Cylindrical resonator filled with four alternating sectors of DPS and DNG materials

Cylindrical resonator filled with four alternating sectors of DPS and DNG materials

V.G. Daniele*, R. D. Graglia*, G. Lombardi*, P. L. E. Uslenghi†

Abstract — **A circular cylindrical metallic resonator with four alternating sectors of DPS and DNG materials is analyzed, in the frequency domain. The two materials are linear, lossless, homogeneous, and anti-isorefractive to each other. The electric field is assumed to be parallel to the cylinder axis. It is also shown that the resonator's dispersion relation is always satisfied for particular geometries. Numerical results for several sectors' combinations will be presented and discussed at the conference.**

1 INTRODUCTION

A circular cylindrical resonator with metallic (PEC) walls and filled with four sectors containing alternatively double-positive (DPS) material and double-negative (DNG) metamaterial is considered.

The two DPS regions are filled with a linear, uniform and isotropic material characterized by a real positive electric permittivity ε and a real positive magnetic permeability μ, or alternatively by a real positive wavenumber $k = \omega\sqrt{\varepsilon\mu}$ and a real positive intrinsic impedance $Z = \sqrt{\mu/\varepsilon}$, where ω is the angular frequency. The two DNG regions are filled with a linear, uniform and isotropic material characterized by a real negative permittivity $-\varepsilon$ and a real negative permeability $-\mu$, or alternatively by a real negative wavenumber $-k$ and a real positive intrinsic impedance Z. Thus, the DPS and DNG regions of the resonator are filled with materials having real refractive indexes of opposite sign and the same real intrinsic impedance.

The concept of utilizing DNG metamaterial inclusions to render a resonator size-independent was introduced by Engheta [1]-[2] for a one-dimensional structure, to show that it is possible to build resonators that functions independently of dimensions at those frequencies for which the metamaterial behaves as postulated. This concept was later extended to fully three dimensional cavity resonators by Couture et al. [3]-[4] and by Uslenghi [5]-[7]. Recently, Daniele et al. [8]-[11] studied in detail a cylindrical resonator sectorally filled with metamaterial; in particular, their analysis showed that phase compensation leading to size independence is possible only if the cylinder is half-filled with metamaterial. In the present work we extend the results obtained for structures with two sectors to structures with four sectors.

*DET, Politecnico di Torino, C.so Duca degli Abruzzi 24, 10129 Torino, Italy, e-mail: vito.daniele, roberto.graglia, guido.lombardi @polito.it

†Department of ECE, University of Illinois at Chicago 851 S. Morgan M/C 154, Chicago, IL, USA, e-mail: uslenghi@uic.edu

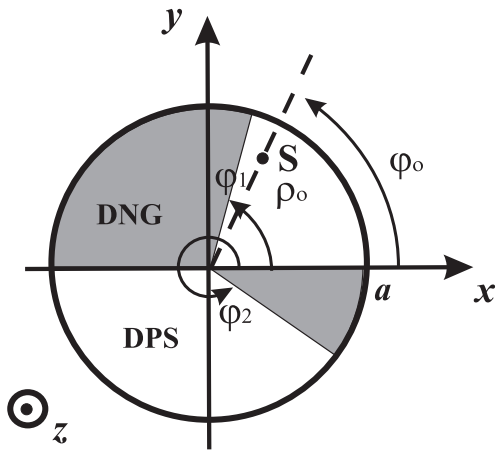

Figure 1: Cross section of the resonator. First sector $0 < \varphi < \varphi_1$ and third sector $\pi < \varphi < \varphi_2$ are filled with DPS material, while second sector $\varphi_1 < \varphi < \pi$ and fourth sectors $\varphi_2 < \varphi < 2\pi$ are filled with DNG material. The electric line source is located at (ρ_0, φ_0) inside the first DPS region, $(0 < \varphi_0 < \varphi_1)$.

The analysis is conducted in phasor domain with time-dependence factor $exp(+j\omega t)$. The electric field is assumed to be parallel to the axis of the cylinder, so that the boundary conditions on the bases of the resonator are satisfied and the height of the cylinder does not appear in the analysis. It is shown that the resonator's dispersion relation is always satisfied for particular geometries. Several sectors' combinations will be also considered, in particular the case of four geometrically identical sectors. The structure will be excited by a line source parallel to the axis. Numerical results for several sectors' combinations will be presented and discussed at the conference.

2 GEOMETRY OF THE PROBLEM

With reference to cylindrical coordinates (ρ, φ, z), the metallic cylindrical resonator has the z axis as symmetry axis, the inner radius of its circumference in any plane $z = constant$ is a, and the length of the resonator in the z direction does not come into play because the resonator is assumed to be excited by an electric line source parallel to its axis, leading to an electric field that is everywhere parallel to z, so that the boundary conditions at the two circular bases of the cylinder are

always satisfied.

With reference to Fig.1, the DPS material fills the cross-sectional area $0 \leq \rho < a$, $(0 < \varphi < \varphi_1) \bigcup (\pi < \varphi < \varphi_2)$, whereas the DNG material fills the cross-sectional area $0 \leq \rho < a$, $(\varphi_1 < \varphi < \pi) \bigcup (\varphi_2 < \varphi < 2\pi)$.

The electric line source $\mathbf{J_0}$ located inside the DPS region at $\boldsymbol{\rho_0}$ ($\rho = \rho_0, \varphi = \varphi_0$) is assumed to be:

$$\mathbf{J_0} = I\delta(\boldsymbol{\rho} - \boldsymbol{\rho_0})\hat{z} = I\frac{1}{\rho_0}\delta(\rho - \rho_0)\delta(\varphi - \varphi_0)\hat{z} \quad (1)$$

where I is the intensity (in A), $\hat{z}$ is a unit vector parallel to the z axis, and δ is the delta function.

3 SOLUTION PROCEDURE

Since we excite the structure with the line source (1), we limit our considerations to electric fields parallel to the axis z of the resonator, the boundary-value problem is two-dimensional and, the z coordinate and the length d of the cylinder do not play any role in our analysis. In this case only TM_{mn0} modes (typical of cylindrical cavities) are compatible where m,n, 0 are respectively the indices related to φ, ρ, z. The longitudinal component E_z of the electric field must satisfy the following equation:

$$\left(\frac{\partial^2}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial}{\partial\rho} + \frac{1}{\rho^2}\frac{\partial^2}{\partial\varphi^2} + k^2\right) E_z(\rho,\varphi) = -j\omega\mu_o J_z \quad (2)$$

with boundary conditions at the DPS/DNG interface and at the cylindrical border. From the homogeneous version of (2) we obtain the resonator modes $\Phi_{mn}(\rho,\varphi)$, that satisfy the completeness equation

$$\frac{1}{\rho_o}\delta(\rho-\rho_o)\delta(\varphi-\varphi_o) = \sum_{m,n} w(\varphi_o)\Phi^*_{mn}(\rho_o,\varphi_o)\Phi_{mn}(\rho,\varphi) \quad (3)$$

where $w(\varphi)$ is a suitable function. Eq. (3) allows to represent the solution of (2) in terms of the basis $\Phi_{mn}(\rho,\varphi)$:

$$E_z(\rho,\varphi) = -j\frac{1}{2}\omega\mu I_o \sum_{m,n} w(\varphi_o)\frac{\Phi^*_{mn}(\rho_o,\varphi_o)}{k^2 - \tau^2_{mn}}\Phi_{mn}(\rho,\varphi) \quad (4)$$

To get $\Phi_{mn}(\rho,\varphi)$ we resort to separation of variables and we define each $\Phi_{mn}(\rho,\varphi)$ as piece-wise functions for each sector ($\ell = 1,2,3,4$) as numbered in Fig. 1:

$$\Phi^{(\ell)}_{mn}(\rho,\varphi) = F^{(\ell)}_{\nu_m}(\varphi)\frac{\sqrt{2}}{a\,J_{\nu_m+1}(z_n(\nu_m))}J_{\nu_m}((-1)^{\ell+1}\tau_{\nu_m n}\,\rho) \quad (5)$$

where ν_m is a countable parameter related to φ variation, $z_n(\nu_m)$ is the n-th zero of the Bessel function of order ν_m and, $\tau_{\nu_m n} = z_n(\nu_m)/a$.

While the ρ variation in (5) is obtained via the classical expression of the completeness relation in radial direction, we focus the attention on terms related to the φ variation

$$\left(\frac{d^2}{d\varphi^2} + \nu^2\right) F_\nu(\varphi) = 0 \quad (6)$$

We apply the Green resolvent technique (also used in [12],[8]-[11] for Sturm-Liouville problem with classical boundary conditions) in a generalized form as reported in [13] where mixed type boundary conditions are considered. In fact our problem shows φ periodicity and it is not defined in a limited interval as in classical Sturm-Liouville problem.

Once the Green function is obtained, its denominator determines the dispersion relation (7) for the problem under investigation and thus a set of countable eigenvalues ν^2_m (8)

$$\sin[\nu(2\pi - \varphi_1 - \varphi_2)] = 0 \quad (7)$$

$$\nu_m = \frac{m\pi}{2\pi - (\varphi_1 + \varphi_2)} \quad (8)$$

from which it is possible to define a set of ortho-normal basis functions to represent $F_\nu(\varphi)$. The procedure described in [13] is fundamental to build an organized algorithm for ortho-normal bases in mixed type boundary condition problems.

If $\varphi_1 + \varphi_2 = 2\pi$, the dispersion equation (7) is satisfied for any ν. In this case the Green function does not exist [13] and a different approach to obtain the solution must be implemented.

In the case of four geometrically identical sectors, we build the solution using the image theory with a particular distribution of image sources. A similar heuristic approach has already been used for a different problem in [11]. Further details and numerical results will be presented at the conference.

Acknowledgment

This work was sponsored in part by the Italian Ministry of Education, University and Research (MIUR) under PRIN grant 20097JM7YR, and in part by the College of Engineering in the University of Illinois at Chicago.

References

[1] N. Engheta, "Compact cavity resonators using metamaterials with negative permittivity and permeability," *Proc. Intl. Conf. Electromagnetics in Advanced Applications (ICEAA'01),* Torino, Italy, Sept. 2001, pp. 739–742.

[2] N. Engheta, "An idea for thin subwavelength cavity resonators using metamaterials with negative permittivity and permeability," *IEEE Antennas Wireless Propag. Lett.,* vol. 1, pp. 10–13, 2002.

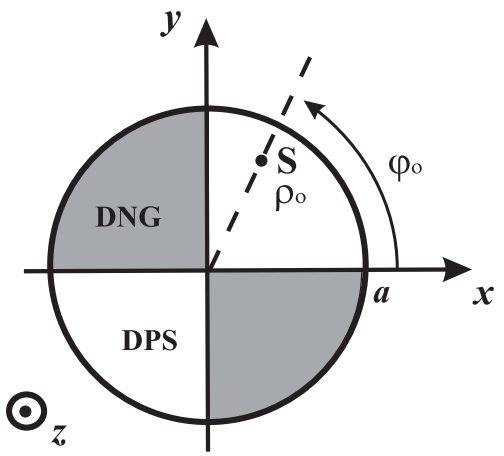

Figure 2: Cross section of the resonator filled with four geometrically identical sectors with alternating DPS and DNG materials. The electric line source is located at (ρ_0, φ_0) inside the DPS region.

[3] S. Couture, A. Parsa and C. Caloz, "Design of a size-independent zero-permittivity rectangular resonator and possible antenna applications," *14th Intl. Symposium on Antenna Technology and Applied Electromagnetics (ANTEM),* Ottawa, Canada, July 2010, pp. 1–4.

[4] S. Couture, A. Parsa and C. Caloz, "Size-independent zeroth-order electric plasmonic cavity resonator," *Microwave Opt. Tech. Lett.,* vol. 53, pp. 927–932, April 2011.

[5] P.L.E. Uslenghi, "A miniaturized parallelepipedal cavity resonator," *IEEE Antennas and Wireless Propag. Lett.,* vol. 9, pp. 807–808, 2010.

[6] P.L.E. Uslenghi, "Size-Independent Prism Resonator Partially Filled With DNG Metamaterial," *IEEE Antennas Wireless Propag. Lett.,* vol. 10, pp.746–747, 2011.

[7] P.L.E. Uslenghi, "A fully three-dimensional size-independent parallelepipedal resonator partially filled with DNG metamaterial," *International Conference on Electromagnetics in Advanced Applications (ICEAA)*, 2-7 Sept. 2012, Cape Town, South Africa, pp. 11-14, doi: 10.1109/ICEAA.2012.6328575

[8] V.G. Daniele, R. D. Graglia, G. Lombardi and P. L. E. Uslenghi, "Cylindrical resonators partially filled with a DNG metamaterial sector," *International Conference on Electromagnetics in Advanced Applications (ICEAA)*, 2-7 Sept. 2012, Cape Town, South Africa, pp. 1295-1297, doi: 10.1109/ICEAA.2012.6328828

[9] V.G. Daniele, R. D. Graglia, G. Lombardi and P. L. E. Uslenghi, "Cylindrical Resonator Sectorally Filled With DNG Metamaterial and Excited by a Line Source," *IEEE Antennas Wireless Propag. Lett.,*, vol.11, pp.1060-1063, 2012, doi: 10.1109/LAWP.2012.2215300

[10] V.G. Daniele, R. D. Graglia, G. Lombardi, P.L.E. Uslenghi, "Size-independent Cylindrical Resonator Half-filled with DNG Metamaterial," *2013 URSI International Symposium on Electromagnetic Theory (EMTS)*, May 20-23, 2013, Hiroshima, Japan, paper n. 21PM2D_03, 3 pages.

[11] V.G. Daniele, R. D. Graglia, G. Lombardi and P. L. E. Uslenghi, "Size-independent Cylindrical Resonator Half-filled with DNG Metamaterial and Excited by a Line Source," *IEEE Antennas Wireless Propag. Lett.,* in press, 2013, doi: 10.1109/LAWP.2013.2270944

[12] L.B. Felsen and N. Marcuvitz, *Radiation and Scattering of Waves.* New York: IEEE Press, 1994.

[13] B. Friedman, *Principles and Techniques of Applied Mathematics,* chapter 3. New York: Dover, 1990.

[14] V. Daniele, M. Gilli, R.D. Graglia, "Alternative representation of dyadic Green's functions for circular cylindrical cavities with applications to the EMC characterization of space station modules," *Radio Sci.,* vol. 35, pp. 621-637, 2000.