Modelling of junctions with edges in surface integral equations

Original

Modelling of junctions with edges in surface integral equations / Lombardi, Guido; Graglia, Roberto. - ELETTRONICO. - unico:(2013), pp. 920-921. (Intervento presentato al convegno 2013 IEEE International Symposium on Antennas and Propagation and USNC-URSI National Radio Science Meeting tenutosi a Orlando, FL nel July 7- 13, 2013) [10.1109/APS.2013.6711119].

Availability:

This version is available at: 11583/2518679 since: 2022-04-04T12:59:31Z

Publisher:

IEEE - INST ELECTRICAL ELECTRONICS ENGINEERS INC

Published

DOI:10.1109/APS.2013.6711119

Terms of use:

openAccess

This article is made available under terms and conditions as specified in the corresponding bibliographic description in the repository

Publisher copyright

IEEE postprint/Author's Accepted Manuscript

©2013 IEEE. Personal use of this material is permitted. Permission from IEEE must be obtained for all other uses, in any current or future media, including reprinting/republishing this material for advertising or promotional purposes, creating new collecting works, for resale or lists, or reuse of any copyrighted component of this work in other works.

(Article begins on next page)

Modelling of junctions with edges in surface integral equations

Guido Lombardi

Dipartimento di Elettronica e Telecomunicazioni
Politecnico di Torino
Torino, Italy
guido.lombardi@polito.it

Roberto D. Graglia

Dipartimento di Elettronica e Telecomunicazioni
Politecnico di Torino
Torino, Italy
roberto.graglia@polito.it

Abstract— When the surface integral equation method is applied to metallic structures with junctions, a special procedure to handle the basis functions and the unknowns is needed. In this summary paper we present the possibility to combine the capability of modeling edges/wedges with patch junctions. Numerical simulations will illustrate the validity of the proposed method as well as some applications will be presented in terms of evaluation of current density and RCS in PEC structures.

I. INTRODUCTION

The Method of Moments (MoM) has been successfully validated for modeling 3D sharp-wedge structures using Surface Integral Equations (SIE) [1].

From the other side, the procedure for the handling junctions in SIE are successfully implemented in several papers [2] in structures composed by piecewise inhomogeneous geometries, in particular the use of Rao-Wilton-Glisson (RWG) basis functions has been dealt in junctions with PEC and dielectric regions.

In this summary we consider the modeling capability of singular divergence-conforming vector bases with the procedure for handling surface junctions of PEC structures.

Numerical simulations will illustrate the validity of the proposed method as well as some applications will be presented in terms of evaluation of current density and RCS in PEC structures.

II. SINGULAR DIVERGENCE-CONFORMING VECTOR BASES

As proposed in [3] we have defined singular divergence-conforming vector based that models 3D sharp-wedge structures. These functions are able to model singularities in curvilinear patches of triangular and quadrilateral shapes. Moreover the bases are complete to arbitrarily high order and they constitute an additive subset to regular high order bases that are commonly used in MoM applications[4]. The singular subset are located at the edge profile as shown in Fig. 1 and in the case of triangular discretization of the geometry they are of two kinds: edge singular elements and vertex singular elements.

Figure 1. Edge singularity quadrilaterals and edge (e) and vertex (v) singularity triangles with local edge numbering scheme.

III. JUNCTIONS IN JOINED ELECTRICALLY CONDUCTING PLATES

Without loss of generality, we focus the attention on joined electrically conducting plates, see Fig. 2 the case of tri-plate.

Figure 2. Edge singularity quadrilaterals and edge (e) and vertex (v) singularity triangles with local edge numbering scheme

the a perfect electric conductor (PEC) wedge lying on a dielectric half-space, see Fig. 1, *i.e.* a PEC wedge with a face

tangent to the dielectric half-space. The PEC wedge is with aperture angle $\pi - \Phi_e$ with $\Phi_e > \pi/2$.

Without loss of generality, the structure is illuminated by an E-polarized plane wave with direction φ_o ($0 \leq \varphi_o \leq \Phi_e$) at normal incidence (1) with $k_o = \omega\sqrt{\mu_o \epsilon_o}$, located in the upper half-space constituted by free-space (permittivity ϵ_o and permeability μ_o). The lower half-plane is constituted by dielectric region with permittivity $\epsilon = \epsilon_r \epsilon_o$ and permeability $\mu = \mu_r \mu_o$.

$$E_z^i = E_o e^{jk_o \rho \cos(\varphi - \varphi_o)} \quad (1)$$

The skew incidence case will not introduce conceptual difficulties but it will double the number of equations to be solved.

The literature on this problem is apparently very poor. Particular cases of a wedge immersed in a stratified medium were studied by Bertoncini and al. [1-2] by using the Uniform Theory of Diffraction (UTD). However the application of this method is limited to edges not close to the stratified regions.

Our approach is based on a formulation that uses the generalized Wiener-Hopf equations (GWHE) developed in [3-12] for different kind of wedge problems, *i.e.* scattering and diffraction by impenetrable wedge and penetrable wedge illuminated by plane waves.

Figure 1. Perfect electric conductor (PEC) wedge lying on a dielectric half-space.

This technique has been recently extended to investigate wedge over a stratified medium [13].

Since closed form solutions of classical/generalized Wiener-Hopf equations are possible only for a small class of problems, we have introduced a semi-analytical method based on the reduction of GWHE to Fredholm integral equations (FIE) [13-17] to obtain solutions for more general problems.

The aim of this summary paper is to successfully introduce the extension of the application of Wiener-Hopf techniques to the present problem.

IV. FORMULATION

Following the procedure presented in [13] we formulate the problem in terms of radial transmission line [3,18,5] in the upper free-space region and in terms of longitudinal transmission line in the lower dielectric region.

With reference to Fig. 1, we only consider time harmonic electromagnetic fields with a time dependence specified by the factor $e^{j\omega t}$ which is omitted in the following.

We define the following Wiener-Hopf spectral unknowns in the η -plane in the upper region for ($y > 0, 0 \leq \varphi \leq \Phi_e$)

$$\begin{aligned} V_+(\eta, \varphi) &= \int_0^\infty E_z(\rho, \varphi) e^{j\eta \rho} d\rho \\ I_+(\eta, \varphi) &= \int_0^\infty H_\rho(\rho, \varphi) e^{j\eta \rho} d\rho \end{aligned} \quad (2)$$

and in the lower region for ($y < 0$)

$$\begin{aligned} v_\eta(y) &= \int_{-\infty}^\infty E_z(x, y) e^{j\eta x} dx \\ i_\eta(y) &= \int_{-\infty}^\infty H_x(x, y) e^{j\eta x} dx \end{aligned} \quad (3)$$

By recalling the boundary conditions on the PEC and dielectric faces, the radial transmission line ($y > 0, 0 \leq \varphi \leq \Phi_e$) is modeled by the following GWHE

$$Y_c V_+(\eta) - I_+(\eta) = -I_{+(-m)} \quad (4)$$

where $Y_c = \frac{\xi}{\omega \mu_o}$, $\xi = \xi(\eta) = \sqrt{k_o^2 - \eta^2} = \tau_o$, $m = -\eta \cos \Phi_e + \xi \sin \Phi_e$.

The longitudinal transmission line ($y < 0$) is modeled by the following classical Wiener-Hopf equation

$$Y_d V_+(\eta) + I_+(\eta) = -I_-(\eta) \quad (5)$$

where $Y_d = \frac{\xi_d}{k_d Z_d}$, $\xi = \xi(\eta) = \sqrt{k_d^2 - \eta^2}$, $k_d = k_o \sqrt{\epsilon_r}$.

III. STEPS FOR THE SOLUTION

Starting from the system of Wiener-Hopf equations (4) and (5) we apply a generalized version of the Fredholm factorization method [14-17] as reported in [13] yielding:

$$Y_c(\eta) V_+(\eta) - I_+(\eta) + \frac{1}{2\pi j} \left[\int_{-\infty}^\infty \frac{Y_c(\eta')}{\alpha(\eta') - \alpha(\eta)} \alpha'(\eta') - \frac{Y_c(\eta)}{\eta' - \eta} \right] V_+(\eta') d\eta' = f(\eta) \quad (6)$$

where $f(\eta)$ is a source term extracted from the Wiener-Hopf unknowns as offending pole. This term is related to the Geometrical Optics field (GO).

The numerical discretization of (6) [19] yields the solution in terms of $V_+(\eta)$.

Using the inverse Fourier transform and asymptotic evaluation it is possible to evaluate the GTD diffraction coefficient and the total far-fields.

III. CONCLUSION

The method proposed in this summary paper shows the possibility of the GWHE technique to solve the problem of evaluating the electromagnetic field of a perfect electric conductor wedge lying on a dielectric half-space.

ACKNOWLEDGEMENT

This work was sponsored in part by the Italian Ministry of Education, University and Research (MIUR) under PRIN grant 20097JM7YR.

REFERENCES

- [1] F. Bertoncini, R.G. Kouyoumjian, G. Manara and P. Nepa, "High-Frequency Scattering by objects buried in lossy media," *IEEE Trans. Antennas and Propagat.*, vol. 49, pp. 1649-1656, 2001.
- [2] F. Bertoncini, R.G. Kouyoumjian, G. Manara and P. Nepa, "EM Scattering by a wedge buried in a lossy medium: a UTD solution for the field in the lossy half-plane," *URSI EMTS 2004*, pp. 1035-1037, 2004.
- [3] V.G. Daniele, "New analytical Methods for wedge problems," *Proceedings of the 2001 International Conference on Electromagnetics in Advanced Applications (ICEAA01)*, Torino, Italy, 10-14 September, 2001, pp. 385-393.
- [4] V.G. Daniele, "The Wiener-Hopf technique for impenetrable wedges having arbitrary aperture angle," *SIAM Journal of Applied Mathematics*, vol. 63, pp. 1442-1460, 2003.
- [5] V.G. Daniele, G. Lombardi, "The Wiener-Hopf technique for impenetrable wedge problems," invited paper at plenary session, *Proceedings of Days on Diffraction 2005*, Saint Petersburg, Russia, pp. 50-61, June 28-July 1, 2005, doi: 10.1109/DD.2005.204879
- [6] V.G. Daniele; G. Lombardi, "Wiener-Hopf Solution for Impenetrable Wedges at Skew Incidence," *IEEE Transactions on Antennas and Propagation*, vol.54, no.9, pp.2472-2485, Sept. 2006, doi: 10.1109/TAP.2006.880723
- [7] V. Daniele, G. Lombardi, "The Wiener-Hopf method applied to multiple angular region problems: The penetrable wedge case," in *Proc. 2009 IEEE International Symposium on Antennas and Propagation and USNC/URSI National Radio Science Meeting*, Charleston, SC, 2009, pp. 1-4, doi: 10.1109/APS.2009.5172090.
- [8] V.G. Daniele, "The Wiener-Hopf formulation of the dielectric wedge problem. Part I," *Electromagnetics*, vol.30, n.8, pp.625-643, 2010.
- [9] V.G. Daniele, "The Wiener-Hopf formulation of the dielectric wedge problem. Part II," *Electromagnetics*, vol.31, n.1, pp.1-17, 2011.
- [10] V.G. Daniele, "The Wiener-Hopf Formulation of the Penetrable Wedge Problem. Part III: The skew incidence case," *Electromagnetics*, vol.31,n.8 : pp.550-570, 2011.
- [11] V. Daniele, G. Lombardi, "The Wiener-Hopf Solution of the Isotropic Penetrable Wedge Problem: Diffraction and Total Field," *IEEE Transactions on Antennas and Propagation*, vol.59, no.10, pp.3797-3818, Oct. 2011, doi: 10.1109/TAP.2011.2163780
- [12] G. Lombardi, "Skew Incidence on Concave Wedge With Anisotropic Surface Impedance," *IEEE Antennas and Wireless Propagation Letters*, vol.11, no., pp.1141-1145, 2012, doi: 10.1109/LAWP.2012.2219845
- [13] V.G. Daniele, "Electromagnetic fields for PEC wedge over stratified media. Part I,," submitted to *Electromagnetics*.
- [14] N.P. Vekua, *Systems of Singular Integral Equations*, Gronigen: P. Noordhoff, 1967.
- [15] V.G. Daniele, An introduction to the Wiener-Hopf technique for the solution of electromagnetic problems. Internal Report ELT-2004-1. Available on line at <http://personal.delen.polito.it/vito.daniele/>
- [16] V.G. Daniele; G. Lombardi, "Fredholm Factorization for Wedge Problems," *Antennas and Propagation Society International Symposium 2006, IEEE*, pp. 2478-2481, 9-14 July 2006, Albuquerque, NM, USA, doi: 10.1109/APS.2006.1711100
- [17] V. Daniele, and G. Lombardi, "Fredholm factorization of Wiener-Hopf scalar and matrix kernels," *Radio Sci.*, 42, RS6S01, 2007, doi:10.1029/2007RS003673
- [18] V.G. Daniele, The Wiener-Hopf technique for wedge problems, Internal Report ELT-2004-2. Available on line at <http://personal.delen.polito.it/vito.daniele/>
- [19] L.V. Kantorovich, V.I. Krylov, *Approximate methods of higher analysis*, Groningen: P.Noordhoff, 1964