Noise compliant macromodel synthesis for RF and Mixed-Signal applications

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***Abstract*—This paper proposes a compact synthesis approach for reduced-order behavioral macromodels of linear circuit blocks for RF and Mixed-Signal design. The proposed approach revitalizes the classical synthesis of lumped linear and time-invariant multiport networks by reactance extraction, which is here exploited to obtain reduced-order equivalent SPICE netlists that can be used in any type of system-level simulations, including transient and noise analysis. The effectiveness of proposed approach is demonstrated on a real design application.**

I. Introduction

Design and Analysis of System on Chip (SoC) and System in Package (SiP) components for mobile applications becomes more challenging from day to day. Parasitic effects from on-chip, package and Printed Circuit Board (PCB) multi-layer interconnect stacks are increasingly relevant for performance assessment and verification and need to be carefully evaluated in pre-tapeout (Chip, SoC) and module (Package, PCB) design phases to avoid complete system fails or even yield losses in the production ramp-up phase. In such complex systems, many lossy passive devices are already integrated in the metal stacks, like shielded transmission lines, integrated coils, transformers, inductors and capacitors. All these passive devices and interconnect structures are lossy, that means they have beneath distributed capacitance C/m and distributed inductance L/m also a distributed resistivity R/m per unit length. This internal device resistivity per unit length significantly contributes to the overall noise behaviour of the communication system and must be thoroughly taken in account.

Nowadays, it is a common practice to assess system performances by means of Computer Aided Design (CAD) simulation methodologies. To cope with the complexity of system level simulations, divide and conquer approaches prove quite effective. The whole system is first decomposed into macro-blocks. By clever partitioning, the designer can split the system into mainly nonlinear (active) circuit blocks and linear (passive) circuit blocks interacting through well defined ports. For the linear building blocks, Linear Time Invariant (LTI) models with low complexity can be extracted by means of standard techniques [1] and synthesized into a reduced-order, linear network using resistors (R), capacitors (C) and controlled sources (CS) [2]. This technique leads to tremendous complexity reduction and can be readily extended to model soft non-linear devices under suitable biasing [3], [4]. The main drawback of this methodology lies in the noise analysis simulation.

The RCCS synthesis from [2] is quite popular in Model Order Reduction applications because it mimics accurately the input-output response of the LTI model using a small number of network elements. Unfortunately, simulated noise spectra are often orders of magnitude away from the real physical behaviour of system under analysis, mainly because the resistors extracted by the RCCS synthesis of black-box or behavioral macromodels are not "physical", but only dummy elements used to cast the state-space equations of the macromodel in a SPICE-compatible form. Therefore, any physics-based noise model applied to such components is intrinsically ill-defined. Moreover, dependent sources do not have an associated noise model and inevitably lead to incorrect results in circuit-based noise characterization.

In order to extract proper resistors preserving the noise behaviour of the system, the RCCS synthesis from [2] is replaced in this work by a classical RLCT (Resistors, Inductors, Capacitors and ideal Transformers) synthesis from [5]. This work demonstrates in practice that this synthesis method not only reproduces accurately the input-output response of the LTI system but also preserves the noise response of the original system extracting the resistors associated to the unique passive invariants of the reciprocal lumped linear network [11].

This noise-preserving synthesis is very important for a widespread application of LTI models in RF and Mixed-Signal (MS) applications. The relevance of the RLCT synthesis was underestimated for several years due to the need of ideal transformers. As it was demonstrated by McMillan [6] the usage of transformers can not be avoided in the synthesis of LTI networks. Thus, the RLCT synthesis is used here to exploit the benefits of this classic method to automatically construct reduced-order LTI macromodels for RF applications, which can be applied in all SPICE analysis types (DC, AC TRAN, NOISE, HBB, PSS) preserving the correct physical noise behaviour. The availability of a noise compliant synthesis opens the way for notable simplifications and improvements in model-based analysis and design verification methodology.

II. Preliminaries

Consider a given linear and passive circuit block. It is common practice to model this network via frequency-dependent scattering parameters, which are easily extracted by common EDA tools from layout data. A system level circuit analysis however requires the conversion of such representation into an equivalent circuit which can be directly employed in transient analysis using solvers of the SPICE class. This conversion, usually denoted as *macromodeling*, amounts to fitting the scattering parameter data with a rational function of frequency specified in terms of poles and residue matrices. This task is easily achieved by widespread tools such as Vector Fitting (VF) [1]. Once available, the rational macromodel can be converted to state-space form

$$\dot{x}(t) = \mathbf{A}x(t) + \mathbf{B}u(t), \tag{1}$$
$$y(t) = \mathbf{C}x(t) + \mathbf{D}u(t), \tag{2}$$

with $\mathbf{A} \in \mathbb{R}^{n\times n}$, $\mathbf{B} \in \mathbb{R}^{n\times p}$, $\mathbf{C} \in \mathbb{R}^{p\times n}$ and $\mathbf{D} \in \mathbb{R}^{p\times p}$, which in turn is readily synthesized into an equivalent netlist made of resistors, capacitors, and controlled sources [2].

The above process can be directly applied to admittance or impedance data. In the following, the models are assumed to be in the impedance input-output representation, so that

$$\mathbf{Z}(s) = \mathbf{C}(s\mathbf{I}-\mathbf{A})^{-1}\mathbf{B}+\mathbf{D} \leftrightarrow \left(\begin{array}{c|c}\mathbf{A} & \mathbf{B}\\ \hline \mathbf{C} & \mathbf{D}\end{array}\right), \tag{3}$$

where s is the complex frequency (Laplace) variable.

The McMillan degree [12] of $\mathbf{Z}(s)$ is equal to n (the size of $\mathbf{A}$) when the state-space realization (3) is minimal, i.e.,when the system is both controllable and observable. Although state-space realizations are not unique, two minimal state-space realizations of the same system

$$\left(\begin{array}{c|c}\mathbf{A} & \mathbf{B}\\ \hline \mathbf{C} & \mathbf{D}\end{array}\right) \leftrightarrow \left(\begin{array}{c|c}\tilde{\mathbf{A}} & \tilde{\mathbf{B}}\\ \hline \tilde{\mathbf{C}} & \tilde{\mathbf{D}}\end{array}\right) \tag{4}$$

are equivalent to each other through a change of basis in the state space [12], applied as a similarity transformation as

$$\tilde{\mathbf{A}} = \mathbf{T}^{-1}\mathbf{A}\mathbf{T}, \qquad \tilde{\mathbf{B}} = \mathbf{T}^{-1}\mathbf{B}, \tag{5}$$
$$\tilde{\mathbf{C}} = \mathbf{C}\mathbf{T}, \qquad \tilde{\mathbf{D}} = \mathbf{D}, \tag{6}$$

with $\mathbf{T} \in \mathbb{R}^{n\times n}$ invertible.

A minimal system is passive [8] if and only if the Kalman-Yakubovich-Popov (KYP), also known as Positive-Real Lemma (PRL) [9] holds,

$$\exists\ \mathbf{P} = \mathbf{P}^T \succ 0 \text{ s.t.} \begin{bmatrix}\mathbf{A}^T\mathbf{P}+\mathbf{P}\mathbf{A} & \mathbf{P}\mathbf{B}-\mathbf{C}^T\\ \mathbf{B}^T\mathbf{P}-\mathbf{C} & -\mathbf{D}-\mathbf{D}^\mathbf{T}\end{bmatrix} \preceq 0 \tag{7}$$

If (7) is not fulfilled after the rational fitting process, a passivity enforcement algorithm can be applied, see [10], [16] and references therein.

III. Synthesis by Reactance Extraction

The RLCT macromodel synthesis used in this work is based on the well-known procedure of *reactance extraction* [5]. Suppose to start with a LTI multiport structure $\mathcal{N}$ with p ports and n internal dynamic elements. This structure is interpreted as the connection of a purely adynamic part $\mathcal{N}_t$ with $p+n$ ports, connected to a purely dynamic part $\mathcal{N}_d$ including all capacitors, inductors and mutual couplings, see Fig. 1.

Each port of the dynamic part $\mathcal{N}_d$ is formed by a single capacitor or inductor. This implies that the hybrid matrix of $\mathcal{N}_d$ is simply $s\mathbf{\Lambda}$, where all ports connected to a capacitor (resp. inductor) are defined as voltage-controlled (resp. current-controlled), and where $\mathbf{\Lambda}$ is a block-diagonal matrix, which reduces to a diagonal matrix in case of no inductive mutual coupling elements.

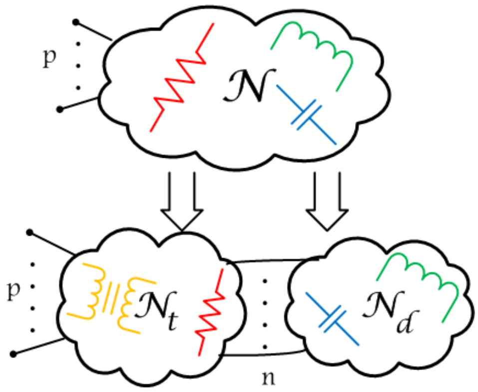

Fig. 1. Decomposition of a general p-port LTI network $\mathcal{N}$ into an adynamic $(p+n)$-port subnetwork $\mathcal{N}_t$ connected to a dynamic n-port (lossless) part $\mathcal{N}_d$ through n internal ports.

The constant non-dynamic network $\mathcal{N}_t$ can be described by its hybrid matrix

$$\mathbf{M} = \begin{bmatrix}\mathbf{M}_{1,1} & \mathbf{M}_{1,2}\\ \mathbf{M}_{2,1} & \mathbf{M}_{2,2}\end{bmatrix} \tag{8}$$

with $\mathbf{M} \in \mathbb{R}^{(p+n)\times(p+n)}$, $\mathbf{M}_{1,1} \in \mathbb{R}^{p\times p}$ and $\mathbf{M}_{2,2} \in \mathbb{R}^{n\times n}$. The first block of p ports of $\mathbf{M}$ are defined as current-controlled, whereas the second block of n ports connected to $\mathcal{N}_d$ are defined as voltage- or current-controlled, according to their corresponding definition in $\mathbf{\Lambda}$.

These premises imply that the impedance matrix observed at the input p ports of $\mathcal{N}_t$ when loaded by $\mathcal{N}_d$ reads

$$\mathbf{Z}(s) = \mathbf{M}_{1,1} - \mathbf{M}_{1,2}(s\mathbf{\Lambda}+\mathbf{M}_{2,2})^{-1}\mathbf{M}_{2,1}. \tag{9}$$

Assume now that the circuit block is known via a state-space realization of its impedance matrix (3). A direct comparison between (9) and (3) suggests that the circuit synthesis can be performed by defining

$$\mathbf{M} = \begin{bmatrix}\mathbf{D} & -\mathbf{C}\\ \mathbf{B} & -\mathbf{A}\end{bmatrix}, \qquad \mathbf{\Lambda} = \mathbf{I}_n \tag{10}$$

and by interpreting $\mathbf{M}$ as the constant hybrid matrix of a $p+n$ adynamic multiport element, and $s\mathbf{I}_n$ as the hybrid matrix of a dynamic (lossless) multiport element constructed by unitary capacitances and inductances directly connected at its n ports. The synthesis of $\mathbf{M}$ using a network of controlled sources is straightforward and well known from network theory [11].

The main objective is here to synthesize the state-space system (3) into an equivalent circuit which can be used in noise analysis. This requires that, due to the limitations imposed by SPICE-like solvers, the only elements that should be responsible for power dissipation, hence noise generation, should be resistors. In fact, only resistors are equipped by a suitable noise model within standard SPICE solvers, whereas other components such as controlled sources, although they may dissipate power, unfortunately do not have a corresponding noise model. For this reason controlled sources are not allowed in our synthesis, and the attention is restricted to a RLCT synthesis, where the T stands for ideal (multiport) *transformers*. Since the latter are neutral elements for what concerns power [11], their contribution to a noise analysis is null. Since inductors and capacitors do not contribute as noise sources, the overall RLCT netlist is thus guaranteed to provide consistent noise results.

Any RLCT network is passive and reciprocal. Therefore, the proposed approach will be applicable only to reciprocal systems, for which

$$\mathbf{Z}(s) = \mathbf{Z}(s)^T \,. \tag{11}$$

Since $\mathbf{\Lambda}$ in (10) represents a passive (lossless) and reciprocal network by construction, passivity and reciprocity of $\mathbf{Z}(s)$ obtained by reactance extraction is guaranteed by the passivity and reciprocity of its adynamic part $\mathcal{N}_t$. These conditions are guaranteed when

$$\mathbf{M} + \mathbf{M}^T \succeq 0 \tag{12}$$

$$\widehat{\mathbf{\Sigma}}\mathbf{M} = \mathbf{M}^T\widehat{\mathbf{\Sigma}}, \tag{13}$$

where $\widehat{\mathbf{\Sigma}} = \text{blkdiag}\{\mathbf{I}_p, \mathbf{\Sigma}\}$, and where $\mathbf{\Sigma} \in \mathbb{R}^{n\times n}$ is the so-called *reactance signature* matrix [5], basically a diagonal matrix of 1 and -1 placed in correspondence of the current-controlled and voltage-controlled ports, respectively. Since a general state-space realization (3) does not satisfy those constraints, the main enabling factor for this synthesis process is to find a transformation matrix $\mathbf{T}$ that converts via (4) a given state-space system into an equivalent realization for which (12)-(13) hold. This transformation is detailed next, following [12].

Before discussing further details, we should emphasize that the main drawback of the RLCT synthesis, despite being canonical [12], lies in the complexity of the resulting netlist, intended as number of primitive network elements, which scales as $\mathcal{O}(n^2p^2)$. For the RCCS synthesis complexity scales as $\mathcal{O}(np)$. As a consequence, this method is of practical relevance only for models with small or medium number of ports and dynamical order.

IV. Reciprocal Positive Real Realizations

We start from the generic (passive and reciprocal) state-space realization $\{\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}\}$ in (3), and we explicitly solve the Positive Real Lemma (7) for $\mathbf{P}$. Also, we form the dual system $\{\mathbf{A}^T, \mathbf{C}^T, \mathbf{B}^T, \mathbf{D}^T\}$ and we solve the PRL for the corresponding matrix $\mathbf{Q}$. Restricting now the analysis to the case $\mathbf{R} = \mathbf{D} + \mathbf{D}^T \succ 0$ (corresponding to asymptotic strict dissipativity), it follows that the matrices $\mathbf{P}$ and $\mathbf{Q}$ can be found by solving the Continuous Algebraic Riccati Equations (CARE) [13]

$$\mathbf{A}^T\mathbf{P} + \mathbf{P}\mathbf{A} + \left(\mathbf{P}\mathbf{B} - \mathbf{C}^T\right)\mathbf{R}^{-1}\left(\mathbf{B}^T\mathbf{P} - \mathbf{C}\right) = 0, \tag{14}$$

$$\mathbf{A}\mathbf{Q} + \mathbf{Q}\mathbf{A}^T + \left(\mathbf{Q}\mathbf{C}^T - \mathbf{B}\right)\mathbf{R}^{-1}\left(\mathbf{C}\mathbf{Q} - \mathbf{B}^T\right) = 0, \tag{15}$$

with $\mathbf{P} = \mathbf{P}^T \succ 0$ and $\mathbf{Q} = \mathbf{Q}^T \succ 0$. This calculation can be performed through the Laub's method [14], based on the evaluation of the invariant subspaces of the Hamiltonian matrices associated to (14) and (15).

The next step is to compute the Cholesky factorization [15] of $\mathbf{P}$ and $\mathbf{Q}$

$$\mathbf{P} = \mathbf{F}^T\mathbf{F}, \tag{16}$$

$$\mathbf{Q} = \mathbf{G}^T\mathbf{G}, \tag{17}$$

with $\mathbf{F}, \mathbf{G} \in \mathbb{R}^{n\times n}$ triangular matrices. The matrix product $\mathbf{F}\mathbf{G}^T$ is then subject to a Singular Value Decomposition [15]

$$\mathbf{F}\mathbf{G}^T = \mathbf{U}\mathbf{S}\mathbf{V}^T, \tag{18}$$

with $\mathbf{U}, \mathbf{V} \in \mathbb{R}^{n\times n}$ orthogonal, where the diagonal matrix $\mathbf{S} \in \mathbb{R}^{n\times n}$ stores the singular values in decreasing order on its main diagonal. An invertible similarity transformation matrix $\mathbf{T}$ is now defined as

$$\mathbf{T} = \mathbf{G}^T\mathbf{V}\mathbf{S}^{-1/2} \tag{19}$$

and applied as in (4). A complete proof that the resulting state-space realization verifies (12)-(13) can be found in [7].

At this point the synthesis of the constant hybrid matrix $\mathbf{M}$ (10) associated to the network $\mathcal{N}_t$ can be performed according to [11]. The outer diagonal blocks in (10), i.e. $\mathbf{M}_{1,2}$ and $\mathbf{M}_{2,1}$, are directly synthesized via multiport ideal transformers. The main diagonal blocks, i.e. $\mathbf{M}_{1,1}$ and $\mathbf{M}_{2,2}$, are first diagonalized through their orthogonal eigenvector matrices [15], and then synthesized via multiport ideal transformers and unitary resistors. The connection of n internal ports of $\mathcal{N}$ on unitary inductors and capacitors, as depicted in Fig. 1, concludes the synthesis process.

V. Example

Several tests were performed to assess the reliability of the proposed RLCT synthesis for RF and MS applications. A representative case is illustrated below, namely the centrally involved LC-tank coil of a single-coil Digitally Controlled Oscillator (DCO). DCOs can be tuned very accurately: their noise behaviour is a key figure of merit and requires therefore accurate noise modeling of all involved design parts. Thus modeling of the centrally involved LC-tank coil is a good benchmark for the noise compliant synthesis.

The Scattering parameters obtained from full-wave simulations of the centrally involved LC-tank coil of real DCO for RF applications have been used as starting point for derivation and synthesis of a macromodel. An initial macromodel was obtained by the Vector Fitting scheme [1]. This macromodel was already passive, as confirmed by a check via [16]. The port count for this example is 25 and the McMillan degree of the resulting model is 350. The state-space transformation (5) discussed in this work was then performed, leading to a synthesized RLCT netlist in SPICE form. A frequency (AC analysis) sweep of the macromodel in SPICE led to the frequency responses depicted and compared to the original data in Fig. 2. The frequency dependent noise analysis results

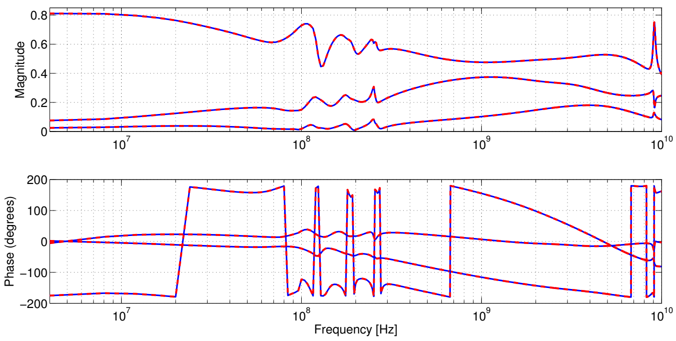

Fig. 2. Selected S-parameters from the centrally involved LC-tank coil of a real RF Digitally Controlled Oscillator. The blue continuous lines are the S-parameters obtained from full-wave simulation; the red dashed lines are obtained by SPICE simulation of the RLCT synthesized macromodel.

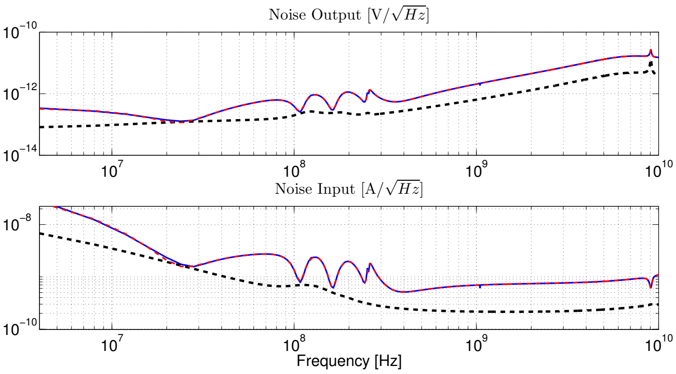

Fig. 3. Port 1 frequency dependent input-output noise spectral density. The input noise is depicted in the bottom plot while the output noise is in the top plot. The blue continuous lines are the noise results obtained using the S-parameter raw data [17]; the red dashed lines are obtained by SPICE simulation of the RLCT synthesized macromodel, while the black dashed lines come from the SPICE simulation of the RCCS synthesized macromodel.

are depicted in Fig. 3. This figures confirms the excellent accuracy of the macromodel.

The RLCT macromodel was used to perform a noise analysis of the structure, whose results are reported in Table I. The Table also reports the results from the same noise simulation using directly the S-parameters [17], as well as the results obtained using the macromodel synthesized as a RCCS network. As expected, the RLCT synthesis produces the same results that can be obtained directly from the S-parameters analysis, while the RCCS provides wrong answers.

TABLE I
DC INPUT NOISE SPECTRAL DENSITY $[A/\sqrt{Hz}]$ AND OUTPUT NOISE SPECTRAL DENSITY $[V/\sqrt{Hz}]$ FOR AN RF SINGLE COIL DIGITALLY CONTROLLED TRANSFORMER.

Port		SP data		RLCT synth		RCCS synth	
In	Out	Input	Output	Input	Output	Input	Output
1	1	4.7e-8	3.6e-13	4.7e-8	3.6e-13	1.1e-8	7.7e-14
1	2	4.8e-8	3.4e-13	4.8e-8	3.4e-13	1.1e-8	7.7e-14
2	1	4.8e-8	3.4e-13	4.8e-8	3.4e-13	1.1e-8	7.7e-14
2	2	1.1e-7	1.4e-13	1.1e-7	1.4e-13	1.1e-8	7.7e-14

VI. CONCLUSION

A noise compliant RLCT synthesis method for linear behavioral macromodels based on the classical reactance extraction technique is presented. Relying solely on the use of network elements possessing a proper noise model in SPICE based solvers, the proposed strategy is able to reproduce properly the noise behaviour of the system. The accuracy of the results obtained from the noise analysis is assessed by comparing the proposed synthesis with standard methods [17], [2].

The availability of a noise compliant network synthesis can be of paramount importance in analog behavioural modeling for devices and complete building blocks. Noise-preserving modeling is a must for simulation-based design and design verification purposes of complex analog systems. Unfortunately the RLCT synthesis requires a number of networks elements which scales as $\mathcal{O}(n^2p^2)$. Further investigations are needed in order to reduce the complexity of the synthesized network while preserving the noise behaviour.

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