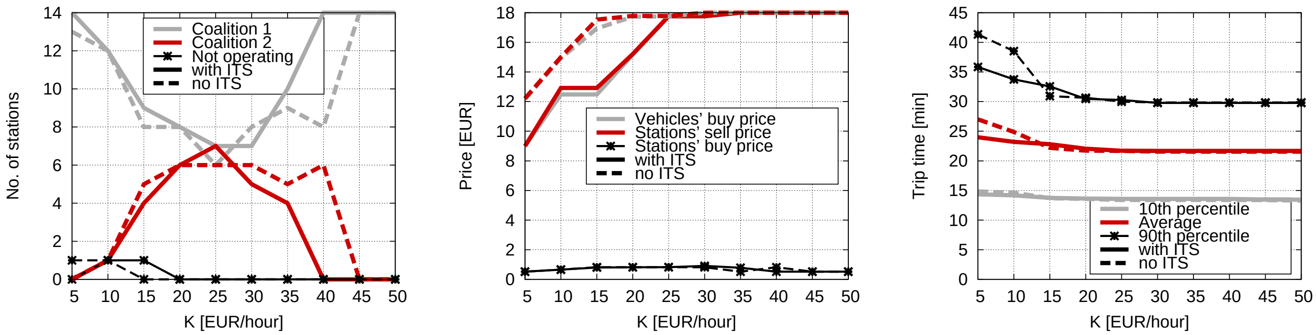
In (8), the parameter $L \in \mathbb{R}$ expresses how strong the incentive to create a coalition is; we set its default value to $L = 1$. As a consequence, in the default settings, the buy prices approximately range between 0.5 and 3. Finally, we set the rationality parameter to $r = 20$.

VII. Results

For each case study, we let the value of the coefficient K vary between 5 and 50, with the latter depicting very impatient drivers, and the former representing the lowest value drivers could give to their time (at least in popular culture [22]).

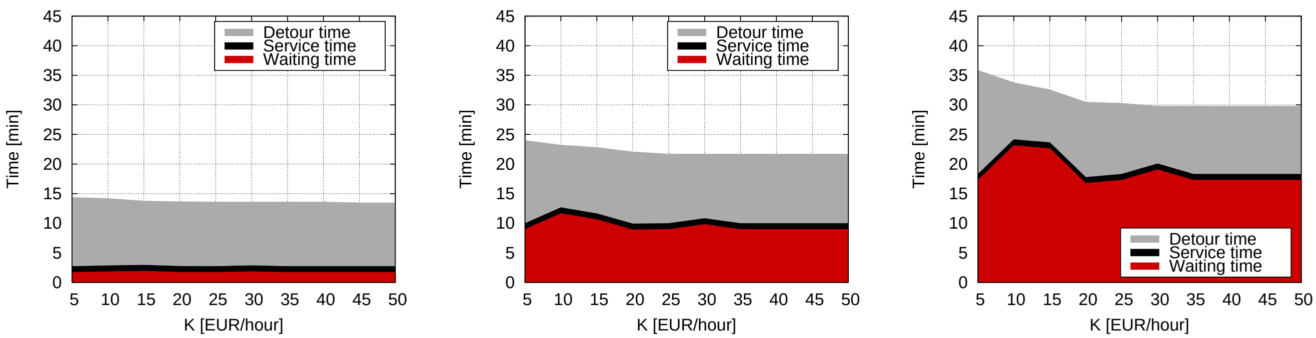
Also, for each case study, we compare the cases in which an ITS system is available, i.e., vehicles can correctly estimate w_c, and in which vehicles have no ITS support, i.e., will always assume $w_c = 0$.

We start from the baseline scenario described in Sec. VI, and investigate the prices paid by the vehicles and the coalitions formed by charging stations. Then, we look at how anti-cooperative behaviors from the charging stations can be countered by acting on the number and size of the coalitions (Sec. VII-B) and on the coalition incentive L (Sec. VII-C). For each of these anti-trust strategies, we are also interested



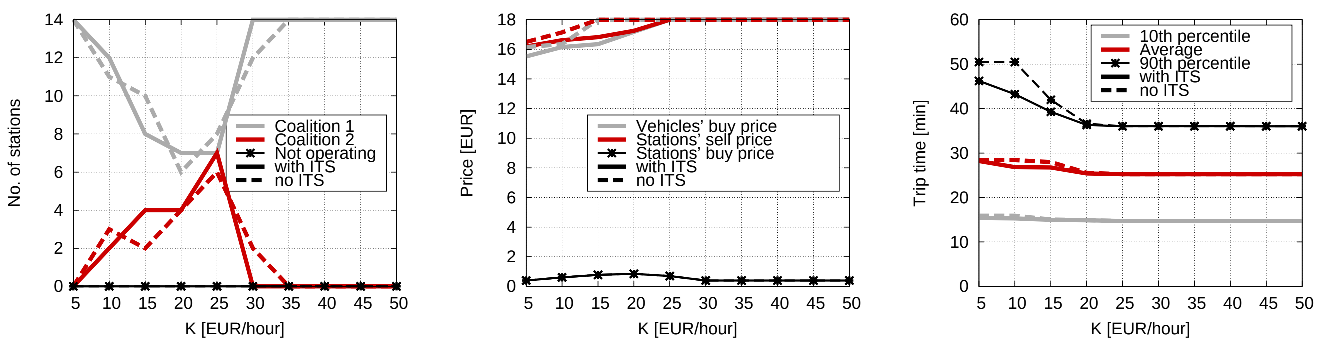
(a) (b) (c)

Fig. 2. Baseline scenario: coalitions size (a), prices (b), vehicle trip time (c), with and without ITS support.



(a) (b) (c)

Fig. 3. Baseline scenario with ITS support. Breakdown of trip times for the 10% vehicles with longest trip time (a), the average (b) and the 10% vehicles with shortest trip time (c).



(a) (b) (c)

Fig. 4. Peak-time scenario: coalitions size (a), prices (b), vehicle trip time (c), with and without ITS support.

to check whether the availability of ITS support has any impact on its effectiveness.

Fig. 2 depicts the behavior of the system under our baseline scenario. The first aspect of interest is the way coalitions are formed, shown in Fig. 2(a). We can see that the incentive to form coalitions given by $L = 1$ is quite effective. There are never more than two coalitions. More interestingly, when the vehicles give to their time a very high or a very low value, there is only one coalition, attaining the lowest possible buy price b_c. Thanks to the large number of vehicles existing in the topology, no charging station decides not to operate, i.e., to join coalition k_0, except when $K \leq 10$.

Such a behavior, highlighted by Fig. 2(a), can be explained by looking at the prices portrayed in Fig. 2(b). When K is low, vehicles tend to select the cheapest station, (almost) regardless of its distance. Therefore, stations will react by selecting the lowest possible price and forming a single coalition so as to enjoy the lowest buy price b_c. Then, since all stations have the same price, vehicles will optimize their objective, as in (1), by selecting their closest station. It follows that stations in disadvantageous locations will go out of business, being unable to further reduce their prices to attract more vehicles. In other words, they will join coalition k_0.

As the value of time K increases, we can see another interesting effect: two coalitions are formed, as shown in Fig. 2(a), and the average sell and buy prices tend to remain constant (red and gray curves in Fig. 2(b)). Indeed, in these cases, vehicles always select the closest charging station and the prices tend to reach their maximum value.

Also, by comparing red and gray curves in Fig. 2(b), we observe that the average price paid by vehicles[3], i.e., $\frac{1}{|\mathcal{V}|}\sum_{v\in\mathcal{V}} s_v$, can be lower than the average price charged by stations, i.e., $\frac{1}{|\mathcal{C}|}\sum_{c\in\mathcal{C}} s_c$. Clearly, stations charging a cheaper price attract more vehicles.

As K further increases, we enter a regime where vehicles have a strong tendency to select the closest station, no matter which price they have to pay. Stations react by charging the maximum possible prices (Fig. 2(b)), and again forming a single coalition (Fig. 2(a)) to minimize the buy price b_c and, thus, maximize their profit.

Finally, let us look at the trip times portrayed in Fig. 2(c). The plot shows the average, as well as the 10th and the 90th percentiles, of vehicle trip times. This includes:

- detour time from the original route to the charging station and back;
- waiting time in line at the charging station;
- service time, i.e., the time required for battery replacement.

In spite of the variability due to the differences in the routes, the trend is clear. As K grows to 25, the trip times decrease. For higher values of K, stations always charge the highest possible price, and vehicles always select the closest station, thus attaining the lowest possible trip time.

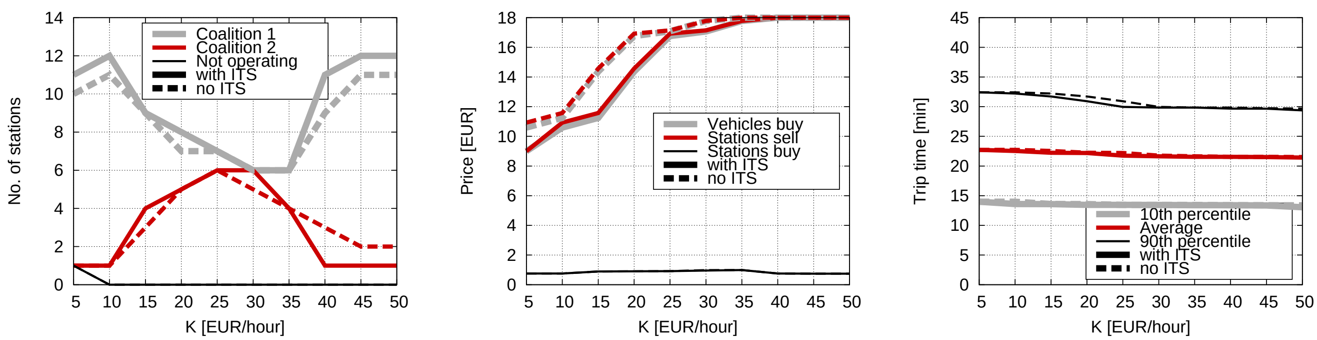
The role of ITS: In all the above cases, we can see that the availability of ITS support for vehicles consistently implies lower prices and shorter trip times. While the fact that ITS support shortens trip times is quite obvious, the impact of such a support on prices is not.

The intuition is that when vehicles underestimate their waiting times, they are more likely to select a more expensive charging station, as their value of objective (1) is still low. Surprisingly enough, if vehicles with no ITS support always assume that waiting times are very long, prices are still higher than in the case with ITS support. The reason is as follows: vehicles select more expensive charging stations in order to avoid further increasing their objective values. Therefore, any incorrect estimation of waiting times yields higher trip times *and* higher prices – that knowledge is power has long been known in fields other than networking or game theory.

In Fig. 3, we focus on the case with ITS and we seek to understand the relevance of each of the trip time components, for vehicles with short, average, and long total trip times. First, we recall that the service time is constant and quite short; this is due to the fact that batteries are replaced and not recharged. Also, the detour time varies little as K increases. Indeed, waiting times are a very important part of the total trip time, and they represent the most significant difference between vehicles with short and long trip times. Finally, lower values of K correspond to longer waiting times.

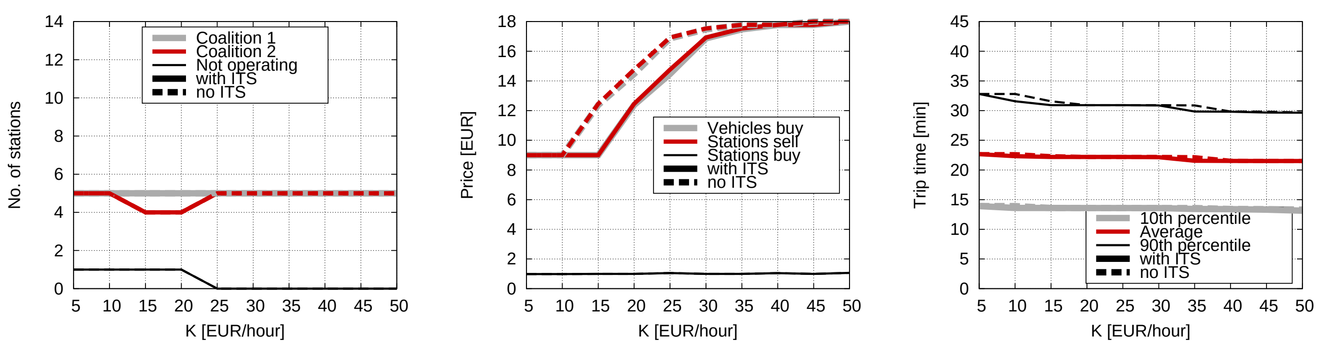
As mentioned, our model can account for those scenarios in which batteries are charged on the spot instead of being

[3] We abuse the notation, indicating by s_v the price paid by vehicle v, i.e., the sell price s_c charged by station c selected by vehicle v.



(a) (b) (c)

Fig. 5. At least three coalitions enforced: coalitions size (a), prices (b), vehicle trip time (c), with and without ITS support.



(a) (b) (c)

Fig. 6. At most five members per coalitions permitted: coalitions size (a), prices (b), vehicle trip time (c), with and without ITS support.

replaced. In that case, service times (the black area in Fig. 3) would be longer and, more importantly, not uniform as they would depend on factors like the charge level of each vehicle.

A. *Peak-time scenario*

We all know that prices are formed through the balance of demand and offer. In the following, we explore what happens if such a balance is changed, e.g., during peak traffic hours. Specifically, we increase the number of vehicles that need to replace their battery to $|\mathcal{V}| = 500$.

The effect, shown in Fig. 4, is quite clear. Firstly, a single, giant coalition forms sooner, i.e., for lower values of K. Secondly, costs and trip times both increase (notice the different scale in Fig. 4(c)). Thirdly, the impact of the presence of ITS is less significant. Intuitively, in this scenario there are so many vehicles that charging stations can enforce whatever pricing policy they see fit, and still have customers. Finally, looking at Fig. 4(b), we note that there is a wider spread between sell and buy prices.

B. *Countering trusts: number and size of coalitions*

As we have seen from Fig. 2(a), the system naturally drifts towards very few, big coalitions, which can easily maximize their profit by increasing the prices. Many countries deter such aggressive cartels through anti-trust regulations, forbidding large aggregations of subjects operating in the same market. We model these regulations by mandating that at least three coalitions (not including k_0) be formed, for all values of K. The results are summarized in Fig. 5.

The first thing Fig. 5(a) highlights is that the third coalition rarely includes more than one station: charging stations behave as if they were trying to elude anti-trust regulations – no surprise there. Nonetheless, the presence of the extra coalition has a significant impact on prices: as shown in Fig. 5(b), the regime in which all stations charge the maximum price is reached only for $K \geq 40$.

As far as trip times are concerned, Fig. 5(c) shows that they tend to be slightly lower, and to decrease more steadily, than in the baseline case (Fig. 2(c)).

Even better results are obtained, as we can see from Fig. 6, if at most five members per coalition are permitted. In this case, the average of the sell prices s_c, hence the average prices at which vehicles buy, is lower (see Fig. 6(b)). Such price reduction leads to no degradation in terms of trip time (see Fig. 6(c)). Furthermore, looking at Fig. 6(a), we observe that we always have two coalitions for any value of K.

Finally, it is important to note that none of the above settings reduces the penalty – in terms of energy prices – that vehicles incur if no ITS system is available.

C. *Countering trusts: removing and reversing the coalition incentive*

In some cases, it may prove difficult to directly enforce a minimum number of coalitions, or a maximum size of the coalitions. Thus, here we try to act upon the coalition incentive L with the goal of obtaining lower prices for end users. We start by setting $L = 0$, i.e., we remove any incentive for charging stations to form big coalitions.

A first, unexpected result can be seen from Fig. 7(a): even if the incentive represented by a positive value of L is removed, coalitions are still formed. This is due to a secondary effect of belonging to a coalition, i.e., the fact that prices are decided coalition-wise. It follows that two charging stations that would otherwise be competing against each other can agree on a common price policy. Note, however, that the curves in Fig. 7(a) do not sum up to $|\mathcal{C}| = 14$, as the composition of several small coalitions that originate is not reported in the plot. Also, from Fig. 7(a) we observe that now there is a high number of non-operating charging stations. Indeed, due to the higher buy prices b_c (and the lower sell prices s_c), these stations cannot make a profit, especially when K is low (Fig. 7(b)).

By looking at Fig. 7(b) and Fig. 2(b), we can see that sell prices s_c are comparatively lower. Indeed, there are always some stations belonging to minor coalitions (see Fig. 7(a)) that compete with larger coalitions by lowering their sell prices.

Finally, observe from Fig. 7(c) that trip times tend to be longer than in the previous cases, especially for low values of K, as a consequence of the lower number of operating stations.

These results motivate us to ask a further question: what if we *reverse* the coalition incentive, i.e., set L to a negative value? Such a scenario could model those cases in which a high energy consumption is discouraged rather than encouraged, e.g., due to environmental concerns. Indeed, this is exactly the way residential consumers are currently billed by electric companies. The results are summarized in Fig. 8.

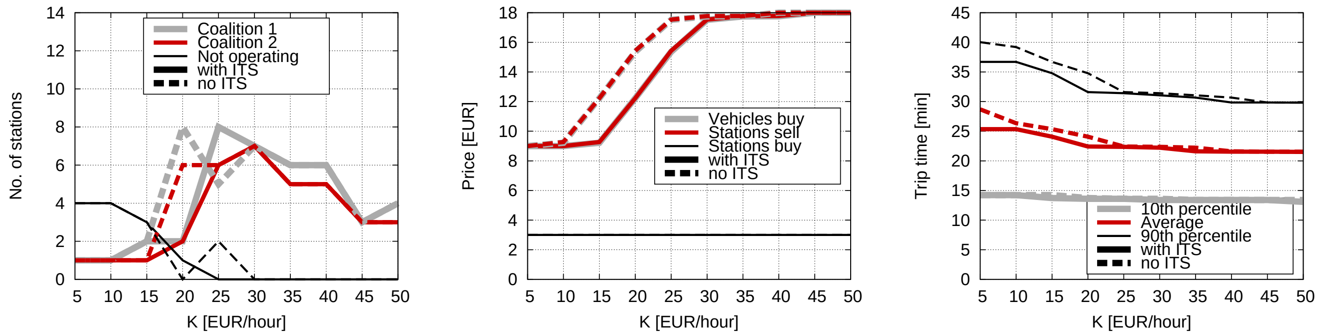
We can see the direct effect of having $L < 0$ by looking at Fig. 8(b): prices are remarkably higher than in the previous scenarios. Surprisingly, in spite of this effect, Fig. 8(a) shows that a sizable coalition, as big as 10 stations for some values of K, is nonetheless formed . Similarly to the case $L = 0$, this is due to stations that find forming a coalition more profitable than competing against each other. Indeed, our model proves to be remarkably good at capturing the anti-competitive behavior that charging stations are likely to mutuate from traditional ones. Also notice, from Fig. 8(a), that there is a higher number of charging stations that do not find it convenient to operate when K is small, essentially due to the higher energy costs to offset. Consistently, trip times (Fig. 8(c)) tend to be longer than in the other cases.

Again, no matter the anti-trust mechanism in place, ITS support is always associated to lower prices.

D. *Rationality*

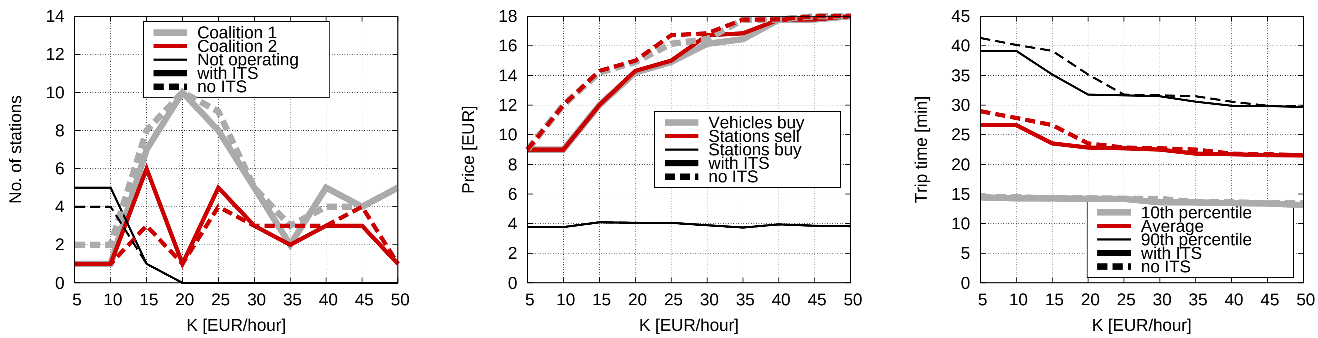
At last, we look at the number of rounds that, as described in Section V, are needed to reach an equilibrium. Recall that we have limited their number through the rationality parameter $r = 20$.

Table I shows that the number of iterations is always significantly smaller than the limit represented by r. This has two important consequences. First, assuming a bounded rationality has no impact on the results for the scenario under



(a) (b) (c)

Fig. 7. Scenario with $L = 0$: coalitions size (a), prices (b), vehicle trip time (c), with and without ITS support.



(a) (b) (c)

Fig. 8. Scenario with $L = -0.5$: coalitions size (a), prices (b), vehicle trip time (c), with and without ITS support.

study, i.e., keeping the traditional assumption of unbounded rationality would have yielded the same results.

More importantly, these figures show that, through our model, we manage to find an equilibrium for a realistic scenario with a limited number of iterations, hence in a short time. This achievement is important in light of an on-line usage of the system.

Also notice that extreme values of K typically yield a smaller number of iterations. This is due to vehicle behavior being less likely to change as a consequence of price or coalition-related decisions, which are in turn less likely to be reverted.

VIII. Conclusion

We presented a comprehensive model for networks of electric vehicles, accounting for the most relevant players (electric vehicles, charging stations and the coalitions they form). We also accounted for the way players' actions influence each other and for the possible intervention of anti-trust regulators. We described an algorithm to find a Nash equilibrium for our model, and discussed its complexity. Finally, we applied our model to a set of realistic scenarios, highlighting some non-obvious aspects of the behavior that can be expected from charging station owners. The value assigned by vehicle drivers to their time proved to be a fundamental parameter of the system, which has a relevant impact not only on vehicle trip times but also on the prices charged by stations and the way the latter group into coalitions. Finally, equilibria have been reached with quite a small number of iterations, always smaller than the rationality limit r. Due to its flexibility, our model can be adapted with trivial modifications to virtually any market situation. For instance, future research could further investigate interventions by market regulators (e.g., city or local authorities) on anticompetitive behaviors of charging stations. Also, the case in which fixed costs depend upon the coalition size can be examined.

The first aspect we looked at was the impact on the availability of ITS on the prices paid by the vehicles and the coalitions formed by the charging stations. We found that when no ITS support is available, charging stations have an incentive to increase their prices. Another interesting observation concerns coalitions: they exhibit a clear tendency to form coalitions, with one of such coalitions aggregating most of the active stations. This happens even if energy prices are determined in order to discourage such a behavior. Indeed, the main advantage that charging stations obtain from forming a coalition is not represented by lower energy prices, but by the possibility of enacting a common price policy – i.e., to form a trust. In all cases, the availability of ITS support invariably

TABLE I
NUMBER OF ITERATIONS NEEDED TO REACH AN EQUILIBRIUM

Scenario	$K = 5$	$K = 25$	$K = 50$
Baseline	2	7	1
$L = 0$	2	10	2
$L = -0.5$	3	13	5
Anti-trust	2	12	9

results not only in shorter trip and waiting times for vehicles, but also in lower prices.

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APPENDIX

THE WAITING AND SERVICE TIME w_c

In the following, we give a closed-form expression of the waiting/service time w_c.

Let us consider a generic vehicle arriving at station c, which is equipped with σ replacement stalls. Let τ be the replacement time, and n be the number of vehicles already waiting at the charging station before the tagged vehicle. The waiting and service time w_c is given by:

$$w_c = \begin{cases} \tau & \text{if } n < \sigma \\ \frac{\tau}{2} + \tau\frac{n-\sigma}{\sigma} + \tau & \text{otherwise.} \end{cases} \tag{9}$$

The first line of (9) applies when the arriving vehicle finds a free stall; in this case, its battery is immediately replaced in time τ. If all stalls are busy, instead, the vehicle has to wait in line. It will have to wait for the vehicles being served to finish ($\tau/2$ on average), then for the full service time τ of the $(n-\sigma)/\sigma$ other vehicles currently in line. Finally, the tagged vehicle will be served in time τ.

It is very important to stress that the expression in (9) also holds if batteries are recharged on the spot, and not replaced – the only difference being a higher, and possibly vehicle-specific, value of τ.

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