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1 **Methods for traceability in food production processes involving**
2 **bulk products**

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13
14 **Keywords:** traceability, batch dispersion, food processing, compartmental models

15
16 **Abstract**

17 In food processing plants, raw materials are fed to the system in different *supply-lots* of
18 product, and are processed through different stages. In these stages, raw or intermediate
19 materials are mixed or combined together, and physical-chemical and/or microbiological
20 processes such as heating, concentration, pasteurisation etc. take place. In this setting,
21 traceability consists in the ability to determine for each portion of intermediate or final
22 product, in any part of the plant, its relative composition in terms of supply-lots fed into the
23 system as well as of new lots generated during the production process.

24 The traceability problem becomes particularly difficult in the very-frequent case when bulk
25 products, such as liquids or grains, are involved in the production chain. Current traceability
26 practices are in most cases unable to directly deal with bulk products, and typically resort to
27 the definition of very large lots to compensate the lack of knowledge about lot composition.
28 As demonstrated even in very recent food crisis, this over-bounding approach has shown its
29 weakness in the identification of the interested products immediately after risk assessment,
30 leading to unavoidably wide, expensive and very impacting recalls.

31 Motivated by these considerations, this paper presents a novel approach to manage
32 traceability of bulk products in production, storage and delivering phases that provides a tight
33 definition of lots in terms of their composition and size, thus allowing a strict control of the
34 production and supply chains.

35 **1. Introduction**

36 The problem addressed in this paper refers to the traceability of food products in processing
37 plants, or part thereof, in which raw materials to be processed are fed to the system in
38 different supply-lots of bulk product, with specific attention to the frequent case when bulk
39 products are involved in the production phase. Indeed, many ingredients used in food
40 industries are liquids (milk, vegetal oils, etc.), powders (cocoa, powdered milk, flour, yeast
41 etc.), crystals (e.g. sugar, salt) or grains. These products are stored, in many cases, in huge
42 silos or tanks, which are very rarely completely emptied, so that many lots are
43 contemporarily kept in the same container. Throughout the plant, the supplied material is
44 processed in one or more production lines until one or more final products are created,
45 packed, and stored ready for sale.

46 Typically, the production process consists of different stages, which are usually carried out in
47 different production stations. Some stages involve different raw or intermediate materials that
48 are mixed or combined together, while in other stages physical-chemical and/or
49 microbiological processes such as, for instance, heating, cooling, concentration, and
50 pasteurisation, take place. Thus the production process generates different production lots.

51 In food processing plants, supplied raw materials as well as intermediate products are usually
52 stored in silos, tanks or other suitable containers, before being processed or during the
53 process itself in between different production stages. In general, the material stored in one
54 container is delivered to it in different batches, each one possibly constituted of material
55 coming from different supply or production lots. Whenever the stored material is drawn from
56 a container in order to be delivered to a production station or to a new storage container, the
57 retrieved material results in a combination of material from the different batches that have
58 been previously fed into the container.

59 In this setting, traceability consists in the ability to determine for each portion of intermediate
60 or final product, at any time and in any part of the plant, its composition in terms of supply-
61 lots fed into the system. This information is indeed crucial for identifying the amount and
62 location of product portions affected by possible deficiencies caused by a defect of the
63 material delivered in one of the supply-lots. The event of food recalls due to unforeseen
64 problems is becoming more and more frequent: for instance, the web-site provided by the US
65 Food and Drugs Administration, which gathers information from press releases and other

66 public notices about certain recalls of FDA-regulated products, listed more than seventy cases
67 of recalls for the first two months of 2013 (US Food and Drugs Administration, 2013).

68 One possible approach to minimize recalls consists in maintaining the lots as much as
69 possible separated in the plant. In the case of fluids, for instance, the use of different
70 containers and cleaning between two product batches is a viable solution to allow distinct
71 separated batches identities. In particular, *cleaning-in-place* procedures, which involve
72 pumping water and detergent through the production equipment, besides guaranteeing high
73 hygienic standards, is foreseen as the good procedure to strictly guarantee that the different
74 batches cannot contaminate each other. However, these cleaning procedures, besides
75 representing a high cost for the company in terms of energy, manpower, and cleaning agents,
76 can become undesirable in the case of continuous production systems (such as, e.g., milk
77 production in a dairy) where *continuous flow*, without even minimal interruptions, of
78 liquid/granular raw material is necessary to maintain the production.

79 In these cases, the currently adopted solution consists in defining large lots, mainly referred
80 to production periods rather than to their precise composition of the lots. For instance, lots
81 based on the production day (or even a whole week) are typically encountered. This rather
82 conservative approach, based on the definition of very large lots, has shown its weaknesses in
83 recent cases of food recall, when the lack of knowledge in the identification of the interested
84 products immediately after risk assessment, has unavoidably led to wide, expensive and very
85 impacting recalls.

86 Moreover, to most types of bulk products, it is very difficult to associate any kind of label,
87 marker or identifier that could directly identify the different lots. Recently, some markers
88 based on RFID technology have been developed for the case of continuous granular flows
89 (specifically, iron pellets) by Kvarnström et al. (2011). These allow on-line traceability of
90 continuous flows, thus improving upon previous off-line solutions based on the introduction
91 of specific tracers into the grains, such as chemical compounds or radioactive tracers; see
92 Kvarnström and Oghazi (2008) and Lee et al., (2010) for detailed discussion and references
93 on these techniques. The situation is complicated by the obvious requirement that the markers
94 should not compromise by any means the integrity and quality of the food and must be not
95 dangerous for the consumer. Thus, any RFID-based traceability system would require the
96 development of a device for safely removing the tracing devices from the final product (e.g.
97 before grain grinding). In this regard, some interesting solutions have been proposed in Lee et
98 al. (2010) and Liang et al. (2012) for the specific case of grains, which involve particular pill-

99 sized food-grade tracer particles to be inserted directly into grain during harvest. These
100 tracers have printed with food-grade ink a miniaturized data-matrix code carrying identity
101 information related to product origins, and are composed of materials that can be safely eaten
102 such as sugar or cellulose. Anyway, these solutions remain principally an off-line approach,
103 suitable for modelling and validation purposes, since collecting and identifying the tracers
104 would usually still require interrupting the production.

105 The problem of the traceability of fluid products in the case of continuous processing has
106 been, to our best knowledge, addressed first by Skoglund and Dejmek (2007), who used
107 dynamic models and simulations to identify the changeover of lots of liquid product in a pipe.
108 The presence of portions of product deriving from the partial mixing of two subsequent lots
109 led to the introduction of the concept of *fuzzy traceability*. An interesting approach has been
110 also proposed by Bollen et al. (2007) and by Riden and Bollen (2007), who considered the
111 case of apples processed in a packhouse. Apples, supplied to the packhouse in bulk bins, are
112 moved in a bulk flow (water dump) up to the grader that handles individual fruits and directs
113 them into packaging lines. At the end of these lines the fruits are placed into homogeneous
114 packs (in terms of colour or size). During their flow in the water dump and then in the
115 packaging lines, some mixing among different lots of apples occurs. Even if apples are
116 discrete items, their fluidized flow can be assimilated to the flow of small particles. In their
117 first paper, Bollen et al. (2007) developed and validated a set of statistical models using the
118 measured arrival sequence of 100 blue marker balls. The proposed models are able to assign
119 a probability of bin origin to any individual fruit in the final packs.

120 The performances of a traceability system can be identified with the skill of limiting the
121 quantity of final product to be recalled in the averted case of food safety crisis (Dabbene &
122 Gay, 2011). However, at present methods to precisely estimate the amount of product that has
123 to be discarded in the case of a recall are available only for the case in which discrete lots of
124 product are processed (Dabbene, Gay, & Tortia, 2013; Dabbene & Gay, 2011; Dupuy, Botta-
125 Genoulaz, & Guinet, 2005). The quantity of product to be recalled, to which a *recall cost* is
126 associated, may depend on many factors, among which the size of the batches that have been
127 individually tracked and managed by the traceability system (and hence the skill of the firm
128 in managing and maintaining segregated different batches of product), and the way the
129 batches of different components have been mixed to obtain the final product.

130 These methods have been applied to many different supply chains, e.g. for fruits (Bollen et
131 al., 2007; Riden & Bollen, 2007), meat (Barge P., Gay P., Merlino V., & Tortia C., 2013;

132 Donnelly, Karlsen, & Olsen, 2009; Dupuy et al., 2005), fish (Karlsen, Donnelly, & Olsen,
133 2011; Randrup et al., 2008), grains (Thakur & Donnelly, 2010; Thakur & Hurburgh, 2009;
134 Thakur, Wang, & Hurburgh, 2010), chocolate (Saltini & Akkerman, 2012), perishable
135 products (Li, Kehoe, & Drake, 2005; Rong & Grunow, 2010; Wang X., Li D., & O'Brien C.,
136 2009) etc.

137 To allow traceability of bulk products, a convenient model of the production plant is needed.
138 This model should provide a description of the production process in terms of mass transfer
139 and storage at a lot level of detail, in order to enable an accurate prediction of the dynamics
140 of each supply-lot that can therefore be conveniently tracked.

141 The paper is structured as follows: in Section 2 a thorough theoretical analysis is carried out
142 and a modelling framework based on compartmental models is derived. Section 3 addresses
143 the problem of the determination of specific models of the two basic cases of uniform-mixing
144 and FIFO tanks. A simulation case study, showing the effectiveness of the proposed
145 methodology, is proposed in Section 4. Finally, conclusions are drawn in Section 5.

146 **2. Definitions and problem formulation**

147 The first step for developing the framework introduced in this work consists in providing a
148 formal definition of lots and of lot homogeneity.

149 **Definition 2.1** (Lot) *A lot is defined as a set of units of a product that are homogeneous in*
150 *terms of composition and processing history.*

151 This definition is coherent with the one reported in ISO 22005 (2008), where a lot is defined
152 as “*set of units of a product which have been produced and/or processed or packaged under*
153 *similar circumstances*”, and it extends to some degree the concept of traceable unit (TRU)
154 introduced by Kim et al., 1999. It should be noted that at this point the notions of
155 *homogeneity* and *composition* considered in Definition 2.1 are still rather vague, and need a
156 rigorous formalization to be of practical value. To this end, the concept of S-lot (supply lot) is
157 explicitly defined next

158 **Definition 2.2** (S-lot) *An S-lot is defined as a set of units of homogeneous raw materials that*
159 *enter the system from outside.*

160 More specifically, S-lots represent raw-materials or semi-processed products fed into the
161 system by a supplier as a unique lot. At each instant, the traceability system should be able to

162 determine the relative composition, in terms of S-lots, of the material present in the different
 163 intermediate production stages, with specific attention to the composition of the final
 164 products leaving the production chain.

165 To exemplify, consider the case in which two different raw-materials are fed into the system
 166 and are labelled for simplicity as ‘A’ and ‘B’. Then, the relative composition of a final
 167 product X leaving the chain is given by the percentages $\gamma^A(X)$ and $\gamma^B(X)$ of materials ‘A’
 168 and ‘B’ present¹ in X . More generally, the composition of a product can be defined as follows

169 **Definition 2.3** (Composition) *Let $\mathcal{L} = \{ 'A', 'B', 'C', \dots \}$ denote the (ordered) list of possible S-*
 170 *lots entering the system. Then, the (relative) composition of a product X is defined as the*
 171 *vector of percentages of the different S-lots composing X , that is,*

$$c(X) \doteq [\gamma^A(X) \ \gamma^B(X) \ \gamma^C(X) \ \dots]^T. \quad (1)$$

172 The above definition is instrumental to a rigorous definition of homogeneous materials, in
 173 terms of composition, which in turn represents a fundamental step towards a rigorous
 174 treatment of the traceability problem for the case of bulk materials. To this end, the
 175 *composition-distance* between two products X and Y is introduced as follows

$$d(X, Y) \doteq \|c(X) - c(Y)\|_\infty, \quad (2)$$

176 where $\|x\|_\infty \doteq \max_{L=1, \dots} x_L$ denotes the ℓ_∞ -norm of vector x . Note that composition-
 177 distances different than (2) can be introduced: for instance a *weighted-norm* version, with
 178 $\|x\|_\infty^W \doteq \max_{L=1, \dots} w_L x_L$, can be considered in order to take into account the different risks
 179 associated with the different S-lots. In this case, the larger is the *risk-factor* w_L , the more
 180 importance is given to S-lot L . The concept of composition distance $d(X, Y)$ allows the
 181 following rigorous formalization of homogeneity.

182 **Definition 2.4** (Homogeneous products) *Given a threshold level δ , two products X and Y are*
 183 *said to be homogeneous in composition (up to accuracy ρ) if their composition-distance is*
 184 *less than δ , i.e.*

$$d(X, Y) < \delta. \quad (3)$$

185 Note that this definition does not take into account processing history. Clearly, a
 186 homogeneous-in-composition lot processed in $m > 1$ sessions splits in m ‘production’ lots

¹ Here $\gamma^A(X)$ has to be interpreted as the percentage of product coming from S-lot ‘A’ present in X . A formal definition is given in Section 3.

187 characterized by the same composition vector. The handling of these production lots can be
188 performed in a completely analogous way to the one discussed in this paper, and it is not
189 considered in the present work for sake of simplicity.

190 Note also that the introduction of the quantization level δ is absolutely necessary when
191 dealing with bulk products, since in principle the relative composition of the materials can
192 vary with continuity. This approach, based on a threshold level, reflects what proposed in the
193 EC Regulation No 1829/2003 (European Commission, 2003) for genetic modified (GM) and
194 non-GM grains labelling. In this case, for the consumer information, these regulations
195 guarantee that any food containing material that contains more than 0.9% of GM would be
196 labelled as “contains GM”.

197 It immediately follows from Definition 2.4 that two materials whose composition-distance is
198 greater than δ cannot belong to the same lot (according to Definition 2.1). Consequently,
199 every time two products in the supply chain assume a composition-distance greater than the
200 considered threshold, the traceability system should be able to detect this event and keep
201 trace of the two products (and their specific composition) separately. Hence, this framework
202 provides a direct and natural way of discriminating final products and, possibly, to divide
203 them into homogeneous lots.

204 Like the already mentioned case of GM and non-GM grains, there are other situations related
205 to ethical, organic, low carbon footprint, issued or subject to disciplinary, as well as to
206 religious constraints, where lots should be maintained as much as possible separated and
207 facilities and logistics have to be designed and planned accordingly. Different management
208 strategies have been proposed to cope with this problem and these are typically based on
209 separation of products *in space*, allocating specific collecting units (e.g. silos) for any
210 different lot, or on separation *in time*, where different lots are processed in successive
211 sessions, separated by suitable cleaning cycles (see e.g. Coléno, 2008; Maier, 2006).

212

213 In this work, to derive accurate methods for tracing the composition of the product in terms
214 of S-lots are derived using specific *compartmental models*. Compartmental models are
215 mathematical models widely used to describe the way in which materials and/or energies are
216 transferred among (and stored within) the different parts of a physical system (Godfrey,
217 1983). Although compartmental models have been primarily developed in biomedical
218 engineering (the interested reader can refer to Rescigno (2001) for a short overview and a

219 critical analysis of their use), they have been also used recently by Comba et al. (2011) to
220 describe heat-transfer phenomena in food plants characterized by mixed
221 continuous/discontinuous flow food plants of materials.

222 Indeed, in principle, a food production plant can be modelled as a set of storage
223 compartments, each one corresponding to a storage container or to a batch processing station.
224 Examples of compartment are tanks, vats, silos but also grain dryers, mixers, chocolate
225 conching machine, cheese-vats etc. Material is transferred from a compartment to another
226 either by flows, that in most cases are discontinuous (in time), or in batches. The description
227 of these phenomena is usually simple and quite precise, since flows between compartments
228 and masses of batches are known with good precision, and mass transfer equations are
229 accurate. This information can be easily acquired from the plant itself, by monitoring the
230 states of valves, pumps, conveyors, and, in general, any device that controls the flow of the
231 material in the plant. Then, assuming that the relative composition of flows and batches in
232 terms of S-lots is properly known, also the dynamics of such lots, in connection with the
233 mass transfers among tanks, can be accurately determined (see, for instance, Skoglund and
234 Dejmek, 2007 for the case of liquid products).

235 The crucial point is indeed to know such relative composition, which is not always an easy
236 task. In order to better understand this point, the behaviour of the compartments used to
237 describe the production plant should be analysed, since any product flow or product batch
238 transferring masses from compartment to compartment can be regarded as the output of a
239 specific compartment. Only the inflow into the system of S-lots cannot be regarded as the
240 output of a compartment, but the composition of such flow (or batch delivery) in terms of S-
241 lots is indeed well known.

242 Any compartment, whether it represents a storage unit, like a silo, or a processing station, like
243 a mixer, a concentrator, a heater, etc., is itself a dynamic system. As a matter of fact it can
244 store some amount of mass delivered to it over time through one or more inputs and each one
245 of its output flows is a suitable combination of the masses stored in it.

246 Assuming that the relative composition of input flows in the compartment (or batch deliveries
247 to it) in terms of S-lots is perfectly known, then the relative composition of the outputs can be
248 accurately computed only if the storage mechanism in the compartment is accurately known
249 together with the laws supervising the way in which output flows are formed from the stored
250 material.

251 There are at least two important and representative cases in which this happens. The first case
252 is when all the material delivered to a compartment is instantaneously and uniformly mixed.
253 Under this condition, referred to as *uniform-mixing (UM) compartment*, the relative
254 composition of the material in the compartment in terms of S-lots is perfectly known at any
255 time from the knowledge of the composition of the input flows (or batch deliveries). Hence,
256 the relative composition of the output flow at any time is the same of the material in the tank
257 at the same time.

258 The second case is when a single-input-single-output compartment behaves as a first-in-first-
259 out (FIFO) buffer in which, however, input and output mass flows do not need to share the
260 same intensity-time profiles. This second condition is referred to as *FIFO compartment*.

261 Remark that if a plant can be fully described using only UM or FIFO compartments, then the
262 relative composition of any lot in the plant can be accurately derived, as detailed in Sections
263 3 and 4, and thus lot traceability can be conveniently implemented.

264

265 **3. Modelling uniform-mixing and FIFO compartments**

266 In this section, the two important cases of UM and FIFO compartments, schematizing storage
267 units or processing stations in food processing plants, are analysed, and specific models are
268 derived.

269 In the following, it is assumed that a total of ℓ different S-lots are available, belonging to the
270 set of labels $\mathcal{L} = \{ 'A', 'B', 'C', \dots \}$, with $\text{card}(\mathcal{L}) = \ell$. Moreover, for the sake of simplicity
271 and without loss of generality, it is assumed that any mass that is fed to the production chain
272 belongs to one and only one S-lot at the time it enters the system.

273 The case of n interconnected tanks is considered, with material flowing from the outside and
274 between them. Considering a generic compartment i , it follows that there are possibly up to n
275 different mass inflows $q_{ij}(t)$, $j = 0, \dots, n, j \neq i$ entering compartment i from other $n -$
276 1 compartments, or from outside the system. So, $q_{ij}(t)$ represents the mass flow leaving
277 compartment j and entering compartment i , while $q_{i0}(t)$ represents the flow entering the i -th
278 compartment from outside the system, and $q_{0j}(t)$, represents the flow leaving the system
279 from the i -th compartment. Remark that the flows $q_{ij}(t)$ are bounded to be positive or zero,
280 and can never assume negative values. In particular, if no flow exists from compartment j to
281 compartment i , then we assume $q_{ij}(t) = 0$. Hence, we can define the following *flow matrix*

$$Q \doteq [q_{ij}]_{i,j=0,\dots,n}. \quad (4)$$

282 Formally, the matrix $Q \in \mathbb{R}^{n+1,n+1}$ coincides with the adjacency matrix of the weighted
 283 graph representing the interconnections between compartments; see for instance (Godsil &
 284 Royle, 2001). Note that, by construction, the matrix Q is square with zero diagonal elements.

285

286 3.1. Compartments ensuring uniform mixing

287 Hereafter the case in which compartments schematizing a storage container or a processing
 288 station ensure uniform (instantaneous) mixing of their content is considered first. Note that
 289 this kind of assumption is rather common for several modelling problems, in particular when
 290 compartmental models are used (Godfrey, 1983). Moreover, the assumption of uniform and
 291 instantaneous mixing appears quite reasonable in several processes typically encountered in
 292 the food processing industry. Indeed, inside the different compartments in which the process
 293 stages are carried on, the processed material is usually mixed in a continuous way in order to
 294 avoid settling phenomena, and to suppress possible thermal or concentration gradients. This
 295 is sometimes also the case of many storage devices, for instance whenever the processed
 296 material is liquid, so that diffusion and convection motions lead over time to a uniform
 297 mixing. Clearly, in real systems the mixing is never really instantaneous. However, it is in
 298 general rather fast, and the mixing time-constants are usually faster than those governing the
 299 process itself. On top of this, it should be noted that a non-uniform mixing would mainly
 300 induce errors only in the relative composition of the outflow from the compartment. Hence,
 301 whenever inflows and outflows are discontinuous and do not occur at the same time, truly
 302 uniform mixing actually occurs also in a real plant.

303 In order to describe the dynamics governing the different lots, a compartmental model is
 304 introduced, where each compartment coincides with a tank in the system. First, to describe
 305 the dynamic behaviour of a generic compartment i a set of suitable *state variables* that fully
 306 account for its status at any time is chosen.

307 To this regard, denote by $m_i(t)$ the total mass available in compartment i at time t . This
 308 mass can be divided into ℓ different sub-masses $m_i^L(t)$, one for every $L \in \mathcal{L}$, representing the
 309 fraction of the mass $m_i(t)$ containing material from S-lot L . The masses $m_i^L(t)$, $L \in \mathcal{L}$ are the
 310 *state variables* that fully describe the dynamics of compartment i .

311 Then, the following quantities are defined

$$\gamma_i^L(t) \doteq \frac{m_i^L(t)}{m_i(t)}, \text{ for } i = 1, \dots, n \text{ and } L \in \mathcal{L}, \quad (5)$$

312 denoting the fraction of S-lot L contained in compartment i at time t . Obviously, by
 313 definition, it holds that $\sum_{L \in \mathcal{L}} \gamma_i^L(t) = 1$. Notice also that, again by definition, the quantity

$$\gamma_i(t) \doteq [\gamma_i^A(t) \ \gamma_i^B(t) \ \gamma_i^C(t) \ \dots]^T, \quad (6)$$

314 coincides with the *instantaneous composition* of the material present in compartment i at
 315 time t .

316 At any time, the mass flow $q_{ij}(t)$ is composed by masses belonging to different S-lots. In
 317 particular, it can be easily seen that the relative fraction of $q_{ij}(t)$ which is constituted by a
 318 mass-flow belonging to the S-lot L is given by $\gamma_j^L(t)q_{ij}(t)$.

319 The quantities previously defined allow to compactly write the state equations of the mass
 320 exchange in the i -th compartment as follows

$$\dot{m}_i^L(t) = \sum_{j=0}^n \gamma_j^L(t) q_{ij}(t) - \gamma_i^L(t) \sum_{i=0}^n q_{ji}(t), \text{ for } L \in \mathcal{L}, \quad (7)$$

321 where $\dot{m}_i^L(t) = \frac{dm_i^L(t)}{dt}$ denotes the time variation of mass $m_i^L(t)$. The first summation on the
 322 right-hand side of equation (7) represents the total inflow of material belonging to S-lot L
 323 entering compartment i , while the second term is the total outflow of material belonging to S-
 324 lot L leaving the compartment. Under the assumption that a uniform and instantaneous
 325 mixing takes place in all compartments of the production chain, then the whole system can be
 326 easily described by means of n different sets of equations (7), one for each compartment.

327 To show the behaviour of the introduced model in this case of completely uniform mixing, an
 328 illustrative example is introduced next.

329 *Example 1 (Completely uniform mixing).* In order to clarify the concepts previously
 330 presented, a simple system depicted in Fig. 1 is introduced. Focusing on the first part of the
 331 plant, constituted by the cascade of two storage compartments (Tank 1 and Tank 2)
 332 characterized by uniform mixing, we consider the following situation: At initial time $t_0 = 0$
 333 s, Tank 1 is filled with 100 kg of mass belonging to S-lot 'A'. Then, at time $t_1 = 10$ s a flow
 334 of 1 kgs⁻¹ is transferred into Tank 2 for a duration of 60 seconds. Subsequently, at time
 335 $t_2 = 80$ s, an outflow of 0,5 kgs⁻¹ starts from Tank 2. At $t_3 = 90$ s, additional 70 kg of mass

336 belonging to S-lot 'B' are added to Tank 1. Finally, at time $t_4 = 100$ s, a flow of 1 kgs^{-1} is
337 again transferred into Tank 2 for other 100 seconds. Values of the mass flows between the
338 three tanks over the time interval $[0,300]$ s are plotted in Fig. 2.

339 Assuming that the material is uniformly mixed in the first two compartments, the masses
340 $m_1^A(t)$, $m_1^B(t)$, $m_2^A(t)$, $m_2^B(t)$ of material belonging to S-lots 'A' and 'B' in Tank 1 and
341 Tank 2 are reported in Fig. 3 and Fig. 4, respectively, over the interval $[0,300]$ s. Fig. 5 and 6
342 report the fractions $\gamma_1^A(t)$, $\gamma_1^B(t)$, $\gamma_2^A(t)$, and $\gamma_2^B(t)$, describing the relative composition in
343 terms of S-lots 'A' and 'B' of the two flows $q_{21}(t)$ and $q_{32}(t)$, respectively. In particular, in
344 Fig. 6 it can be seen that the composition of the flow from Tank 2 to Tank 3 is continuously
345 varying, with the percentage of material belonging to S-lot 'B' increasing and the one from
346 S-lot 'A' decreasing. The blue vertical lines in Fig. 6 refer to the introduction of quantization
347 levels, which is discussed in the next section.

348

349 3.2. Compartments behaving as FIFO buffer

350 The case in which a generic i -th compartment behaves like a first-in-first-out buffer is surely
351 more complex, and is discussed hereafter. Note that the FIFO model can represent several
352 practical situations encountered in real production lines when dealing with bulk solids and
353 powders. Indeed, there is a growing research pursued by the industrial technology in
354 designing specific devices and tank configurations that ensure plug-flow. Plug flow (referred
355 also as *mass flow*) silos are frequently used in industrial processing because of some
356 beneficial properties. Plug flow is the most productive flow, it eliminates problems like
357 channelling, hang-ups, flooding of powders, prevents stagnant regions formation, while
358 caking, degrading and segregation phenomena are minimized. In silos and hoppers filled with
359 a densely packed product, upon opening of the outlet, a narrow plug-type zone of flowing
360 material establishes and propagates upward. Except in the proximity of the outlet, the
361 boundaries of the plug-flow zone are nearly vertical, and the zone widens laterally and may
362 reach eventually the walls (Waters & Drescher, 2000). The main disadvantage in designing
363 plug-flow silos is that a steep hopper angle is required, making the silo relatively tall.
364 Moreover, flowability characteristics of granular solids and powders depends on many
365 factors, among which moisture content, temperature, particle size, compacting pressure,
366 relative humidity of the interstitial and head space air and the addition of flow conditioners
367 and anticaking agents that can vary (Ganesan, Rosentrater, & Muthukumarappan, 2008).

368 Some general solutions to facilitate plug flow in grain handling and drying are, for example,
 369 the use of inserts to improve material flow patterns (Wójcik, Tejchman, & Enstad, 2012), the
 370 adoption of revolving extracting screws (see e.g. Borghi, 2012; Mulmix, (2012)) and blade
 371 extractors for homogeneous bin emptying and powered grain spreaders to evenly fill the silos.
 372 Different techniques are nowadays available to measure and verify if flow conditions
 373 corresponds to manufacturer's claims. See, for instance, the case of the application of RDIF
 374 tags (Chen, Rotter, Ooi, & Zhong, 2007) or of specific tracers (Job, Dardenne, & Pirard,
 375 2009), directly introduced at the top of the silo.

376 A FIFO compartment can be schematically represented as a vertical cylinder of constant
 377 cross-section S_i , in which the outflow is at the bottom, i.e. at height $h = 0$, while the material
 378 inflowing the compartment enters the silo or tank from above and it is uniformly deposited at
 379 height $H_i(t)$ on top of the material that is already stored. Notice that the total level $H_i(t)$ of
 380 material stored in the pipe is in general time-varying: if the total inflow is larger than the total
 381 outflow it increases in time, while it decreases if the outflow is larger than the inflow.
 382 Obviously, it results that $H_i(t) \geq 0$ for all t and the mass stored in this i -th compartment at
 383 any time t is equal to $m_i(t) = \rho S_i H_i(t)$, where ρ is the density of the material contained in
 384 the FIFO compartment. In order to ensure a purely FIFO behaviour for compartment i , it is
 385 assumed that all the material stored in the compartment strictly moves only downwards and
 386 at the same speed, which is equal to $q_{OUT,i}(t)/(\rho S_i)$, where the total inflow to compartment i
 387 is defined as follows $q_{OUT,i}(t) \doteq \sum_{j=0}^n q_{ji}(t)$. Similarly, the total inflow to compartment i is
 388 defined as $q_{IN,i}(t) \doteq \sum_{j=0}^n q_{ij}(t)$.

389 Then, relative fraction of flow *entering* compartment i at time t that is constituted of material
 390 belonging to S-lot L only can be written as follows

$$\gamma_{IN,i}^L(t) \doteq \frac{q_{IN,i}^L(t)}{q_{IN,i}(t)} = \frac{\sum_{j=0}^n \gamma_j^L(t) q_{ij}(t)}{\sum_{j=0}^n q_{ij}(t)}, \text{ for } L \in \mathcal{L}. \quad (8)$$

391 Obviously, it holds that $\sum_{L \in \mathcal{L}} \gamma_{IN,i}^L(t) = 1$. Also, the following vector can be introduced

$$\gamma_{IN,i}(t) \doteq [\gamma_{IN,i}^A(t) \ \gamma_{IN,i}^B(t) \ \gamma_{IN,i}^C(t) \ \cdots]^T, \quad (9)$$

392 which can be interpreted as the instantaneous composition of the inflow into compartment
 393 i at time t .

394 It follows then that also for the material stored in this compartment it is possible to derive ℓ
 395 functions $\gamma_i^\ell(h, t)$ that provide, at any cross-section at height h in the pipe, the relative

396 fraction of material belonging to each S-slot L at time t . Note that these functions vary
 397 continuously with respect to the height h . The total percentage of S-slot L contained in tank i at
 398 time t can be computed integrating $\gamma_i^L(h, t)$ in the interval $[0, h]$, that is

$$\gamma_i^L(t) = \int_0^{H_i(t)} \gamma_i^L(h, t) dh, \text{ for } L \in \mathcal{L}. \quad (10)$$

399 Similarly, the total mass of material belonging to S-slot L contained in tank i at time t can be
 400 obtained as $m_i^L(t) = \gamma_i^L(t)m_i(t)$, for $L \in \mathcal{L}$.

401 Notice that the functions $\gamma_i^L(h, t)$, $L \in \mathcal{L}$, fully describe the state of the tank i with FIFO
 402 behaviour, which turns out being a dynamic system with an *infinite dimensional* state vector.
 403 The dynamics of the tank can therefore be precisely represented only by partial differential
 404 equations. The integration of such equations, however, is usually performed numerically by
 405 approximating the system with discrete or finite elements techniques, which provide
 406 approximating models with a finite dimensional state vector (González-Montellano, Gallego,
 407 Ramírez-Gómez, & Ayuga, 2012; Ketterhagen et al., 2007).

408 In our case this task can be easily done directly approximating the functions $\gamma_{IN,i}^L(t)$, $L \in \mathcal{L}$,
 409 by quantizing them over a given number of levels. It means that the inflow relative
 410 composition is assumed to be constant over time as long as its composition does not vary
 411 more than given thresholds. Obviously the same holds also for the outgoing flow leaving the
 412 tank.

413 In the sequel, adopting a compartmental models terminology, the amount of material with a
 414 homogeneous composition (up to quantization level δ), in terms of share of S-slots, that enters
 415 or leaves a compartment is called a *cohort*. The status of the i -th compartment with first-in-
 416 first-out behaviour is then fully described by the ordered list of the cohorts that are stored in
 417 it. Formally, the i -th compartment is hence completely described by the list

$$\begin{bmatrix} TOP \\ queued_v \\ \vdots \\ queued_1 \\ BOTTOM \end{bmatrix}_i \quad (10)$$

418 of its contained cohorts. To each of these cohorts is associated the information relative to its
 419 total mass and its composition.

420 For what concerns the i -th compartment, if at time t the composition-distance between the
421 inflow IN, i entering the compartment at time t and the material already present in the top
422 cohort of the compartment TOP, i is greater than the given quantization level, so that
423 $d(\gamma_{IN, i}(t), \gamma_{TOP, i}(t)) > \delta$, then a new cohort is created. This new generated cohort, with all
424 the information that fully describes its composition, is then piled in the FIFO array. For the
425 sake of clarity, the algorithm is schematized in Fig. 7. In particular, the differential equations
426 in (7) are simulated (step 4) until a new event, such as a valve opening/closing or a pump
427 start/stop, occurs.

428 In order to clarify the impact of using cohorts, the dynamics of the scheme introduced in
429 Example 1 is now analysed focusing on the third tank, schematized as a FIFO container.

430 *Example 2.* The analysis is carried out twice, using two different quantization levels,
431 $\delta_1 = 0.1$ and $\delta_2 = 0.02$, so that the influence of quantization levels can be considered as
432 well. In the time instant $t_1 = 80$ s the valve on the connection between Tank 2 and Tank 3 is
433 opened and the flow of product $q_{32}(t)$ that is established is equal to 0.5 kgs^{-1} . Tank 3 starts
434 to release product out of the system at $t_2 = 110$ s, with a flow rate $q_{03}(t) = 0.2 \text{ kgs}^{-1}$, as
435 shown in Fig. 2. The threshold δ_1 to generate new cohorts in Tank 3 is applied on the
436 composition of flow $q_{32}(t)$, whose relative amount of S-Lot ‘A’ and S-Lot ‘B’ is represented
437 in Fig. 6, using $\gamma_{IN, 2}^A(t)$ and $\gamma_{IN, 2}^B(t)$ indexes. The time instants in which one of the $\gamma_{IN, 2}^L(t)$
438 crosses a quantization levels, with a threshold set of $\delta_1 = 0.1$, are reported in Fig. 6 with
439 vertical lines. Masses $m_3^A(t)$ and $m_3^B(t)$ of material belonging to S-Lot ‘A’ and to S-Lot ‘B’,
440 and the overall mass $m_3(t)$, in Tank 3 are shown in Fig. 9, that however lacks of information
441 about the cohorts that have been generated during the filling phase with $q_{32}(t)$. For this
442 reason, Fig. 10 is reported, in which mass content of Tank 3 is represented in six different
443 time instants. Each cohort is characterized by a different colour, related to the relative
444 composition in terms of S-Lot ‘A’ and ‘B’. The influence of product quantization in Tank 3
445 on the outflow $q_{03}(t)$ can be seen in Fig. 8, where $\gamma_{OUT, 2}^A(t)$ and $\gamma_{OUT, 2}^B(t)$ indexes are
446 plotted over the time. Results obtained setting a threshold δ_2 equal to 0.02, are reported in
447 Fig. 11 and 12. Note that the generated cohorts are in this case smaller and more
448 homogeneous. The movie of this simulation example is recorded in MPEG files S1 and S2,
449 for thresholds δ_1 and δ_2 , respectively.

450

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452 4. A case study: plant with both UM and FIFO tanks

453

454 In order to clarify the concepts and the procedures introduced in previous sections, a case
455 study, consisting in seven interconnected tanks depicted in Fig. 13, is now presented. In this
456 example, all compartments behave as FIFO buffers, with the exception of Tank 6, where an
457 agitator ensures a uniform mixing of processed products. At time $t=0$ s, Tanks 1 to 4 are
458 filled with homogeneous raw material. More in detail, 100 kg of S-lot 'A' and 200 kg of 'B'
459 are stored into Tank 1, Tank 2 is filled with 50 kg of S-lot 'C' and 200 kg of 'D', Tank 3 with
460 200 kg of S-lot 'E', and finally 300 kg of S-lot 'F' and 100 kg of 'G' are stored in Tank 4.
461 Valves opening at time $t_1=60$ s allow product flows $q_{51}(t) = 0.32 \text{ kgs}^{-1}$ and $q_{52}(t) = 0.2$
462 kgs^{-1} from Tanks 1 and 2 to Tank 5. At time $t_2=120$ s, flows $q_{63}(t)=0.18 \text{ kgs}^{-1}$ and
463 $q_{64}(t)=0.28 \text{ kgs}^{-1}$ start from Tanks 3 and 4 to Tank 6, where the incoming products are
464 continuously mixed. Then, at time $t_3=300$ s the product in Tanks 5 and 6 starts flowing into
465 Tank 7 with a rate of $q_{75}(t)$ and $q_{76}(t)$ equal to 0.3 kgs^{-1} . Fig. 14 shows the evolution of the
466 flows between storage units and processing stations over time. Adopting a quantization level
467 δ equal to 0.05, six cohorts of final product, characterized by different percentages of S-Lots
468 'A' to 'G', are generated. The simulation movie of the working plant is reported in MPEG
469 file S3. The way in which the S-lots spread into the plant and mix to produce the six cohorts
470 of final product in Tank 7 is schematized in the graph of Fig. 15 where the composition of
471 each cohort is directly reported in the node. Note that this dispersion graph can be directly
472 used to measure (and possibly to optimize) the performances of the traceability system as
473 proposed in (Dabbene & Gay, 2011). As already remarked, the level of detail of the
474 traceability, and therefore the number of generated cohorts, depends on the choice of the
475 threshold δ . Simulations were performed at different values of δ ranging from 10^{-3} to 10^{-1} .
476 Fig. 16 (left) shows how the number of generated cohorts considerably increases for
477 decreasing values of threshold δ . As expected, at increasing number of cohorts, it correspond
478 smaller average cohort sizes (Fig. 16, right) and more homogeneous compositions. Figure 16
479 shows also masses of the largest and smallest cohort generated in each simulation of the set.
480 These figures show that there exists a clear trade-off between the quantization level δ and the
481 number of different lots generated. This trade-off should be taken in due consideration by the
482 supply chain manager in designing and optimizing the traceability system.

483

484 5. Conclusions and future directions

485

486 The paper proposed a methodology for efficiently tackling the problem of traceability when
487 continuously processing and storing bulk materials. In particular, the introduced framework is
488 particularly suitable for the management of the *internal traceability*, i.e. during the
489 production processes within a company. According to the key advantages provided by
490 internal traceability, as discussed in Moe (1998), this methodology makes it possible
491 monitoring (and avoiding) uneconomic mixing of high and low-quality raw materials and
492 ingredients, and gives the basis for the adoption of efficient recall procedures to minimize
493 losses, at present available only for the processing of discrete lots of products. In particular,
494 this method allows the proper identification and definition of batches of homogeneous
495 product, without resorting to the nowadays often-adopted operation of oversizing the lots. In
496 particular, the availability of precise information about the composition, in terms of lots of
497 raw ingredients, introduces the possibility to correlate product data with raw materials and
498 then to optimize the recipes for each final product type. In the same way, the availability of
499 this information can be exploited to design improved process control strategies.

500 The present work analysed two representative cases of product containers, namely the
501 uniform-mixing and the first-in-first-out compartments. It is however important to notice that
502 the approach introduced in the paper can be extended to the more general case of storage
503 compartments that do not show a UM or FIFO behaviour. Fundamental in this case is the
504 availability of an accurate description of the dynamics governing the way the material
505 delivered to the compartment is stored within its volume, and of the laws by which such
506 material is recombined into the output flow. The problem of experimentally determining such
507 laws has been the subject of growing interest in the literature. See, for instance, the recent
508 works of Ganesan et al. (2008), González-Montellano et al. (2011), Mellmann et al. (2011),
509 Sielamowicz & Czech (2010), and Sielamowicz et al., (2011), which applied finite/discrete
510 elements techniques to describe tank filling/emptying dynamics. Indeed, once the laws
511 governing the storing and mixing phenomena taking place in the tanks are adequately
512 modelled, these mathematical models can be directly integrated in the framework discussed
513 so far, since compartmental models are well-suited to cope with such situations. Specific
514 cases are currently under study, and will be the subject of further works.

515 Finally, in the context of the present work, the fraction of the inflow allocated to each S-lots
516 has been considered exactly known. However, it appears possible to consider instead the case
517 when such fraction is subject to uncertainty. For instance, this could account for situations in
518 which the UM or FIFO models are not sufficiently accurate in describing the real behaviour
519 of the processes or some uncertainties affect flow dynamics (for example in the case in which
520 the flow is dependent on some product conditions like temperature, moisture content etc.). In
521 such case, the knowledge of the real composition of the outflow is not precise, and it can be
522 determined only up to a given tolerance. Hence, it could be important to develop a framework
523 able to determine the maximum amount of each S-lot that could be present in each
524 compartment as well as in each flow.

525

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Parameter	Meaning
$\mathcal{L} = \{ 'A', 'B', 'C', \dots \}$	Ordered list of possible S-lots entering the system
ℓ	Cardinality of \mathcal{L} (number of S-lot)
n	Number of compartments
t	Time variable [s]
$c(X)$	Composition of product X
$\gamma^A(X)$	Percentage of product coming from S-lot A present in product X
$d(X, Y)$	Composition-distance between products X and Y
δ	Threshold level of homogeneity
$\ x\ _\infty$	ℓ_∞ -norm of vector x
$\ x\ _\infty^W$	Weighted ℓ_∞ -norm of vector x
w_L	Risk-factor
$m_i(t)$	Mass in the i -th compartment at time instant t [kg]
$m_i^L(t)$	Fraction of the mass $m_i(t)$ containing material from S-lot L [kg]
$\dot{m}_i^L(t)$	Time derivative of the mass fraction $m_i^L(t)$ [kgs^{-1}]
$q_{ij}(t)$	Mass flow from compartment j to compartment i at time t [kgs^{-1}]
$q_{i0}(t)$	Mass flow entering the i -th compartment at time t [kgs^{-1}]
$q_{0j}(t)$	Mass flow leaving the j -th compartment at time t [kgs^{-1}]
$q_{IN,i}(t)$	Sum of incoming mass flow $q_{ij}(t)$ in compartment i at time t
$q_{OUT,i}(t)$	Total outflow from the i -th compartment
Q	Flow matrix collecting the q_{ij} 's
S_i	Cross-section of compartment i
$H_i(t)$	Height of material in compartment i at time t
ρ	Product density [kgm^{-3}]
$\gamma_i^L(t)$	Percentage of S-lot L contained in compartment i at time t
$\gamma_i(t)$	Instantaneous composition of the material present in compartment i at time t
$\gamma_{IN,i}^L(t)$	Relative fraction of flow entering compartment i at time t that is constituted of material belonging to S-lot L only, at time t

$\gamma_i^L(h, t)$	Relative fraction of material in compartment i belonging to S-lot L at a cross-section at height h , at time t
t_{next_event}	Time of the occurrence of the next event in the algorithm for creation of homogeneous cohorts in FIFO compartments
t_{end}	Simulation end time of algorithm for homogeneous cohorts creation in FIFO compartments
Δt	Simulation time interval for the algorithm for homogeneous cohorts creation in FIFO compartments

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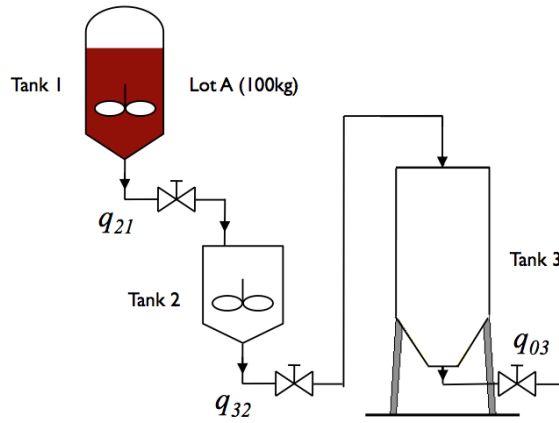


Figure 1. Scheme of the plant in examples 1 and 2

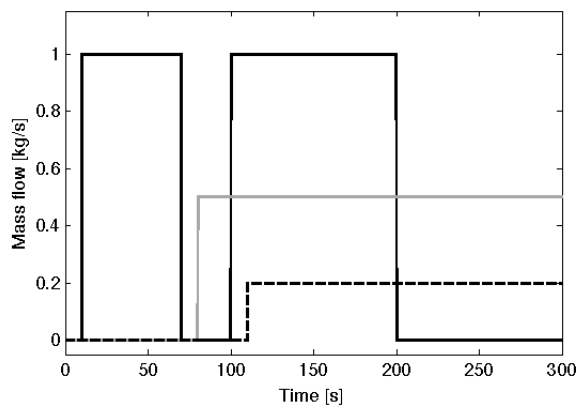


Figure 2. Mass flow q_{21} (black solid) from Tank 1 to Tank 2, q_{32} (gray solid) from Tank 2 to Tank 3, and q_{03} (black dashed) from Tank 3 out of the system

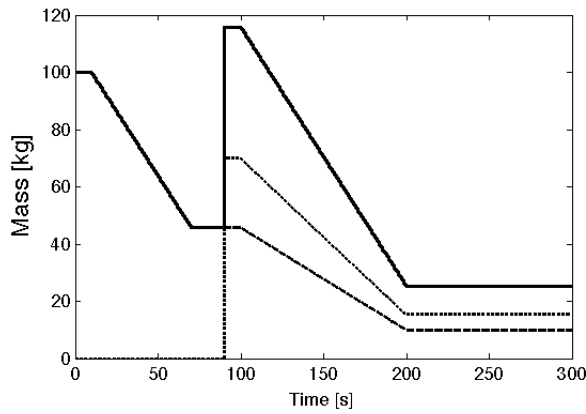


Figure 3. Mass of product belonging to S-lot A (black dashed), S-lot B (black dotted), and overall mass amount in Tank 1 (black solid).

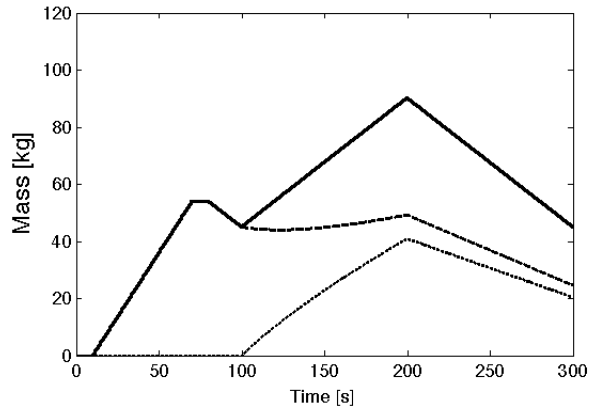


Figure 4. Mass of product belonging to S-lot A (black dashed), S-lot B (black dotted), and overall mass amount in Tank 2 (black solid).

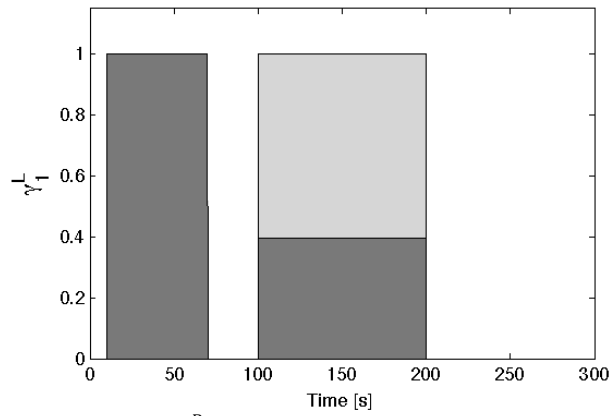


Figure 5. Relative fractions $\gamma_1^A(t)$ (dark gray) and $\gamma_1^B(t)$ (light gray) of q_{21} flow constituted by mass belonging to lot of product A and B respectively. The sum of $\gamma_1^A(t)$ and $\gamma_1^B(t)$ is always equal to 1.

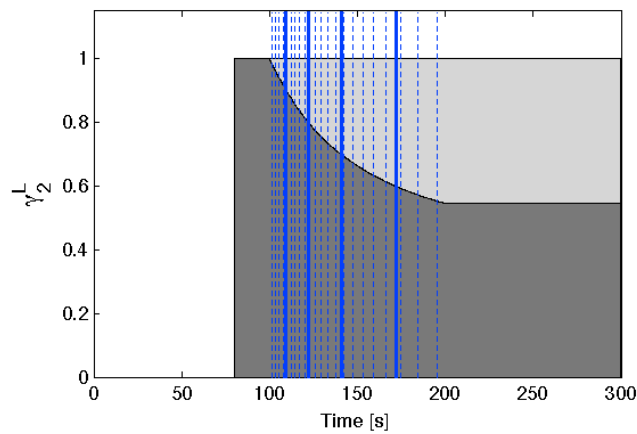


Figure 6. Relative fractions $\gamma_2^A(t)$ (dark gray) and $\gamma_2^B(t)$ (light gray) of flow q_{02} constituted of mass belonging to S-lot A and B respectively. The sum of $\gamma_2^A(t)$ and $\gamma_2^B(t)$ is always equal to 1. Time instants in which a new cohort is generated inside Tank 3 are represented by vertical solid and dashed lines, for the two cases of quantization level δ equal to 0.1 and 0.02, respectively.

```

1:  $j \leftarrow 0$ 
2: Do
3:    $t_{j+1} \leftarrow \min(t_j + \Delta t, t_{next\_event})$ 
4:   Simulate  $\gamma_{IN,i}(t)$  for  $t \in [t_j, t_{j+1}]$ 
5:   For  $t = t_j$  to  $t_{j+1}$  do
6:     If  $d(\gamma_{IN,i}(t), \gamma_{IN,i}(t_j)) > \delta$  then
7:        $j \leftarrow j + 1$ 
8:        $t_j \leftarrow t$ 
9:        $v \leftarrow v + 1$ 
10:       $queued_v \leftarrow TOP$ 
10:      Create new  $TOP$  cohort
11:    Goto 3
12:   End
13:    $j \leftarrow j + 1$ 
14: While  $t < t_{end}$ 
15: End

```

Figure 7. Algorithm for the creation of homogeneous cohorts in a FIFO compartment. Simulation parameters: t_{next_event} - time of the occurrence of the next event after t ; t_{end} - end time of the simulation; Δt arbitrary time interval.

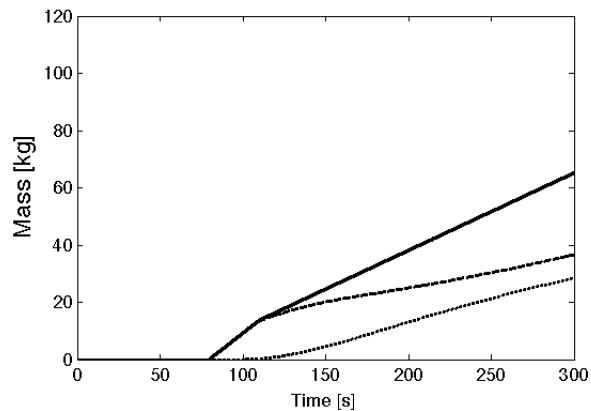


Figure 8. Mass of product belonging to S-lot A (black dashed), S-lot B (black dotted), and overall mass amount in Tank 3 (black solid).

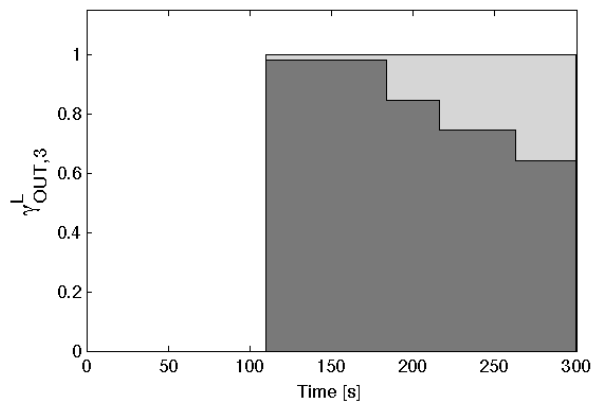


Figure 9. Relative fractions $\gamma_{OUT,3}^A(t)$ (dark gray) and $\gamma_{OUT,3}^B(t)$ (light gray) of flow q_{03} constituted of mass belonging to S-lot A and B respectively, in the case of quantization level δ equal to 0.1

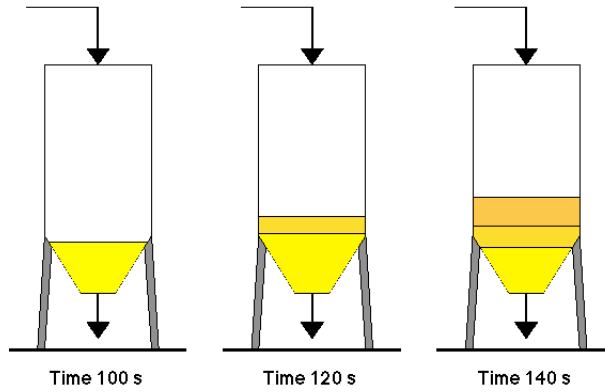


Figure 10. Tank 3 content at $t=100, 120$ and 140 seconds, in the case of quantization level δ equal to 0.1 . Different cohorts are represented with color hues proportional to the % of product belonging to S-Lot A and S-Lob B.

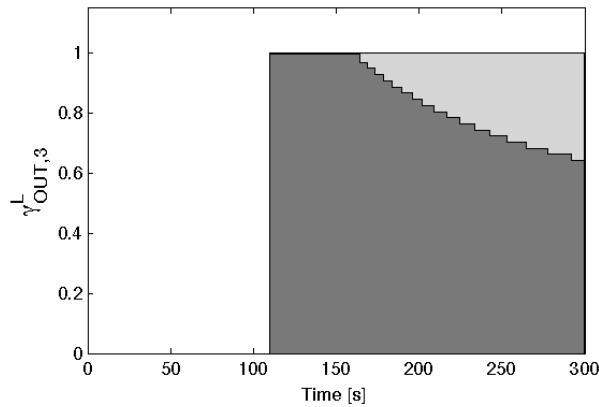


Figure 11. Relative fractions $\gamma_{OUT,3}^A(t)$ (dark gray) and $\gamma_{OUT,3}^B(t)$ (light gray) of flow q_{03} constituted of mass belonging to S-lot A and B respectively, in the case of quantization level δ equal to 0.02 .

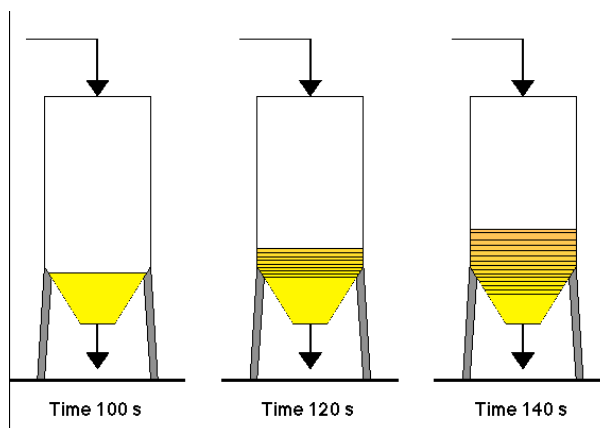


Figure 12. Tank 3 content at $t=100, 120$ and 140 seconds, in the case of quantization level δ equal to 0.02 . Different cohorts are represented with color hues proportional to the % of product belonging to S-Lot A and S-Lob B.

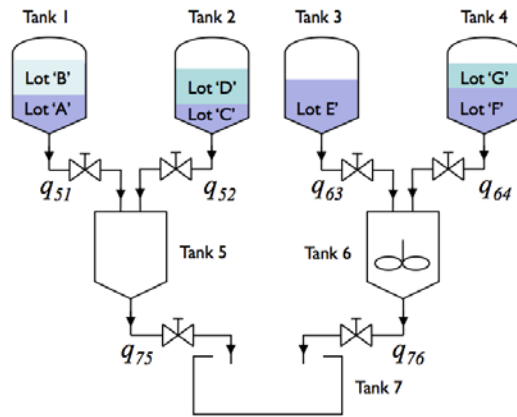


Figure 13. Scheme of the plant in the case study at time $t = 0$

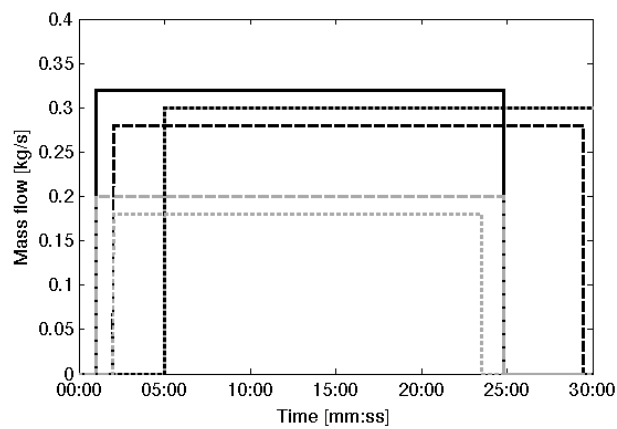


Figure 14. Mass flows $q_{51}(t)$ (black solid) from Tank 1 to 5, $q_{52}(t)$ (gray dashed) from Tank 2 to 5, $q_{63}(t)$ (gray dotted) from Tank 3 to 6, $q_{64}(t)$ (black dashed) from Tank 4 to 6, $q_{75}(t)$ (gray solid) from Tank 5 to 7, and $q_{76}(t)$ (black dotted) from Tank 6 to 7.

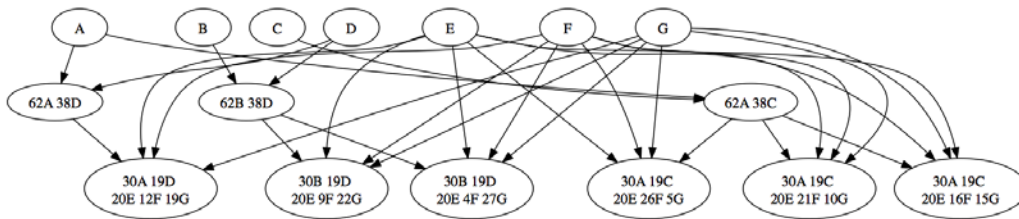


Figure 15. Graph of the composition of the six cohorts. The label of each node in the graph reports the composition of the cohort, where the numbers express the percentage of the different S-lots (A to G).

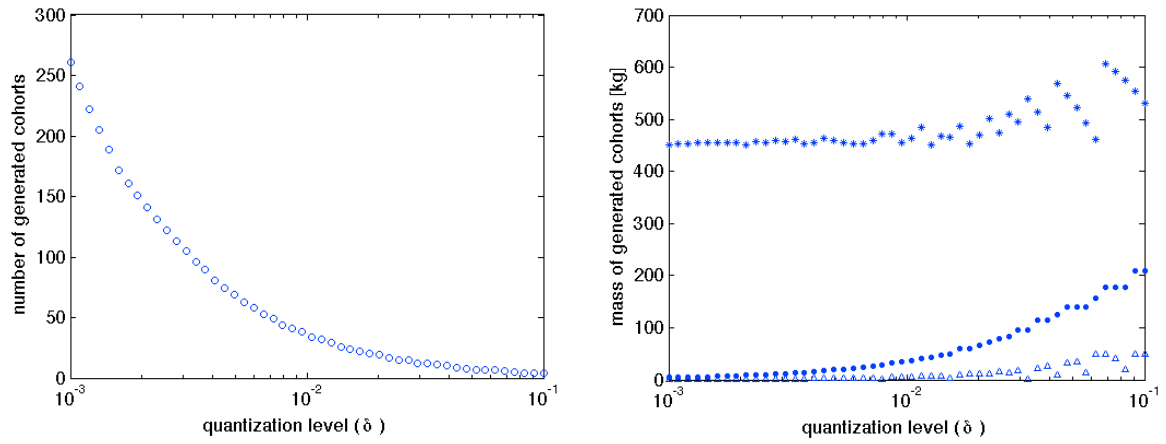


Figure 16. Number (on the left) and average mass (dotted, on the right) of generated cohorts obtained with different quantization levels δ ranging from 10^{-3} to 10^{-1} in Example 3. On the right, masses of the biggest (*) and smallest (Δ) cohort are also reported.