Size-Independent Cylindrical Resonator Half-Filled With DNG Metamaterial and Excited by a Line Source

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Vito G. Daniele, Roberto D. Graglia, *Fellow, IEEE*, Guido Lombardi, *Senior Member, IEEE*, and Piergiorgio L. E. Uslenghi, *Life Fellow, IEEE*

***Abstract*—A circular cylindrical resonator with metallic (PEC) walls is half-filled with double-negative (DNG) metamaterial that is anti-isorefractive to the double-positive (DPS) material filling the remaining volume of the resonator. The resonance condition of the structure is studied and it is shown that the resonator may perform independently of diameter size. The problem of an electric line source parallel to the axis and located anywhere inside the DPS region is solved exactly. Numerical results are presented and discussed.**

***Index Terms*—Cavity resonator, electromagnetic theory, metamaterial.**

I. Introduction

THE concept of utilizing double-negative (DNG) metamaterial inclusions to render a resonator size-independent was introduced by Engheta [1]-[2] for a one-dimensional structure, to show that it is possible to build resonators that function independently of dimensions at those frequencies for which the metamaterial behaves as postulated. Three dimensional cavity resonators were investigated by Couture et al. [3]-[4] and by Uslenghi [5]-[7].

Recently, Daniele et al. [8]-[9] studied in detail a cylindrical resonator sectorally filled with metamaterial; in particular, their analysis showed that phase compensation leading to size independence is possible only if the cylinder is half-filled with metamaterial. This is the case examined in the present work, in which the excitation is provided by an electric line source located anywhere inside the double-positive (DPS) half cylinder. The technique proposed in [9] was based on the resolvent technique [10] and the evaluation of the one dimensional characteristic Green function in the azimuthal direction. This technique fails when DPS and DNG subvolumes are semi-cylinders of equal size since it is not possible to define the characteristic Green function.

The proposed solution of the present problem is based on a heuristic interpretation of the physical problem and on the definition of the Green's function using a radial representation as done in [11]. Preliminary analyses of this problem and some progress are reported in abstracts by the authors of this paper at two symposia [12]-[13].

V.G. Daniele, R.D. Graglia and G. Lombardi are with Dipartimento di Elettronica e Telecomunicazioni, Politecnico di Torino, 10129 Torino, Italy e-mails: vito.daniele@polito.it, roberto.graglia@polito.it, guido.lombardi@polito.it.

P.L.E. Uslenghi is with the Department of Electrical and Computer Engineering and the Department of Physics, University of Illinois at Chicago, Illinois 60607, USA e-mail: uslenghi@uic.edu.

Manuscript received April -, 2013.

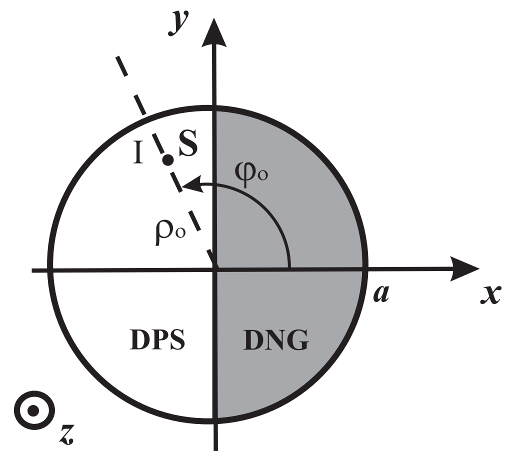

Fig. 1. Cross section of the resonator. The electric line source is located at (ρ_0, φ_0) inside the DPS region.

The geometry of the problem is described in Section II. The condition that must be satisfied for resonance to occur is given in Section III. The procedure for the solution while the resonator is excited by an electric line source parallel to the axis of the cylinder and located anywhere inside the DPS volume is solved analytically in Section IV, and some numerical results based on this exact solution are shown and discussed in Section V.

II. Geometry of the Problem

With reference to rectangular coordinates (x, y, z), the metallic (PEC) cylindrical resonator has the z axis as symmetry axis, the inner radius of its circumference in any plane $z = constant$ is a, and the length of the resonator in the z direction does not come into play because the resonator is assumed to be excited by an electric line source parallel to its axis (1), leading to an electric field that is everywhere parallel to z, so that the boundary conditions at the two circular bases of the cylinder are always satisfied. The half-cylinder occupying the volume $x < 0$ is filled with DPS material characterized by a real positive permittivity ε and a real positive permeability μ, *i.e.* by a wavenumber $k = \omega\sqrt{\varepsilon\mu}$ and an intrinsic impedance $Z = \sqrt{\frac{\mu}{\varepsilon}}$. The half-cylinder occupying the volume $x > 0$ is filled with a DNG material characterized by a real negative permittivity $-\varepsilon$ and a real negative permeability $-\mu$, whose wavenumber $-k$ is the opposite of the wavenumber in the DPS region, and whose intrinsic impedance has the same value Z of the DPS region. The planar interface $x = 0$ separates the DPS and DNG regions.

With reference to circular cylindrical coordinates (ρ, φ, z), the DPS material fills the cross-sectional area ($0 \le \rho \le a$, $\pi/2 \le \varphi \le 3\pi/2$), whereas the DNG material fills the cross-sectional area ($0 \le \rho \le a$, $-\pi/2 \le \varphi \le \pi/2$). The electric line source $\mathbf{J_0}$ located inside the DPS region at $\boldsymbol{\rho_0}$ ($\rho = \rho_0$, $\varphi = \varphi_0$) is assumed to be:

$$\mathbf{J_0} = I\delta(\boldsymbol{\rho} - \boldsymbol{\rho_0})\hat{z} = I\frac{1}{\rho_0}\delta(\rho - \rho_0)\delta(\varphi - \varphi_0)\hat{z} \quad (1)$$

where I is the intensity (in A), $\hat{z}$ is a unit vector parallel to the z axis, and δ is the delta function.

A cross section of the resonator in a plane perpendicular to its axis is shown in Fig. 1.

III. Resonance Condition

The resonance condition of a cylindrical resonator sectorally filled with metamaterial was obtained in [9] by assuming the electric field in the cylindrical cavity to be of the form:

$$\mathbf{E}^{\pm} = \hat{z}E_z^{\pm}(\rho, \varphi) = \hat{z}J_\nu(\pm k\rho)[A_\nu^{\pm}\sin(\nu\varphi) + B_\nu^{\pm}\cos(\nu\varphi)] \quad (2)$$

where the upper (lower) sign applies to the DPS (DNG) subvolume and where J_ν is the Bessel function of first kind and of order ν. Expressions for the magnetic field components are available through Maxwell equations. Imposition of the boundary conditions across the faces of the wedge separating the DPS and DNG regions, *i.e.* the continuity of E_z and H_ρ across the interfaces, leads to an algebraic system of four homogenous equations for the four unknown constants $A_\nu^{\pm}$ and $B_\nu^{\pm}$. For nonzero fields to exist, the determinant of the coefficients must be zero, yielding:

$$\sin[(\pi - 2\alpha)\nu] = 0, \quad (3)$$

where α is the semiaperture angle of the DNG wedge. The same relation (3) is obtainable from the dispersion relation given by Osipov for any homogeneous penetrable wedge [14]. As shown by Osipov [14], such a relation always results in the product of trigonometric functions being equal to zero. In the particular case of a wedge of semi-aperture angle α, whose faces separate two regions of space that are anti-isorefractive to each other, the dispersion relation takes the simple form (3).

In the present work we have $\alpha = \pi/2$ and therefore equation (3) is satisfied for any value of ν. From (2) it follows that the boundary condition on the PEC wall of the resonator

$$J_\nu(ka) = 0 \quad (4)$$

yields the allowed values of ν for any given frequency, that is, the cylinder resonates at all frequencies for which the DNG material behaves as postulated, and therefore the resonator can be miniaturized.

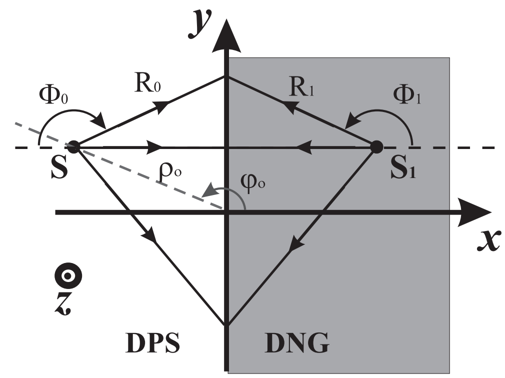

Fig. 2. DNG and DPS half spaces with line sources $\mathbf{J_0}$ and $\mathbf{J_1}$ respectively located at S and S_1 and with intensity I and $-I$. The arrows show the direction of the phase velocity. Local coordinate systems (R_0, Φ_0) and (R_1, Φ_1) are introduced for each line source.

IV. Solution for Line Source Excitation

The proposed solution of the present problem is based on an heuristic interpretation of the physical problem and on the definition of the Green's function using a radial representation as proposed in [11].

A. Line Source Excitation in half space

First, let us study a simpler problem by supposing $a \to \infty$, see Fig. 2. The figure shows two half spaces filled respectively by a DNG metamaterial and a DPS medium. Inside the DPS medium is located a physical line source $\mathbf{J_0}$ in S (ρ_0, φ_0) as defined in (1). The arrows reported in the figure are related to the phase velocity of different propagating rays, in accordance to theory described by Veselago [15]: in a DNG medium the phase velocity (the wave vector) are anti-parallel to the direction of Poynting vector (group velocity).

Using symmetry, we introduce an image electric line source $\mathbf{J_1}$ inside the DNG medium located at the image point identified by the vector $\boldsymbol{\rho_1}$ ($\rho = \rho_1 = \rho_0$, $\varphi = \varphi_1 = \pi - \varphi_0$) in cylindrical coordinates:

$$\mathbf{J_1} = -I\delta(\boldsymbol{\rho} - \boldsymbol{\rho_1})\hat{z} = -I\frac{1}{\rho_0}\delta(\rho - \rho_0)\delta(\varphi - \pi + \varphi_0)\hat{z} \quad (5)$$

where I is the intensity (in A) of the physical line source, $\hat{z}$ is a unit vector parallel to the z axis, and δ is the delta function. The expression of the field component due to the two line source $\mathbf{J_0}$ and $\mathbf{J_1}$ are respectively:

$$\begin{cases} \mathbf{E}^{(0)} = E_z^{(0)}\hat{z} \\ \mathbf{H}^{(0)} = -\frac{1}{jkZ}\frac{\partial E_z^{(0)}(\rho,\varphi)}{\rho\partial\varphi}\hat{\rho} + \frac{1}{jkZ}\frac{\partial E_z^{(0)}(\rho,\varphi)}{\partial\rho}\hat{\varphi} \end{cases} \quad (6)$$

and

$$\begin{cases} \mathbf{E}^{(1)} = E_z^{(1)}\hat{z} \\ \mathbf{H}^{(1)} = \frac{1}{jkZ}\frac{\partial E_z^{(1)}(\rho,\varphi)}{\rho\partial\varphi}\hat{\rho} - \frac{1}{jkZ}\frac{\partial E_z^{(1)}(\rho,\varphi)}{\partial\rho}\hat{\varphi} \end{cases} \quad (7)$$

where the propagation constants in the DPS and DNG medium are respectively k and $-k$. $E_z^{(0)}$ and $E_z^{(1)}$ assume the classic expression of the electric field radiated by a line source in a homogenous medium and they are given in terms of Hankel function of second kind and zero order $\mathcal{H}_0^{(2)}$:

$$E_z^{(q)}(\rho,\varphi) = -\frac{kZ}{4} I\mathcal{H}_0^{(2)}(k\left|\boldsymbol{\rho}-\boldsymbol{\rho}_q\right|), \quad q=0,1 \tag{8}$$

where q identifies the medium (0=DPS, 1=DNG). To deduce (8) we have taken into account the properties of Hankel function and the properties of the two materials. Using (6) and (7) it is straightforward to prove that the solution of the problem of Fig. 2 is given by:

$$\mathbf{E} = \mathbf{E}^{(0)} p_o + \mathbf{E}^{(1)} p_1 \tag{9}$$

where p_0 and p_1 are window functions vanishing respectively in the DNG space and in the free space. In fact (9) satisfies all the boundary conditions for the E and H field components at the interface $x=0$. Introducing local cylindrical coordinate systems (R_q,Φ_q) where $R_q = |\boldsymbol{\rho}-\boldsymbol{\rho_q}|$ for each source S_q with $q=0,1$; from (7) and (8) we obtain at the interface $x=0$

$$H_{R_q}^{(q)}(R_q,\Phi_q) = 0 \tag{10}$$

$$H_{\Phi_q}^{(q)}(R_q,\Phi_q) = \mp\frac{j}{4} kI\mathcal{H}_1^{(2)}(kR_q) \tag{11}$$

for $q=0,1$.

B. The problem under investigation

We extend the procedure of the previous subsection to the problem under investigation. The main difficulty is to obtain an expression of $E_z^{(0)}$ and $E_z^{(1)}$ for the cylindrical resonator half-filled with DNG metamaterial. We resort to the definition of the Green's function using a radial representation as done in [11]. It yields:

$$E_z^{(q)}(\rho,\varphi) = jkZIg_q(\boldsymbol{\rho},\boldsymbol{\rho}_q),\ q=0,1 \tag{12}$$

where q identifies the medium (0=DPS, 1=DNG) and

$$g_q(\boldsymbol{\rho},\boldsymbol{\rho}_q) = \frac{1}{2\pi}\sum_{m=0}^{\infty}\alpha_m \hat{g}_{qm}(\rho,\rho_q)\cos[m(\varphi-\varphi_q)] \tag{13}$$

with $\alpha_0=1$, $\alpha_m=2$ for $m>0$ and $\varphi_1=\pi-\varphi_0$. The functions g_q are defined as

$$\hat{g}_{qm}(\rho,\rho_q) = \frac{\pi J_m(\pm k\rho_<)D_m(\pm k\rho_>)}{2J_m(\pm ka)} \tag{14}$$

with

$$D_m(k\rho) = J_m(k\rho_>)Y_m(ka) - J_m(ka)Y_m(k\rho_<) \tag{15}$$

and $\rho_<$ $(\rho_>)$ is the smaller (larger) between ρ and ρ_0. In (14)-(15) J_m and Y_m are respectively the Bessel functions of the first kind and second kind. It is easily verified that the boundary conditions at the interface between DPS and DNG regions are satisfied.

V. Numerical results

The expressions of the electromagnetic fields inside the structure are given in (12) and (6)-(7).

As an example, we consider a line source of intensity $I=1A$ located at ($\rho_0=0.7a$, $\varphi_0=5\pi/8$) with $Z=377\Omega$.

The field excited by the line source assumes the expression reported in (12) that shows a discrete spectrum, although we have shown in Section III that any ν is with compatible with the structure at issue. The discrete spectrum is characterized by Bessel functions of first and second kind and integer order m, in contrast to TM modes in resonators filled completely by free space that only require functions of the first kind.

Fig. 3 shows the contour plot of the E_z component for different values of ka respectively for the classic circular resonator completely filled by free space (case a,c,e) and for the DNG half-filled circular resonator (case b,d,f). We note that: case a) and b) are with $ka=1$ (*i.e.* below the first resonance of the corresponding circular resonator completely filled by free space), case c) and d) are with $ka=4.5$ therefore only TM_{01} and TM_{11} modes are propagating in the corresponding unfilled circular resonator, case e) and f) are with $ka=8$ therefore 7 modes are above cut-off in the corresponding unfilled circular resonator. Fig. 4 shows the vector plots of the transverse component of the magnetic field $\boldsymbol{H_t}$ for the circular resonator half filled with DNG and excitation $I=1A$, $\rho_o=0.7a$, $\varphi_o=5\pi/8$ with $ka=4.5$

The numerical results are obtained using the following arrest criterion in the summation (13):

$$\frac{\left|\frac{1}{2\pi}\hat{g}_{qm+1}(\rho,\rho_q)\right|}{\left|g_q^{(M)}(\boldsymbol{\rho},\boldsymbol{\rho}_q)\right|} < 10^{-N} \tag{16}$$

where $g_q^{(M)}(\boldsymbol{\rho},\boldsymbol{\rho}_q)$ is the summation (13) with a finite number M of terms and $q=0,1$. Fig. 3 and 4 are obtained using $N=3$.

Discussion and Conclusion

In this work we have examined two interrelated problems: the resonance condition of the resonator, and the fields present when the primary field exciting the resonator is an electric line source. In contrast to the case examined in [9], the resonance condition (3) is here satisfied identically, meaning that the wavenumber inside the resonator is not quantized, but belongs to a continuum spectrum. Thus, any single mode such as (2) may exist independently of frequency, provided that the constant ν satisfies (4). When the resonance condition (3) is identically satisfied, as is the case in the present work, the straightforward manner of finding the Green function for a specified source, as outlined in [10], does not work (this is clearly explained e.g. in [16]) and an ad-hoc approach must be developed to solve the boundary-value problem analytically. In our case, this ad-hoc approach is provided by the method of images. Therefore, it should be noted that although the geometry of the structure in the present work is a particular case of the geometry studied in [9], the method of solution of the problem for line source excitation is radically different in the two cases.

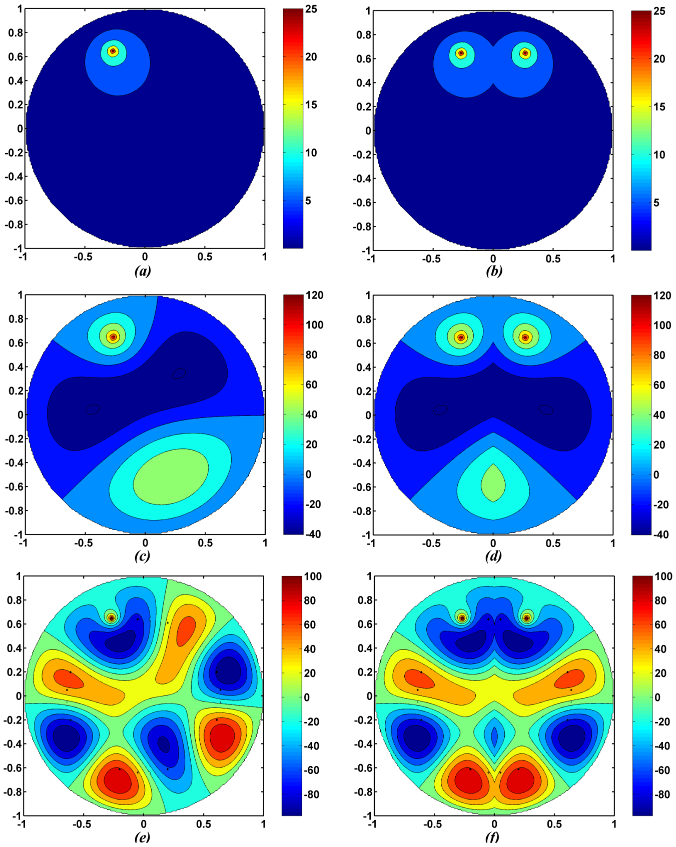

Fig. 3. Contour plots of the amplitudes of E_z for $I = 1A$, $\rho_o = 0.7a$, $\varphi_o = 5\pi/8$. Plots (a),(c) and (e) are for a resonator filled with DPS material. Plots (b),(d) and (f) are for the resonator half-filled with DNG material. Plots (a),(b) are for $ka = 1$; plots (c) and (d) are for $ka = 4.5$; plots (e) and (f) are for $ka = 8$.

ACKNOWLEDGMENT

This work was sponsored in part by the Italian Ministry of Education, University and Research (MIUR) under PRIN grant 20097JM7YR, and in part by the College of Engineering in the University of Illinois at Chicago.

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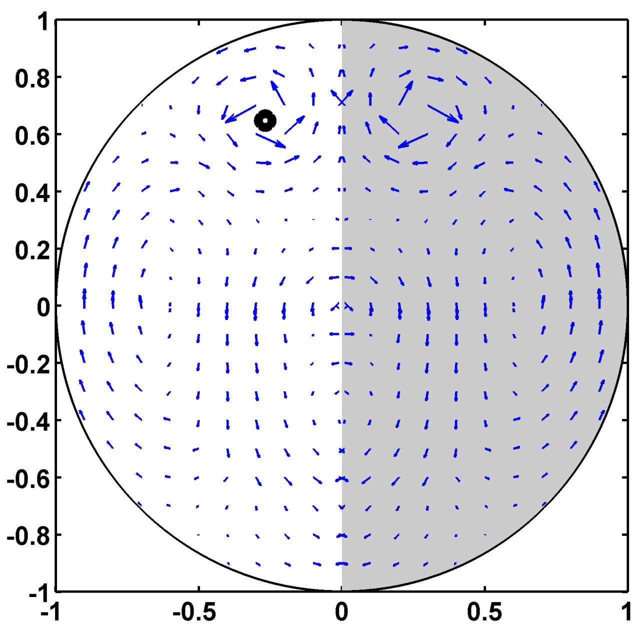

Fig. 4. Vector plots of $\boldsymbol{H_t}$ for the circular resonator half filled with DNG and excitation $I = 1A$, $\rho_o = 0.7a$, $\varphi_o = 5\pi/8$ with $ka = 4.5$.