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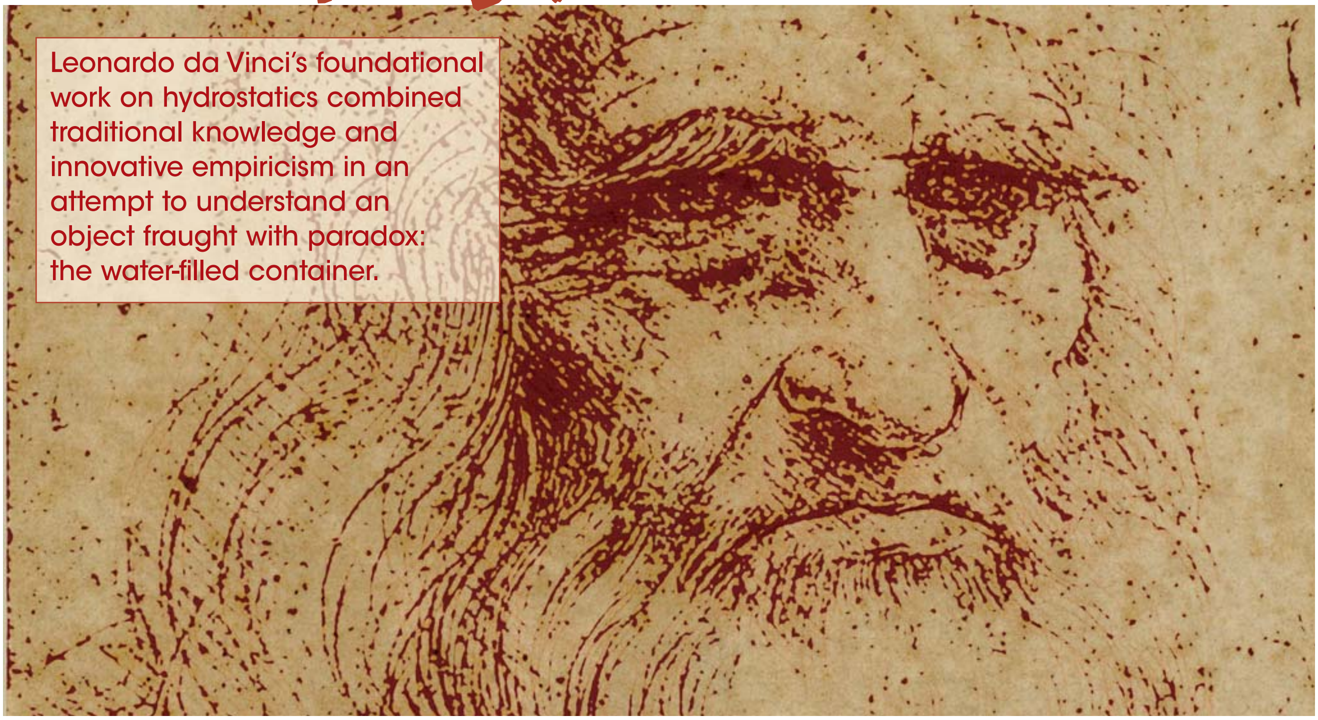
Leonardo da Vinci's foundational work on hydrostatics combined traditional knowledge and innovative empiricism in an attempt to understand an object fraught with paradox: the water-filled container.



Self-portrait by Leonardo da Vinci. (Copy from *The Drawings of Leonardo da Vinci and His Circle in the Royal Library of Turin*, Giunti, Florence, Italy, 1990. Used with permission.)

Water was always a favorite subject of investigation for Leonardo da Vinci (1452–1519). It fascinated him so much that he imagined a "Book of Water" to appear first among all the treatises he wanted to compile and publish to describe his research. The Book of Water never came to life, but da Vinci did leave us numerous manuscripts that exist as notebooks and bound codices, many of which are located in European libraries and private collections; together they contain hundreds of notes and drawings on water that give a sense of the extent and novelty of Leonardo's research into hydraulics.

In Leonardo's day, natural philosophers generally justified their speculations with theoretical and philosophical models. No doubt Leonardo was influenced by the classical physical theories reworked during the Middle Ages.[1] But his ventures into the practically unexplored field of fluid mechanics employed a methodology characterized by a continual confrontation between received wisdom and insights coming from observations of natural phenomena and experimental activities. That da Vinci method, applied to hydrostatics, yielded original and remarkable results.

In the course of his investigations, Leonardo faced—probably for the first time in the modern era[2]—the problem of the weight of water: What exactly does the "weight" of water specify, and how, if at all, is that weight affected by water's proximity to other elements? Leonardo tried to answer the question, offering his own interpretation of some properties and phenomena nowadays described in terms of hydrostatic force and pressure, in an intellectual journey that mixed inspired intuitions, changes of mind, and contradictions.

Archimedes in an Aristotelian world

Leonardo inherited a vision of the physical world whose basis was set by Aristotle in the treatise *On the Heavens*, which dates from the fourth century BC. Like Aristotle, Leonardo thought that each of the four elements—earth, water, air, and fire—moves according to its innate tendency to occupy its natural place. The Aristotelian theory states that the natural place of earth and water is the bottom, so those two elements move downward and are deemed heavy. The natural place of air and fire is the top, so they move upward and are designated

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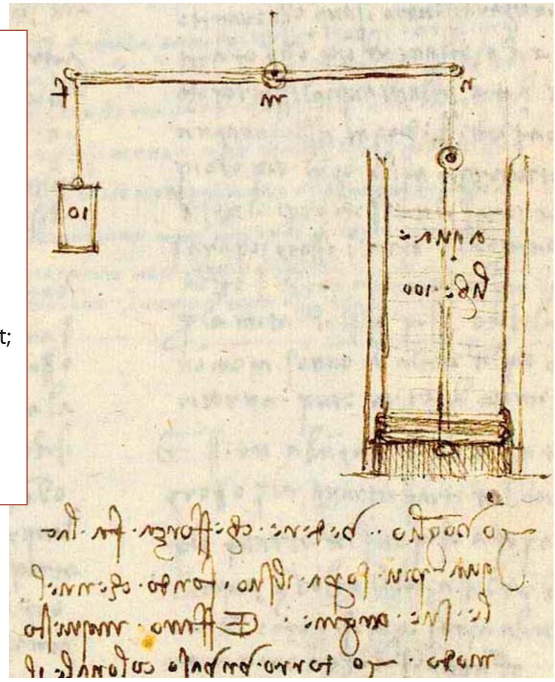

Figure 1. "I want to see how much more force water exerts on its bottom than on its banks." So Leonardo da Vinci wrote in the Codex Madrid I, folio 150 recto, around 1495. "And I will do it as follows." A mobile base is jointed to the walls of a vessel by means of a strip of leather and then is suspended from the arm of a balance. In Leonardo's words, "The weight that counter-balances water in the vessel is the weight of water that rests on the bottom. All the weight that doesn't rest on the base falls on the walls of the container." The passage suggests that Leonardo never performed the experiment; otherwise, he would probably have realized that the weight resting on the bottom is actually the entire weight of water. Note Leonardo's characteristic backwards writing in this and the other figures. (© National Library of Spain. Used with permission.)

as light. Earth is considered to be heavier than water, and fire is lighter than air.[3]

Unlike Aristotle, however, who did not suggest an unambiguous way to compare the weights of two different bodies, Leonardo thought that the notions of heavier and lighter implied a comparison via a common property. In a note on folio 33 recto of the Codex Madrid I, for example, Leonardo described a method for comparing the weight of a lead cylinder with that of an identical volume of water, which is obtained by means of a wax mold of the lead cylinder itself.

The core concept of Leonardo's note—what today we would call specific gravity—probably came to Leonardo via a Latin text that began to circulate in Europe between the 12th and the 13th centuries, the *Liber Archimedis de insidentibus in humidum* (*Archimedean Book on Floating Bodies*).[4] The opus derived from Arab translations and amalgamations of Greek source works belonging to the Archimedean tradition. Scholars still debate whether Leonardo had direct access to the text or had heard of its contents from the engineers and scholars whom he met during his stays at the main Italian courts. In any case, within the treatise, the specific gravity of a body (*gravitas secundum speciem*) is clearly defined with respect to a reference volume, while the total weight of the body (*gravitas secundum numerositatem*) refers to the object in and of itself.

Weight, weight, don't tell me

The influence of *De insidentibus in humidum* on Leonardo's thoughts, however, was not limited to the idea of specific gravity. Traces of at least two passages from the apocryphal Archimedean text appear among the da Vinci notes. The first refers to

Box 1. An Archimedean masterpiece

The first Latin translation of Archimedes's treatise *On Floating Bodies* was part of a book completed in 1269 by William of Moerbeke, a Flemish monk, at the papal court in Viterbo, Italy. Moerbeke produced his translation from two now-vanished Greek manuscripts. Those works, known as Codices A and B, contained copies of Archimedean texts; *On Floating Bodies* appeared in Codex B.

Although some copies of Moerbeke's book were made in the 14th century, it remained nearly inaccessible until 1503, when part of it was edited and printed by Italian astrologer and mathematician Luca Gaurico. The translation of *On Floating Bodies*, however, appeared in print only later, in editions by Niccolò Tartaglia (1543), Curzio Troiano Navò (1564), and Federico Commandino (1565).

Meanwhile, Italian humanist and mathematician Jacopo da Cremona produced a translation of Codex A in 1450. Thanks to German mathematician Regiomontanus, a reworked version of da Cremona's translation found its way to Germany in 1468. The Regiomontanus book was also the nucleus of the first complete printed collection of the Greek and Latin versions of all Archimedes's works, the so-called *editio princeps*.

Leonardo da Vinci tried to get hold of *On Floating Bodies*,[9] but most Renaissance historians have concluded that he did not succeed in directly accessing Archimedes's masterpiece on hydrostatics. A single note on folios 413 recto and verso of the Codex Atlanticus quotes a fragment from book 2 of *On Floating Bodies* in Moerbeke's translation, but it is not in Leonardo's handwriting. No other trace of Archimedes's mathematical and theoretical treatise appears in the extant da Vinci manuscripts. Moreover, in referring to the force that water exerts on a submerged body, Leonardo never used wording that was identical or equivalent to the Archimedean terminology. He probably encountered the Archimedean tradition only through direct or indirect access to the contents of the *Liber Archimedis de insidentibus in humidum*, a derivative text that was readily available during his lifetime.

proposition 5 from book 1 of the authentic *On Floating Bodies* by Archimedes (see box 1), which states that any floating object displaces its own weight of fluid. The second passage, in contrast with Aristotle, affirms that a body is neither heavy nor light when it is surrounded by a substance made of the same element.

The first passage was quoted by Leonardo in a few places, always with more or less the same words. For example, a note dating from the 1490s on folio 563 recto of the Codex Atlanticus says that "the weight of the ship equals exactly the weight of the water that the ship displaces from its own site." (The translations here and below are our own.) Leonardo's few notes, however, do not address the cause of buoyancy, which Archimedes had identified with a force whose intensity precisely equals the weight of the water displaced by a floating body.

In many of Leonardo's notes, the second passage, too, appears in nearly identical form. The first of those notes dates from the 1490s, when Leonardo observed, "No element weighs in its own sphere" (Codex Arundel, folio 181 recto). Later in the decade he stated that "the weight is a certain push or desire, or wish to escape, that arises when an element is shifted into another one" (Codex Madrid I, folio 145 verso).

In the first decade of the 16th century, Leonardo noted that "no part of an element has heaviness or lightness in its own element. Move a vessel of water in air: All the water weighs in air and no part of this water has heaviness in itself. Move air under the water: All the air is light and it has no lightness or heaviness in itself" (Codex Atlanticus, folio 515 verso). Similarly, he wrote on folio 255 verso of the Codex Arundel that "no element weighs in itself, but it weighs above a lighter one, and it weighs less above a heavier one." When a body is immersed in a material with a smaller specific gravity, the body does not receive a boost sufficient to prevent it from moving downward; it therefore increases its heaviness. On the other hand, when the same body is immersed in a substance with a greater specific gravity, it is pushed upward and increases its lightness. The absence of heaviness or lightness of a body immersed in itself is a concept similar to the present notion of neutral buoyancy.

In Leonardo's conception, heaviness or lightness was not an absolute property of an element, as claimed by Aristotle; instead, heaviness and lightness depended on the matter from which a body is made and the substance surrounding the body. Furthermore, Leonardo considered heaviness and lightness as coexisting in a body, "like unequal weights on a balance" (Codex Arundel, folio 264 recto), until an equilibrium is restored when the body is brought back to its own element.

Leonardo's theory of weight, in brief, incorporates pseudo-Archimedean notions to extend, or at least reinterpret, the Aristotelian concepts of heaviness and lightness. It has much in common with the present understanding of what happens to a body when it is immersed in a substance with a different specific weight, but Leonardo did not provide an explanation of buoyancy similar or equivalent to Archimedes's principle.

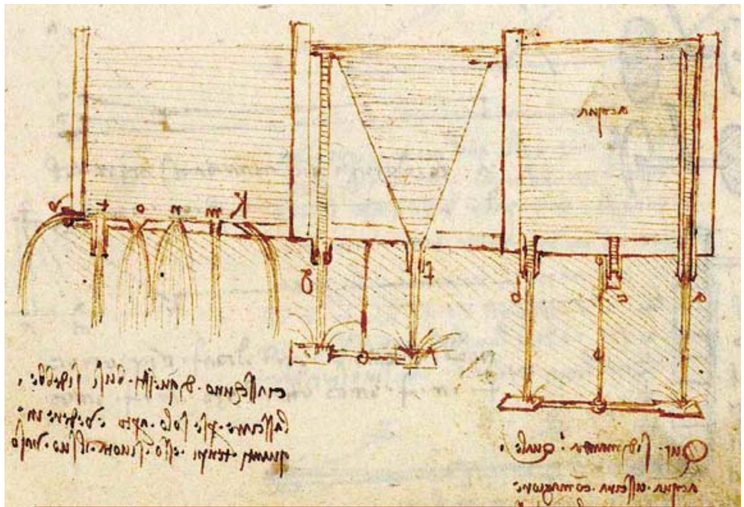

Figure 2. "What water will pour with the greatest violence, that from the nozzle a, b, or c?" Leonardo da Vinci's question and the drawing reproduced here come from the Codex Madrid I, folio 152 recto. The panel on the right represents an experiment designed to test whether water's weight on the bottom of a container varies from point to point. The "violence" of water pouring from different nozzles is compared by means of a suspended scale whose plates are struck by the jets. Since the plates are represented in balance, Leonardo seemed to have realized that water's weight on the bottom doesn't change from point to point. With the experiment depicted in the central panel, Leonardo tested whether the action that water exerts on a point at its bottom depends on the volume of the container. The picture demonstrates a good understanding of the hydrostatic paradox: The action depends only on the vertical distance from the bottom to the surface of the liquid, not on the volume of the liquid itself. In many cases, Leonardo probably did not perform the experiments he drew. But the renditions here are so detailed and accurate that they suggest actual investigations. (© National Library of Spain. Used with permission.)

Does water weigh on its bottom?

Leonardo's theory ran into difficulties when he considered the weight of water inside a container. Assuming that water has weight only when it is moved in or rests above an element with a smaller specific gravity—and, in particular, that it does not have weight when surrounded by more water—Leonardo initially concluded that water does not weigh on the bottom of its container, because the container is made of a heavier element (Codex Madrid I, folio 145 verso). Some natural phenomena apparently confirm that hypothesis. In many notes Leonardo recalled that mud does not compact on the bottom of a pond, as it would do if the weight of water were to tamp it down, or that aquatic plants wave freely as if they were not burdened by any weight. (See, for example, the Codex Arundel, folio 266 verso.)

The way some instruments work, however, evidently contradicts those observations. As an example, Leonardo observed that in some kinds of reciprocating pumps, a valve is located at the bottom of the pump cylinder, and it opens only when the external force acting on the pump prevails over the

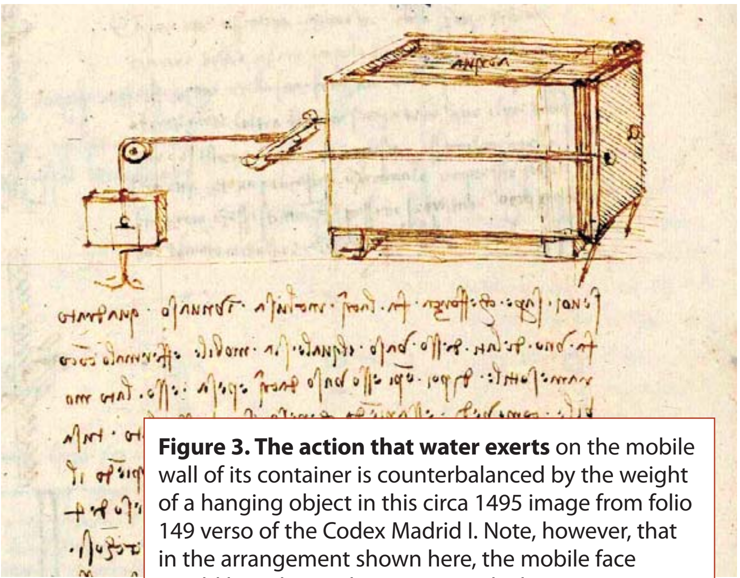

Figure 3. The action that water exerts on the mobile wall of its container is counterbalanced by the weight of a hanging object in this circa 1495 image from folio 149 verso of the Codex Madrid I. Note, however, that in the arrangement shown here, the mobile face would be subjected to a counterclockwise torque. On folio 127 verso of the codex, Leonardo da Vinci had noted that "the center of gravity of each pyramid is at one third of its length from its base," but apparently he did not yet appreciate that the action of water increases linearly with depth. (© National Library of Spain. Used with permission.)

weight of water that rests on the valve (Manuscript H, folio 73 verso). Moreover, experience suggests that water also exerts a push on the walls of its container, as demonstrated by the jets that spring from holes in vessel walls; Leonardo depicted that phenomenon on folio 151 recto of the Codex Madrid I. The notes from the first half of the 1490s reveal that the problem of water's weight was one that Leonardo found to be difficult and frustrating.

In about 1495 a compromise solution emerged in a note on folio 74 recto of Manuscript H. There Leonardo affirmed that the weight of water resting on the bottom of its container is reduced by the weight of water that rests on the walls. With that idea in mind, he came up with experiments for determining how much of water's weight rests on the container bottom—he called them "ways of weighing water" (folio 146 recto of the Codex Madrid I). The most significant of them, which is depicted on folio 150 recto of the Codex Madrid I, is a balance attached to the movable bottom of a container; figure 1 is a reproduction of Leonardo's sketch of the experiment. A similar experiment is depicted by a drawing from around the same time that appears on folio 1023 verso of the Codex Atlanticus. For that drawing, however, the related note is not about the weight of water on the bottom of the container but rather about the weight that rests on a hole in the bottom. Leonardo wrote, "If you want to know whether or not the water that perpendicularly stays over the hole in the bottom of the vessel weighs on the hole itself, that is to say if water partly pushes on the walls of the vessel, [perform the] experiment as illustrated above."

Historians are still debating whether Leonardo ever quantified the weight of water that rests on the bottom of a vessel or eventually abandoned the idea of a subdivision of weight between the bottom and the walls of a container. In a note on folio 219 recto of the Codex Atlanticus dating from 1510, Leonardo raised the divided-weight hypothesis one more time; he probably never came to a definitive conclusion about its validity.

Notwithstanding his indecision vis-à-vis the role of a container's walls, Leonardo stated in many notes that the magnitude of the action that water exerts on a point at the bottom of a container depends only on the vertical distance between the point and the free surface of water. Leonardo probably came to that conclusion, which shows an understanding of what today is called the hydrostatic paradox, by means of actual experimentation.[5] After all, the experiment depicted in figure 2 (from folio 152 recto of the Codex Madrid I) is rendered with such great accuracy that it almost appears to be a photograph of an experiment in progress. With time, investigations of that sort give Leonardo a great mastery of

Box 2. The power of water

Pressure, as we now conceive it, is absent from Leonardo da Vinci's notes. But it's undeniable that Leonardo often described water properties in terms of a concept similar to the contemporary idea of pressure. The word he most often used in referring to that concept is "power," and in many cases it is related to a consideration of how the action exerted by water on a surface is distributed. Basically by observing fluids pouring from multiple nozzles on the walls of water and wine tanks, Leonardo gradually became aware that water "power" increases in proportion to depth. (See, for example, Manuscript C, folio 7 recto, and the Manuscript Forster II, folio 117 verso.)

Leonardo also realized that heavy objects put on top of flexible containers such as bags or bellows further intensify the jets of water that flow from the vessel openings. In many notes, he considered how an external weight affects the power of water within a container. On folio 148 verso of the Codex Madrid I, Leonardo said the weight that rests on water "pushes proportionally" in all the parts of the container, just as water does by itself. That assertion is wrong, but the issue is not a simple one. Probably by means of continuing experimentation, Leonardo gradually became aware that "every part of the bag is equally affected by the weight that rests on it," as he wrote on folio 169 recto of the Codex Madrid I in about 1495. Some years later, circa 1508, he was fully aware that the weight of an object put on top of a container transmits a "uniform power" to the water within the container and that the uniform power adds to the "unequal power that lives with water by its nature." That understanding is substantially equivalent to Pascal's law, which states that external pressure is transmitted equally throughout a fluid and that earlier pressure variations thus remain unchanged.

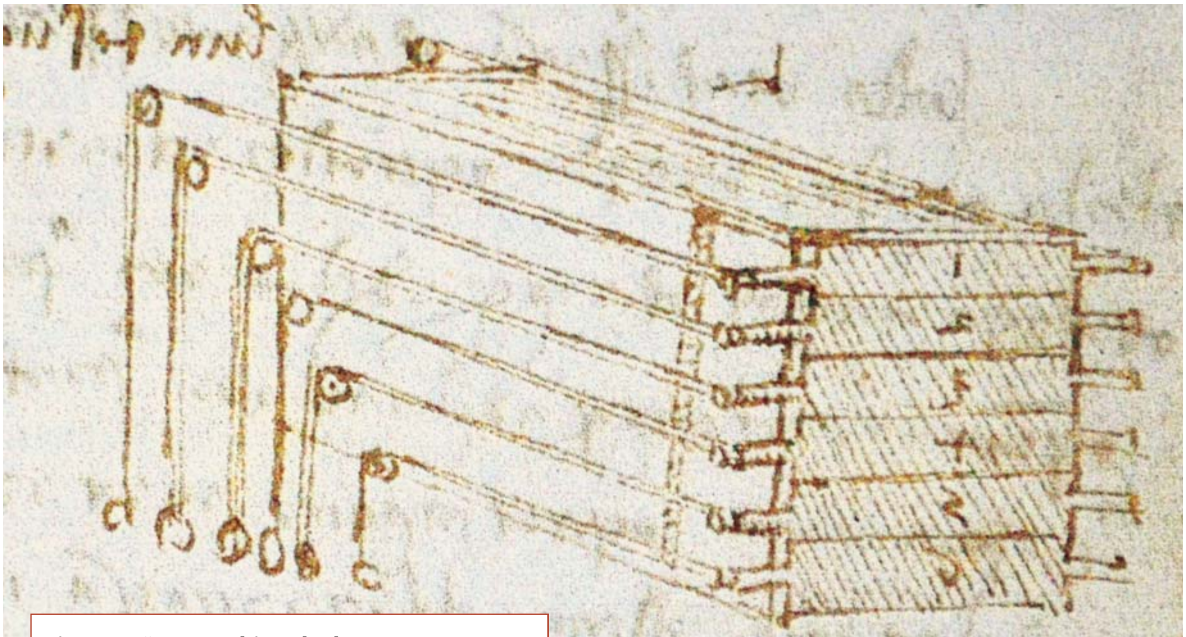

Figure 4. "Approaching the bottom, water presses its banks with increasing force. You can measure the inequality of its power with a container." The container mentioned by Leonardo da Vinci is a tank with a wall replaced by a flexible parchment externally supported by rigid horizontal bands. Each band is kept "in contact with the front edge of the container" by a pair of counterbalancing weights of increasing magnitude, whose sum equals the total "power" acting on the wall. This circa 1508 conception, from the Codex Leicester, folio 6 recto, was most likely not realized in practice. But with it, Leonardo came close to the present understanding of the hydrostatic pressure distribution. (Copy from *The Codex Hammer*, Giunti, Florence, Italy, 1987. Used with permission. When Giunti published the codex, it was owned by Armand Hammer. Bill Gates bought the codex in 1994 and restored the original name, Codex Leicester.)

the hydrostatic paradox. For example, in a note on folio 26 recto of the Codex Leicester dating from 1508, Leonardo considered two pipes with the same diameter: The lengths and inclinations of the pipes may be completely different, but as long as their tops and bottoms are delimited by two parallel planes, they "generate the same weight of water" on their bottoms when filled.

A mechanical discretization

In Leonardo's thinking, the "action" that water exerts on the bottom of a container is related to the action that water exerts on the walls of the same container. With that understanding, he imagined an indirect way to determine how much of water's weight rests on the bottom. Leonardo used the idea of a mobile surface, as illustrated in figure 1, but that time he adapted it to a wall of the container.

Leonardo's interesting and novel experiment is depicted in a drawing (figure 3) dating from around 1495 on folio 149 verso of the Codex Madrid I. The mobile wall of the water-filled tank is kept in place by the weight of an object that hangs from a pulley; the weight of the object measures the total push that the water exerts on the wall. Leonardo stated that by means of "well-done experiments," water's weight on the bottom can be determined by subtracting the four identical weights on the walls from the weight of the whole volume of water.

There is no evidence that Leonardo actually performed the experiment, and his instruction to subtract the four weights shows that in dealing with water, he did not understand what we now call the vectorial nature of forces. Moreover, Leonardo incorrectly positioned the balancing force that acts on the mobile wall. Apparently, at the end of the 15th century, he was not fully aware that the intensity of the action exerted by water on a vertical surface increases according to water depth. To balance the action of the water without torquing the movable wall, the balancing force would need to be applied at one-third of the height of the wall from the bottom, not at the wall's center.

By around 1508, however, Leonardo was aware of the height dependence of water's thrust on a wall. Folio 6 recto of the Codex Leicester deals with an actual hydraulic engineering problem: how to build banks and levees that are strong enough to prevent collapse. In his notes there, Leonardo identified the "level" of water as the only parameter that affects the intensity of water's action. Water indeed "pushes in the height of the bank, from the surface to the bottom, with varying power," and that "variation" is actually "caused by the difference, or inequality, of water height" from point to point. Leonardo correctly deduced that "water gains degrees of power in each degree of its depth," and as a consequence, the bank must also "increase its resistance from its top to its bottom," so that the "resistance" of the bank equals the "power" of water. For more of Leonardo on the power of water, see box 2.

Leonardo did not develop any calculational technique to theoretically determine how the power exerted by water varies, but on the same folio he sketched one of his most original techniques (figure 4) for measuring the "inequality of this power." With such a setup, he could not only measure the magnitude of the whole action that water exerts on the wall, as he proposed to do with the experiment depicted in figure 3, he could also evaluate the magnitude and distribution of the actions that water exerts on all the bands making up the wall. No evidence exists to suggest that Leonardo actually performed the experiment, but in conceptualizing a mechanical discretization, he was probably the first in the modern era to come close to envisioning the present notion of the hydrostatic pressure distribution.[6]

An evolving view

The extant manuscripts of Leonardo da Vinci contain a remarkable and detailed record of a 15th-century scholar's approach to some basic concepts in hydrostatics. Leonardo's intellectual journey through the field, previously unexplored in the modern era, may appear hesitant and affected by sudden transitions between correct and incorrect notions. But a chronological examination of his works reveals an evolving understanding.[7,8] That evolution is the result of Leonardo's very personal research method—one that balanced received wisdom against intuitions, observations, and experiments—that allowed him to move from the technical and theoretical knowledge of his time and to perform his own genuine investigation of the physical nature of water. And it is probably the inventiveness of his method, even more than the remarkable results that he obtained with it, that makes Leonardo the outstanding Renaissance man who, half a millennium later, still fascinates us.

We are deeply grateful to the late Enzo Macagno (1914–2012), whose advice and support were of fundamental importance as we began this article.

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