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*Original*

*Availability:*
This version is available at: 11583/2522891 since:

*Publisher:*
Elsevier

*Published*
DOI:10.1016/j.sbspro.2013.12.838

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Modeling the Retail System Competition

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Abstract

The retail system is a competitive environment and its transformations have a relevant socio-economic impact. In this context, it is important to represent customer-store interactions, and, to this end, literature mostly proposes logit models. It is well-known that these models present some behavioral and structural anomalies (e.g., the Independence-from-Irrelevant-Alternatives) making them hardly applicable to retail system analysis. In this paper, we show that even some alternative approaches (e.g. Nested-logit or Paired-Combinatorial logit models) do not suitably represent the competition between retail stores, and we present a new modeling framework. It aims at overcoming the above limits by two cooperating logit-based models: the first one analyzes customer-store interactions; the second model uses the interaction information to evaluate the impact of some major transformations. The framework has been integrated in a decision support system and used in real-life cases to determine the impact of new stores in some Italian regions.

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Selection and peer-review under responsibility of AIRO.

Keywords: Retail System Competition ; Logit Model ; Spatial Clusters ; Customer-Store Interactions ; Impact Analysis

1. Introduction

The analysis of a retail system consists in studying the demand, the supply and their economic and spatial interactions determined by the customer choice of the stores to patronize. Roughly speaking, the store choice depends, among other factors, on the store type (Gonzalez-Benito, 2005) and customer preferences, which in turn depend on customer socio-economic characteristics. To this end, stores can be clustered into different types characterized by efficiency, competition levels, marketing, and retail strategies. In particular, two main groups can be identified

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• *Traditional* stores (i.e. corner shops): they are small sized shops, with, generally, high price levels, low efficiency and productivity and, often, high product quality. They are related to customers by *neighborhood functions* and personal relationships and they have an important urban and social role;

• *Modern* stores (i.e. super-markets, hyper-markets and shopping centers): they are characterized by modern market strategies, large sale surfaces, high productivity and efficiency, low price levels and access to privileged supply markets. They are related to customers by *marketing functions* and have a high competing power.

Many transformations are taking place in the retail system, where the market competition is intensifying and relevant is the *substitution* of traditional corner shops with modern stores. In order to understand and control these phenomena, it is necessary to model the competition between stores and measure their substitution effects.

In a retail system, the demand can be quantified by customer purchasing expenditures, the supply by store sales and the demand-supply interactions by expenditure flows: the main aim of the retail system analysis is to model such expenditure flows.

Literature mainly suggests logit models to represent interactions in the retail system, deriving them as solution of particular optimization problems (e.g., Fotheringham & O’Kelly, 1989, Train, 2003). Notwithstanding this solid theoretical derivation, the different families of logit models, including some recent extensions, present some behavioral and structural anomalies making them unsuitable to capture the retail system competition.

The aim of this paper is to provide the retail system analyzers and planners with a modeling tool for both the interaction and the impact analysis, related in particular to substitution effects due to new stores location. In Section 3, we propose a new modeling framework able to differentiate interactions by store and customer types, and to take retail market competition into account. The framework is based on two combined logit models and overcomes the anomalies of literature models. The modeling framework has been integrated in a decision support system and actually used by the retail system planners in different urban and regional contexts to analyze the retail system interactions and to evaluate the impact of new stores.

The rest of the paper is organized as follows. In Section 2 the main anomalies of the existing retail models are discussed. In Section 3 a new modeling framework which overcomes such anomalies is presented. In Section 4 some computational results and concluding remarks are given.

2. Anomalies of the state-of-the-art models

Most models in the literature for the analysis of demand-to-supply allocation are *spatial interaction models* and *discrete choice models*. In fact, customer choices derive from the trade-off between the utility related to store attractiveness and the cost incurred to cover the distance between customer and store. A detailed survey can be found, e.g., in Fotheringham & O’Kelly (1989) or Train (2003), together with some applications to retail systems. Logit models are a widespread used family of spatial interaction models having both a macro-economic justification (entropy maximization or information minimization) and a micro-economic derivation (random utility maximization): the origin-destination flows are the macro-economic effect of the individual choices about stores to patronize (Fotheringham, & O’Kelly, 1989). Notwithstanding this solid theoretical outline, at least two main problems are related to using the logit framework for modeling the choice behavior.

The first problem affects the interaction analysis; it comes from the assumption that individuals (customers) choose alternatives (stores) according to a globally-optimal information-processing strategy. In spatial choice situations, with a generally large number of alternatives, individuals are more likely to employ a hierarchical information-processing strategy. That is, they first choose a cluster of alternatives, and then look for an optimal one inside the selected cluster.

The second problem affects the impact analysis and is related to the well-known Independence-from-Irrelevant-Alternatives (IIA) assumption. IIA imposes the restriction of zero covariance between the utilities of
pair of alternatives, implying that the ratio of the probabilities of an individual selecting two alternatives is unaffected by the addition of a third choice. This is very unlikely to occur in practice, in fact the more an alternative is close to the new one, the more it is impacted.

Different frameworks and model extensions have been developed in order to overcome these issues: the main approaches are discussed in the following.

The Nested logit model (McFadden, 1978, Koppelman & Wen, 2001) is based on the assumption that individuals process information hierarchically, according to known-to-the-modeler clusters. The zero-covariance restriction is relaxed for alternatives inside the same cluster, but remains between clusters, so that the IIA anomaly still persists. Even in the Nested consideration logit model extension (Pancras, 2011), where nested restricted choice sets are considered, the application to spatial choice situations is limited, since it assumes an a-priori identification of the (restricted) choice clusters. Actually, the cluster membership of alternatives cannot be rigidly identified, since it depends on spatial variables (for example the travel cost). Hence, only a probability of cluster membership can be recognized and spatial clusters of alternatives have to be defined. They are fuzzy clusters (Zadeh, 1965), since they depend on continuous variables and rigid borders between clusters cannot be identified: an alternative may belong to different clusters, with different probabilities.

The competing-destination approach (Fotheringham, 1983) comes from purely spatial considerations, under the assumption of hierarchical information-processing strategy and the representation of fuzzy clusters of alternatives. The competing-destination model is as follows

\[ p_{ij} = \frac{e^{y_{ij}l_i(j \in m)}}{\sum_j e^{y_{ij}l_i(j' \in m)}} \]  

(1)

where \( p_{ij} \) is the probability for individual \( i \) to choose alternative \( j \), \( l_i(j \in k_i) \) is the likelihood that individual \( i \) perceives alternative \( j \) in the spatial cluster \( k_i \), and \( V_{ij} \) summarizes \( i \)'s propulsion and \( j \)'s attraction.

Thanks to terms \( l_i(j \in k_i) \), the model structure changes, allowing for overcoming the structural problems (like the IIA property) and for representing fuzzy clusters. Fotheringham (1983) proposes a measure of centrality to define \( l \): the closer alternatives to each other, the more likely they are to substitute for one another, i.e., belong to the same fuzzy cluster. The likelihood of an alternative \( j \) to be perceived by individual \( i \) in cluster \( k_i \) is

\[ l_i(j \in k_i) = \left( \frac{1}{n-1} \sum_{j \neq j'} \frac{w_{jj'}}{d_{jj'}} \right)^{\lambda} = c_j^{\lambda} \]  

(2)

where \( w_j \) is the attraction of the alternative \( j \), \( d_{jj'} \) is the distance between alternatives \( j \) and \( j' \), \( n \) is the total number of alternatives and \( \lambda < 0 \) is a parameter reflecting the competition among close alternatives. The likelihood \( l \) can be seen as a measure of the centrality of alternative \( j (c_j) \). Although known modeling anomalies are excluded, the above likelihood definition is independent from individual \( i \), which leads to some other problems with the application of this model to the retail context.

We call the first problem the Competition Spatial Inconsistency (CSI) anomaly and we illustrate it by an example. Let us consider Fig. 1, where a simple retail system is represented: one origin (the individual, denoted by a circle) and three destinations (the alternatives, denoted by squares) are considered, and the related distances are reported. The table shows the probability of choosing alternatives, as determined by (1) and (2). The origin has the same distance from the three destinations and, for the sake of simplicity, we assume that parameter \( V_{ij} \) is equal to the alternative attraction \( w_j \). Even if all the alternatives have the same attraction, A and B have a smaller choice probability, due to their proximity. This is relevant in interaction analysis, since the model is not able to
capture the expected homogeneity of the competition among stores, in this case. The CSI anomaly depends on (2), which involves distances between destinations, rather than between origins and destinations.

![Diagram of the Competition Spatial Inconsistency of the competing-destination logit model](image1)

Fig. 1. The Competition Spatial Inconsistency of the competing-destination logit model

<table>
<thead>
<tr>
<th>Store</th>
<th>( V_{ij} = w_j )</th>
<th>Prob. before</th>
<th>Prob. after</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>log 10</td>
<td>50.00%</td>
<td>42.60%</td>
</tr>
<tr>
<td>B</td>
<td>log 10</td>
<td>50.00%</td>
<td>52.20%</td>
</tr>
<tr>
<td>C</td>
<td>log 2</td>
<td>-</td>
<td>5.10%</td>
</tr>
</tbody>
</table>

![Diagram of the Additive Competition Flow anomaly of the competing-destination logit model](image2)

Fig. 2. The Additive Competition Flow anomaly of the competing-destination logit model

We call the second problem the *Additive Competition Flow* (ACF) anomaly. It is relevant for the impact analysis and is exemplified in Fig. 2. The introduction of a new alternative (C) increases some expenditure flows towards alternatives far away from the new one: the probability of choosing B after the opening of C is greater than before, which is not expected in retail systems. The CSI and the ACF anomalies make the competing-destination model unable to suitably represent the competition between stores in a retail system.

The *Paired Combinatorial logit model* (Koppelman & Wen, 2000) allows different covariance for each couple of alternatives. It uses a *similarity matrix* to represent mutual competition relationships, but some restrictions are necessary in order to estimate the model parameters. Some applications of the Paired Combinatorial logit model are available in transportation analysis for modal choice or route choice (e.g. Koppelman & Wen, 2000). In the retail system, the number of alternatives is very large, making it difficult to impose realistic restrictions to the similarity matrix and, then, to apply the Paired Combinatorial logit model. Some recent approaches can be seen as special paired combinatorial logit models where similarity is defined in terms of store types (González-Benito, 2005, González-Benito, Muñoz-Gallego & Kopalle, 2005), or substitutability between stores (Jun et al. 2012).

### 3. A framework for retail system analysis

In order to overcome the anomalies related to the interaction and the impact analysis, we propose two different integrated models. The *Interaction Model* deals with customer purchasing behavior and its particular logit formulation allows the representation of fuzzy clusters of alternatives, thus overcoming the behavioral anomaly (i.e., the assumption that individuals chose alternatives according to a globally-optimal information-processing strategy) and the CSI anomaly. The *Impact Model* measures the effects of retail system transformations, using the information provided by the first model to evaluate changes in the customer-to-store flows. The Impact Model has a logit structure too, but it is able to overcome both the IIA and the ACF anomaly, since it changes perspective and operates directly on interaction information gathered by the Interaction Model.
3.1. The Interaction Model

The main elements of a retail system are customers, stores and their spatial context.

Customers are located in demand zones which are the origins of the expenditure flows. Their total expenditures and store choices depend on psychological, sociological and economic factors, like education, job type, family structure, age, income rate etc. We group customers according to their socio-economic types.

Stores are located in supply zones which are the destinations of the expenditure flows. Their attraction depends on both economic and structural factors which are summarized by proxy variables called *attraction factors*. Stores are grouped by sale surface: small surfaces identify traditional store groups, larger surfaces correspond to modern store groups.

3.1.1. Origin-destination flows and spatial clusters of stores

Origin-destination flows are the results of the customer choice about the stores to patronize. Customers evaluate the utility deriving from patronizing a store. The utility depends on the customer’s socio-economic type, the store type, the store attraction factors, and spatial factors (summarized by the generalized travel cost).

We recall that the retail-system origin-destination interaction is hierarchical and spatial: stores are grouped into *fuzzy spatial clusters*. This means that clusters are not known a priori, since they have fuzzy boundaries (an alternative belongs to different clusters with different values of membership function), and since individuals in different locations have different perceptions of clusters (cluster membership depends on distance between customers and stores).

In addition, the store clusters features depend on customer socio-economic types and store types. For instance, let us consider an elderly customer and a young one. In order to patronize more attractive stores, the latter is statistically more willing to cover longer distances than the former. From the store point of view, let us consider a traditional store and a modern one. The latter has, in general, a greater attraction than the former and it makes customers cover longer distances. In general, the attraction areas related to different customer and store types have different characteristics, which define the features of the related choice clusters. Fig. 3 illustrates four different kinds of alternative clusters, with reference to (a) high mobility customers and modern stores, (b) high mobility customers and traditional stores, (c) low mobility customers and modern stores, and (d) low mobility customers and traditional stores.

3.1.2. Model formulation

Let it be
interaction, as in the case of increases when the value of the affinity parameters supply the basic data for the impact analysis.

For each store, several attraction factors are defined and quantified by proxy variables, i.e. known measures of some economic (e.g. prices levels, marketing investments) or structural (e.g. sale surface, number of items on sale, parking facilities) characteristics.

We want to determine the most probable expenditure flows from customer (i,m) to store (j,g) (interaction analysis), and detect any changes of these flows, due to some major transformations in the retail system (impact analysis).

The choice mechanism described in Section 3.1.1, including a hierarchical information processing strategy and the spatial clusters of alternatives, is represented by the following model

\[
T_{ij}^{mg} = O_i^m \frac{W_j^g e^{-\beta_{ij} c_{ij}}}{\sum_{g \in I} \sum_{j \in \Pi} W_j^g e^{-\beta_{ij} c_{ij}}}, \forall m \in M, i \in I^m, g \in \Gamma, j \in \Pi^g
\]

where

- \( T_{ij}^{mg} \) is the expenditure flow from customer \((i,m)\) to store \((j,g)\)
- \( W_j^g \) is the deterministic utility of using store \((j,g)\)
- \( c_{ij} \) is a positive parameter related to the affinity between type-\(m\) customers and type-\(g\) stores
- \( \beta_{ij} \) is a positive parameter representing the distance-decay of store attraction and measuring the reluctance of type-\(m\) customers to cover long distances for patronizing type-\(g\) stores.

Factor \( e^{-\beta_{ij} c_{ij}} \) is the likelihood a destination \((j,g)\) belongs to the choice cluster perceived by an origin \((i,m)\), and allows us to represent spatial fuzzy clusters. Large values of \( \beta_{ij} \) let type \(m\) customers include in their choice clusters only type \(g\) stores located at small distances, and vice versa. Note that an exponential function of the generalized cost is used, according to the widespread consensus that exponential function is more appropriate for analyzing short distance interactions, as in the case of retail systems. The cluster membership probability also increases when the value of the affinity parameter \( \beta_{ij} \) is large.

Model (3) is a production constrained model providing a large amount of good quality information (Fotheringham & O’Kelly, 1989). This is mainly contained in the utility factors \( W_j^g \), defined as

\[
W_j^g = \prod_k \left( x_{ikj}^{g} \right)^{\alpha_k}
\]

where \( x_{ikj}^{g} \) is the value of the \(k\)-th attraction factor of store \((j,g)\) and \(\alpha_k\) is the related weight. Weights define the relevance of specific factors and determine their contribution to the overall store attraction.

This information, together with the origin-destination flows, the distance decay parameters and the customer-store affinity parameters, supply the basic data for the impact analysis.
3.2. Impact Analysis

Model (3) has a competing-logit structure and it is not suitable for impact analysis: a different model has to be assessed in order to evaluate the effects of the substitution among stores.

3.2.1. Impact model

We focus on some major transformations of the retail system, like the opening of new stores, which affects the demand/supply allocation and, hence, the expenditure flows \( T_{ij}^{mg} \); the impact model should be able to update such flows. Let us consider the opening of a new store with given expected total sales and assume a constant level of the customer expenditures. We want to determine how the new incomer reduces the current expenditure flows defined by (3). The model acts as follows: the new store will produce its expected amount of sales by reducing some of the origin-destination flows determined by the interaction analysis (see Fig. 4). We may say that the new store has to “select” the flows to reduce, within a spatial choice framework. This choice is represented by the Impact Model, a particular spatial interaction model which takes from the interaction analysis the origin-destination flows \( T_{ij}^{mg} \) (which are the choice alternatives) and the attraction factors’ weights \( \alpha_k \).

![Fig. 4. Impact of a new opening on expenditure flows](image)

Basically, given an expenditure flow \( T_{ij}^{mg} \), its reduction depends on:

- the attraction of the new store (the higher the attraction, the larger the reduction);
- the generalized travel cost from the origin \((i,m)\) to the old destination \((j,g)\) (the higher the cost, the larger the reduction);
- the attraction of the old destination (the smaller the attraction, the larger the reduction);
- the distance of the new store (the smaller the distance, the larger the reduction).

This can be captured by the following singly constrained logit model, which determines the impact on previous origin-destination flows of a new store \((J,G)\) with expected total sales \( D_j^G \).

\[
P_{ij}^{mg | (J,G)} = D_j^G \frac{T_{ij}^{mg} (W_j^g)^{-\sigma G} e^{-\sigma^G e^{\sum \zeta^{mg} c_{ij}^{mg}}}}{\sum_{m' \in M} \sum_{i' \in I} \sum_{g' \in \Gamma} \sum_{j' \in \Pi} T_{i'j'}^{m'g'} (W_{j'}^{g'})^{-\sigma G} e^{-\sigma^G e^{\sum \zeta^{m'g'} c_{i'j'}^{m'g'}}}}
\]

where
\[ T_{ij}^{mg} = \max \{ T_{ij}^{mg} - p_{ij}^{mg}_{(J,G)}, 0 \} \] (6)

and the flows directed to the new store \((J,G)\) will be

\[ T_{ij}^{mg} = \sum_{g1} \sum_{j \in W} (T_{ij}^{mg} - T_{ij}^{mg}_{(N)}) \] (7)

Finally, the total sales of the new store \((J,G)\) is

\[ D_{J}^{G} = \sum_{m} \sum_{i \in \Phi} T_{ij}^{mg} \] (8)

Note that it may be \(D_{J}^{G}_{(N)} < D_{J}^{G}\), meaning that the expected sales may be overestimated.

The following Properties 1, 2 and 3 show that the modeling framework derived from equations (3) and (5) overcomes the anomalies presented in Section 2 and is suitable for retail system interaction and impact analysis.

**Property 1** The impact model (5) overcomes the IIA anomaly.

**Proof.** Proving the assert is equivalent to showing that the ratio of the flows towards two existing alternatives is affected by the addition of a new one. Let \((J,G)\) be the new opening store. If at least one of the flows is 0, the ratio is 0 or cannot be stated. Otherwise, by (6) we can write the new flow from an origin \((i,m)\) to an existing store \((j,g)\) as \(T_{ij}^{mg} = T_{ij}^{mg} - p_{ij}^{mg}_{(J,G)}\) and, by (5), we have
From an application point of view, a further benefit of the proposed framework is the possibility of performing the necessary parameter estimation by standard procedures for competing-logit models (Fotheringham & O’Kelly, 1989), which are based on observed expenditure-flows (Interaction Model) or flow losses. Further, as gathering the necessary information may be difficult, an alternative parameter estimation procedure has been developed, based on aggregated data like the average travel time and the total store sale and average sale per customer and store types.
The proposed framework has been applied to several real-life cases at both regional and urban levels in the context of research projects coordinated by Scuola Superiore del Commercio Turismo e Servizi (Milan, Italy) and aiming at supporting retail-system decision makers in evaluating the impact of new store openings. In this context the model estimation and evaluation procedures have been integrated in a software tool able to support what-if analysis by representing several performance indicators related to interaction and impact analysis on a GIS-based interface. Results validate the new modeling framework as an appropriate tool to suitably represent customer-store interactions in a competitive environment. Among others, we report in Table 1 an example of impact pattern related to the opening of a new hypermarket in the grocery retail system of the region of Milan (Italy).

Table 1. Impact of a new hypermarket on existing traditional and modern stores

<table>
<thead>
<tr>
<th>Distance up to (minutes)</th>
<th>% Loss for traditional stores</th>
<th>% Loss for modern stores</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>23.30</td>
<td>3.87</td>
</tr>
<tr>
<td>10</td>
<td>11.54</td>
<td>1.13</td>
</tr>
<tr>
<td>15</td>
<td>4.12</td>
<td>0.91</td>
</tr>
<tr>
<td>20</td>
<td>2.78</td>
<td>1.06</td>
</tr>
<tr>
<td>25</td>
<td>0.79</td>
<td>3.58</td>
</tr>
<tr>
<td>30</td>
<td>0.22</td>
<td>3.08</td>
</tr>
</tbody>
</table>

Results shows that, as one may expect, the new store has less impact, in percentage, among existing modern stores than traditional ones, but modern stores located far away are more impacted than corresponding traditional one: in fact the competition of modern stores is much more spread out and generates trade areas which overlap to each other. These results also confirm that the IIA property does not hold anymore, since the loss of different types of stores located at the same distance from the new one are well differentiated.

As future work, other competing-logit models can be derived, in the spirit of the Impact Model, in order to evaluate different kinds of modifications, such us closing of existing stores, and variations of customer expenditures or store sales, thus allowing a comprehensive analysis of the retail system and its transformations.

References


