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LARGE DEVIATIONS OF THE INTERFERENCE IN THE GINIBRE NETWORK MODEL

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Under different assumptions on the distribution of the fading random variables, we derive large deviation estimates for the tail of the interference in a wireless network model whose nodes are placed, over a bounded region of the plane, according to the β-Ginibre process, $0 < \beta \leq 1$. The family of β-Ginibre processes is formed by determinantal point processes, with different degree of repulsiveness. As $\beta \to 0$, β-Ginibre processes converge in law to a homogeneous Poisson process. In this sense the Poisson network model may be considered as the limiting uncorrelated case of the β-Ginibre network model. Our results indicate the existence of two different regimes.

When the fading random variables are bounded or Weibull superexponential, large values of the interference are typically originated by the sum of several equivalent interfering contributions due to nodes in the vicinity of the receiver. In this case, the tail of the interference has, on the log-scale, the same asymptotic behavior for any value of $0 < \beta \leq 1$, but it differs from the asymptotic behavior of the tail of the interference in the Poisson network model (again on a log-scale) [14].

When the fading random variables are exponential or subexponential, instead, large values of the interference are typically originated by a single dominating interferer node and, on the log-scale, the asymptotic behavior of the tail of the interference is insensitive to the distribution of the nodes, as long as the number of nodes is guaranteed to be light-tailed.

1. Introduction. An important performance index in a wireless network is the so-called outage (or success) probability, which measures the reliability degree of communications channels established between each transmitter and its associated receiver. The outage probability is mainly determined by the mutual interference among simultaneous transmissions over the same physical channel [20, 23, 29, 34, 35]. In the last years a huge effort has been devoted to characterize the interference produced by transmitting nodes operating over the same channel [3, 4, 5, 12, 13, 14, 15, 16, 17, 18, 21, 24, 28, 30, 33]. Most of these works, however, focused on networks in which

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transmitting nodes are either distributed according to a homogeneous Poisson process or, in a few cases, located on a perfectly regular grid.

Although the Poisson assumption offers many analytical advantages, it appears rather unrealistic in many cases, since it neglects the correlations among the positions of different transmitters, possibly resulting from the application of smart scheduling policies or intelligent network planning techniques. The assumption that transmitting nodes are located on a perfectly regular grid is unrealistic too, since it does not capture the effects of environmental constraints that prevent network planners from placing wireless access points regularly spaced.

In many practical situations, the set of nodes that transmit simultaneously over the same channel may be thought as a point process of repulsive nature, i.e. a point process whose points are negatively correlated. However, only very recently, the research community has started investigating the mathematical properties of wireless network models in which transmitting nodes are distributed according to general point processes [1, 16, 15, 17, 18, 24, 28, 30].

Under various assumptions on the distribution of the fading random variables (i.e. signal powers) and on the attenuation function, a first attempt to analyze the performance of a network in which nodes locations are modeled as a general stationary and isotropic point process has been carried out in [16, 17] and [18]. In [16] and [18] the authors study the asymptotic behavior of the outage probability as the intensity of the nodes goes to zero. In [17], instead, the outage probability of the network is approximated using the factorial moment expansion of functionals of point processes and the proposed moment expansion can be successfully applied when the joint intensities of the underlying point process can be efficiently computed. In [1, 24, 28], the authors propose different methodologies to estimate the outage probability of networks in which the nodes are distributed according to a Matérn hardcore process. At last, in [15] authors characterize the outage probability of wireless networks in which nodes are distributed according to attractive Poisson cluster processes, such as Neyman-Scott, Thomas and Matérn point processes and fading variables are exponentially distributed.

This paper may be considered as a natural extension of the study started in [14], where large deviation estimates for the interference in the Poisson network model have been provided, under various assumptions on the distribution of the fading random variables. Here we move a step forward targeting networks in which the nodes are placed according to repulsive point processes. Our main findings can be summarized as follows. When the fading random variables are bounded or Weibull superexponential and the nodes are placed according to the β-Ginibre process, $0 < \beta \leq 1$, we derive the large deviations of the interference by relating the tail of the interference with the number of points falling in the proximity of the receiver. Our results show that, on the log-scale, the tail of the interference exhibits the same asymptotic behavior for any value of $\beta \in (0, 1]$. At the same time, our results indicate that, on the log-scale, the asymptotic behavior of the tail of the interference in the β-Ginibre network model, $0 < \beta \leq 1$, and the asymptotic behavior of the tail of the interference in the Poisson network model are different. Since the Poisson process is the weak limit of the β-Ginibre process, as $\beta \to 0$, this enlightens a discontinuous behavior of the tail of the interference with respect to the convergence in law. When the fading random variables are exponential or subexponential, we prove that, on the log-scale, the asymptotic behavior of the tail of the interference is insensitive to the distribution of the nodes, as long as the number of nodes is guaranteed to be light-tailed. Such insensitivity property descends from the fact that large values of the interference are typically originated by a single dominant interferer node.

From a mathematical point of view, the analysis of the β-Ginibre network model, $0 < \beta \leq 1$, carried out in this paper differs from the analysis of the Poisson network model studied in [14], since we can not anymore resort on the independence properties of the Poisson process. This difficulty is circumvented by combining ad hoc arguments, that leverage the specific structure of the β-Ginibre process, $0 < \beta \leq 1$, and the properties of subexponential distributions.

The paper is organized as follows. In Section 2 we describe the system model. In Section 3 we give some preliminaries on large deviations, determinantal processes and β-Ginibre processes, $0 < \beta \leq 1$. The statistical assumptions on the model are provided in Section 4. In Sections 5 and 6 we derive the large deviations of the interference in the β-Ginibre network model, $0 < \beta \leq 1$, when the fading random variables are bounded and Weibull superexponential, respectively. In Section 7 we provide the large deviations of the interference in more general network models when the signal powers are exponential or subexponential. In Section 8 we summarize the main findings of this paper. We include an Appendix where some technical results are proved.

2. The system model. We consider the following simple model of wireless network, which accounts for interference among different simultaneous transmissions. Transmitting nodes (antennas) are distributed according to a simple (i.e. without multiple points) point process $\mathbf{N} \equiv \{Y_i\}_{i \geq 1}$ on the plane. One of the points of $\mathbf{N}$ is placed at the origin, say O. A tagged receiver is then added at $y \in \mathbb{R}^2$.

We suppose that the useful signal emitted by the node at the origin is received at y with power $Z_0 L(y)$, where $L : \mathbb{R}^2 \to (0, \infty)$ is a non increasing function called attenuation function, and Z_0 is a random term modeling the effects of the fading. Similarly, we assume that the interfering signal emitted by the node at $Y_i \neq O$ is received at y with power $Z_i L(y - Y_i)$. We suppose that the fading random variables Z_i are non-negative, independent and identically distributed and independent of $\{Y_i\}_{i \geq 1}$. Finally, we denote by $w > 0$ the average thermal power noise at the receiver.

Let $\{X_i\}_{i \geq 1}$ denote the points of the point process $\mathbf{N} \setminus \{O\} \,|\, O \in \mathbf{N}$ (the law of this process is the so-called reduced Palm probability of $\mathbf{N}$ at the origin, see e.g. [10].) We shall analyze the interference due to simultaneous transmissions of nodes falling in a measurable and bounded region Λ of the plane that contains both O and y in its interior. Assuming that all the random quantities considered above are defined on the same probability space $(\Omega, \mathcal{F}, \mathbb{P})$, we define the interference by

$$I_\Lambda = \sum_{i \geq 1} Z_i L(y - X_i) \mathbb{1}_\Lambda(X_i)$$

where, with a slight abuse of notation, we have still denoted by Z_i the fading random variable associated to the transmission of the node at X_i. Here the symbol $\mathbb{1}_\Lambda$ denotes the indicator function of the set Λ.

The tail of the interference is tightly related to the probability of successfully decoding the signal from the transmitter at the origin. Indeed, depending on the adopted modulation and encoding scheme, the receiver at y can successfully decode the signal from the transmitter at O if the Signal to Interference plus Noise Ratio (SINR) at the receiver is greater than a given threshold, say $\tau > 0$ (which depends on the adopted scheme.) In other words, the success probability is given by

$$\mathbb{P}(\mathrm{SINR} > \tau) \qquad \text{where} \qquad \mathrm{SINR} = \frac{Z_0 L(y)}{w + I_\Lambda}.$$

The relationship between the tail of I_Λ and the success probability is highlighted by the following relation

$$\mathbb{P}(\mathrm{SINR} > \tau \mid Z_0 = z) = \mathbb{P}\left(I_\Lambda < \frac{z L(y)}{\tau} - w \right).$$

3. Preliminaries. In this section, first we recall the notion of large deviation principle and subexponential distribution (the reader is directed to [11] for an introduction to large deviations theory and to [2] for more insight into heavy-tailed random variables), second we recall the definition of determinantal process, explain its repulsive nature and provide the definition of β-Ginibre process, $0 < \beta \leq 1$ (the reader is referred to [9, 10] and [27] for notions of point processes theory, to [22] for more insight into determinantal processes and to [6] and [7] for notions of functional analysis.)

3.1. *Large deviation principles.* A family of probability measures $\{\mu_\varepsilon\}_{\varepsilon>0}$ on $([0,\infty), \mathcal{B}([0,\infty)))$ obeys a large deviation principle (LDP) with rate function I and speed v if $I : [0,\infty) \to [0,\infty]$ is a lower semi-continuous function, $v : (0,\infty) \to (0,\infty)$ is a measurable function which diverges to infinity at the origin, and the following inequalities hold for every Borel set $B \in \mathcal{B}([0,\infty))$:

$$- \inf_{x \in B^\circ} I(x) \leq \liminf_{\varepsilon \to 0} \frac{1}{v(\varepsilon)} \log \mu_\varepsilon(B) \leq \limsup_{\varepsilon \to 0} \frac{1}{v(\varepsilon)} \log \mu_\varepsilon(B) \leq - \inf_{x \in \overline{B}} I(x),$$

where B° denotes the interior of B and $\overline{B}$ denotes the closure of B. Similarly, we say that a family of $[0,\infty)$-valued random variables $\{V_\varepsilon\}_{\varepsilon>0}$ obeys an LDP if $\{\mu_\varepsilon\}_{\varepsilon>0}$ obeys an LDP and $\mu_\varepsilon(\cdot) = P(V_\varepsilon \in \cdot)$. We point out that the lower semi-continuity of I means that its level sets:

$$\{x \in [0,\infty) : I(x) \leq a\}, \quad a \geq 0,$$

are closed; when the level sets are compact the rate function I is said to be good.

In this paper we shall use the following criterion to provide the large deviations of a non-negative family of random variables. Although its proof is quite standard, we give it in the Appendix for the sake of completeness.

PROPOSITION 3.1. *Let $I : [0,\infty) \to [0,\infty)$ be an increasing function which is continuous on $(0,\infty)$ and such that $I(0) = 0$ and let $v : (0,\infty) \to (0,\infty)$ be a measurable function which diverges to infinity at the origin. If $\{V_\varepsilon\}_{\varepsilon>0}$ is a family of non-negative random variables such that $V_\varepsilon \downarrow 0$ and, for any $x \geq 0$,*

$$\limsup_{\varepsilon \to 0} \frac{1}{v(\varepsilon)} \log \mathbb{P}(V_\varepsilon \geq x) \leq -I(x)$$

and

$$\liminf_{\varepsilon \to 0} \frac{1}{v(\varepsilon)} \log \mathbb{P}(V_\varepsilon > x) \geq -I(x),$$

then the family of random variables $\{V_\varepsilon\}_{\varepsilon>0}$ obeys an LDP on $[0,\infty)$ with speed v and rate function I.

A random variable Z is called subexponential if it has support on $(0,\infty)$ and

$$\lim_{x\to\infty}\frac{\overline{F^{*2}}(x)}{\overline{F}(x)}=2,$$

where $F(x)=\mathbb{P}(Z\leq x)$, $\overline{F}(x)=\mathbb{P}(Z>x)$ and F^{*2} is the two-fold convolution of F.

Finally, we fix some notation. Let f and g be two real-valued functions defined on some subset of $\mathbb{R}$. We write $f(x)=O(g(x))$ if there exist constants $M>0$ and $x_0\in\mathbb{R}$ such that $|f(x)|\leq M|g(x)|$ for all $x>x_0$. We write $f(x)=o(g(x))$ if for any $\varepsilon>0$ there exists $x_0\in\mathbb{R}$ such that $|f(x)|\leq\varepsilon|g(x)|$ for all $x>x_0$. We write $f(x)\sim g(x)$ if $\lim_{x\to\infty}f(x)/g(x)=1$. For any complex number $z\in\mathbb{C}$, we denote by $\overline{z}$ its complex conjugate. For any $x_0\in\mathbb{R}^2$ or $\mathbb{C}$, we denote by $b(x_0,r)$ the closed ball in $\mathbb{R}^2$ or $\mathbb{C}$ of radius $r>0$ centered at x_0. For any $x\geq 0$, we denote by $[x]$ the biggest integer not exceeding x.

3.2. *Determinantal processes and their repulsive nature.* We start recalling the notion of joint intensities (or kth order product density functions) of a point process on the complex field. Let $S\subseteq\mathbb{C}$ be a measurable set, λ a Radon measure on S and $\mathbf{N}\equiv\{Y_i\}_{i\geq 1}$ a simple point process on S. The joint intensities of $\mathbf{N}$ with respect to λ are measurable functions (if any exist) $\rho^{(k)}:S^k\to[0,\infty)$, $k\geq 1$, such that for any family of mutually disjoint subsets $\Lambda_1,\dots,\Lambda_k$ of S

$$\mathbb{E}\left[\prod_{j=1}^{k}\left(\sum_{i\geq 1}\mathbb{1}_{\Lambda_j}(Y_i)\right)\right]=\int_{\prod_{j=1}^{k}\Lambda_j}\rho^{(k)}(x_1,\dots,x_k)\,\lambda(\mathrm{d}x_1)\dots\lambda(\mathrm{d}x_k).$$

In addition, we require that $\rho^{(k)}(x_1,\dots,x_k)$ vanishes if $x_h=x_k$ for some $h\neq k$. Intuitively, for any pairwise distinct points $x_1,\dots,x_k\in S$, $\rho^{(k)}(x_1,\dots,x_k)\,\lambda(\mathrm{d}x_1)\dots\lambda(\mathrm{d}x_k)$ is the probability that, for each $i=1,\dots,k$, $\mathbf{N}$ has a point in an infinitesimally small region around x_i of volume $\lambda(\mathrm{d}x_i)$. If $\rho^{(1)}$ and $\rho^{(2)}$ exist, we may consider the following second order summary statistic of $\mathbf{N}$ (called pair correlation function)

$$g(x_1,x_2)=\frac{\rho^{(2)}(x_1,x_2)}{\rho^{(1)}(x_1)\rho^{(1)}(x_2)}\qquad\text{for }\rho^{(1)}(x_1)>0,\rho^{(1)}(x_2)>0$$

$g(x_1,x_2)=0$ when either $\rho^{(1)}(x_1)=0$ or $\rho^{(1)}(x_2)=0$.

Due to the interpretation of the joint intensities, if $g\leq 1$ $\lambda^{\otimes 2}$-a.e. then the points of $\mathbf{N}$ repel each other (indeed the process is negative correlated and has an anti-clumping behavior.)

$\mathbf{N}$ is said to be a determinantal process on S with kernel $K : S \times S \to \mathbb{C}$ and reference measure λ if

$$\rho^{(k)}(x_1, \dots, x_k) = \det(K(x_i, x_j))_{1 \le i,j \le k},$$

where $\det(K(x_i, x_j))_{1 \le i,j \le k}$ is the determinant of the $k \times k$-matrix with ij-entries $K(x_i, x_j)$. From now on, we assume that K is locally square integrable on $S \times S$ with respect to $\lambda^{\otimes 2}$ and let

$$\mathcal{K} f(x) = \int_S K(x, y) f(y)\, \lambda(\mathrm{d}y), \qquad f \in L^2(S, \lambda).$$

be the integral operator with kernel K and reference measure λ. Here $L^2(S, \lambda)$ is the space of functions $f : S \to \mathbb{C}$ which are square integrable with respect to λ. In the sequel, for a compact set $\Lambda' \subset S$, we denote by $\mathcal{K}_{\Lambda'}$ the restriction of $\mathcal{K}$ to Λ'. If the operator $\mathcal{K}_{\Lambda'}$ is positive, we denote by $\mathrm{Tr}(\mathcal{K}_{\Lambda'})$ the trace of $\mathcal{K}_{\Lambda'}$. To guarantee the existence and uniqueness (in law) of a determinantal process with a given kernel K and reference measure λ one assumes

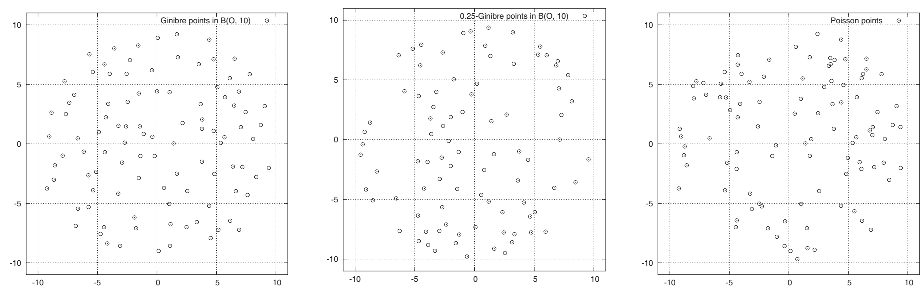
- $\mathcal{K}$ is Hermitian, i.e. $K(x_i, x_j) = \overline{K(x_j, x_i)}$, $\lambda^{\otimes 2}$-a.e.
- The spectrum of $\mathcal{K}$ is contained in $[0, 1]$.
- $\mathcal{K}$ is locally of trace class, i.e. $\mathrm{Tr}(\mathcal{K}_{\Lambda'}) < \infty$ for any compact $\Lambda' \subset S$.

By the spectral theorem for compact and Hermitian operators, under the above assumptions, for any fixed compact $\Lambda' \subset S$, there exists an orthonormal basis $\{\varphi_{n,\Lambda'}\}_{n \ge 1}$ of $L^2(\Lambda', \lambda)$ of eigenfunctions of $\mathcal{K}_{\Lambda'}$. We denote by $\{\kappa_n(\Lambda')\}_{n \ge 1}$ the corresponding eigenvalues, i.e. $\mathcal{K}_{\Lambda'} \varphi_{n,\Lambda'} = \kappa_n(\Lambda') \varphi_{n,\Lambda'}$, $n \ge 1$. Note that $\kappa_n(\Lambda') \in [0, 1]$ for any $n \ge 1$, because the spectrum of $\mathcal{K}$ is contained in $[0, 1]$. Note also that the above conditions imply $K(x, x) \ge 0$, λ-a.e.

We remark that for a determinantal process $\mathbf{N}$ on S with kernel K and reference measure λ we have

$$\begin{aligned} g(x_1, x_2) &= \frac{K(x_1, x_1)K(x_2, x_2) - K(x_1, x_2)K(x_2, x_1)}{K(x_1, x_1)K(x_2, x_2)} \\ &= 1 - \frac{K(x_1, x_2)K(x_2, x_1)}{K(x_1, x_1)K(x_2, x_2)} \\ &= 1 - \frac{|K(x_1, x_2)|^2}{K(x_1, x_1)K(x_2, x_2)} \le 1, \quad \lambda^{\otimes 2}\text{-a.e.} \end{aligned} \tag{1}$$

which shows the repulsiveness of determinantal processes. Here, in (1) one uses first the Hermitianity of $\mathcal{K}$ and second that $K(x, x) \ge 0$ λ-a.e.



(a) Ginibre (b) β-Ginibre (c) Poisson

FIG 1. *Realizations of the Ginibre process, the β-Ginibre process with $\beta = 0.25$ and the homogeneous Poisson process of intensity $1/\pi$ within the ball $b(O, 10)$.*

In this paper, we shall consider the Ginibre and more generally the β-Ginibre process. The Ginibre process is a determinantal process on $S = \mathbb{C}$ with kernel K and reference measure λ defined respectively by

$$K(x, y) = \mathrm{e}^{x\overline{y}} \qquad \text{and} \qquad \lambda(\mathrm{d}x) = \frac{1}{\pi}\mathrm{e}^{-|x|^2}\,\mathrm{d}x.$$

Here $\mathrm{d}x$ denotes the Lebesgue measure on $\mathbb{C}$. The β-Ginibre process, $0 < \beta \leq 1$, is the point process obtained by retaining, independently and with probability β, each point of the Ginibre process and then scaling by $\sqrt{\beta}$ the remaining points. Note that the 1-Ginibre process is the Ginibre process and that the β-Ginibre process converges weakly to the homogeneous Poisson process of intensity $1/\pi$, as $\beta \to 0$ (this latter fact may be easily checked proving that the Laplace functional of the β-Ginibre process converges to the Laplace functional of the Poisson process of intensity $1/\pi$, as $\beta \to 0$; see e.g. Theorem 4 in [8].) In other words the β-Ginibre processes, $0 < \beta < 1$, constitute an intermediate class between the homogeneous Poisson process of intensity $1/\pi$ and the Ginibre process. We remark that the β-Ginibre processes, $0 < \beta \leq 1$, are still determinantal processes and satisfy the usual conditions of existence and uniqueness (see e.g. [19].) Figures 1(a) and 1(b) show, respectively, a realization of the Ginibre processs and of the β-Ginibre process with $\beta = 0.25$ within the ball $b(O, 10)$. For comparison, a realization of the homogeneous Poisson process of intensity $1/\pi$ within the ball $b(O, 10)$ is reported in the Figure 1(c). Note that the points of the Ginibre process exhibit the highest degree of regularity, while the points of the Poisson process exhibit the lowest degree of regularity.

4. Statistical assumptions. Throughout this paper we assume that the signal power is attenuated according to the ideal Hertzian law, i.e.

$$L(x) = \max\{R, |x|\}^{-\alpha}, \quad R > 0,\ \alpha > 2.$$

We recall that the simple point process $\mathbf{N} = \{Y_i\}_{i\geq 1}$ denotes the locations of the nodes and $\{X_i\}_{i\geq 1}$ are the points of the reduced Palm version at the origin of $\mathbf{N}$, i.e. $\mathbf{N} \setminus \{O\} \,|\, O \in \mathbf{N}$. In the following, any time we refer to a determinantal process we identify the plane with $\mathbb{C}$.

LEMMA 4.1. *Let $\{X_i\}_{i\geq 1}$ be a reduced Palm version at the origin of a β-Ginibre process, $\{V_i\}_{i\geq 1}$ a Ginibre process and G a centered complex Gaussian random variable with $\mathbb{E}[|G|^2] = 1$. The point process which is obtained by an independent thinning of $\{\sqrt{\beta}V_i\}_{i\geq 1}$ with retention probability β has the same law of the point process which is obtained by adding to $\{X_i\}_{i\geq 1}$ the point $\sqrt{\beta}G$ with probability β.*

Given a measurable and bounded subset Λ' of the plane, we denote by $N(\Lambda')$ the number of points $\{X_i\}_{i\geq 1}$ in Λ'.

LEMMA 4.2. *(i) Let $\{V_i\}_{i\geq 1}$ be a Ginibre process and $\{A_i\}_{i\geq 1}$ a sequence of independent and identically distributed events, independent of $\{V_i\}_{i\geq 1}$. For any fixed $r \in (0, \infty)$ and $x_0 \in \mathbb{C}$,*

$$\mathbb{P}\left(\sum_{i\geq 1} \mathbb{1}_{b(x_0,r)}(V_i)\mathbb{1}_{A_i} \geq m\right) = \mathrm{e}^{-\frac{1}{2}m^2 \log m(1+o(1))}, \quad \text{as } m \uparrow \infty. \tag{2}$$

(ii) Let $\{X_i\}_{i\geq 1}$ be a reduced Palm version at the origin of a β-Ginibre process. For any fixed $r \in (0, \infty)$ and $x_0 \in \mathbb{C}$,

$$\mathbb{P}(N(b(x_0, r)) \geq m) = \mathrm{e}^{-\frac{1}{2}m^2 \log m(1+o(1))}, \quad \text{as } m \uparrow \infty.$$

LEMMA 4.3. *Let $\{X_i\}_{i\geq 1}$ be a reduced Palm version at the origin of a β-Ginibre process. For any compact $\Lambda' \subset \mathbb{C}$,*

$$\mathbb{E}[N(\Lambda')] \leq \sum_{n\geq 1} \kappa_n(\Lambda'/\sqrt{\beta}) < \infty \tag{3}$$

and

$$\mathbb{E}[\mathrm{e}^{\theta N(\Lambda')}] \leq \prod_{n\geq 1}(1 + (\mathrm{e}^{\theta} - 1)\kappa_n(\Lambda'/\sqrt{\beta})) < \infty, \qquad \theta \geq 0. \tag{4}$$

Here

$$\Lambda'/\beta = \{x \in \mathbb{C} : \; x = y/\sqrt{\beta} \text{ for some } y \in \Lambda'\}$$

and $\kappa_n(\Lambda'/\beta)$ *are the eigenvalues of the integral operator, restricted to* Λ'/β, *of the* 1-*Ginibre process.*

Lemma 4.1 is a straightforward consequence of Remark 24 in [19] (see Theorem 1 in [19] for the case $\beta = 1$.) The proofs of Lemmas 4.2 and 4.3 are given in the Appendix. Lemmas 4.1, 4.2 and 4.3 will come in handy in Sections 5 and 6.

In Section 7, we consider a general simple point process $\mathbf{N}$ on the plane satisfying one of the following two light-tail conditions:

- when the fading is exponentially distributed (see Subsection 7.1) we assume that

$$\mathbb{E}[\mathrm{e}^{\theta N(\Lambda)}] < \infty \text{ for any } \theta > 0; \tag{5}$$

- when the fading is subexponential (see Subsection 7.2) we assume that

$$\exists\; a > 0 \text{ such that } \mathbb{E}[\mathrm{e}^{\theta N(\Lambda)}] < \infty \;\forall\; \theta < a. \tag{6}$$

Note that Conditions (5) and (6) are fairly general. The homogeneous Poisson process and the β-Ginibre process, $0 < \beta \le 1$, represent just two particular point processes satisfying (5), and therefore (6). This is a simple consequence of the Slivnyak Theorem and Lemma 4.3.

5. Large deviations of the interference: Bounded fading. The standing assumptions of this section are: $\mathbf{N}$ is the β-Ginibre process, $0 < \beta \le 1$; the fading random variables Z_i, $i \ge 1$, have bounded support with supremum $B > 0$.

THEOREM 5.1. *Under the foregoing assumptions, the family of random variables* $\{\varepsilon I_\Lambda\}_{\varepsilon>0}$ *obeys an LDP on* $[0,\infty)$ *with speed* $\frac{1}{\varepsilon^2}\log\frac{1}{\varepsilon}$ *and good rate function* $I_1(x) = \frac{R^{2\alpha}x^2}{2B^2}$.

The proof of this theorem is based on the following lemmas whose proofs are given below.

LEMMA 5.2. *Under the foregoing assumptions, for any* $x \ge 0$,

$$\limsup_{\varepsilon\to 0} \frac{\varepsilon^2}{\log(1/\varepsilon)} \log \mathbb{P}(\varepsilon I_\Lambda \ge x) \le -I_1(x).$$

LEMMA 5.3. *Under the foregoing assumptions, for any $x \geq 0$,*

$$\liminf_{\varepsilon \to 0} \frac{\varepsilon^2}{\log(1/\varepsilon)} \log \mathbb{P}(\varepsilon I_\Lambda > x) \geq -I_1(x).$$

PROOF OF THEOREM 5.1. The claim follows by Proposition 3.1 and Lemmas 5.2 and 5.3. ☐

PROOF OF LEMMA 5.2. The claim is clearly true if $x = 0$. We prove the claim when $x > 0$. Since $\max\{R, |X_i - y|\} \geq R$ we have $L(X_i - y) \leq R^{-\alpha}$, $i \geq 1$, and so

$$\mathbb{P}\left(\varepsilon I_\Lambda \geq x\right) \leq \mathbb{P}\left(R^{-\alpha}\varepsilon \sum_{i\geq 1} Z_i \mathbb{1}_\Lambda(X_i) \geq x\right), \quad \varepsilon > 0 \tag{7}$$

(it is worthwhile to remark that due to its generality this bound will be used later on even to derive large deviation upper bounds in the case of signals not necessarily bounded and nodes not necessarily distributed as the reduced Palm version at the origin of a β-Ginibre process.) Since Λ is bounded and $y \in \Lambda^\circ$ there exists $\widetilde{R} > 0$ so that $b(y, \widetilde{R}) \supseteq \Lambda$. Combining this with (7) and the assumption on the support of the signals, for any $\varepsilon > 0$, we have

$$\mathbb{P}(\varepsilon I_\Lambda \geq x) \leq \mathbb{P}\left(\sum_{i\geq 1} \mathbb{1}_{b(y,\widetilde{R})}(X_i) \geq \frac{R^\alpha x}{B\varepsilon}\right) = \mathbb{P}\left(N(b(y,\widetilde{R})) \geq \frac{R^\alpha x}{B\varepsilon}\right). \tag{8}$$

By this inequality and Lemma 4.2(*ii*) we then have

$$\begin{aligned}\limsup_{\varepsilon\to 0} \frac{\varepsilon^2}{\log(1/\varepsilon)} \log \mathbb{P}(\varepsilon I_\Lambda \geq x) &\leq \limsup_{\varepsilon\to 0} \frac{\varepsilon^2}{\log(1/\varepsilon)} \log \mathbb{P}\left(N(b(y,\widetilde{R})) \geq \frac{R^\alpha x}{B\varepsilon}\right) \\ &= -\frac{R^{2\alpha}x^2}{2B^2},\end{aligned}$$

and the proof is completed (note that in the latter equality one makes use of the elementary relation $\lim_{\varepsilon\to 0} \frac{\log(c/\varepsilon)}{\log(1/\varepsilon)} = 1$, for any positive constant $c > 0$.) ☐

PROOF OF LEMMA 5.3. The idea is to produce a suitable lower bound for the quantity $\mathbb{P}(\varepsilon I_\Lambda > x)$ by a thinning argument. For this we shall combine Lemma 4.1 and Lemma 4.2(*i*). The claim of the lemma is clearly true if

$x = 0$ and so we consider $x > 0$. Since $y \in \Lambda^\circ$, there exists $r \in (0, R)$ such that $b(y,r)^\circ \subset \Lambda$. So, for any $\varepsilon > 0$, we have

$$\mathbb{P}(\varepsilon I_\Lambda > x) \geq \mathbb{P}(\varepsilon I_{b(y,r)^\circ} > x) = \mathbb{P}\left(\sum_{i\geq 1} Z_i \mathbb{1}_{b(y,r)^\circ}(X_i) > \frac{R^\alpha x}{\varepsilon}\right), \tag{9}$$

where the equality is a consequence of the fact that $r \in (0, R)$. Letting $\{U\} \cup \{U_i\}_{i\geq 1}$ denote a sequence of independent random variables uniformly distributed on $[0, 1]$ and Z denote a random variable distributed as Z_1, and assuming that the random variables $\{U, Z\} \cup \{U_i\}_{i\geq 1}$ are independent of all the other random quantities, we have

$$\begin{aligned}
&\mathbb{P}\left(\sum_{i\geq 1} Z_i \mathbb{1}_{b(y,r)^\circ}(X_i) > \frac{R^\alpha x}{\varepsilon}\right) \\
&\quad = \mathbb{P}\Bigg(\sum_{i\geq 1} Z_i \mathbb{1}_{b(y,r)^\circ}(X_i) + Z\mathbb{1}_{b(y,r)^\circ}(\sqrt{\beta}G)\mathbb{1}\{U < \beta\} \\
&\qquad\qquad > \frac{R^\alpha x}{\varepsilon} + Z\mathbb{1}_{b(y,r)^\circ}(\sqrt{\beta}G)\mathbb{1}\{U < \beta\}\Bigg) \\
&\quad \geq \mathbb{P}\left(\sum_{i\geq 1} Z_i \mathbb{1}_{b(y,r)^\circ}(X_i) + Z\mathbb{1}_{b(y,r)^\circ}(\sqrt{\beta}G)\mathbb{1}\{U < \beta\} > \frac{R^\alpha x}{\varepsilon} + B\right) && (10)\\
&\quad = \mathbb{P}\left(\sum_{i\geq 1} Z_i \mathbb{1}_{b(y,r)^\circ}(\sqrt{\beta}V_i)\mathbb{1}\{U_i < \beta\} > \frac{R^\alpha x}{\varepsilon} + B\right), && (11)
\end{aligned}$$

where (10) follows by the upper bound

$$Z\mathbb{1}_{b(y,r)^\circ}(\sqrt{\beta}G)\mathbb{1}\{U < \beta\} \leq B$$

and (11) is consequence of Lemma 4.1. Since Z_1 has bounded support with supremum $B > 0$, for arbitrarily small $\delta \in (0, 1)$ there exists $p_\delta > 0$ such that $\mathbb{P}(Z_1 > (1 - \delta)B) = p_\delta$. Using the elementary relations

$$1 \geq \mathbb{1}_{((1-\delta)B,\infty)}(Z_i), \quad \mathbb{1}_{b(y,r)^\circ}(\sqrt{\beta}V_i) = \mathbb{1}_{b(y/\sqrt{\beta},r/\sqrt{\beta})^\circ}(V_i)$$

we have

$$\begin{aligned}
&\mathbb{P}\left(\sum_{i\geq 1} Z_i \mathbb{1}_{b(y,r)^\circ}(\sqrt{\beta}V_i)\mathbb{1}\{U_i < \beta\} > \frac{R^\alpha x}{\varepsilon} + B\right) \\
&\quad \geq \mathbb{P}\left(\sum_{i\geq 1} Z_i \mathbb{1}_{b(y/\sqrt{\beta},r/\sqrt{\beta})^\circ}(V_i)\mathbb{1}\{U_i < \beta\}\mathbb{1}_{((1-\delta)B,\infty)}(Z_i) > \frac{R^\alpha x}{\varepsilon} + B\right).
\end{aligned} \tag{12}$$

Note also that

$$
\begin{aligned}
&\mathbb{P}\left(\sum_{i\geq 1} Z_i \mathbb{1}_{b(y/\sqrt{\beta},r/\sqrt{\beta})^\circ}(V_i)\mathbb{1}\{U_i<\beta\}\mathbb{1}_{((1-\delta)B,\infty)}(Z_i) > \frac{R^\alpha x}{\varepsilon}+B\right)\\
&\geq \mathbb{P}\left((1-\delta)B\sum_{i\geq 1} \mathbb{1}_{b(y/\sqrt{\beta},r/\sqrt{\beta})^\circ}(V_i)\mathbb{1}\{U_i<\beta\}\mathbb{1}_{((1-\delta)B,\infty)}(Z_i) > \frac{R^\alpha x}{\varepsilon}+B\right)\\
&= \mathbb{P}\left(\sum_{i\geq 1} \mathbb{1}_{b(y/\sqrt{\beta},r/\sqrt{\beta})^\circ}(V_i)\mathbb{1}\{U_i<\beta\}\mathbb{1}_{((1-\delta)B,\infty)}(Z_i) > \frac{R^\alpha x}{(1-\delta)B\varepsilon}+\frac{1}{1-\delta}\right)\\
&\geq \mathbb{P}\left(\sum_{i\geq 1} \mathbb{1}_{b(y/\sqrt{\beta},r/\sqrt{\beta})^\circ}(V_i)\mathbb{1}\{U_i<\beta\}\mathbb{1}_{((1-\delta)B,\infty)}(Z_i) > \left[\frac{R^\alpha x}{(1-\delta)B\varepsilon}+\frac{1}{1-\delta}\right]+1\right)\ ,
\end{aligned}
\tag{13}
$$

where the latter inequality follows by the definition of $[x]$ (i.e. the biggest integer not exceeding x.) Collecting (9), (11), (12) and (13) we deduce

$$
\begin{aligned}
&\mathbb{P}(\varepsilon I_\Lambda > x)\\
&\geq \mathbb{P}\left(\sum_{i\geq 1} \mathbb{1}_{b(y/\sqrt{\beta},r/\sqrt{\beta})^\circ}(V_i)\mathbb{1}\{U_i<\beta\}\mathbb{1}_{((1-\delta)B,\infty)}(Z_i) > \left[\frac{R^\alpha x}{(1-\delta)B\varepsilon}+\frac{1}{1-\delta}\right]+1\right)\ .
\end{aligned}
\tag{14}
$$

By this inequality and (2), we have

$$
\begin{aligned}
\liminf_{\varepsilon\to 0}\frac{\varepsilon^2}{\log(1/\varepsilon)}\log\mathbb{P}(\varepsilon I_\Lambda > x) &\geq -\frac{1}{2}\lim_{\varepsilon\to 0}\frac{\varepsilon^2}{\log(1/\varepsilon)}\left[\frac{R^\alpha x}{(1-\delta)B\varepsilon}\right]^2\log\left[\frac{R^\alpha x}{(1-\delta)B\varepsilon}\right]\\
&= -\frac{1}{2}\frac{R^{2\alpha}x^2}{(1-\delta)^2B^2}\,.
\end{aligned}
$$

The claim follows letting δ tend to zero. □

We conclude this section stating the following immediate corollary of Theorem 5.1.

COROLLARY 5.4. *Under the assumptions of Theorem 5.1,*

$$
\lim_{x\to\infty}\frac{\log\mathbb{P}(I_\Lambda \geq x)}{x^2\log x} = -\frac{1}{2}\frac{R^{2\alpha}}{B^2}. \tag{15}
$$

The proof of Theorem 5.1 suggests that large values of the interference are typically obtained as the sum of the signals coming from a large number of interfering nodes. This interpretation follows by inequalities (8) and (14).

Now we can compare (15) against its analogue for Poisson networks derived in [14] and here repeated (see also Proposition 5.1 in [33]):

PROPOSITION 5.5. *If* $\mathbf{N}$ *is a homogeneous Poisson process and the fading random variables have bounded support with supremum* $B > 0$,

$$\lim_{x\to\infty} \frac{\log \mathbb{P}(I_\Lambda \geq x)}{x \log x} = -\frac{R^\alpha}{B}.$$

We conclude that: i) on the log-scale, the asymptotic behavior of the tail of the interference is insensitive to the choice of the particular β-Ginibre network model (it does not depend on $0 < \beta \leq 1$), as a consequence of the fact that the tail of the number of points falling in a ball has the same asymptotic behavior for any value of $\beta \in (0, 1]$ (see Lemma 4.2); ii) the tail of the interference in the β-Ginibre network model is significantly lighter than the tail of the interference in the Poisson network model. This is a direct consequence of the repulsiveness of the β-Ginibre process, $0 < \beta \leq 1$. Since the β-Ginibre process converges weakly to the homogeneous Poisson process with intensity $1/\pi$, as $\beta \to 0$, the tail of the interference exhibits a discontinuous behavior with respect to the convergence in law.

From an application point of view, our results lead to the following conclusion: when transmissions are marginally affected by fading such as in outdoor scenarios with (almost) line of sight transmissions, the impact of the node placement can be significant. Network planners should place network nodes as regularly as possible, avoiding concentration of nodes in small areas.

6. Large deviations of the interference: Weibull superexponential fading. The standing assumptions of this section are: $\mathbf{N}$ is the β-Ginibre process, $0 < \beta \leq 1$; the fading random variables Z_i, $i \geq 1$, are Weibull superexponential in the sense that $-\log \mathbb{P}(Z_1 > z) \sim cz^\gamma$, for some constants $c > 0$ and $\gamma > 1$.

Hereafter, for a constant $\mu \in \mathbb{R}$ and $x > 0$ we use the standard notation $\log^\mu x = (\log x)^\mu$.

THEOREM 6.1. *Under the foregoing assumptions, the family of random variables* $\{\varepsilon I_\Lambda\}_{\varepsilon>0}$ *obeys an LDP on* $[0, \infty)$ *with speed* $\frac{1}{\varepsilon^{2\gamma/(\gamma+1)}} \log^{(\gamma-1)/(\gamma+1)}(\frac{1}{\varepsilon})$ *and good rate function*

$$I_2(x) = \frac{1}{2} R^{2\alpha\gamma/(\gamma+1)} \left(\frac{\gamma}{\gamma-1}\right)^{(\gamma-1)/(\gamma+1)} (c(\gamma+1))^{2/(\gamma+1)} x^{2\gamma/(\gamma+1)}.$$

The proof of this theorem is based on the following lemmas whose proofs are given below.

LEMMA 6.2. *Under the foregoing assumptions, for any $x \geq 0$,*

$$\limsup_{\varepsilon \to 0} \frac{\varepsilon^{2\gamma/(\gamma+1)}}{\log^{(\gamma-1)/(\gamma+1)}(1/\varepsilon)} \log \mathbb{P}(\varepsilon I_\Lambda \geq x) \leq -I_2(x).$$

LEMMA 6.3. *Under the foregoing assumptions, for any $x \geq 0$,*

$$\liminf_{\varepsilon \to 0} \frac{\varepsilon^{2\gamma/(\gamma+1)}}{\log^{(\gamma-1)/(\gamma+1)}(1/\varepsilon)} \log \mathbb{P}(\varepsilon I_\Lambda > x) \geq -I_2(x).$$

PROOF OF THEOREM 6.1. The claim follows by Proposition 3.1 and Lemmas 6.2 and 6.3. □

PROOF OF LEMMA 6.2. The claim is clearly true if $x = 0$. We prove the claim when $x > 0$ in four steps. In the first step we provide a general upper bound for $\mathbb{P}(\varepsilon I_\Lambda \geq x)$, $\varepsilon > 0$, by applying the Chernoff bound (it is worthwhile to remark that due to its generality the bound obtained in this step will be used later on even to derive large deviation upper bounds in the case of exponential signals and nodes not necessarily distributed as the reduced Palm version at the origin of a β-Ginibre process.) In the second step, using the determinantal structure of the Ginibre process and the bound derived in Step 1, we give a further upper bound for $\mathbb{P}(\varepsilon I_\Lambda \geq x)$. In the third step we show how the conclusion can be derived by the bound proved in Step 2. This is done up to a technical point which is addressed in the subsequent Step 4.

Step 1: An upper bound for $\mathbb{P}(\varepsilon I_\Lambda \geq x)$. Let Λ' be a bounded set of the complex plane such that $\Lambda' \supseteq \Lambda$ and let $\theta > 0$ be an arbitrary positive constant. By the Chernoff bound and the independence, we deduce

$$\begin{aligned} \mathbb{P}\left(\varepsilon R^{-\alpha} \sum_{i=1}^{N(\Lambda')} Z_i \geq x\right) &\leq \exp\left(-\theta x + \log \mathbb{E}\left[\mathrm{e}^{\theta\varepsilon R^{-\alpha} \sum_{i=1}^{N(\Lambda')} Z_i}\right]\right) \\ &= \exp\left(-\theta x + \log \mathbb{E}\left[\mathbb{E}\left[\mathrm{e}^{\theta\varepsilon R^{-\alpha} Z_1}\right]^{N(\Lambda')}\right]\right). \end{aligned} \tag{16}$$

Combining (7) and (16), we deduce

$$\mathbb{P}(\varepsilon I_\Lambda \geq x) \leq \exp\left(-\theta x + \log \mathbb{E}\left[\mathbb{E}\left[\mathrm{e}^{\theta\varepsilon R^{-\alpha} Z_1}\right]^{N(\Lambda')}\right]\right) \tag{17}$$

(note that by the assumption on the distribution of Z_1 one has $\mathbb{E}[\mathrm{e}^{\delta Z_1}] < \infty$ for any $\delta > 0$ and so the bound is finite.)

Step 2: A further upper bound for $\mathbb{P}(\varepsilon I_\Lambda \geq x)$. Let $\widetilde{R} > 0$ be such that $b(O, \widetilde{R}) \supseteq \Lambda$ and set $R' = \widetilde{R}/\sqrt{\beta}$. Using (4) we deduce

$$\begin{aligned} \log \mathbb{E}\left[\mathbb{E}\left[\mathrm{e}^{\theta\varepsilon R^{-\alpha} Z_1}\right]^{N(b(O,\widetilde{R}))}\right] &\leq \log \prod_{n\geq 1}\Big(1 + \Big(\mathbb{E}\left[\mathrm{e}^{\theta\varepsilon R^{-\alpha} Z_1}\right] - 1\Big)\,\kappa_n(b(O,R'))\Big) \\ &= \sum_{n\geq 1} \log\Big(1 + (\mathbb{E}[\mathrm{e}^{\theta\varepsilon R^{-\alpha} Z_1}] - 1)\kappa_n(b(O,R'))\Big). \end{aligned} \tag{18}$$

Combining (17) with $\Lambda' = b(O, \widetilde{R})$ and (18), for any $\varepsilon, x > 0$, we have

$$\begin{aligned} \mathbb{P}\left(\varepsilon I_\Lambda \geq x\right) &\leq \exp\left(-\theta x + \log \mathbb{E}\left[\mathbb{E}\left[\mathrm{e}^{\theta\varepsilon R^{-\alpha} Z_1}\right]^{N(b(O,\widetilde{R}))}\right]\right) \\ &\leq \exp\left(-\theta x + \sum_{n\geq 1} \log\Big(1 + (\mathbb{E}[\mathrm{e}^{\theta\varepsilon R^{-\alpha} Z_1}] - 1)\kappa_n(b(O,R'))\Big)\right). \end{aligned} \tag{19}$$

Step 3: Conclusion of the proof. By (19), for any $0 < \varepsilon < \min\{1, x\}$, we have

$$\begin{aligned} &\frac{\varepsilon^{2\gamma/(\gamma+1)}}{\log^{(\gamma-1)/(\gamma+1)}(1/\varepsilon)} \log \mathbb{P}\left(\varepsilon I_\Lambda \geq x\right) \\ &\leq -\frac{\varepsilon^{2\gamma/(\gamma+1)}\theta x}{\log^{(\gamma-1)/(\gamma+1)}(1/\varepsilon)} \\ &\quad + \frac{\varepsilon^{2\gamma/(\gamma+1)}}{\log^{(\gamma-1)/(\gamma+1)}(1/\varepsilon)} \sum_{n\geq 1} \log\Big(1 + (\mathbb{E}[\mathrm{e}^{\theta\varepsilon R^{-\alpha} Z_1}] - 1)\kappa_n(b(O,R'))\Big). \end{aligned} \tag{20}$$

From now on we take

$$\theta = \frac{R^\alpha \tilde{\gamma}}{\varepsilon}\left(\frac{x}{\varepsilon}\log\frac{x}{\varepsilon}\right)^{(\gamma-1)/(\gamma+1)},$$

where

$$\tilde{\gamma} = \frac{1}{2}\left(\frac{R^\alpha \gamma}{\gamma - 1}\right)^{(\gamma-1)/(\gamma+1)} (c(\gamma+1))^{2/(\gamma+1)}.$$

Note that

$$\lim_{\varepsilon\to 0} \frac{\varepsilon^{2\gamma/(\gamma+1)}\theta x}{\log^{(\gamma-1)/(\gamma+1)}(1/\varepsilon)} = R^\alpha \tilde{\gamma} x^{2\gamma/(\gamma+1)}. \tag{21}$$

We shall show in the next step that

$$\lim_{\varepsilon\to 0} \frac{\varepsilon^{2\gamma/(\gamma+1)}}{\log^{(\gamma-1)/(\gamma+1)}(1/\varepsilon)} \sum_{n\geq 1} \log\Big(1 + (\mathbb{E}[\mathrm{e}^{\theta\varepsilon R^{-\alpha} Z_1}] - 1)\kappa_n(b(O,R'))\Big) = 0. \tag{22}$$

The claim follows taking the $\limsup$ as $\varepsilon \to 0$ in the inequality (20) and using (21) and (22).

Step 4: Proof of (22). We start recalling that by Lemma 8 in [14] we have

$$\lim_{\theta\to\infty} \frac{\log \mathbb{E}[e^{\theta Z_1}]}{\gamma' \theta^{\gamma/(\gamma-1)}} = 1, \tag{23}$$

where $\gamma' = (\gamma-1)\gamma^{-\gamma/(\gamma-1)} c^{-1/(\gamma-1)}$. Since the eigenvalues $\kappa_n(b(O,R'))$ belong to $[0,1]$, by (23) we deduce

$$\begin{aligned}
0 &\le \limsup_{\varepsilon\to 0} \frac{\varepsilon^{2\gamma/(\gamma+1)}}{\log^{(\gamma-1)/(\gamma+1)}(1/\varepsilon)} \log\left(1 + (\mathbb{E}[e^{\theta\varepsilon R^{-\alpha} Z_1}] - 1)\kappa_n(b(O,R'))\right)\\
&\le \limsup_{\varepsilon\to 0} \frac{\varepsilon^{2\gamma/(\gamma+1)}}{\log^{(\gamma-1)/(\gamma+1)}(1/\varepsilon)} \log \mathbb{E}[e^{\theta\varepsilon R^{-\alpha} Z_1}]\\
&= \gamma' \tilde{\gamma}^{\gamma/(\gamma-1)} \lim_{\varepsilon\to 0} \frac{\varepsilon^{2\gamma/(\gamma+1)}}{\log^{(\gamma-1)/(\gamma+1)}(1/\varepsilon)} \left(\frac{x}{\varepsilon}\log\frac{x}{\varepsilon}\right)^{\gamma/(\gamma+1)} = 0.
\end{aligned}$$

So, for (22) we only need to check that we can interchange the limit with the infinite sum. To this aim, we shall prove that there exists a right neighborhood of zero, say $\mathcal{N}_0$, such that

$$\sum_{n\ge 1} \sup_{\varepsilon\in\mathcal{N}_0} \frac{\varepsilon^{2\gamma/(\gamma+1)}}{\log^{(\gamma-1)/(\gamma+1)}(1/\varepsilon)} \log\left(1 + (\mathbb{E}[e^{\theta\varepsilon R^{-\alpha} Z_1}] - 1)\kappa_n(b(O,R'))\right) < \infty.$$

By (23), for any $\delta > 0$ there exists $\varepsilon_\delta \in (0, \min\{1, x\})$ such that for any $\varepsilon \in (0, \varepsilon_\delta)$

$$\mathbb{E}[e^{\theta\varepsilon R^{-\alpha} Z_1}] \le \exp\left(C_\delta \left(\frac{x}{\varepsilon}\log\frac{x}{\varepsilon}\right)^{\gamma/(\gamma+1)}\right)$$

where $C_\delta = (1+\delta)\gamma'\tilde{\gamma}^{\gamma/(\gamma-1)}$. Therefore, for all $\varepsilon \in (0, \varepsilon_\delta)$, we have

$$\begin{aligned}
&\frac{\varepsilon^{2\gamma/(\gamma+1)}}{\log^{(\gamma-1)/(\gamma+1)}(1/\varepsilon)} \log\left(1 + (\mathbb{E}[e^{\theta\varepsilon R^{-\alpha} Z_1}] - 1)\kappa_n(b(O,R'))\right)\\
&\le \frac{\varepsilon^{2\gamma/(\gamma+1)}}{\log^{(\gamma-1)/(\gamma+1)}(1/\varepsilon)} \log\left(1 + \left(\exp\left(C_\delta\left(\frac{x}{\varepsilon}\log\frac{x}{\varepsilon}\right)^{\gamma/(\gamma+1)}\right) - 1\right)\kappa_n(b(O,R'))\right).
\end{aligned} \tag{24}$$

Consequently, it suffices to prove that there exists a right neighborhood of zero contained in $(0, \varepsilon_\delta)$, say $\mathcal{N}_0'$, such that

$$\sum_{n\ge 1} \sup_{\varepsilon\in\mathcal{N}_0'} \frac{\varepsilon^{2\gamma/(\gamma+1)}}{\log^{(\gamma-1)/(\gamma+1)}(1/\varepsilon)} \log\left(1 + \left(\exp\left(C_\delta\left(\frac{x}{\varepsilon}\log\frac{x}{\varepsilon}\right)^{\gamma/(\gamma+1)}\right) - 1\right)\kappa_n(b(O,R'))\right) < \infty. \tag{25}$$

The first derivative (with respect to ε) of the term in the right-hand side of (24) is equal to

$$\frac{2\gamma}{\gamma+1}\varepsilon^{(\gamma-1)/(\gamma+1)}\frac{\log\left(1+\left(\exp\left(C_\delta\left(\frac{x}{\varepsilon}\log\frac{x}{\varepsilon}\right)^{\gamma/(\gamma+1)}\right)-1\right)\kappa_n(b(O,R'))\right)}{\log^{(\gamma-1)/(\gamma+1)}(1/\varepsilon)}$$
$$+\frac{\gamma-1}{\gamma+1}\frac{\varepsilon^{(\gamma-1)/(\gamma+1)}\log\left(1+\left(\exp\left(C_\delta\left(\frac{x}{\varepsilon}\log\frac{x}{\varepsilon}\right)^{\gamma/(\gamma+1)}\right)-1\right)\kappa_n(b(O,R'))\right)}{\log^{2\gamma/(\gamma+1)}(1/\varepsilon)}$$
$$-\frac{\frac{\gamma}{\gamma+1}x^{\gamma/(\gamma+1)}C_\delta\exp\left(C_\delta\left(\frac{x}{\varepsilon}\log\frac{x}{\varepsilon}\right)^{\gamma/(\gamma+1)}\right)\kappa_n(b(O,R'))}{1+\left(\exp\left(C_\delta\left(\frac{x}{\varepsilon}\log\frac{x}{\varepsilon}\right)^{\gamma/(\gamma+1)}\right)-1\right)\kappa_n(b(O,R'))}$$
$$\times\frac{1}{\varepsilon^{1/(\gamma+1)}}\frac{\log^{\gamma/(\gamma+1)}(x/\varepsilon)}{\log^{(\gamma-1)/(\gamma+1)}(1/\varepsilon)}\left(1+\frac{1}{\log^{\gamma/(\gamma+1)}(x/\varepsilon)\log^{1/(\gamma+1)}(1/\varepsilon)}\right).$$

This quantity is bigger than or equal to zero if and only if

(26)
$$2\gamma\,\varepsilon^{\gamma/(\gamma+1)}\frac{\log\left(1+\left(\exp\left(C_\delta\left(\frac{x}{\varepsilon}\log\frac{x}{\varepsilon}\right)^{\gamma/(\gamma+1)}\right)-1\right)\kappa_n(b(O,R'))\right)}{\log^{\gamma/(\gamma+1)}(1/\varepsilon)}$$

(27)
$$+(\gamma-1)\frac{\varepsilon^{\gamma/(\gamma+1)}\log\left(1+\left(\exp\left(C_\delta\left(\frac{x}{\varepsilon}\log\frac{x}{\varepsilon}\right)^{\gamma/(\gamma+1)}\right)-1\right)\kappa_n(b(O,R'))\right)}{\log^{(2\gamma+1)/(\gamma+1)}(1/\varepsilon)}$$
$$\geq\gamma\frac{x^{\gamma/(\gamma+1)}C_\delta\exp\left(C_\delta\left(\frac{x}{\varepsilon}\log\frac{x}{\varepsilon}\right)^{\gamma/(\gamma+1)}\right)\kappa_n(b(O,R'))}{1+\left(\exp\left(C_\delta\left(\frac{x}{\varepsilon}\log\frac{x}{\varepsilon}\right)^{\gamma/(\gamma+1)}\right)-1\right)\kappa_n(b(O,R'))}$$
$$\times\frac{\log^{\gamma/(\gamma+1)}(x/\varepsilon)}{\log^{\gamma/(\gamma+1)}(1/\varepsilon)}\left(1+\frac{1}{\log^{\gamma/(\gamma+1)}(x/\varepsilon)\log^{1/(\gamma+1)}(1/\varepsilon)}\right)$$

Since $\kappa_n(b(O,R'))\in[0,1]$, we have

$$\gamma\frac{x^{\gamma/(\gamma+1)}C_\delta\exp\left(C_\delta\left(\frac{x}{\varepsilon}\log\frac{x}{\varepsilon}\right)^{\gamma/(\gamma+1)}\right)\kappa_n(b(O,R'))}{1+\left(\exp\left(C_\delta\left(\frac{x}{\varepsilon}\log\frac{x}{\varepsilon}\right)^{\gamma/(\gamma+1)}\right)-1\right)\kappa_n(b(O,R'))}$$
$$\times\frac{\log^{\gamma/(\gamma+1)}(x/\varepsilon)}{\log^{\gamma/(\gamma+1)}(1/\varepsilon)}\left(1+\frac{1}{\log^{\gamma/(\gamma+1)}(x/\varepsilon)\log^{1/(\gamma+1)}(1/\varepsilon)}\right)$$
$$\leq J(\varepsilon):=\gamma\,x^{\gamma/(\gamma+1)}C_\delta\frac{\log^{\gamma/(\gamma+1)}(x/\varepsilon)}{\log^{\gamma/(\gamma+1)}(1/\varepsilon)}\left(1+\frac{1}{\log^{\gamma/(\gamma+1)}(x/\varepsilon)\log^{1/(\gamma+1)}(1/\varepsilon)}\right).$$

Therefore, the first derivative of the term in the right-hand side of (24) is bigger than or equal to zero if

$$H^{(1)}(\varepsilon,\kappa_n(b(O,R')))+H^{(2)}(\varepsilon,\kappa_n(b(O,R')))\geq J(\varepsilon),$$

where, for ease of notation, we denoted by $H^{(1)}(\varepsilon, \kappa_n(b(O,R')))$ the term in (26) and by $H^{(2)}(\varepsilon, \kappa_n(b(O,R')))$ the term in (27). By Remark 3.3 in [32] we have $\kappa_n(b(O,R')) = \mathbb{P}(\mathrm{Po}(R'^2) \geq n+1)$, where $\mathrm{Po}(R'^2)$ is a Poisson random variable with mean R'^2. So the sequence $\{\kappa_n(b(O,R'))\}_{n\geq 1}$ is decreasing (and decreases to zero.) Hence

$$\begin{aligned}
&\lim_{\varepsilon\to 0}\sup_{n\geq 1}(H^{(1)}(\varepsilon, \kappa_n(b(O,R'))) + H^{(2)}(\varepsilon, \kappa_n(b(O,R')))) \\
&\quad = \lim_{\varepsilon\to 0}(H^{(1)}(\varepsilon, \kappa_1(b(O,R'))) + H^{(2)}(\varepsilon, \kappa_1(b(O,R')))) \\
&\quad = 2\gamma\, x^{\gamma/(\gamma+1)} C_\delta.
\end{aligned} \tag{28}$$

Furthermore,

$$\lim_{\varepsilon\to 0} J(\varepsilon) = \gamma\, x^{\gamma/(\gamma+1)} C_\delta. \tag{29}$$

Let $\eta > 0$ be such that $\gamma\, x^{\gamma/(\gamma+1)} C_\delta > 2\eta$. By (28) and (29), there exists $\varepsilon_\eta > 0$ such that for all $0 < \varepsilon < \min\{\varepsilon_\delta, \varepsilon_\eta\}$

$$\begin{aligned}
\sup_{n\geq 1}(H^{(1)}(\varepsilon, \kappa_n(b(O,R'))) + H^{(2)}(\varepsilon, \kappa_n(b(O,R')))) &> 2\gamma\, x^{\gamma/(\gamma+1)} C_\delta - \eta \\
&> \gamma\, x^{\gamma/(\gamma+1)} C_\delta + \eta > J(\varepsilon).
\end{aligned}$$

This guarantees that the function of ε in the right-hand side of (24) is non-decreasing on $(0, \min\{\varepsilon_\delta, \varepsilon_\eta\})$. Consequently, setting $\bar\varepsilon := \min\{\varepsilon_\delta, \varepsilon_\eta\}$ and $\mathcal{N}_0' = (0, \bar\varepsilon)$ we have

$$\begin{aligned}
&\sum_{n\geq 1}\sup_{\varepsilon\in\mathcal{N}_0'} \frac{\varepsilon^{2\gamma/(\gamma+1)}}{\log^{(\gamma-1)/(\gamma+1)}(1/\varepsilon)} \log\left(1 + (\mathbb{E}[\mathrm{e}^{\theta\varepsilon R^{-\alpha} Z_1}] - 1)\kappa_n(b(O,R'))\right) \\
&\leq \sum_{n\geq 1} \frac{\bar\varepsilon^{2\gamma/(\gamma+1)}}{\log^{(\gamma-1)/(\gamma+1)}(1/\bar\varepsilon)} \log\left(1 + \left(\exp\left(C_\delta\left(\frac{x}{\bar\varepsilon}\log\frac{x}{\bar\varepsilon}\right)^{\gamma/(\gamma+1)}\right) - 1\right)\kappa_n(b(O,R'))\right) \\
&\leq \frac{\bar\varepsilon^{2\gamma/(\gamma+1)}}{\log^{(\gamma-1)/(\gamma+1)}(1/\bar\varepsilon)} \exp\left(C_\delta\left(\frac{x}{\bar\varepsilon}\log\frac{x}{\bar\varepsilon}\right)^{\gamma/(\gamma+1)}\right)\sum_{n\geq 1}\kappa_n(b(O,R')) < \infty,
\end{aligned}$$

where the latter inequality follows by $\log(1+x) \leq x$, $x > -1$, and Lemma 4.3. The proof is completed. □

Proof of Lemma 6.3. Since the claim is true if $x = 0$, we take $x > 0$. Since $y \in \Lambda^\circ$, there exists $r \in (0, R)$ such that $b(y,r)^\circ \subset \Lambda$. For all $\varepsilon > 0$

and $n \geq 1$, we have

$$
\begin{aligned}
\mathbb{P}(\varepsilon I_\Lambda > x) \geq \mathbb{P}(\varepsilon I_{b(y,r)^\circ} > x) &= \mathbb{P}\left(\sum_{i\geq 1} Z_i \mathbb{1}_{b(y,r)^\circ}(X_i) > \frac{R^\alpha x}{\varepsilon}\right) \\
&\geq \mathbb{P}\left(\sum_{i\geq 1} Z_i \mathbb{1}_{b(y,r)^\circ}(X_i) > \frac{R^\alpha x}{\varepsilon}, N(b(y,r)^\circ) \geq n\right).
\end{aligned} \tag{30}
$$

Define the event

$$
A_\varepsilon^{(n)} := \left\{\min\{Z_1, \dots, Z_n\} > \frac{R^\alpha x}{n\varepsilon}, N(b(y,r)^\circ) \geq n\right\}. \tag{31}
$$

Since

$$
\begin{aligned}
&\mathbb{P}\left(\sum_{i\geq 1} Z_i \mathbb{1}_{b(y,r)^\circ}(X_i) > \frac{R^\alpha x}{\varepsilon}, N(b(y,r)^\circ) \geq n\right) \\
&\geq \mathbb{P}\left(\sum_{i=1}^{n} Z_i > \frac{R^\alpha x}{\varepsilon}, N(b(y,r)^\circ) \geq n\right) \\
&\geq \mathbb{P}\left(A_\varepsilon^{(n)}\right),
\end{aligned} \tag{32}
$$

combining (30) and (32) and using the independence and that the signals are identically distributed, we have

$$
\mathbb{P}(\varepsilon I_\Lambda > x) \geq \mathbb{P}(A_\varepsilon^{(n)}) = \mathbb{P}(N(b(y,r)^\circ) \geq n)\mathbb{P}\left(Z_1 > \frac{R^\alpha x}{n\varepsilon}\right)^n. \tag{33}
$$

For $0 < \varepsilon < 1$, define the integer

$$
n = \left[\frac{\kappa}{\varepsilon^{\gamma/(\gamma+1)} \log^{1/(\gamma+1)}(1/\varepsilon)}\right], \tag{34}
$$

where $\kappa > 0$ is a constant which will be specified later. By Lemma 4.2(ii), as $\varepsilon \to 0$, we deduce

$$
\begin{aligned}
&-\log \mathbb{P}\left(N(b(y,r)^\circ) \geq n\right) \\
&\sim \frac{1}{2} \frac{\kappa^2}{\varepsilon^{2\gamma/(\gamma+1)} \log^{2/(\gamma+1)}(1/\varepsilon)} \log\left(\frac{1}{\varepsilon^{\gamma/(\gamma+1)} \log^{1/(\gamma+1)}(1/\varepsilon)}\right) \\
&\sim \frac{\gamma}{2(\gamma+1)} \frac{\kappa^2}{\varepsilon^{2\gamma/(\gamma+1)}} \log^{(\gamma-1)/(\gamma+1)}(1/\varepsilon).
\end{aligned} \tag{35}
$$

Here, for the latter relation we used the following elementary computation

$$\begin{aligned}
&\frac{1}{\log^{2/(\gamma+1)}(1/\varepsilon)} \log\left(\frac{1}{\varepsilon^{\gamma/(\gamma+1)}\log^{1/(\gamma+1)}(1/\varepsilon)}\right)\\
&= \frac{\log(1/\varepsilon^{\gamma/(\gamma+1)})}{\log^{2/(\gamma+1)}(1/\varepsilon)} + \frac{\log\left(1/\log^{1/(\gamma+1)}(1/\varepsilon)\right)}{\log^{2/(\gamma+1)}(1/\varepsilon)}\\
&= \frac{\gamma}{\gamma+1}\frac{\log(1/\varepsilon)}{\log^{2/(\gamma+1)}(1/\varepsilon)} + \frac{\log\left(1/\log^{1/(\gamma+1)}(1/\varepsilon)\right)}{\log^{2/(\gamma+1)}(1/\varepsilon)}\\
&= \frac{\gamma}{\gamma+1}\log^{(\gamma-1)/(\gamma+1)}(1/\varepsilon) - \frac{1}{\gamma+1}\frac{\log\log(1/\varepsilon)}{\log^{2/(\gamma+1)}(1/\varepsilon)}\\
&\sim \frac{\gamma}{\gamma+1}\log^{(\gamma-1)/(\gamma+1)}(1/\varepsilon).
\end{aligned}$$

Since the fading is Weibull superexponential we have

$$\begin{aligned}
-n\log\mathbb{P}\left(Z_1 > \frac{R^\alpha x}{n\varepsilon}\right) &\sim \frac{c\kappa}{\varepsilon^{\gamma/(\gamma+1)}\log^{1/(\gamma+1)}(1/\varepsilon)}\left(\frac{R^\alpha x\log^{1/(\gamma+1)}(1/\varepsilon)}{\kappa\varepsilon^{1/(\gamma+1)}}\right)^\gamma\\
&= \frac{c(R^\alpha x)^\gamma}{\kappa^{\gamma-1}}\frac{\log^{(\gamma-1)/(\gamma+1)}(1/\varepsilon)}{\varepsilon^{2\gamma/(\gamma+1)}}.
\end{aligned} \tag{36}$$

Combining (33), (35) and (36) we have

$$\liminf_{\varepsilon\to 0}\frac{\varepsilon^{2\gamma/(\gamma+1)}}{\log^{(\gamma-1)/(\gamma+1)}(1/\varepsilon)}\log\mathbb{P}(\varepsilon I_\Lambda > x) \geq -\frac{\gamma\kappa^2}{2(\gamma+1)} - \frac{c(R^\alpha x)^\gamma}{\kappa^{\gamma-1}}. \tag{37}$$

The maximum value of the lower bound is attained at

$$\kappa = \left(\frac{c(\gamma^2-1)(R^\alpha x)^\gamma}{\gamma}\right)^{1/(\gamma+1)}.$$

The claim follows by a straightforward computation substituting this value of κ in (37). □

We conclude this section stating the following immediate corollary of Theorem 6.1.

Corollary 6.4. *Under the assumptions of Theorem 6.1,*

$$\begin{aligned}
&\lim_{x\to\infty}\frac{\log\mathbb{P}(I_\Lambda \geq x)}{x^{2\gamma/(\gamma+1)}\log^{(\gamma-1)/(\gamma+1)}x}\\
&= -\frac{1}{2}R^{2\alpha\gamma/(\gamma+1)}\left(\frac{\gamma}{\gamma-1}\right)^{(\gamma-1)/(\gamma+1)}(c(\gamma+1))^{2/(\gamma+1)}.
\end{aligned} \tag{38}$$

In this case huge values of the interference are typically obtained as the sum of a large number of interfering nodes with large signals. This interpretation follows from the proof of Theorem 6.1, which establishes that the event $A_\varepsilon^{(n)}$ defined by (31) with n defined as in (34) is a dominating event, as $\varepsilon \to 0$.

Again, we can compare (38) against its analogue for Poisson networks derived in [14] and here repeated (see also Proposition 5.2 in [33]):

PROPOSITION 6.5. *If* $\mathbf{N}$ *is a homogeneous Poisson process and the fading random variables are Weibull superexponential as in Theorem 6.1,*

$$\lim_{x\to\infty} \frac{\log \mathbb{P}(I_\Lambda \geq x)}{x \log^{(\gamma-1)/\gamma} x} = -\gamma(\gamma-1)^{-(\gamma-1)/\gamma} c^{1/\gamma} R^\alpha. \tag{39}$$

We conclude that also when the fading is Weibull superexponential the tail of the interference can be significantly reduced by carefully placing transmitting nodes as regularly as possible. Note that the differences between the terms in (38) and (39) vanish as $\gamma \to 1$. This is hinting at the fact that for exponential or subexponential fading random variables, on the log-scale, the asymptotic behavior of the tail of the interference becomes insensitive to the node placement process. This issue will be investigated in Section 7.

7. Large deviations of the interference: Exponential and subexponential fading.

7.1. *Exponential fading.* The standing assumptions of this subsection are: (5) and the fading random variables Z_i, $i \geq 1$, are exponential in the sense that $-\log \mathbb{P}(Z_1 > z) \sim cz$, for some constant $c > 0$.

THEOREM 7.1. *Under the foregoing assumptions, the family of random variables* $\{\varepsilon I_\Lambda\}_{\varepsilon>0}$ *obeys an LDP on* $[0,\infty)$ *with speed* $\frac{1}{\varepsilon}$ *and good rate function* $I_3(x) = cR^\alpha x$.

The proof of this theorem is based on the following lemmas whose proofs are given below.

LEMMA 7.2. *Under the foregoing assumptions, for any* $x \geq 0$,

$$\limsup_{\varepsilon\to 0} \varepsilon \log \mathbb{P}(\varepsilon I_\Lambda \geq x) \leq -I_3(x).$$

LEMMA 7.3. *Under the foregoing assumptions, for any* $x \geq 0$,

$$\liminf_{\varepsilon\to 0} \varepsilon \log \mathbb{P}(\varepsilon I_\Lambda > x) \geq -I_3(x).$$

Proof of Theorem 7.1. The claim follows by Proposition 3.1 and Lemmas 7.2 and 7.3. □

Proof of Lemma 7.2. Since the claim is true if $x = 0$, we take $x > 0$. By the assumption on the tail of Z_1, one may easily realize that $\mathbb{E}[\mathrm{e}^{\delta Z_1}] < \infty$, for any $\delta < c$. We note here that the inequality (17) holds indeed for general positive random variables Z_i, $i \geq 1$, (not necessarily Weibull distributed), a general point process $\{X_i\}_{i\geq 1}$ (not necessarily a reduced Palm version at the origin of a β-Ginibre process), any $\varepsilon, \theta > 0$ and any bounded set Λ' such that $\Lambda' \supseteq \Lambda$. Setting $\Lambda' = \Lambda$ and $\theta = (c-\delta)R^\alpha/\varepsilon$ in (17), we deduce

$$\mathbb{P}\left(\varepsilon I_\Lambda \geq x\right) \leq \exp\left(-(c-\delta)R^\alpha x/\varepsilon + \log \mathbb{E}\left[\mathbb{E}\left[\mathrm{e}^{(c-\delta)Z_1}\right]^{N(\Lambda)}\right]\right).$$

Therefore by assumption (5) we have

$$\limsup_{\varepsilon\to 0} \varepsilon \log \mathbb{P}\left(\varepsilon I_\Lambda \geq x\right) \leq -(c-\delta)R^\alpha x.$$

The claim follows letting δ tend to zero. □

Proof of Lemma 7.3. Since the claim is true if $x = 0$, we take $x > 0$. Since $y \in \Lambda^\circ$, there exists $r \in (0, \min\{1, R\})$ such that $b(y,r)^\circ \subset \Lambda$. For all $\varepsilon > 0$, we have

$$\begin{aligned} \mathbb{P}(\varepsilon I_\Lambda > x) \geq \mathbb{P}(\varepsilon I_{b(y,r)^\circ} > x) &= \mathbb{P}\left(\sum_{i\geq 1} Z_i \mathbb{1}_{b(y,r)^\circ}(X_i) > \frac{R^\alpha x}{\varepsilon}\right) \\ &\geq \mathbb{P}\left(Z_1 > \frac{R^\alpha x}{\varepsilon}, N(b(y,r)^\circ) \geq 1\right) \\ &= \mathbb{P}\left(Z_1 > \frac{R^\alpha x}{\varepsilon}\right)\mathbb{P}(N(b(y,r)^\circ) \geq 1), \end{aligned} \tag{40}$$

where the latter equality follows by the independence of $N(b(y,r)^\circ)$ and $\{Z_i\}_{i\geq 1}$.

The claim follows by the exponential decay of the tail of Z_1, taking first the logarithm on the above inequality, multiplying then by ε and finally letting ε tend to zero. □

We conclude this section stating the following immediate corollary of Theorem 7.1.

COROLLARY 7.4. *Under the assumptions of Theorem 7.1,*

$$\lim_{x\to\infty} \frac{\log \mathbb{P}(I_\Lambda \geq x)}{x} = -cR^\alpha. \tag{41}$$

The fact that under the exponential fading the tail of the interference is given by (41), for any point process satisfying condition (5), can be explained by observing that large values of the interference are typically originated by a single strong interfering contribution (by (40) clearly emerges that $\{Z_1 > R^\alpha x/\varepsilon\}$ is the dominating event, as $\varepsilon \to 0$.) In view of these premises, it is reasonable to expect a similar result also when the distribution of the fading is heavier than the exponential law. This issue is analyzed for a family of subexponential fading random variables in the Subsection 7.2.

7.2. *Subexponential fading.* The standing assumptions of this subsection are: (6) and the fading random variables Z_i, $i \geq 1$, are subexponential and such that

$$\text{For any } \sigma > 0,\ \lim_{z\to\infty} \frac{\log \overline{F}(\sigma z)}{\log \overline{F}(z)} = \sigma^\gamma, \text{ for some constant } \gamma \geq 0. \tag{42}$$

In particular, note that the above condition is satisfied if Z_1 is subexponential and such that $-\log \overline{F}(z) \sim cz^\gamma$ (Weibull subexponential fading) or $-\log \overline{F}(z) \sim c \log z$ (Pareto fading), for some constants $c > 0$ and $\gamma \in (0, 1)$.

THEOREM 7.5. *Under the foregoing assumptions, the family of random variables $\{\varepsilon I_\Lambda\}_{\varepsilon>0}$ obeys an LDP on $[0,\infty)$ with speed $-\log \overline{F}(\frac{1}{\varepsilon})$ and rate function $I_4(0) = 0$ and $I_4(x) = R^{\alpha\gamma} x^\gamma$, $x > 0$.*

The proof of this theorem is based on the following lemmas whose proofs are given below.

LEMMA 7.6. *Under the foregoing assumptions, for any $x \geq 0$,*

$$\limsup_{\varepsilon\to 0} -\frac{1}{\log \overline{F}\left(\frac{1}{\varepsilon}\right)} \log \mathbb{P}(\varepsilon I_\Lambda \geq x) \leq -I_4(x).$$

LEMMA 7.7. *Under the foregoing assumptions, for any $x \geq 0$,*

$$\liminf_{\varepsilon\to 0} -\frac{1}{\log \overline{F}\left(\frac{1}{\varepsilon}\right)} \log \mathbb{P}(\varepsilon I_\Lambda > x) \geq -I_4(x).$$

PROOF OF THEOREM 7.5. The claim follows by Proposition 3.1 and Lemmas 7.6 and 7.7. □

PROOF OF LEMMA 7.6. Since the claim is true if $x = 0$, we take $x > 0$. By assumption $N(\Lambda)$ has a convergent Laplace transform in a right neighborhood of zero, therefore since Z_1 is subexponential by e.g. Lemma 2.2 p. 259 in [2] it follows

$$\mathbb{P}\left(\sum_{i=1}^{N(\Lambda)} Z_i \geq x\right) \sim \mathbb{E}[N(\Lambda)]\overline{F}(x), \quad \text{as } x \to \infty. \tag{43}$$

We note here that the inequality (7) holds indeed for general positive random variables Z_i, $i \geq 1$, (not necessarily with bounded support), a general point process $\{X_i\}_{i\geq 1}$ (not necessarily a reduced Palm version at the origin of a β-Ginibre process) and any $\varepsilon, x > 0$. By (7) and (43) easily follows that

$$\begin{aligned}
&\limsup_{\varepsilon\to 0} -\frac{1}{\log \overline{F}(1/\varepsilon)} \log \mathbb{P}(\varepsilon I_\Lambda \geq x) \\
&\leq \limsup_{\varepsilon\to 0} -\frac{1}{\log \overline{F}(1/\varepsilon)} \log \mathbb{P}\left(\sum_{i\geq 1} Z_i \mathbb{1}_\Lambda(X_i) \geq \frac{R^\alpha x}{\varepsilon}\right) \\
&= \limsup_{\varepsilon\to 0} -\frac{1}{\log \overline{F}(1/\varepsilon)} \log\left(\mathbb{E}[N(\Lambda)]\overline{F}\left(\frac{R^\alpha x}{\varepsilon}\right)\right) = -R^{\alpha\gamma}x^\gamma,
\end{aligned}$$

where the latter equality is consequence of condition (42). □

PROOF OF LEMMA 7.7. Since the claim is true if $x = 0$, we take $x > 0$. Arguing as in the proof of Lemma 7.3 we have the inequality (40). The claim follows by the subexponential decay of the tail of Z_1, taking first the logarithm on the inequality (40), multiplying then by $-\frac{1}{\log \overline{F}(1/\varepsilon)}$ and finally letting ε tend to zero. □

We conclude this section stating the following immediate corollary of Theorem 7.5.

COROLLARY 7.8. *Under the assumptions of Theorem 7.5,*

$$\lim_{x\to\infty} \frac{\log \mathbb{P}(I_\Lambda \geq x)}{\log \overline{F}(x)} = R^{\alpha\gamma}.$$

Note that also when the fading is subexponential large values of the interference are due to a single strong interfering node, for any point process which satisfies (6).

TABLE 1
LDPs of the family $\{\varepsilon I_\Lambda\}$, when the nodes are distributed according to a β-Ginibre process, $0 < \beta \le 1$

Fading distribution	Speed	Rate function
Bounded	$\frac{1}{\varepsilon^2}\log\left(\frac{1}{\varepsilon}\right)$	$\frac{R^{2\alpha}x^2}{2B}$
Weibull superexponential	$\varepsilon^{-\frac{2\gamma}{\gamma+1}}\log^{\frac{\gamma-1}{\gamma+1}}\left(\frac{1}{\varepsilon}\right)$	$\frac{1}{2}R^{\frac{2\alpha\gamma}{\gamma+1}}\left(\frac{\gamma}{\gamma-1}\right)^{\frac{\gamma-1}{\gamma+1}}(c(\gamma+1))^{\frac{2}{\gamma+1}}x^{\frac{2\gamma}{\gamma+1}}$

TABLE 2
LDPs of the family $\{\varepsilon I_\Lambda\}$, when the nodes are distributed according to a Poisson process

Fading distribution	Speed	Rate function
Bounded	$\frac{1}{\varepsilon}\log\left(\frac{1}{\varepsilon}\right)$	$\frac{R^\alpha x}{B}$
Weibull superexponential	$1/\varepsilon\log^{1-\frac{1}{\gamma}}\left(\frac{1}{\varepsilon}\right)$	$\gamma(\gamma-1)^{\frac{1}{\gamma}-1}c^{\frac{1}{\gamma}}R^\alpha x$

TABLE 3
LDPs of the family $\{\varepsilon I_\Lambda\}$, when the number of nodes is light-tailed. Here $\overline{F} = 1 - F$, being F the distribution function of the fading and $\sigma > 0$, $\gamma \ge 0$

Fading distribution	Speed	Rate function
Exponential	$1/\varepsilon$	$cR^\alpha x$
$\log\overline{F}(\sigma x) \sim \sigma^\gamma \log\overline{F}(x)$	$-\log\overline{F}(1/\varepsilon)$	0 if $x = 0$; $R^{\alpha\gamma}x^\gamma$ if $x > 0$

8. Conclusions. The results of this paper contribute to better understand the reliability of large scale wireless networks. We proved asymptotic estimates, on the log-scale, for the tail of the interference in a network whose nodes are placed according to a β-Ginibre process (with $0 < \beta \le 1$) and the fading random variables are bounded or Weibull superexponential. We gave also asymptotic estimates, on the log-scale, for the tail of the interference in a network whose nodes are placed according to a general point process and the fading random variables are exponential or subexponential. The results, summarized in Tables 1 and 3, show the emergence of two different regimes (for the ease of comparison results for the Poisson model under bounded or Weibull superexponential fading are reported in Table 2). When the fading variables are bounded or Weibull superexponential, the tail of the interference heavily depends on the node spatial process. Instead, when the fading variables are exponential or subexponential, the tail of the interference is essentially insensitive to the distribution of nodes, as long as the number of nodes is guaranteed to be light-tailed.

APPENDIX

Proof of Proposition 3.1. Let F be a closed subset of $[0, \infty)$ and let x denote the infimum of F. Since I is increasing, $I(x) = \inf_{y\in F} I(y)$. Since F is contained in $[x, \infty)$, by the large deviation upper bound for closed half-intervals $[x,\infty)$ we deduce

$$\begin{aligned}\limsup_{\varepsilon\to 0}\frac{1}{v(\varepsilon)}\log\mathbb{P}(V_\varepsilon\in F) &\leq \limsup_{\varepsilon\to 0}\frac{1}{v(\varepsilon)}\log\mathbb{P}(V_\varepsilon\geq x)\\ &\leq -I(x)=-\inf_{y\in F}I(y).\end{aligned}$$

This establishes the large deviation upper bound for arbitrary closed sets.

Now, let G be an open subset of $[0,\infty)$. Suppose first that $0\notin G$. Since $\inf_{y\in G}I(y)<\infty$, for arbitrary $\delta>0$, we can find $x\in G$ such that $I(x)\leq \inf_{y\in G}I(y)+\delta$. Since G is open, we can also find $\eta>0$ such that $(x-\eta, x+\eta)\subset G$. By the large deviation bounds on half-intervals we have

$$\liminf_{\varepsilon\to 0}\frac{1}{v(\varepsilon)}\log P(V_\varepsilon>x-\eta)\geq -I(x-\eta)$$

and

$$\limsup_{\varepsilon\to 0}\frac{1}{v(\varepsilon)}\log\mathbb{P}(V_\varepsilon\geq x+\eta)\leq -I(x+\eta),$$

and by the monotonicity of I we deduce $I(x-\eta)\leq I(x+\eta)$. Consequently, after an easy computation we get

$$\liminf_{\varepsilon\to 0}\frac{1}{v(\varepsilon)}\log(\mathbb{P}(V_\varepsilon>x-\eta)-\mathbb{P}(V_\varepsilon\geq x+\eta))\geq -I(x-\eta).$$

Note that

$$\mathbb{P}(V_\varepsilon\in G)\geq\mathbb{P}(V_\varepsilon\in(x-\eta,x+\eta))=\mathbb{P}(V_\varepsilon>x-\eta)-\mathbb{P}(V_\varepsilon\geq x+\eta),$$

and so

$$\liminf_{\varepsilon\to 0}\frac{1}{v(\varepsilon)}\log\mathbb{P}(V_\varepsilon\in G)\geq -I(x-\eta).$$

Since I is continuous on $(0,\infty)$, by letting η tend to zero we get

$$\liminf_{\varepsilon\to 0}\frac{1}{v(\varepsilon)}\log\mathbb{P}(V_\varepsilon\in G)\geq -I(x)\geq -\inf_{y\in G}I(y)-\delta,$$

where the latter inequality follows by the choice of x. The large deviation lower bound for arbitrary open sets not containing the origin follows letting δ tend to zero. If $0\in G$, then, since G is open, there is an $\eta>0$ such that $[0,\eta)\subset G$. Hence,

$$\mathbb{P}(V_\varepsilon\in G)\geq 1-\mathbb{P}(V_\varepsilon\geq\eta).$$

By similar arguments to the above, we can show that

$$\liminf_{\varepsilon\to 0}\frac{1}{v(\varepsilon)}\log\mathbb{P}(V_\varepsilon\in G)\geq 0.$$

Since I is increasing we have $\inf_{y\in G}I(y)=I(0)=0$, and the proof is completed.

Proof of Lemma 4.2. *Proof of (i)* By Theorem 6 in [26], for any fixed $r>0$ and $x_0\in\mathbb{C}$, we have

$$\mathbb{P}\left(\sum_{i\geq 1}\mathbb{1}_{b(x_0,r)}(V_i)\geq m\right)=\mathrm{e}^{-\frac{1}{2}m^2\log m(1+o(1))} \tag{44}$$

(note that the processes $\{X_i\}_{i\geq 1}$ and $\{V_i\}_{i\geq 1}$ are different and so a priori one can not say that the tails of $\mathbb{P}(\sum_{i\geq 1}\mathbb{1}_{b(x_0,r)}(V_i)\geq m)$ and $\mathbb{P}(N(b(x_0,r))\geq m)$ are equal.) Since the Ginibre process is stationary, so is the independently thinned process and thus it suffices to check (2) with $x_0=O$. By (44) we have

$$\mathbb{P}\left(\sum_{i\geq 1}\mathbb{1}_{b(O,r)}(V_i)\mathbb{1}_{A_i}\geq m\right)\leq \mathrm{e}^{-\frac{1}{2}m^2\log m(1+o(1))}.$$

It remains to check the matching lower bound. The function

$$r\mapsto\mathbb{P}\left(\sum_{i\geq 1}\mathbb{1}_{b(O,r)}(V_i)\mathbb{1}_{A_i}\geq m\right)$$

is clearly nondecreasing. Since we are going to check the lower bound, we may assume $0<r<1$. We have

$$\begin{aligned}\mathbb{P}\left(\sum_{i\geq 1}\mathbb{1}_{b(O,r)}(V_i)\mathbb{1}_{A_i}\geq m\right)&\geq\mathbb{P}\left(\mathbb{1}_{b(O,r)}(V_i)\mathbb{1}_{A_i}=1,\ \forall\, i=1,\dots,m\right)\\&=\mathbb{P}\left(|V_i|<r,\ A_i,\ \forall\, i=1,\dots,m\right)\\&=\mathbb{P}(A_1)^m\mathbb{P}(|V_i|<r,\ \forall\, i=1,\dots,m).\end{aligned} \tag{45}$$

By Theorem 1.1 in [25] (see also Theorem 4.7.3 p. 73 in [22]) the set $\{|V_i|\}_{i\geq 1}$ has the same distribution as the set $\{\rho_i\}_{i\geq 1}$, where the random variables ρ are independent and ρ_i^2 has the Gamma(i,1) distribution for every $i\geq 1$. Hence ρ_i^2 has the same distribution of $\xi_{i1}+\dots+\xi_{ii}$, where the random variables $\{\xi_{jk}\}_{j,k\geq 1}$ are independent and have the Exponential(1) distribution. So

$$\begin{aligned}\mathbb{P}\left(|V_i|<r,\ \forall\, i=1,\dots,m\right)&=\mathbb{P}\left(\rho_i^2<r^2,\ \forall\, i=1,\dots,m\right)\\&=\mathbb{P}\left(\sum_{k=1}^{i}\xi_{ik}<r^2,\ \forall\, i=1,\dots,m\right)\\&=\prod_{i=1}^{m}\mathbb{P}\left(\sum_{k=1}^{i}\xi_{ik}<r^2\right)\end{aligned} \tag{46}$$

$$\geq \prod_{i=1}^{m} \mathbb{P}\left(\xi_{ik} < r^2/i,\ \forall\, k = 1, \dots, i\right)$$

$$= \prod_{i=1}^{m} \prod_{k=1}^{i} \mathbb{P}\left(\xi_{ik} < \frac{r^2}{i}\right) = \prod_{i=1}^{m} \left(1 - \mathrm{e}^{-\frac{r^2}{i}}\right)^i \tag{47}$$

$$\geq \prod_{i=1}^{m} \left(\frac{r^2}{2i}\right)^i, \tag{48}$$

where (46) and the first equality in (47) follow by the independence of the random variables ξ_{jk} and the inequality (48) is a consequence of the fact that $0 < r^2/i < 1$ for any $i = 1, \dots, m$ and $1 - \mathrm{e}^{-x} \geq x/2$ for $0 < x < 1$. Combining (45) and (48) and using the elementary inequality

$$\mathbb{P}(A_1)^m \geq \mathbb{P}(A_1)^{m(m+1)/2} = \prod_{i=1}^{m} \mathbb{P}(A_1)^i$$

we have

$$\mathbb{P}\left(\sum_{i \geq 1} \mathbb{1}_{b(O,r)}(V_i)\mathbb{1}_{A_i} \geq m\right) \geq \prod_{i=1}^{m} \left(\frac{\mathbb{P}(A_1) r^2}{2i}\right)^i. \tag{49}$$

A straightforward computation shows that

$$\begin{aligned}
\prod_{i=1}^{m} \left(\frac{\mathbb{P}(A_1) r^2}{2i}\right)^i &= \left(\frac{\mathbb{P}(A_1) r^2}{2}\right)^{\frac{m(m+1)}{2}} \exp\left(-\sum_{i=1}^{m} i \log i\right) \\
&\geq \left(\frac{\mathbb{P}(A_1) r^2}{2}\right)^{\frac{m(m+1)}{2}} \exp\left(-\frac{1}{2}(m+1)^2 \log(m+1) + \frac{(m+1)^2}{4} - \frac{1}{4}\right) \qquad (50) \\
&= \mathrm{e}^{-\frac{1}{2} m^2 \log m (1 + o(1))},
\end{aligned}$$

where the inequality in (50) follows by the elementary relation:

$$\sum_{i=1}^{m} i \log i \leq \frac{1}{2}(m+1)^2 \log(m+1) - \frac{(m+1)^2}{4} + \frac{1}{4}, \qquad m \geq 1.$$

The proof is completed.

Proof of (ii) Letting $\{U\} \cup \{U_i\}_{i \geq 1}$ denote a sequence of independent random variables uniformly distributed on $[0, 1]$ and Z denote a random variable distributed as Z_1, and assuming that the random variables $\{U, Z\} \cup$

$\{U_i\}_{i\geq 1}$ are independent of all the other random quantities. For any bounded and measurable set $\Lambda' \subset \mathbb{C}$, by Lemma 4.1 we have

$$\begin{aligned} N(\Lambda') + \mathbb{1}_{\Lambda'}(\sqrt{\beta}G)\mathbb{1}\{U < \beta\} &\overset{law}{=} \sum_{i\geq 1} \mathbb{1}_{\Lambda'}(\sqrt{\beta}V_i)\mathbb{1}\{U_i < \beta\} \\ &= \sum_{i\geq 1} \mathbb{1}_{\Lambda'/\sqrt{\beta}}(V_i)\mathbb{1}\{U_i < \beta\}, \end{aligned} \tag{51}$$

where the symbol $\overset{law}{=}$ denotes the identity in law. Note that

$$\begin{aligned} &\mathbb{P}\left(N(b(x_0,r)) + \mathbb{1}_{b(x_0,r)}(\sqrt{\beta}G)\mathbb{1}\{U < \beta\} \geq m+1\right) \\ &\quad \leq \mathbb{P}\Big(N(b(x_0,r)) + \mathbb{1}_{b(x_0,r)}(\sqrt{\beta}G)\mathbb{1}\{U < \beta\} \geq m \\ &\qquad + \mathbb{1}_{b(x_0,r)}(\sqrt{\beta}G)\mathbb{1}\{U < \beta\}\Big) \\ &\quad = \mathbb{P}\left(N(b(x_0,r)) \geq m\right). \end{aligned}$$

Combining (51) (with $\Lambda' = b(x_0,r)$) and this latter relation, we have

$$\begin{aligned} &\mathbb{P}\left(\sum_{i\geq 1} \mathbb{1}_{b(x_0/\sqrt{\beta}, r/\sqrt{\beta})}(V_i)\mathbb{1}\{U_i < \beta\} \geq m+1\right) \\ &\leq \mathbb{P}(N(b(x_0,r)) \geq m) \\ &\leq \mathbb{P}(N(b(x_0,r)) + \mathbb{1}_{b(x_0,r)}(\sqrt{\beta}G)\mathbb{1}\{U < \beta\} \geq m) \\ &\leq \mathbb{P}\left(\sum_{i\geq 1} \mathbb{1}_{b(x_0/\sqrt{\beta}, r/\sqrt{\beta})}(V_i)\mathbb{1}\{U_i < \beta\} \geq m\right). \end{aligned}$$

The claim follows by (2).

Proof of Lemma 4.3. By (51) we have

$$\begin{aligned} \mathbb{E}[N(\Lambda')] &\leq \mathbb{E}\left[N(\Lambda') + \mathbb{1}_{\Lambda'}(\sqrt{\beta}G)\mathbb{1}\{U < \beta\}\right] \\ &= \mathbb{E}\left[\sum_{i\geq 1} \mathbb{1}_{\Lambda'/\sqrt{\beta}}(V_i)\mathbb{1}\{U_i < \beta\}\right] \\ &\leq \mathbb{E}\left[\sum_{i\geq 1} \mathbb{1}_{\Lambda'/\sqrt{\beta}}(V_i)\right] \\ &= \sum_{n\geq 1} \kappa_n(\Lambda'/\sqrt{\beta}) < \infty, \end{aligned} \tag{52}$$

where (52) follows by e.g. Proposition 2.3 in [32] and formula (3.41) in [31]. Now we prove (4). Let $\theta \geq 0$ be arbitrarily fixed. We start checking that

$$\prod_{n\geq 1}(1+(\mathrm{e}^{\theta}-1)\kappa_n(\Lambda'/\sqrt{\beta}))<\infty.$$

We have

$$\begin{aligned}\log\prod_{n\geq 1}(1+(\mathrm{e}^{\theta}-1)\kappa_n(\Lambda'/\sqrt{\beta})) &= \sum_{n\geq 1}\log(1+(\mathrm{e}^{\theta}-1)\kappa_n(\Lambda'/\sqrt{\beta}))\\ &\leq (\mathrm{e}^{\theta}-1)\sum_{n\geq 1}\kappa_n(\Lambda'/\sqrt{\beta})<\infty,\end{aligned} \tag{53}$$

where in (53) we used the inequality $x \geq \log(1+x)$, $x \geq 0$, and (3). Finally, we prove the first inequality in (4). Using again (51), for any $\theta \geq 0$, we have

$$\begin{aligned}\mathbb{E}[\mathrm{e}^{\theta N(\Lambda')}] \leq \mathbb{E}\left[\mathrm{e}^{\theta(N(\Lambda')+\mathbb{1}_{\Lambda'}(\sqrt{\beta}G)\mathbb{1}\{U<\beta\})}\right] &\leq \mathbb{E}\left[\exp\left(\theta\sum_{i\geq 1}\mathbb{1}_{\Lambda'/\sqrt{\beta}}(V_i)\right)\right]\\ &= \prod_{n\geq 1}(1+(\mathrm{e}^{\theta}-1)\kappa_n(\Lambda'/\sqrt{\beta})),\end{aligned}$$

where the latter equality follows by e.g. Proposition 2.2 in [32]. The proof is completed.

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