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Nonlinear nuclear equation of state and thermodynamical instabilities in warm and dense nuclear matter

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Abstract.
We study a nonlinear nuclear equation of state in the framework of a relativistic mean field theory. We investigate the possible thermodynamic instability in a warm and dense asymmetric nuclear medium where a phase transition from nucleonic matter to resonance dominated Δ matter can take place. Such a phase transition is characterized by both mechanical instability (fluctuations on the baryon density) and by chemical-diffusive instability (fluctuations on the isospin concentration) in asymmetric nuclear matter. Similarly to the liquid-gas phase transition, the nucleonic and the Δ-matter phase have a different isospin density in the mixed phase. In the liquid-gas phase transition, the process of producing a larger neutron excess in the gas phase is referred to as isospin fractionation. A similar effects can occur in the nucleon-Δ matter phase transition due essentially to a negative Δ-particles excess in asymmetric nuclear matter. In this context, we investigate also the effects of power law effects, due to the possible presence of nonextensive statistical mechanics effects.

1. Introduction

Because nuclei are made of neutrons and protons, the nuclear liquid-gas phase transition is in a binary system where one has to deal with two independent proton and neutron chemical potentials for baryon number and electric charge conservation. Taking into account of this important property, a very detailed study of Müller and Serot [1] focused on the main thermodynamic properties of asymmetric nuclear matter in the framework of a relativistic mean field model.

A relevant aspect of a system with two conserved charges (baryon and isospin numbers) is that the phase transition is of second order from the viewpoint of Ehrenfests definition. At variance with the so-called Maxwell construction for one conserved charge, the pressure is not constant in the mixed phase and therefore the incompressibility does not vanish [1, 2]. Such feature plays a crucial role in the structure and in the possible hadron-quark phase transition in compact star objects [3]. Moreover, for a binary system with two phases, the binodal coexistence surface is two dimensional and the instabilities in the mixed liquid-gas phase arise from fluctuations in the proton concentration (chemical instability) and in the baryon density (mechanical instability).

In this article, we study a nonlinear hadronic EOS at finite temperature and density by means of a relativistic mean-field model with the inclusion Δ-isobars and by requiring the

Gibbs conditions on the global conservation of baryon number and net electric charge. In this context, let us observe that, for the range of temperatures and baryon densities considered in this investigation ($T \leq 50$ and $\rho_B \leq 3\rho_0$), the contribution of strange hadron particles can be neglected in a good approximation due to their very low concentration.

The main goal of this paper is twofold. First, we are going to show that, for an asymmetric warm and dense nuclear medium, the possible Δ-matter phase transition is characterized by mechanical and chemical-diffusive instabilities. Similarly to the liquid-gas phase transition, chemical instabilities play a crucial role in the characterization of the phase transition and can imply a very different electric charge fraction Z/A in the coexisting phases during the phase transition. Second, we study the influence of nonextensive statistical effects on the thermodynamical instabilities in warm and asymmetric nuclear matter and we investigate how the phase diagram of the nuclear liquid-gas phase transition can be modified in the framework of nonextensive statistical mechanics [4].

2. Nonlinear hadronic equation of state

The relativistic mean-field model (RMF) is widely successful used for describing the properties of finite nuclei as well as hot and dense nuclear matter [3, 5, 6].

In the RMF model the Lagrangian density for nucleons can be written as

$$\begin{aligned}\mathcal{L}_{\rm N} &= \overline{\psi}_N \left[i\,\gamma_\mu\,\partial^\mu - (M_N - g_{\sigma N}\,\sigma) - g_{\omega N}\,\gamma_\mu\,\omega^\mu - g_{\rho N}\,\gamma_\mu\,\vec{t}\cdot\vec{\rho}^{\,\mu}\right]\psi_N + \frac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2)\\ &\quad -U(\sigma) + \frac{1}{2}\,m_\omega^2\,\omega_\mu\omega^\mu + \frac{1}{4}\,c\,(g_{\omega N}^2\,\omega_\mu\omega^\mu)^2 + \frac{1}{2}\,m_\rho^2\,\vec{\rho}_\mu\cdot\vec{\rho}^{\,\mu} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}\vec{G}_{\mu\nu}\vec{G}^{\mu\nu}\,,\end{aligned} \tag{1}$$

where $M_N = 939$ MeV is the nucleon vacuum mass and $\vec{t}$ is the isospin operator which acts on the nucleon. The field strength tensors for the vector mesons are given by the usual expressions $F_{\mu\nu} \equiv \partial_\mu\omega_\nu - \partial_\nu\omega_\mu$, $\vec{G}_{\mu\nu} \equiv \partial_\mu\vec{\rho}_\nu - \partial_\nu\vec{\rho}_\mu$, and $U(\sigma)$ is the nonlinear potential of σ meson

$$U(\sigma) = \frac{1}{3}a(g_{\sigma N}\sigma)^3 + \frac{1}{4}b(g_{\sigma N}\sigma)^4\,, \tag{2}$$

usually introduced to achieve a reasonable compression modulus for equilibrium normal nuclear matter. In the following, the meson-nucleon coupling constants and the other parameters (a, b, c) of the EOS will be fixed to the parameters set marked as TM1 of Ref. [5].

In a regime of finite values of temperature and density, a state of high-density resonance matter may be formed and the $\Delta(1232)$-isobar degrees of freedom are expected to play a central role. In particular, the formation of resonances matter contributes essentially to baryon stopping, hadronic flow effects and enhanced strangeness [7].

The Lagrangian density concerning the Δ-isobars can be then expressed as [8]

$$\mathcal{L}_\Delta = \overline{\psi}_{\Delta\,\nu}\left[i\gamma_\mu\partial^\mu - (M_\Delta - g_{\sigma\Delta}\sigma) - g_{\omega\Delta}\gamma_\mu\omega^\mu\right]\psi_\Delta^\nu\,, \tag{3}$$

where ψ_Δ^ν is the Rarita-Schwinger spinor for the Δ-isobars (Δ^{++}, Δ^+, Δ^0, Δ^-). Due to the uncertainty on the meson-Δ coupling constants, we limit ourselves to consider only the coupling with the σ and ω meson fields, more of which are explored in the literature.

Because we are going to describe a finite temperature and density asymmetric nuclear matter, we have to require the conservation of two "charges": baryon number (B) and electric charge (C) (as already remarked, we neglect the contribution of strange hadrons, because a tiny amount of strangeness can be produced in the range of temperature and density explored in this study). As a consequence, the system is described by two independent chemical potentials: μ_B and

μ_C, the baryon and the electric charge chemical potential, respectively. Therefore, the chemical potential of particle of index i can be written as

$$\mu_i = b_i\,\mu_B + c_i\,\mu_C\,, \tag{4}$$

where b_i and c_i are, respectively, the baryon and the electric charge quantum numbers of the ith hadron.

The thermodynamical quantities can be obtained from the baryon grand potential Ω_B in the standard way. More explicitly, the baryon pressure $P_B = -\Omega_B/V$ and the energy density can be written in the following nonlinear equations

$$\begin{aligned} P_B &= \frac{1}{3}\sum_i \gamma_i \int \frac{\mathrm{d}^3k}{(2\pi)^3}\,\frac{k^2}{E_i^*(k)}\,[n_i(k)+\overline{n}_i(k)] - \frac{1}{2}\,m_\sigma^2\,\sigma^2 - U(\sigma) && (5)\\ &+ \frac{1}{2}\,m_\omega^2\,\omega^2 + \frac{1}{4}\,c\,(g_{\omega N}\,\omega)^4 + \frac{1}{2}\,m_\rho^2\,\rho^2, \\ \epsilon_B &= \sum_i \gamma_i \int \frac{\mathrm{d}^3k}{(2\pi)^3}\,E_i^*(k)\,[n_i(k)+\overline{n}_i(k)] + \frac{1}{2}\,m_\sigma^2\,\sigma^2 + U(\sigma) && (6)\\ &+ \frac{1}{2}\,m_\omega^2\,\omega^2 + \frac{3}{4}\,c\,(g_{\omega N}\,\omega)^4 + \frac{1}{2}\,m_\rho^2\,\rho^2\,. \end{aligned}$$

Let us observe that in the presence of nonlinear statistical mechanics effects, the nuclear equation of state will be modified as discussed in Ref. [9].

3. Phase transition and stability conditions

We are dealing with the study of a multi-component system at finite temperature and density with two conserved charges: baryon number and electric charge. For such a system, the Helmholtz free energy density F can be written as

$$F(T,\rho_B,\rho_C) = -P(T,\mu_B,\mu_C) + \mu_B\rho_B + \mu_C\rho_C\,, \tag{7}$$

with

$$\mu_B = \left(\frac{\partial F}{\partial \rho_B}\right)_{T,\rho_C}\,, \quad \mu_C = \left(\frac{\partial F}{\partial \rho_C}\right)_{T,\rho_B}\,. \tag{8}$$

In a system with N different particles, the particle chemical potentials are expressed as the linear combination of the two independent chemical potentials μ_B and μ_C and, as a consequence, $\sum_{i=1}^N \mu_i\rho_i = \mu_B\rho_B + \mu_C\rho_C$.

Assuming the presence of two phases (denoted as I and II, respectively), the system is stable against the separation in two phases if the free energy of a single phase is lower than the free energy in all two phases configuration. The phase coexistence is given by the Gibbs conditions

$$\mu_B^I = \mu_B^{II}\,, \qquad \mu_C^I = \mu_C^{II}\,, \tag{9}$$
$$P^I(T,\mu_B,\mu_C) = P^{II}(T,\mu_B,\mu_C)\,. \tag{10}$$

Therefore, at a given baryon density ρ_B and at a given net electric charge density $\rho_C = y\,\rho_B$ (with $y = Z/A$), the chemical potentials μ_B are μ_C are univocally determined. An important feature of this conditions is that, unlike the case of a single conserved charge, the pressure in the mixed phase is not constant and, although the total ρ_B and ρ_C are fixed, baryon and charge densities can be different in the two phases. For such a system in thermal equilibrium, the possible phase transition can be characterized by mechanical (fluctuations in the baryon density) and chemical instabilities (fluctuations in the electric charge density). As usual the condition of the mechanical stability implies

$$\rho_B \left(\frac{\partial P}{\partial \rho_B}\right)_{T,\,\rho_C} > 0\,. \tag{11}$$

By introducing the notation $\mu_{i,j} = (\partial\mu_i/\partial\rho_j)_{T,P}$ (with $i, j = B, C$), the chemical stability for a process at constant P and T can be expressed with the following conditions [10]

$$\rho_B\, \mu_{B,B} + \rho_C\, \mu_{C,B} = 0\,, \tag{12}$$

$$\rho_B\, \mu_{B,C} + \rho_C\, \mu_{C,C} = 0\,. \tag{13}$$

Whenever the above stability conditions are not respected, the system becomes unstable and the phase transition take place. The coexistence line of a system with one conserved charge becomes in this case a two dimensional surface in (T, P, y) space, enclosing the region where mechanical and diffusive instabilities occur.

By increasing the temperature and the baryon density during the high energy heavy ion collisions ($T \approx 50$ MeV and $\rho_B \geq \rho_0$), a multi-particle system with Δ-isobar and pion degrees of freedom may take place.

In analogy with the liquid-gas case, we are going to investigate the existence of a possible phase transition in the nuclear medium by studying the presence of instabilities (mechanical and/or chemical) in the system. The chemical stability condition is satisfied if [10]

$$\left(\frac{\partial \mu_C}{\partial y}\right)_{T,P} > 0 \quad \text{or} \quad \begin{cases} \left(\dfrac{\partial \mu_B}{\partial y}\right)_{T,P} < 0\,, & \text{if } y > 0\,, \\[2ex] \left(\dfrac{\partial \mu_B}{\partial y}\right)_{T,P} > 0\,, & \text{if } y < 0\,. \end{cases} \tag{14}$$

4. Results and discussion

4.1. Δ-matter phase transition

In Fig. 1, we report the pressure as a function of the baryon density at $T = 30$ MeV (left panel) and $T = 50$ MeV (right panel), for different values of $y = Z/A$ and $x_{\sigma\Delta} = 1.3$. In this context it is interesting to observe that at $T = 50$ MeV and below $y = 0.3$, the system becomes mechanically stable, but, in a similar manner to the liquid-gas case, is chemically unstable.

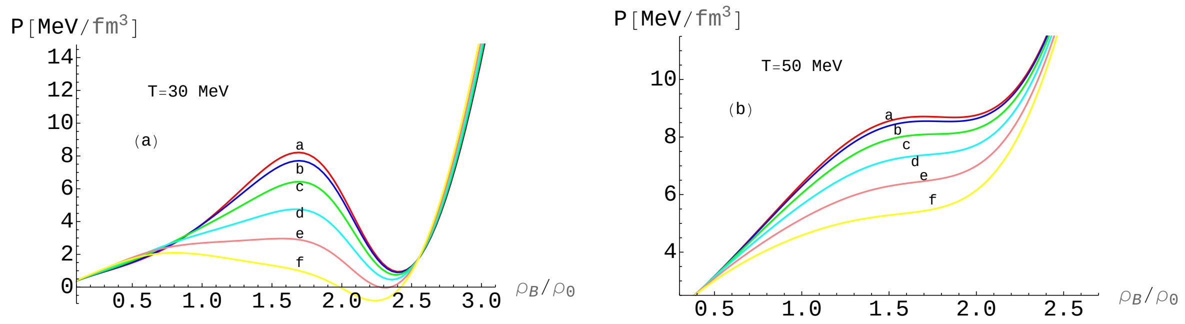

Figure 1. Pressure as a function of baryon density for $T = 30$ MeV (left panel) and $T = 50$ MeV (right panel) with $x_{\sigma\Delta} = 1.3$. Labels a to f correspond to $y = 0.5, 0.4, 0.3, 0.2, 0.1, 0$, respectively.

From the analysis of the chemical potential isobars, we are able to construct the binodal surface relative to the nucleon-Δ matter phase transition.

At lower temperatures, the mixed-phase region becomes more relevant at higher values of y. This feature can be seen in Fig. 2 where the binodal section (left panel) and the isothermal pressure as a function of the baryon density (right panel) is reported at $T = 40$ MeV and $x_{\sigma\Delta} = 1.3$. The Gibbs construction corresponds to the curve from A to C; the isothermal curves in B and D (with $y_B \neq y_D$) are also reported. In this case, we assume that the system is initially prepared in the low-density (nucleonic) phase with $y = 0.3$, corresponding to point A. During the compression each phase evolves following the corresponding curve up to points C and D, where the system leaves the instability region in the Δ-matter phase.

The right branch (at lower density) corresponds to the initial phase (I), where the dominant component of the system is given by nucleons. The left branch (II) is related to the final phase at higher densities, where the system is composed primarily by Δ-isobar degrees of freedom (Δ-dominant phase). In presence of Δ-isobars the phase coexistence region results very different from what obtained in the liquid-gas case, in particular it extends up to regions of negative electric charge fraction.

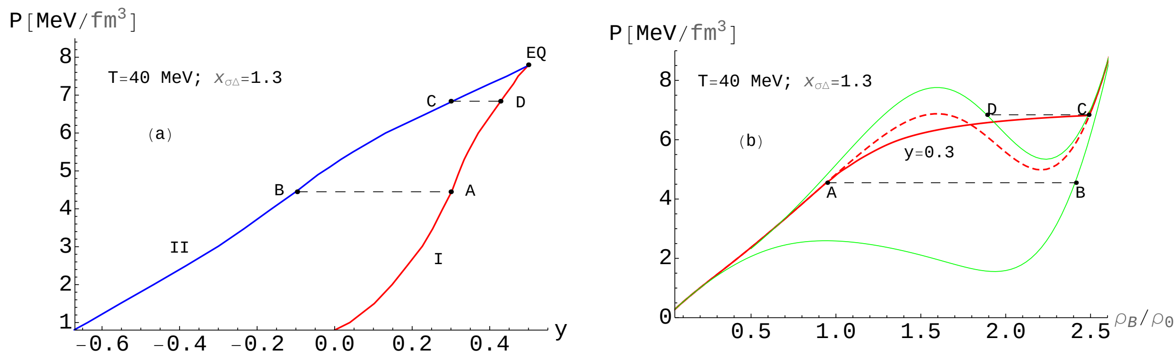

Figure 2. Left panel: binodal section at T= 40 MeV and $x_{\sigma\Delta} = 1.3$, with the point of equal equilibrium in evidence. Right panel: the corresponding isothermal curves, with the Gibbs construction (curve from the point A to C) at $y = 0.3$ and the isotherms of points B and D shown.

Therefore, similarly to the liquid-gas phase transition, the nucleonic and the Δ-matter phase have a different electric charge fraction in the mixed phase. The electric charge fraction in the nucleonic phase reflects a system with higher values of y than the Δ-matter phase. In the liquid-gas phase transition, the process of producing a larger neutron excess in the gas phase is referred to as isospin fractionation. A similar effects can occur in the nucleon-Δ matter phase transition due essentially to a Δ^- excess in the Δ-matter phase with lower values of y.

Many effects discussed in this paper are more evident at low values of y, obtainable, in principle, with radioactive ion beam facilities. On the other hand, it is rather unlikely, at least in the near future, that neutron rich nuclei can be accelerated to energies larger than a few GeV per nucleon. However, some precursor signals of the considered instabilities could be observed even in collisions of stable nuclei at intermediate energies. For example, in Ref. [11], the simulation of the reaction ^{238}U+^{238}U (average $y = 0.39$), at 1 A GeV and semicentral impact parameter $b = 7$ fm, shows that rather exotic nuclear matter can be formed in a transient time of the order of 10 fm/c, with a baryon density up to 3 ρ_0, $T \leq 50 \div 60$ MeV and $y \approx 0.35 \div 0.40$. Such conditions would agree fully with the results for the nucleon-Δ mixed phase region.

4.2. Nonextensive nuclear liquid-gas phase transition

Recently, there has been increasing evidence that the nonlinear and nonextensive statistical mechanics, proposed by Tsallis, can be considered as an appropriate basis to deal with physical phenomena where strong dynamical correlations, long-range interactions and microscopic memory effects take place [4, 12]. A considerable variety of physical applications involve a quantitative agreement between experimental data and theoretical models based on Tsallis thermostatistics. In particular, in the last years there has been a growing interest in high energy physics applications of nonextensive statistics and several authors have outlined the possibility that experimental observations in relativistic heavy ion collisions can reflect nonextensive statistical behaviors [13, 14, 15, 16, 17].

Nonextensive statistical mechanics introduced by Tsallis consists of a generalization of the common Boltzmann-Gibbs statistical mechanics and it is based upon the introduction of entropy [4]

$$S_q[f] = \frac{1}{q-1}\left(1 - \int [f(\mathbf{x})]^q \, d\Omega\right) , \quad \left(\int f(\mathbf{x})\, d\Omega = 1\right) , \tag{15}$$

where $f(\mathbf{x})$ stands for a normalized probability distribution, $\mathbf{x}$ and $d\Omega$ denoting, respectively, a generic point and the volume element in the corresponding phase space (here and in the following we set the Boltzmann and the Planck constant equal to unity). The real parameter q determines the degree of non-additivity exhibited by the entropy form (15) which reduces to the standard Boltzmann-Gibbs entropy in the limit $q \to 1$. By means of maximizing the entropy S_q, under appropriate constraints, it is possible to obtain a probability distribution (or particle distribution) which generalized, in the classical limit, the Maxwell-Boltzmann distribution. The nonextensive classical distribution can be seen as a superposition of the Boltzmann one with different temperatures which has a mean value corresponding to the temperature appearing in the Tsallis distribution [13].

We are going to study the effects of nonextensive statistical effects on the presence of nuclear instabilities at low temperature and subnuclear densities, in the framework of the previous introduced nuclear EOS. In the following we will focus our investigation for small deviations from the standard Boltzmann-Gibbs statistical mechanics and for values $q > 1$, because these values were obtained in several phenomenological studies in high energy heavy ion collisions (see, for example, Ref.s [18, 19, 20, 21, 22]).

In Fig. 3, we report the pressure as a function of baryon density ρ_B (in units of the nuclear saturation density ρ_0) for various values of the electric charge fraction y at fixed temperature $T = 10$ MeV. In the left panel, a comparison of the equation of state in the presence of ($q = 1.05$, dashed lines) and in the absence ($q = 1$, continuous lines) of the nonextensive statistical effects is shown. In the right panel, for $q = 1.05$ only, the continuous lines correspond to the solution obtained with the Gibbs construction in the mixed phase, whereas the dashed lines are without correction.

Let us observe that in presence of small deviation from the standard Boltzmann-Gibbs statistics, the nuclear EOS appears stiffer, with higher values of pressure at fixed baryon density. As we will see, this implies a greater value of the first transition density and a reduction of second transition density in the mixed phase with respect to the standard case ($q = 1$). This feature results in significant changes in the nuclear incompressibility and may be particularly important in identifying the presence of nonextensive effects in heavy-ion collision experiments. Moreover, it appears evident that for a proton fraction $y > 0.2$ a mechanical instability is present, whereas for $y < 0.2$ the system becomes unstable only under chemical-diffusive instability.

Finally, in Fig. 4, we report the phase diagram with evidence of the coexistence regions of the liquid-gas phase transition for $y = 0.3$ and 0.5, in the presence ($q = 1.05$) and in the absence ($q = 1$) of nonextensive statistical effects. As expected, the relevance of nonextensive

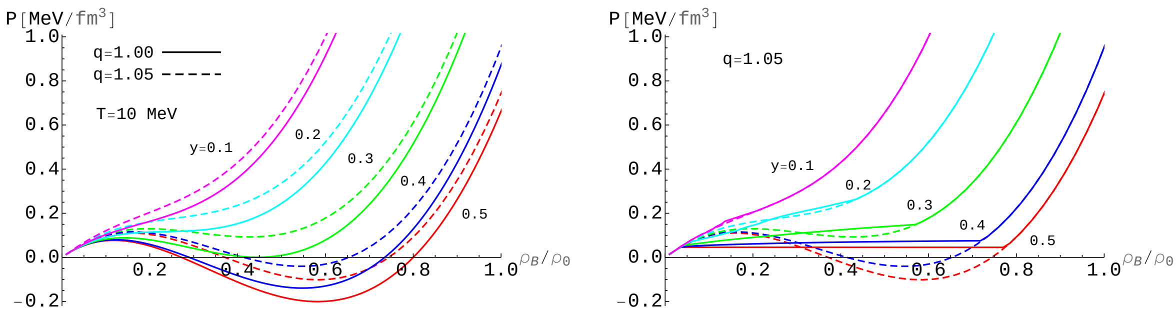

Figure 3. Pressure as a function of baryon density (in units of the nuclear saturation density ρ_0) for various values of the proton fraction y and at $T = 10$ MeV. In the left panel, the dashed lines are related to the entropic value $q = 1.05$ in comparison to the standard case ($q = 1$), corresponding to the continuous line. In the right panel, the continuous (dashed) lines correspond to the solution obtained with (without) the Gibbs construction for $q = 1.05$.

statistical mechanics increases by increasing the temperature and implies a reduction of the critical temperature T_c and a reduction of the baryon density range involved in the mixed phase, with an enhancement of the first critical density and a reduction of the second critical density, with respect to the standard (extensive) case.

Therefore, several features of the nuclear liquid-gas phase transition turn out to be sensibly modified also for small deviations from the standard Boltzmann-Gibbs statistics. In asymmetric nuclear matter, instabilities that produce a liquid-gas phase separation arise from fluctuations in the proton fraction (chemical instability), rather than from fluctuations in the baryon density (mechanical instability). In the presence of nonlinear and nonextensive statistical effects, the relevance of the chemical instability becomes more pronounced; it appears an effective reduction of the dynamical instability region and the so-called isospin fractionation effect turns out to be reduced, with fewer protons in the liquid phase and more proton in the gas phase, with respect to the standard case.

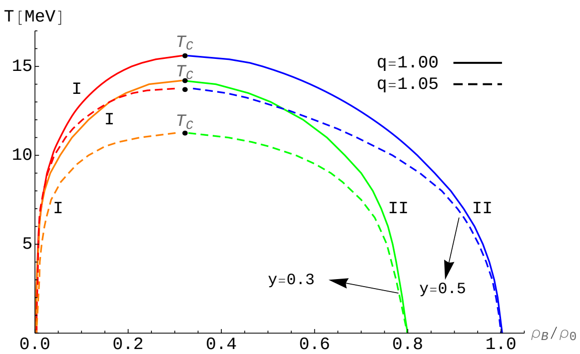

Figure 4. Phase diagram of the liquid-gas phase transition for asymmetric ($y = 0.3$) and symmetric ($y = 0.5$) nuclear matter and for different values of q. The lines labeled with I and II, delimitate the first and second critical densities of the coexistence regions, respectively.

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