

Tuning the accuracy of rational macromodels to nominal load conditions

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Summary. We address the generation of broadband macromodels of complex linear systems via rational curve fitting. We show that standard approaches may not ensure that the macromodel accuracy is preserved in system-level simulations, under loading conditions that are different from the adopted identification settings. Our main contribution is an automated procedure for the definition of a frequency-dependent norm weighting strategy that tunes the macromodel accuracy for a specific nominal termination network, thus improving model robustness under realistic operation.

1 Introduction and problem statement

We consider the situation depicted in Fig. 1. The box on the left represents a complex (large-scale) Linear Time Invariant (LTI) system, that we assume without loss of generality to be known through a set of tabulated frequency samples of its scattering matrix $(\omega_k, \hat{S}_k)$ for $k = 1, \dots, K$. The box on the right represents the nominal termination network that is to be connected to the LTI structure during system-level verification via transient numerical simulation. This termination includes at least one transient source $u(t)$ and at least one output variable $y(t)$ of interest.

Our reference application is Power Integrity (PI) verification of electronic structures, for which the LTI system represents the electrical interconnect network that is responsible for power distribution to the chip through package and board, and the termination network includes: a number of transient current sources (the inputs) representing on-chip switching; several decoupling capacitors; and at least one Voltage Regulator Module (VRM) which defines the nominal supply voltage V_{dd} . All these components or subsystems are connected at suitably defined ports of the Power Distribution Network (PDN). The outputs of interest are the transient voltages at all interface ports. The purpose of PI verification is to make sure that the transient voltage fluctuations due to the parasitics of the PDN are kept below a prescribed design threshold.

System-level verification is usually performed via transient simulation within standard circuit solvers of the SPICE class. Due to the complexity of the PDN structure, it is desirable to compute a reduced-order macromodel described by a state-space realization $\{A, B, C, D\}$, whose frequency response $S(j\omega) = D + C(j\omega I - A)^{-1}B$ matches closely the raw available

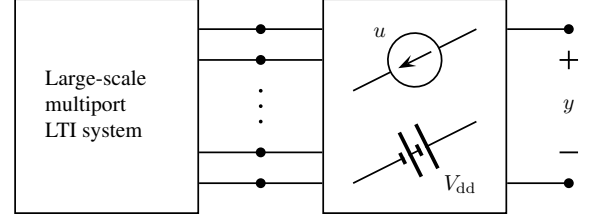


Fig. 1. System configuration under investigation

frequency samples. Once synthesized into a SPICE-compatible netlist, this macromodel allows fast transient analysis and enables simulation-driven design, verification, and optimization flows.

In this work, we concentrate on a black-box macromodeling procedure, which constructs the macromodel in pole-residue form

$$S(s) = R_0 + \sum_{n=1}^N \frac{R_n}{s - p_n} \quad (1)$$

by optimizing poles p_n , residue matrices R_n and direct coupling R_0 so that the following cost function is minimized

$$E^2 = \sum_{k=1}^K \epsilon_k^2 = \sum_{k=1}^K w_k^2 \|S(j\omega_k) - \hat{S}_k\|_F^2, \quad (2)$$

where $\| \cdot \|_F$ denotes the Frobenius norm and w_k is a suitable frequency-dependent weighting scheme. The standard practice is to set $w_k = 1$. The well-known Vector Fitting (VF) algorithm [1] computes a solution to the above optimization problem through a so-called iterative pole relocation process. The VF algorithm is the de facto standard rational curve fitting tool in signal and power integrity modeling, due to its excellent robustness and scalability properties. Therefore, we will use this scheme as the main identification engine.

Suppose now that the termination network (Fig. 1, right box) is known exactly, and let us consider the frequency samples of the transfer function $H(s)$ between some input u and some output y . The error between the exact transfer function $\hat{H}_k$ evaluated using the raw scattering samples $\hat{S}_k$ and the approximate samples $H(j\omega_k)$ evaluated using the macromodel can be defined as

$$\Delta^2 = \sum_{k=1}^K \delta_k^2 = \sum_{k=1}^K \|H(j\omega_k) - \hat{H}_k\|_F^2 \quad (3)$$

The real objective is to control the target error Δ , since this is the error that is observed when running a system-level simulation using the macromodel instead of an exact PDN model. The question arises whether we can control Δ by minimizing E . Our main objective is therefore to design optimal weighting coefficients w_k that, when used in the minimization of (2) through VF, will guarantee a small target error $\Delta < \Delta_{\max}$.

2 Formulation

The main idea is to embed the minimization of (2) within an outer loop that optimizes the weights w_k through iterations. Denoting the outer iteration index with μ , we setup the following scheme:

1. set $\mu = 0$ and initialize weights $w_k^0 = 1$ for all k ;
2. compute by VF a macromodel $S^\mu(s)$ by minimizing (2) based on the current weights w_k^μ ;
3. compute the resulting target errors δ_k and Δ based on (3); if $\Delta < \Delta_{\max}$, stop;
4. update the weights as $w_k^{\mu+1} = w_k^\mu \cdot f_k(\delta)$, where δ is a vector collecting all δ_k , and where f_k is a smoothing filter centered at the k -th sample;
5. set $\mu \leftarrow \mu + 1$ and go to step 2.

The above scheme updates the weighting coefficients in step 4, based on the frequency-dependent target error δ_k of the current macromodel. This allows to emphasize those frequencies for which there is a large sensitivity of the target error due to the feedback mechanism induced by the terminations onto the macromodel. The smoothing filter f is used to reduce the influence of noise.

Several improvements are possible and have been successfully implemented and tested. For instance, step 1 can start with an initial set of weights determined by a first-order sensitivity analysis (either numerical or analytical) of the error transformation $\delta_k(\varepsilon_k)$, see [2]. Alternatively, an elementwise or matrix-based relative instead of absolute error metric can be used in (2) and (3). This choice depends on the particular application at hand. We finally remark that, once a final set of weights w_k is available, they can be used to define a cost function to be minimized within standard passivity enforcement schemes [3], should the computed macromodel be affected by passivity violations.

3 Results

We demonstrate our proposed scheme on two PDN examples having 18 and 11 ports, respectively. Both cases correspond to industrial chip-package structures and are known through tabulated frequency samples of their scattering matrix, obtained numerically from a full-wave solver. In both cases one of the ports is

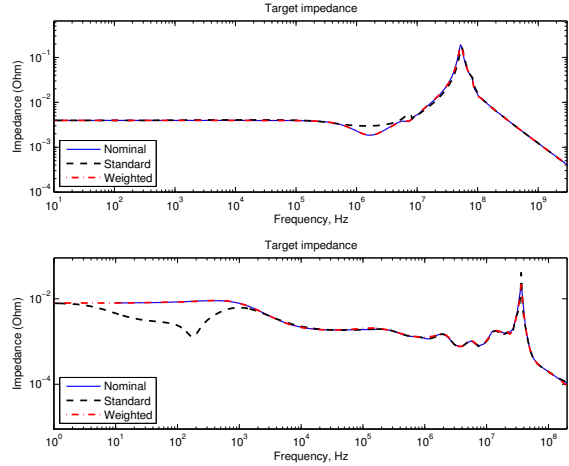


Fig. 2. Comparison between standard and proposed macro-models (see text).

connected to an ideal voltage source (a VRM model), and the remaining ports are connected to either decoupling capacitor models or to core circuit block models. The target transfer function is represented by the transfer impedance that returns the voltage at a prescribed node resulting from a uniformly distributed current excitation at all device ports.

Figure 2 depicts in top and bottom panels the results obtained from the two cases. The thin solid blue line represents the nominal impedance computed from the raw scattering samples describing the PDN. The black dashed line is the target impedance computed using a standard macromodel, obtained with relative weights without applying the proposed strategy. The red dashed line reports the result of our proposed approach, which is observed to match now very closely the nominal impedance. We remark that these responses are resulting from passive macromodels, as processed by the passivity enforcement scheme of [3].

In summary, we have proposed a black-box macro-modeling strategy that optimizes accuracy based on closed-loop nominal operating conditions, and not on standard input-output open-loop representations. The simple proposed approach is able to compensate for the error amplification that occurs when loading the macromodel with termination networks.

References

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