

Guest Editorial of MAMS Special Issue on:
**Applications of Unified Formulation and
Advanced Theories using Several Numerical
Approaches**

Guest-editors

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This special issue of Mechanics of Advanced Materials and Structures (MAMS) collects selected papers on the application of the Carrera Unified Formulation (CUF) and advanced theories for solving different problems with particular emphasis on multilayered structures. CUF is a hierarchical formulation allowing for a straightforward implementation of advanced structural theories and it has acquired particular interest in recent years. This special issue focuses the attention on the use of CUF with the following numerical techniques: Finite Element Method (FEM), Generalized Differential Quadrature (GDQ) method, Radial Basis Function (RBF) method, Dynamic Stiffness Matrix (DSM) method, Ritz and Navier methods.

Seven papers deal with the FEM, that plays a predominant role among the available computational techniques. The free-vibration problem in composite shell structures with double-curvature geometry is discussed by Cinefra in the first paper. A nine-node shell finite element with variable through-the-thickness kinematic, implemented on the basis of CUF and Mixed Interpolation of Tensorial Components (MITC), is employed for the analysis of laminated cylindrical and spherical shells with simply-supported edges and various laminations, orthotropic and thickness ratios. The results, compared with elasticity and analytical solutions in literature, show that the distribution of displacements and stresses along the thickness of the multilayered shell is accurately described. In the second work, Wenzel et alii present the capabilities and limitations of refined two-dimensional (2D) composite plate elements with respect to the stress concentration problem occurring at traction-free edges. Classical displacement-based and advanced partially mixed finite elements are formulated according to CUF. Rectangular laminates are analyzed under extension and bending loading, where the attention is focussed on the local stress response at the free edges. The proposed results are compared with reference ones available in the literature and a three-dimensional (3D) finite element model. In the third article by Biscani et alii, solid and higher-order plate finite elements are coupled for the effective analysis of composite structures. The problem arising by the difficulty in coupling degrees of freedom that cannot be interpreted as displacements or rotations (typical of higher-order plate elements) is tackled by means of the Arlequin method, which allows for coupling incompatible kinematic fields. CUF is here extended by deriving the matrices for the solid and plate elements coupling. Multi-layered composite and sandwich plates are investigated. Coupled multi-models solutions are assessed towards equivalent mono-models and commercial finite element software results. The fourth article by Milazzo focuses on the implementation of layer-wise and equivalent single layer advanced finite elements for the analysis of smart multilayered plates. The proposed modelling strategy reduces the multi-field problems to an effective mechanical plate by the condensation of the electromechanical state into the plate kinematics. CUF is invoked to derive the elemental stiffness and mass matrices and the mechanical and magneto-electric equivalent forces. The obtained smart plate finite element equations involve kinematical variables only and this extends the tools developed for multilayered composite plates to smart laminates. Results for simply-supported magneto-electro-elastic multilayered square plates are presented to validate the proposed modelling approach and related finite elements. The fifth article by Dozio and Alimonti is devoted to a novel and advanced finite

element formulation of the structural acoustic problem involving thin and thick multilayered composite plates coupled with a cavity. By exploiting CUF, many plate and fluid-structure interface elements based on different kinematic models are developed. A large number of vibro-acoustic models are easily obtained according to the accuracy requested by the application. In particular, it is shown that refined models can be adopted in those cases where models relying on traditional or low-order plate theories fail in providing the correct estimation of the fluid-structure coupling. The proposed formulation is also validated with respect to some reference cases available in the literature. Filippi and Carrera investigate the coupled bending-torsion flutter through CUF in the sixth work. The aerodynamic loadings have been determined according to the Theodorsen theory, from which the steady strip formulation can be easily obtained. FEM is used to solve the governing equations. Several wing configurations have been studied, giving great attention to thin-walled box beams made of orthotropic material. The effects of the sweep angle and the lamination scheme on the flutter conditions have also been investigated, and the results are compared with solutions derived from 2D theories, experimental tests and aeroelastic analyses carried out with the doublet lattice method. The last FEM work by Zappino et alii, addresses the dynamic analysis of thin-walled structures reinforced by longitudinal stiffeners using refined one-dimensional (1D) models. The results obtained are compared with those from classical FEM formulations based on 2D plate and shell, 1D beam, and 3D solid elements that are available in commercial software applications. When solid formulation is used to build FEM models, stringers and skin are discretized with only 3D elements whereas, in the 2D-1D models, shell and beam elements are used for skin and stringers, respectively. It is shown that the proposed modelling approach is able of detecting 3D-effects such as very complex shell-like modes, which are typical of thin-walled structures. Moreover, a strong reduction of computational cost in terms of degrees of freedom is obtained by 1D refined analyses. Two of the papers in this special issue focus on the GDQ method. The first one by Tornabene et alii considers the stress and strain recovery procedure applied for solving doubly-curved structures with variable radii of curvature. An equivalent single layer approach based on a general higher-order formulation is used. The theoretical model considers composite structures and employs the differential geometry for the description of doubly-curved, singly-curved, revolution with variable radii of curvature and degenerate shells. The governing static equilibrium equations are solved in their strong form using the local GDQ method. Moreover the Generalized Integral Quadrature (GIQ) is exploited for the evaluation of the stress resultants of the model under study. In the second paper, Tornabene illustrates a general formulation for a higher-order layer-wise theory related to the analysis of the free vibrations of thick doubly-curved laminated composite shells and panels. The theoretical framework regards to the dynamic analysis of shell structures by using a general displacement field based on CUF, including the stretching effect for each layer. The main aim of this work is to determine the explicit fundamental operators that can be used for the layer-wise approach. The free vibration of shell and panel problems are computationally solved using GDQ and GIQ techniques. The numerical results are compared with recent papers in the literature and commercial finite element codes.

Two papers discuss the application of the RBF method. One paper is by Ferreira et alii and it addresses the static and free vibration analysis of doubly-curved laminated shells. The novelty of this work consists in the use of the Reissner-Mixed Variational Theorem (RMVT) through the CUF to derive the equations of motion and the natural boundary conditions of the problem considered. The theory presented accounts for through-the-thickness deformation, and directly computes displacements and transverse stresses in each interface of the laminate. The paper by Pagani et alii investigates the efficiency of the RBF method when applied to higher-order beam theories. The displacement field of the generic-order beam model is expressed by making use of the CUF. The strong form of the principle of virtual displacements is used to obtain the equations of motion of beams in free vibration. Locally supported Wendland's C^6 radial basis functions are subsequently used to approximate the derivatives of the generalized displacements, which are collocated on a number of points (centers) along the beam axis. Several numerical results are proposed including solid structures as well as open and closed thin-walled sections. The solutions by the proposed method are compared both by published literature and by solid/shell models from the commercial code MSC Nastran.

The remaining three papers deal with semi-analytical and analytical exact methods. The paper by Carrera et alii introduces a 1D higher-order exact formulation for linearized buckling analysis of beam-columns. The governing equations and the associated natural boundary conditions are

derived by means of CUF. After the closed form solution is sought, an exact Dynamic Stiffness (DS) matrix is derived by relating the amplitudes of the loads to those of the responses. The global DS matrix is finally processed through the application of the Wittrick-Williams algorithm to extract the buckling loads of the structure. Isotropic solid and thin-walled cross-section beams as well as laminated composite structures are analysed. The validity of the formulation and its broad range of applicability are demonstrated through comparisons with results from the literature and by using commercial finite element codes. Then, the work by Fazzolari considers the linearized buckling analysis of functionally graded material (FGM) isotropic and sandwich plates by means of the hierarchical trigonometric Ritz formulation. Quasi-3D Ritz models based on equivalent single layer and zig-zag plate theories are developed within the framework of CUF. Several in-plane loading conditions accounting for axial, biaxial and shear loadings are taken into account. Parametric studies are carried out in order to evaluate the effects of significant parameters such as volume fraction index, length-to-thickness ratio, sandwich plate type and loading type on the critical buckling loads. Finally, Carrera et alii report Best Theory Diagrams (BTDs) for the static analysis of metallic and laminated composite plates in the last paper. The theories that belong to the BTD have been obtained by means of the axiomatic/asymptotic technique, and a genetic algorithm has been employed to obtain the BTD. Closed-form, Navier-type solutions have been employed and attention has therefore been restricted to simply-supported plates. The influence of various geometries, material properties and layouts has been considered, and their influence on the BTD has been evaluated. Furthermore, some known theories have been evaluated and compared with the BTD curve. The results suggest that the BTD and CUF can be considered as tools to evaluate the accuracy of any structural theory against a reference solution in a systematic manner.

The papers published in this Special Issue discuss only some of the most significant works on the extension of Unified Formulation to different computational methods. However, the included articles present significant and promising contributions.

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