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***Abstract*—Electrodynamic bearings exploit repulsive forces due to eddy currents to produce positive stiffness by passive means without violating the Earnshaw stability criterion. This remarkable characteristic makes this type of bearing a suitable alternative to active magnetic bearings in fields such as kinetic energy storage flywheels, turbo pumps, high speed compressors, among others. However, the suspension can become unstable due to rotating damping.**

To obtain deeper understanding of this instability phenomenon this paper presents the analysis of stability of a four degree of freedom (4dof) rotor supported by electrodynamic bearings. The 4dof rotor model is coupled to the dynamic model of the eddy current forces generated by the electrodynamic bearing and the stability of the complete system is analyzed. This model is used to study the stability of both cylindrical and conical whirling motion of the rotor. In addition to the well known cylindrical whirl instability the possible occurrence of conical instability is demonstrated. Finally the effects of two stabilization strategies are analyzed.

I. Introduction

Magnetic bearings provide contact-less support for rotating machines. They usually exploit reluctance or Lorentz forces to keep a rotor in stable levitation. Reluctance forces can be used to levitate a rotor on active magnetic bearing (AMB) and passive permanent magnet bearing (PMB) configurations. Magnetic bearings exploiting Lorentz forces are called electrodynamic bearings (EDB) and although some layouts can be actively controlled [1], in general they are passive [2].

Different from AMBs and PMBs, that have already been employed in many industrial applications, EDBs have not reached industry yet, remaining at the academic research stage. Despite this fact, the EDB unique characteristic of producing positive stiffness by passive means without violating the Earnshaw criterion has attracted some industrial interest in the last decades. One of the most interesting features of EDBs is the possibility of obtaining stable levitation using standard conductive materials at room temperature, in absence of power electronics and sensors. This makes them an interesting alternative to AMBs for high speed rotors in applications such as energy storage flywheels and turbo pumps, among others.

Nowadays the main challenge precluding industrial application of EDBs remains understanding the stability of rotors supported by them, and how to solve the remaining instability problems. This led most of the research groups working with EDBs to perform detailed studies about rotordynamic stability [3]–[6]. In these works different electromechanical models of EDBs were created and coupled to rotordynamic models. A common feature of these studies was the use of Jeffcott rotor models which, although insightful, neglects gyroscopic effects. Hence, only the instability of cylindrical whirling motion was addressed.

In the present paper a more detailed stability analysis is presented and discussed. To this end a four degree of freedom rotor (4dof) model is used allowing studying the stability of both cylindrical and conical whirling motion. Initially the models of rotor and EDB are described and the dynamic matrix of the complete system is obtained. This model is then used to study the stability of the rotor on EDBs by means of root locus analysis. The root locus analysis evidences the possible occurrence of instability of both cylindrical and conical whirling modes. Finally, two methods for the stabilization of the rotor which were previously presented in the literature are studied and their effectiveness in stabilizing each unstable mode is addressed.

II. Modeling of a rotor supported by EDBs

Due to the nature of the phenomena, studying the dynamics of a rotor on magnetic bearings requires one to consider that the rotor moves relative to the stator. This must be taken into account both in the rotor and bearing models which must then be coupled together to describe the dynamics of the whole system. In this section the models of a gyroscopic rotor and that of an EDB are presented, and the passages to perform the coupling are described.

A. Rotor model

We consider a rotor such as that shown in Fig. 1. The rotor is composed of a gyroscopic rigid body described by its mass m, transversal moment of inertia J_t, and polar moment of inertia J_p. This rigid body is clamped to a massless rigid shaft. This assumption is justified as in general magnetic bearings produce a stiffness which is much lower compared to the stiffness of the shaft.

Two reaction forces generated by the magnetic bearings are represented by two forces F_{b1} and F_{b2} positioned at distance a and b from the rotor center of mass as shown in the figure. The complete equations of motion of the described rotor have six degrees of freedom. Under the hypothesis of small transverse

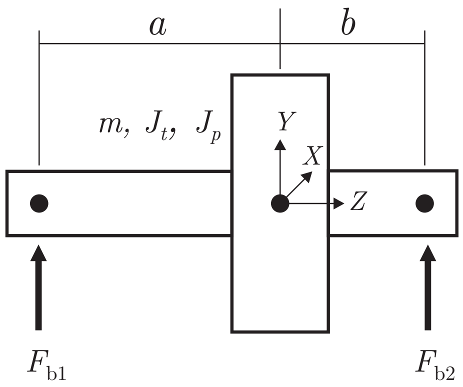


Figure 1. Scheme of the rotor model.

and angular displacements, and restricting our study to the constant rotational speed Ω condition, it can be demonstrated that a model with four degrees of freedom, or two complex degrees of freedom, is enough to study the flexural behavior of the rotor. The complete derivation of the rotor equations is not shown here. An extensive development is given in [7]. The equations describing the dynamic behavior of the four degrees of freedom rotor are the following:

$$\mathbf{M}\begin{Bmatrix}\ddot{q}\\ \ddot{\phi}\end{Bmatrix} - \mathrm{j}\Omega\mathbf{G}\begin{Bmatrix}\dot{q}\\ \dot{\phi}\end{Bmatrix} + \begin{Bmatrix}F_{\mathrm{b}}\\ M_{\mathrm{b}}\end{Bmatrix} = \begin{Bmatrix}F_{\mathrm{ext}}\\ M_{\mathrm{ext}}\end{Bmatrix} \tag{1}$$

where **M** and **G** are the mass and gyroscopic matrices respectively. These are written as:

$$\mathbf{M} = \begin{bmatrix} m & 0\\ 0 & J_t\end{bmatrix} \qquad \mathbf{G} = \begin{bmatrix} 0 & 0\\ 0 & J_p\end{bmatrix}$$

These two equations are written in terms of two complex coordinates q and ϕ that represent the transverse and angular displacements respectively, which are written as:

$$\begin{aligned} q &= X + \mathrm{j}Y\\ \phi &= \phi_y - \mathrm{j}\phi_{X'} \end{aligned} \tag{2}$$

B. Electrodynamic bearing

Electrodynamic bearings exploit repulsive forces due to eddy currents. Many different types of EDBs exist [8], however, here we restrict the analysis to homopolar EDBs which exploit relative motion between a conductor and a magnetic field. As for the rotor model, the complete derivation of the dynamic model is not described here. Only the fundamental equations are given. The extensive discussion of the model used in the present analysis is given in [9].

The dynamic equation of the EDB force can be written as:

$$\dot{F}_{\mathrm{EDB}} = k\dot{q} - \mathrm{j}\Omega k q - \left(\frac{k}{c} - \mathrm{j}\Omega\right) F_{\mathrm{EDB}} \tag{3}$$

The coefficients k and c give the stiffness and damping associated with the eddy current force model [10]. These two coefficients can be identified both by means of finite element simulations [11] or experimentally [6]. The notation used in Eq. 3 is in agreement with Eq. 2, hence, the coupling between rotor and EDB dynamic models is straightforward.

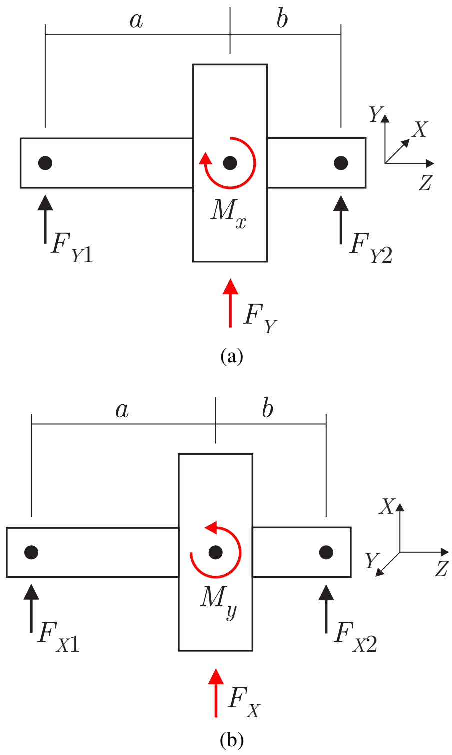


Figure 2. Transformation of reactions from bearing coordinates into center of mass coordinates. The black arrows represent the forces at the bearing and the red arrows represent the forces and moments at the center of mass.

C. Rotor and EDB coupling

In Eq. 1 the reaction forces and moments generated by the two bearings are represented by F_b and M_b whereas the excitation forces and moments are F_{ext} and M_{ext}. Both contributions are considered as acting on the rotor center of mass. Since the bearing's reaction forces are applied away from the center of mass by a and b, it is simple to obtain a relation between reaction forces at the bearing position and force and moment at the center of mass. To do so the schemes in Fig. 2are considered.

Analyzing the scheme of Fig. 2 the forces and moments calculated at the rotor center of mass are equal to:

$$\begin{aligned} F_X &= F_{X1} + F_{X2}\\ M_y &= -aF_{X1} + bF_{X2}\\ F_Y &= F_{Y1} + F_{Y2}\\ M_x &= aF_{Y1} - bF_{Y2} \end{aligned} \tag{4}$$

Positive moments are oriented according to the right hand rule. In matrix format Eq. 4 becomes:

$$\begin{Bmatrix} F_X\\ M_y\\ F_Y\\ M_x \end{Bmatrix} = \mathbf{T}\begin{Bmatrix} F_{X1}\\ F_{Y1}\\ F_{X2}\\ F_{Y2} \end{Bmatrix} \tag{5}$$

where

$$\mathbf{T} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ -a & 0 & b & 0 \\ 0 & 1 & 0 & 1 \\ 0 & a & 0 & -b \end{bmatrix} \tag{6}$$

Using the complex notation for the forces and moments one obtain:

$$\begin{aligned} F_b &= F_X + \mathrm{j}F_Y \\ M_b &= M_y - \mathrm{j}M_x \end{aligned} \tag{7}$$

Equation 6, written using the complex notation, reduces to:

$$\begin{Bmatrix} F_b \\ M_b \end{Bmatrix} = \mathbf{T} \begin{Bmatrix} F_{b1} \\ F_{b2} \end{Bmatrix} \tag{8}$$

where

$$\mathbf{T} = \begin{bmatrix} 1 & 1 \\ -a & b \end{bmatrix} \tag{9}$$

A similar procedure is used to obtain the relation between the displacements at the center of mass and those at the bearing position. The expression is:

$$\begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix} = \mathbf{T}^{\mathrm{T}} \begin{Bmatrix} q \\ \phi \end{Bmatrix} \tag{10}$$

where $\mathbf{T}^{\mathrm{T}}$ is the transpose of $\mathbf{T}$, and q_1 and q_2 are the transverse displacements at the bearings in complex notation as given by Eq. 2.

At this stage it is possible to write a state space model where the model states are given by Eq. 1 and Eq. 3. The state space model of the complete system can be written as:

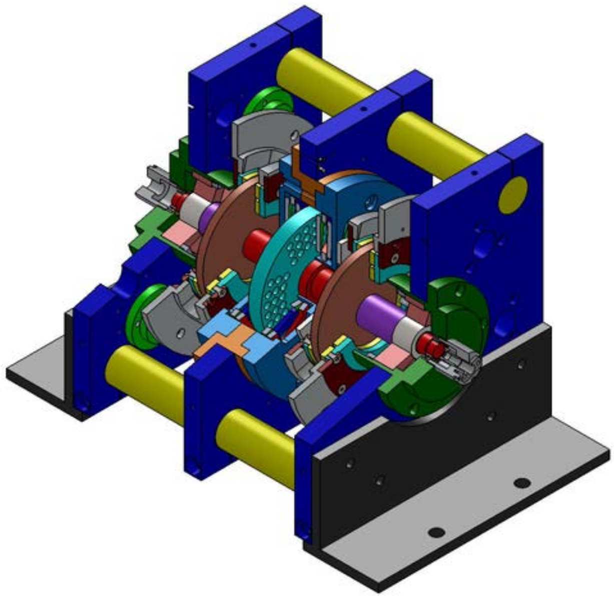
$$\begin{Bmatrix} \ddot{q} \\ \ddot{\phi} \\ \dot{q} \\ \dot{\phi} \\ \dot{F}_{b1} \\ \dot{F}_{b2} \end{Bmatrix} = \mathbf{A} \begin{Bmatrix} \dot{q} \\ \dot{\phi} \\ q \\ \phi \\ F_{b1} \\ F_{b2} \end{Bmatrix} + \mathbf{B} \begin{Bmatrix} F_{\mathrm{ext}} \\ M_{\mathrm{ext}} \end{Bmatrix}. \tag{11}$$

For the stability study only the dynamic matrix $\mathbf{A}$ is required and the poles of the system can be obtained as the eigenvalues of $\mathbf{A}$. The $\mathbf{A}$ matrix is written as:

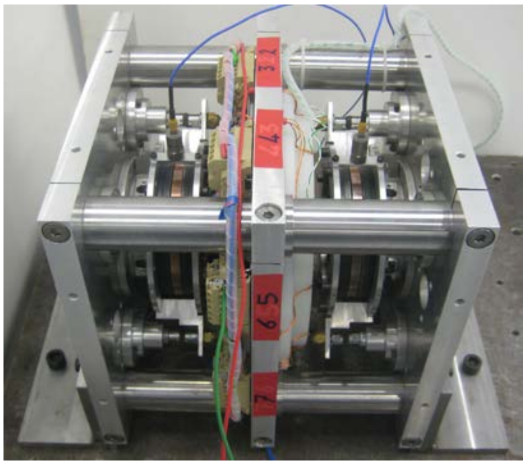
$$\mathbf{A} = \begin{bmatrix} \mathrm{j}\Omega\mathbf{M}^{-1}\mathbf{G} & \mathbf{0}_{2\times2} & -\mathbf{M}^{-1}\mathbf{T} \\ \mathbf{I}_{2\times2} & \mathbf{0}_{2\times2} & \mathbf{0}_{2\times2} \\ \mathbf{K}_{\mathrm{EDB}}\mathbf{T}^{\mathrm{T}} & -\mathrm{j}\Omega\mathbf{K}_{\mathrm{EDB}}\mathbf{T}^{\mathrm{T}} & -\mathbf{C}_{\mathrm{EDB}}^{-1}\mathbf{K}_{\mathrm{EDB}} + \mathrm{j}\Omega \end{bmatrix} \tag{12}$$

The matrices associated with the dynamic behavior of the EDB contain the parameters of the two bearings. They are written as:

$$\mathbf{K}_{\mathrm{EDB}} = \begin{bmatrix} k_1 & 0 \\ 0 & k_2 \end{bmatrix} \qquad \mathbf{C}_{\mathrm{EDB}} = \begin{bmatrix} c_1 & 0 \\ 0 & c_2 \end{bmatrix}$$



(a)



(b)

Figure 3. Test rig of a rotor supported by EDBs. The isometric cross section evidences how the rotor is positioned inside the machine.

where the indexes denote the properties of the EDBs positioned at ends 1 and 2 according to the scheme of Fig. 1.

III. Stability analysis

The present section studies the stability of the 4dof rotor model supported by two EDBs positioned at the two ends of the shaft. This analysis considers the parameters of a real laboratory rotor. The system studied is shown in Fig. 3 and is described in detail in the paper by Impinna et al. published separately in the present conference [12]. The two EDBs have equal properties and are positioned symmetrically with respect to the center of mass. The values of the parameters that characterize the whole suspension model in Eq. 12 are given in Tab. I.

The data from the table is introduced into Eq. 12 and the stability is studied analyzing the eigenvalues calculated for

Table I
PARAMETERS DESCRIBING THE DYNAMICS OF THE 4DOF ROTOR ON EDBS. SOME OF THE DATA REFFERS TO FIG. 6.

Parameter	Symbol	Value	Unit
Rotor mass	m	4.342	kg
Transverse moment of inertia	J_t	0.01982	kgm^2
Polar moment of inertia	J_p	0.00572	kgm^2
EDB 1 distance from center of mass	a	0.067	m
EDB 2 distance from center of mass	b	0.067	m
Damper 1 from center of mass	c	0.12425	m
Damper 2 from center of mass	d	0.12425	m
EDB eddy current stiffness	k_{1-2}	253051.3	Nm^{-1}
EDB eddy current damping	c_{1-2}	363.1	Nsm^{-1}

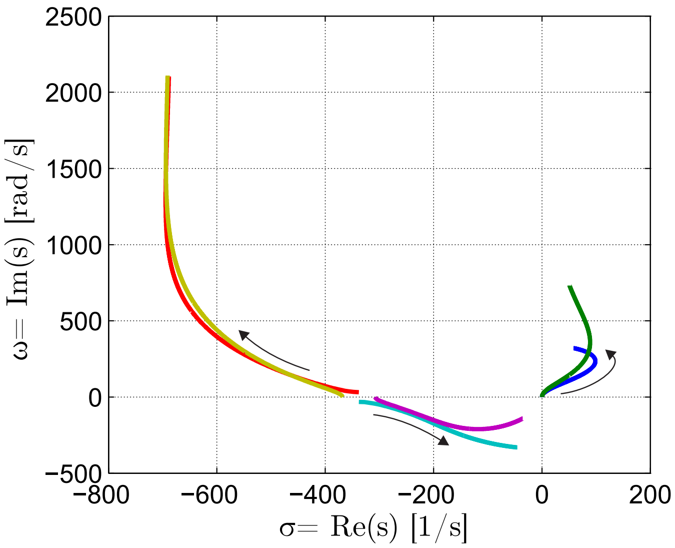


Figure 4. Root locus plot of the 4dof rotor supported by two EDBs. The arrows indicate the direction of the evolution of the poles with increasing rotation speed.

different values of Ω. The root locus plot is obtained plotting the real and imaginary parts of the calculated eigenvalues in the complex plane. Figure 4 shows the root locus of the rotor. Instability is evidenced by the presence of two poles with positive real part. These two unstable poles, illustrated by the blue and green lines, are linked to the cylindrical and conical forward whirl rotor modes.

In absence of external non rotating damping these two modes are unstable for any rotation speed different from zero. This is linked to the dynamic behavior of the EDB and is due to two main aspects. The first important aspect is the fact that the EDBs introduce rotating damping without any contribution of non rotating damping. The second is that EDB stiffness depends on the rotational speed; it does not provide any stiffness at zero rotational speed.

Intuitively one can think that the instability arises from the fact that the system operates always in supercritical regime. It is then clear that rotating damping causes instability since it is known to be destabilizing in these conditions. It is also clear that in case of disc rotors, where $J_p > J_t$, the conical instability does not exist since this mode is subcritical.

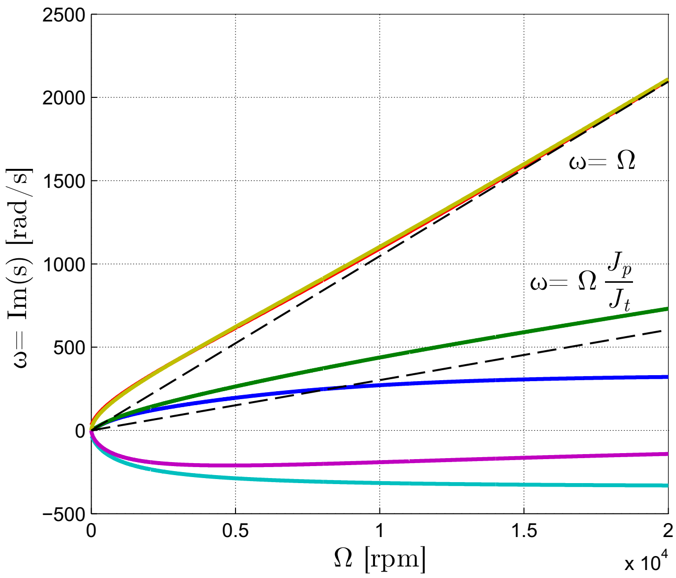


Figure 5. Campbell diagram of the rotor on EDBs. The lines show the evolution with speed of the imaginary part of the poles shown in Fig. 4.

The aforementioned behavior can be better understood observing the Campbell diagram of the system shown in Fig. 5. It can be noticed how the two lines showing the evolution of the two unstable modes are always below the line $\omega = \Omega$, thus these two modes are always supercritical.

IV. STABILIZATION OF ROTORS ON EDBS

In the previous section it was shown that the undamped behavior of a 4dof rotor on EDBs shows two unstable rotor modes. The instability is related to the forward cylindrical and conical whirl modes. In the past, several authors investigated the stability of the cylindrical whirl and two different stabilization methods were proposed [3], [6], [13]. Figure 6 shows the schemes of the two proposed methods.

The first scheme, in Fig. 6a, illustrates the most common stabilization method consisting in the introduction of non rotating damping by electromagnetic means between rotor and casing. In the second method, illustrated in Fig. 6b, non rotating damping is introduced by means of dissipative elements placed between the EDB stator and casing [6]. However, the effectiveness of each method to stabilize the conical mode has not been assessed. In this section the rotor and EDB models presented previously are used to understand the effects of the two stabilization methods upon the stability of the forward conical whirl mode.

A. Stabilization by rotor-casing interaction

The first stabilization method is the most common and straightforward. It consists in placing a permanent magnet attached to the rotor which interacts with a conductive element attached to the casing. Such method is well known and has been studied and employed in the field of passive magnetic bearings even at industrial level [14].

From the modeling point of view this stabilization system consists simply in introducing a damping element associated

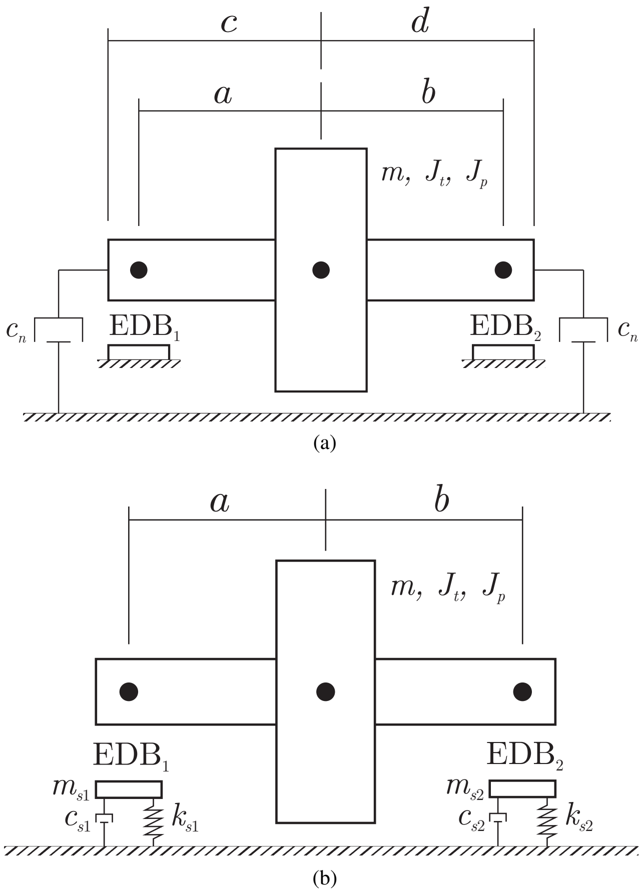


Figure 6. Scheme of the rotor on supported by EDBs with non rotating damping introduced between EDB stator and casing.

with the translation speed of the rotor at the point where the damper is positioned. For the sake of generality, in the scheme of Fig. 6a two dampers are introduced at distances c and d from the center of mass. The damping matrix can be easily obtained using the procedure described in paragraph II-C.

A brief sensitivity analysis can be used to verify the influence of the non rotating damping parameter c_n upon the stability. The results are plotted in Fig. 7. It can be seen that c_n is mainly influential upon the behavior of the forward conical whirl. The poles associated with this mode tend to move towards the left half plane, eventually reaching a condition of stability. On the other hand, the cylindrical mode is much less sensitive to this parameter. Consequently, increasing the non rotating damping introduced between rotor and casing is particularly beneficial for the stability of the conical mode. This is specially related to the fact that a relatively low value of c_n applied far from the rotor center of mass generates a large damping contribution with respect to the rotation degree of freedom ϕ.

B. Stabilization by stator-casing interaction

A different stabilization strategy for rotors on EDBs was proposed by Tonoli et al. [6]. The proposed technique consists in introducing the damping between the non rotating part of the bearing and the casing of the machine. In that work the authors assessed the effectiveness of their technique upon the

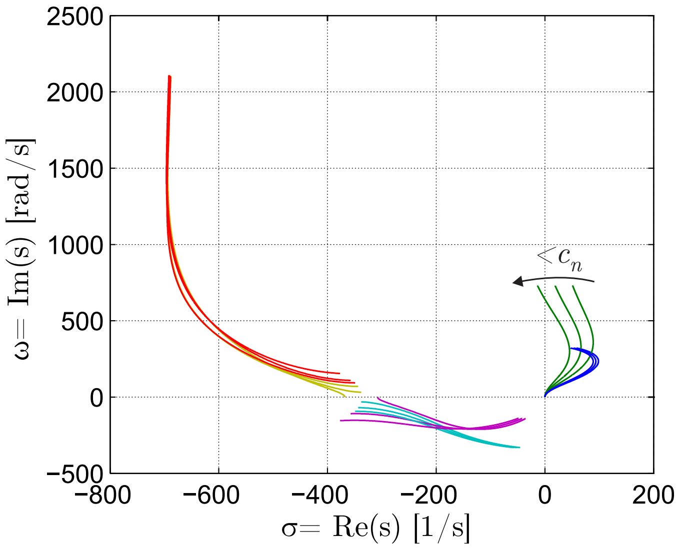


Figure 7. Root locus of the suspension calculated with different values of c_n. The arrow indicates the direction of the motion of the unstable poles when c_n is increased.

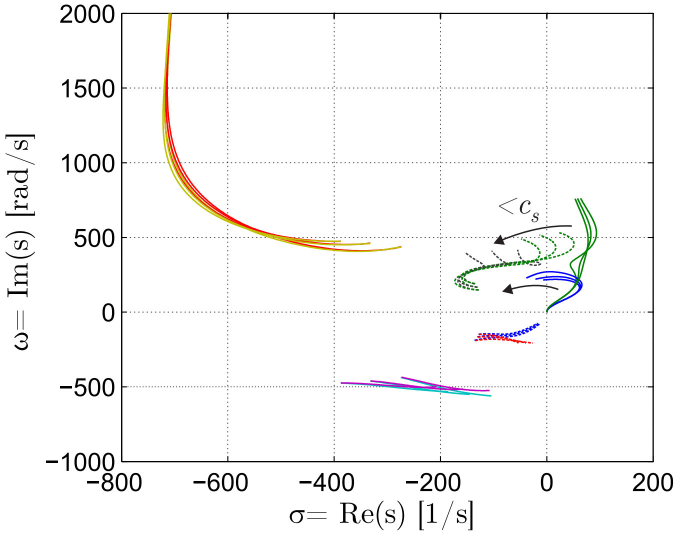


Figure 8. Root locus of the suspension calculated with constant values of m_s and k_s, and different values of c_s. The arrow indicates the direction of the motion of the unstable poles when c_s is increased.

stabilization of the cylindrical forward whirl. However, no mention to the influence upon the conical mode could be made.

Differently from the previous case, instead of introducing the damping on the rotor by means of contactless devices, here the underlying idea is to dissipate energy through a non rotating element. This opens to the possibility of using common damping techniques such as viscoelastic elements or fluid film dampers. Similar to the previous case, also this strategy to introduce non rotating damping is well known. It is widely used with rolling bearings and has been studied for passive [15], [16] and active magnetic bearings [17].

From the modeling point of view this solution is more complex than the previous. Two additional complex degrees of freedom associated with the transverse motion of the EDB stators must be included. The dynamics of the stators is described by simple mass-spring-damper systems.

To understand the influence of the stator damping parameter c_s a procedure similar to that presented in section IV-A is performed. The damping is modified, and the root locus of the whole system is plotted. The results are plotted in Fig. 8. As expected, the graph contains four more poles due to the new stator degrees of freedom. The lines describing the evolution of the stator poles are dashed whereas the rotor and EDB modes are shown in continuous lines. In accordance with the previous graphs, the forward cylindrical whirl is shown in blue and the forward conical is in green. In addition to the unstable rotor modes this new system may present an instability of the stator. Such behavior was verified experimentally by Tonoli et al. [18]. Even if this mode may become unstable and its occurrence should be verified at the design phase, its behavior can be stabilized by the addition of damping and is not of great concern.

For what concerns the rotor stability, it can be observed in the figure how increasing the stator damping is effective for the stabilization of the forward cylindrical mode; however it is not very effective in stabilizing the conical mode.

V. Conclusions

The present paper presented the modeling and stability analysis of a 4 dof rotor supported by electrodynamic bearings. The modeling of the two main subsystems, namely, rotor and EDB, was presented and a method to perform the coupling between the equations was described. Compared to formerly presented studies the present one employed a more complex rotor model. This upgraded rotor model allowed demonstrating that in addition to the well known cylindrical whirl instability, a rotor supported by EDBs may present an unstable conical whirl mode. The occurrence of the conical instability is directly linked to the ratio J_p/J_t. Disc rotors $(J_p/J_t > 1)$ do not present conical instability whereas long rotors $(J_p/J_t < 1)$ do.

The present study also analyzed the effectiveness of two known stabilization methods upon the conical mode stability. The stability analyses were conducted using the root locus method. It was observed that the stabilization method where damping is introduced directly between rotor and casing is particularly effective to stabilize the conical mode as it allows taking advantage of the damper position relative to the rotor center of mass. On the other hand this method is not so effective in stabilizing the forward cylindrical mode. The second stabilization method, based on the introduction of damping between the stator of the bearing and the casing showed to be less effective to stabilize the conical mode, however it is more effective in stabilizing the cylindrical mode. From this analysis it seems evident that the combination of the two methods may prove to be the best solution to guarantee stability of both cylindrical and conical modes for long rotors.

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