Scattering by a PEC wedge on a Dielectric substrate

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Abstract — **A perfect electric conductor wedge lying on a dielectric half-space is analyzed, in the frequency domain. The structure is excited by a plane wave coming from the free space region. The problem is formulated using generalized Wiener-Hopf equations and an approximated solution in terms of spectral quantities is proposed by discretization of Fredholm integral equations. Numerical results for several geometries will be presented and discussed at the conference.**

1 INTRODUCTION

The evaluation of the electromagnetic field in a physical structure constituted by a perfect electric conductor (PEC) wedge lying on a dielectric half-space is analyzed, see Fig. 1. The PEC wedge is with a face tangent to the dielectric half-space and with aperture angle $(\pi - \Phi_e)$ while $\Phi_e > \pi/2$.

Without loss of generality, the structure is illuminated at normal incidence by an E-polarized plane wave (1) coming from the free space region (upper half-space) with direction φ_o $(0 < \varphi_o < \Phi_e)$ and with propagation constant $k_o = \omega\sqrt{\mu_o\,\varepsilon_o}$ (free space permittivity ε_o and permeability μ_o).

$$E_z^i = E_o e^{jk_o\,\rho\cos(\varphi-\varphi_o)} \tag{1}$$

The lower half-space is constituted by a dielectric region with permittivity $\varepsilon = \varepsilon_o\varepsilon_r$ and permeability $\mu = \mu_o\mu_r$.

Apparently, the literature on this problem is very poor. Particular cases of a wedge immersed in a stratified medium were studied by Bertoncini and al. [1]-[2] by using the Uniform Theory of Diffraction (UTD). However the application of this method is limited to edges not close to the stratified regions.

Our approach is based on a formulation that uses the generalized Wiener-Hopf equations developed in [3]-[14] for different kind of wedge problems, i.e. scattering and diffraction by impenetrable wedge and penetrable wedge illuminated by plane waves at normal or skew incidence.

This technique has been recently extended to investigate wedge over a stratified medium [15]. The aim of this summary paper is to successfully introduce the extension of the application of Wiener-Hopf techniques to the present problem.

Since closed form solutions of classical Wiener-Hopf equations (CWHE) and generalized Wiener-Hopf equations (GWHE) are possible only for a small class of problems, we have introduced a semi-analytical method based on the reduction of CWHE and GWHE to Fredholm integral equations (FIE) [15]-[19]. This technique allows to obtain solutions for more general problems and we intend to apply it in the present problem.

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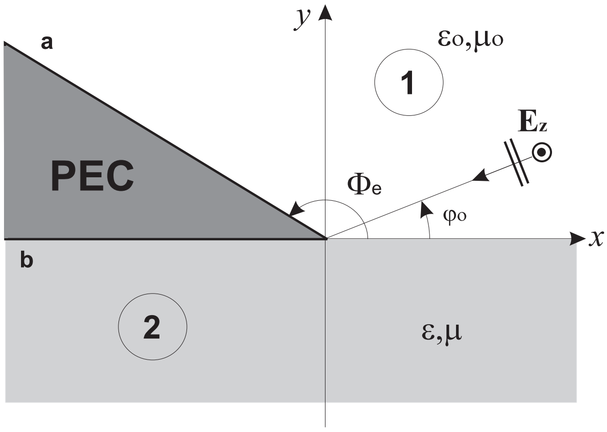

Figure 1: Perfect electric conductor (PEC) wedge aperture angle $(\pi - \Phi_e)$ lying on a dielectric half-space and illuminated by a E-polarized plane wave at normal incidence with direction φ_o.

2 FORMULATION

With reference to the problem described in Fig.1, we only consider time harmonic electromagnetic fields with a time dependence specified by the factor $exp(+j\omega t)$ which is omitted.

Following the procedure presented in [15] we formulate the problem in terms of radial transmission line [3],[20],[6] in the upper free-space region and in terms of longitudinal transmission line in the lower dielectric region.

We define the following Wiener-Hopf spectral unknowns (Laplace transform) in the η-plane respectively for the upper region ($y > 0$, $0 \le \varphi \le \Phi_e$)

$$\begin{aligned} V_+(\eta,\varphi) &= \int_0^\infty E_z(\rho,\varphi)e^{j\eta\,\rho}d\rho \\ I_+(\eta,\varphi) &= \int_0^\infty H_\rho(\rho,\varphi)e^{j\eta\,\rho}d\rho \end{aligned} \tag{2}$$

and for the lower region ($y < 0$)

$$\begin{aligned} v_\eta(y) &= \int\limits_{-\infty}^{\infty} E_z(x,y) e^{j\eta\, x} dx \\ i_\eta(y) &= \int\limits_{-\infty}^{\infty} H_x(x,y) e^{j\eta\, x} dx \end{aligned} \tag{3}$$

In particular we define the axial unknowns

$$\begin{aligned} V_+(\eta) &= V_+(\eta, \varphi = 0) \\ I_+(\eta) &= I_+(\eta, \varphi = 0) \end{aligned}$$

whose physical support is the boundary between the free space region and the dielectric region.

By recalling the boundary conditions on the PEC and dielectric faces, the radial transmission line models electromagnetic problems in angular domains as $(\, y > 0\ ,\ 0 \leq \varphi \leq \Phi_e)$. From the analysis of the the transmission line we obtain the following GWHE

$$Y_c\, V_+(\eta) - I_+(\eta) = -I_{a+}(-m) \tag{4}$$

where

$$\begin{aligned} &Y_c = \tfrac{\xi}{\omega\mu_o} \\ &\xi = \xi(\eta) = \sqrt{{k_o}^2 - \eta^2} = \tau_o \\ &m = -\eta\cos\Phi_e + \xi\sin\Phi_e \end{aligned}$$

The longitudinal transmission line models electromagnetic problems in stratified medium as in particular the dielectric region ($y < 0$). From the analysis of the the transmission line we obtain the following CWHE

$$Y_d\, V_+(\eta) + I_+(\eta) = -I_-(\eta) \tag{5}$$

where

$$\begin{aligned} &Y_d\ = \tfrac{\xi_d}{k_d Z_d} \\ &\xi = \xi(\eta) = \sqrt{{k_d}^2 - \eta^2} \\ &k_d = k_o\sqrt{\varepsilon_r} \end{aligned}$$

3 STEPS FOR THE SOLUTION

Starting from the system of Wiener-Hopf equations (4) and (5) we apply a generalized version of the Fredholm factorization method [16]-[19] as reported in [15]. It yields (6) and (7), reported in the next page, where $f(\eta)$ is a source term extracted from the Wiener-Hopf unknowns as offending pole and it is related to the Geometrical Optics field (GO).

Since (6) and (7) are a system of Fredholm integral equation, their numerical discretization yields fast convergent solutions [21] in terms of $V_+(\eta)$, $I_+(\eta)$. If one is only interested on the spectrum of $V_+(\eta)$, we can eliminate $I_+(\eta)$ by substitution yielding a single integral equation with $V_+(\eta)$ as unique unknown. The properties of convergence are unaffected.

From $V_+(\eta)$, using the inverse Fourier transform and asymptotic evaluation, it is possible to obtain the GTD diffraction coefficient and the total far-fields for the problem under examination.

4 CONCLUSION

The method proposed in this summary paper shows the possibility of the GWHE technique to solve the problem of evaluating the electromagnetic field of a perfect electric conductor wedge lying on a dielectric half-space. Skew incidence case will be considered in future works.

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References

[1] F. Bertoncini, R.G. Kouyoumjian, G. Manara and P. Nepa, "High-Frequency Scattering by objects buried in lossy media," *IEEE Trans. Antennas and Propagat.,* vol. 49, pp. 1649-1656, 2001.

[2] F. Bertoncini, R.G. Kouyoumjian, G. Manara and P. Nepa, "EM Scattering by a wedge buried in a lossy medium: a UTD solution for the field in the lossy half-plane," *URSI EMTS 2004,* pp. 1035-1037, 2004.

[3] V.G.Daniele, "New analytical Methods for wedge problems," *Proceedings of the 2001 International Conference on Electromagnetics in Advanced Applications (ICEAA01),* Torino, Italy, 10-14 September, 2001, pp. 385-393.

[4] V.G. Daniele, "The Wiener-Hopf technique for impenetrable wedges having arbitrary aperture angle," *SIAM Journal of Applied Mathematics,* vol. 63, pp. 1442-1460, 2003.

[5] V. Daniele, G. Lombardi, "Generalized Wiener-Hopf equations for wedge problems involving arbitrary linear media," in *Int. Conf. Electromagn. Adv. Applicat. (ICEAA03),* Torino, Italy, 10-14 September, 2003, pp. 761-764

[6] V.G. Daniele, G. Lombardi, "The Wiener-Hopf technique for impenetrable wedge problems," invited paper at plenary session, *Proceedings of Days on Diffraction 2005,* Saint Petersburg, Russia, pp. 50-61, June 28-July 1, 2005, doi: 10.1109/DD.2005.204879

$$Y_c(\eta)\,V_+(\eta) - I_+(\eta) + \frac{1}{2\pi j}\left[\int_{-\infty}^{\infty} \frac{Y_c(\eta')}{\alpha(\eta') - \alpha(\eta)}\alpha'(\eta') - \frac{Y_c(\eta)}{\eta' - \eta}\right] V_+(\eta')d\eta' = f(\eta) \tag{6}$$

$$Y_d(\eta)V_+(\eta) + I_+(\eta) + \frac{1}{2\pi j}\int_{-\infty}^{\infty} \frac{Y_d(\eta') - Y_d(\eta)}{\eta' - \eta} V_+(\eta')d\eta' = 0 \tag{7}$$

[7] V.G. Daniele, G. Lombardi, "Wiener-Hopf Solution for Impenetrable Wedges at Skew Incidence," *IEEE Transactions on Antennas and Propagation,* vol. 54, no. 9, pp. 2472-2485, Sept. 2006, doi: 10.1109/TAP.2006.880723

[8] V. Daniele, G. Lombardi, "The Wiener-Hopf method applied to multiple angular region problems: The penetrable wedge case," in *Proc. 2009 IEEE International Symposium on Antennas and Propagation and USNC/URSI National Radio Science Meeting,* Charleston, SC, 2009, pp. 1-4, doi: 10.1109/APS.2009.5172090.

[9] V. Daniele, G. Lombardi, "The Wiener-Hopf method applied to dielectric angular regions: The wedge," in *Proc. Int. Conf. on Electromagnetics in Advanced Applications (ICEAA09),* Torino, Italy, Sep. 2009, pp. 489-492, doi: 10.1109/ICEAA.2009.5297388.

[10] V.G. Daniele, "The Wiener-Hopf formulation of the dielectric wedge problem.Part I," *Electromagnetics,* vol. 30, n. 8, pp. 625-643, 2010.

[11] V.G. Daniele, "The Wiener-Hopf formulation of the dielectric wedge problem. Part II," *Electromagnetics,* vol. 31, n. 1, pp. 1-17, 2011.

[12] V.G Daniele, "The Wiener-Hopf Formulation of the Penetrable Wedge Problem. Part III: The skew incidence case," *Electromagnetics,* vol. 31, n. 8, pp. 550-570, 2011.

[13] V. Daniele, G. Lombardi, "The Wiener-Hopf Solution of the Isotropic Penetrable Wedge Problem: Diffraction and Total Field," *IEEE Transactions on Antennas and Propagation,* vol. 59, no. 10, pp. 3797-3818, Oct. 2011, doi: 10.1109/TAP.2011.2163780

[14] G. Lombardi, "Skew Incidence on Concave Wedge With Anisotropic Surface Impedance," *IEEE Antennas and Wireless Propagation Letters,* vol. 11, pp. 1141-1145, 2012, doi: 10.1109/LAWP.2012.2219845

[15] V.G. Daniele, "Electromagnetic fields for PEC wedge over stratified media. Part I.," in press, *Electromagnetics*.

[16] N.P. Vekua, *Systems of Singular Integral Equations,* Gronigen: P. Noordhoff, 1967.

[17] V.G. Daniele, An introduction to the Wiener-Hopf technique for the solution of electromagnetic problems. Internal Report ELT-2004-1. Available on line at http://personal.delen.polito.it/vito.daniele/

[18] V.G. Daniele, G. Lombardi, "Fredholm Factorization for Wedge Problems," *Antennas and Propagation Society International Symposium 2006, IEEE,* pp. 2478-2481, 9-14 July 2006, Albuquerque, NM, USA, doi: 10.1109/APS.2006.1711100

[19] V. Daniele, and G. Lombardi, "Fredholm factorization of Wiener-Hopf scalar and matrix kernels," *Radio Sci.,* 42, RS6S01, 2007, doi: 10.1029/2007RS003673

[20] V.G. Daniele, The Wiener-Hopf technique for wedge problems, Internal Report ELT-2004-2. Available on line at http://personal.delen.polito.it/vito.daniele/

[21] L.V. Kantorovich, V.I. Krylov, *Approximate methods of higher analysis,* Groningen: P.Noordhoff, 1964