# Methods for traceability in food production processes involving bulk products

Lorenzo Comba[a]; Gustavo Belforte[b]; Fabrizio Dabbene[c]; Paolo Gay[a,c]

[a] DI.S.A.F.A. – Università degli Studi di Torino, 44 Via Leonardo da Vinci, 10095 Grugliasco (TO) – Italy, lorenzo.comba@unito.it, paolo.gay@unito.it;

[b] D.AU.IN. - Politecnico di Torino, 24 Corso Duca degli Abruzzi, 10129 Torino – Italy, gustavo.belforte @polito.it;

[c] CNR-IEIIT, 24 Corso Duca degli Abruzzi, 10129 Torino – Italy, fabrizio.dabbene@ieiit.cnr.it, paolo.gay@unito.it

**Corresponding author:** Paolo Gay

Email: paolo.gay@unito.it Tel: +39 011 6708620 Fax: 011 6708591

**Keywords:** traceability, batch dispersion, food processing, compartmental models

.

**Abstract**

In food processing plants, raw materials are fed to the system in different *supply-lots* of product, and are processed through different stages. In these stages, raw or intermediate materials are mixed or combined together, and physical-chemical and/or microbiological processes such as heating, concentration, pasteurisation etc. take place. In this setting, traceability consists in the ability to determine for each portion of intermediate or final product, in any part of the plant, its relative composition in terms of supply-lots fed into the system as well as of new lots generated during the production process.

The traceability problem becomes particularly difficult in the very-frequent case when bulk products, such as liquids or grains, are involved in the production chain. Current traceability practices are in most cases unable to directly deal with bulk products, and typically resort to the definition of very large lots to compensate the lack of knowledge about lot composition. As demonstrated even in very recent food crisis, this over-bounding approach has shown its weakness in the identification of the interested products immediately after risk assessment, leading to unavoidably wide, expensive and very impacting recalls.

Motivated by these considerations, this paper presents a novel approach to manage traceability of bulk products in production, storage and delivering phases that provides a tight definition of lots in terms of their composition and size, thus allowing a strict control of the production and supply chains.

## 1. Introduction

The problem addressed in this paper refers to the traceability of food products in processing plants, or part thereof, in which raw materials to be processed are fed to the system in different supply-lots of bulk product, with specific attention to the frequent case when bulk products are involved in the production phase. Indeed, many ingredients used in food industries are liquids (milk, vegetal oils, etc.), powders (cocoa, powdered milk, flour, yeast etc.), crystals (e.g. sugar, salt) or grains. These products are stored, in many cases, in huge silos or tanks, which are very rarely completely emptied, so that many lots are contemporarily kept in the same container. Throughout the plant, the supplied material is processed in one or more production lines until one or more final products are created, packed, and stored ready for sale.

Typically, the production process consists of different stages, which are usually carried out in different production stations. Some stages involve different raw or intermediate materials that are mixed or combined together, while in other stages physical-chemical and/or microbiological processes such as, for instance, heating, cooling, concentration, and pasteurisation, take place. Thus the production process generates different production lots.

In food processing plants, supplied raw materials as well as intermediate products are usually stored in silos, tanks or other suitable containers, before being processed or during the process itself in between different production stages. In general, the material stored in one container is delivered to it in different batches, each one possibly constituted of material coming from different supply or production lots. Whenever the stored material is drawn from a container in order to be delivered to a production station or to a new storage container, the retrieved material results in a combination of material from the different batches that have been previously fed into the container.

In this setting, traceability consists in the ability to determine for each portion of intermediate or final product, at any time and in any part of the plant, its composition in terms of supply-lots fed into the system. This information is indeed crucial for identifying the amount and location of product portions affected by possible deficiencies caused by a defect of the material delivered in one of the supply-lots. The event of food recalls due to unforeseen problems is becoming more and more frequent: for instance, the web-site provided by the US Food and Drugs Administration, which gathers information from press releases and other public notices about certain recalls of FDA-regulated products, listed more than seventy cases of recalls for the first two months of 2013 (US Food and Drugs Administration, 2013).

One possible approach to minimize recalls consists in maintaining the lots as much as possible separated in the plant. In the case of fluids, for instance, the use of different containers and cleaning between two product batches is a viable solution to allow distinct separated batches identities. In particular, *cleaning-in-place* procedures, which involve pumping water and detergent through the production equipment, besides guaranteeing high hygienic standards, is foreseen as the good procedure to strictly guarantee that the different batches cannot contaminate each other. However, these cleaning procedures, besides representing a high cost for the company in terms of energy, manpower, and cleaning agents, can become undesirable in the case of continuous production systems (such as, e.g., milk production in a dairy) where *continuous flow*, without even minimal interruptions, of liquid/granular raw material is necessary to maintain the production.

In these cases, the currently adopted solution consists in defining large lots, mainly referred to production periods rather than to their precise composition of the lots. For instance, lots based on the production day (or even a whole week) are typically encountered. This rather conservative approach, based on the definition of very large lots, has shown its weaknesses in recent cases of food recall, when the lack of knowledge in the identification of the interested products immediately after risk assessment, has unavoidably led to wide, expensive and very impacting recalls.

Moreover, to most types of bulk products, it is very difficult to associate any kind of label, marker or identifier that could directly identify the different lots. Recently, some markers based on RFID technology have been developed for the case of continuous granular flows (specifically, iron pellets) by Kvarnström et al. (2011). These allow on-line traceability of continuous flows, thus improving upon previous off-line solutions based on the introduction of specific tracers into the grains, such as chemical compounds or radioactive tracers; see Kvarnström and Oghazi (2008) and Lee et al., (2010) for detailed discussion and references on these techniques. The situation is complicated by the obvious requirement that the markers should not compromise by any means the integrity and quality of the food and must be not dangerous for the consumer. Thus, any RFID-based traceability system would require the development of a device for safely removing the tracing devices from the final product (e.g. before grain grinding). In this regard, some interesting solutions have been proposed in Lee et al. (2010) and Liang et al. (2012) for the specific case of grains, which involve particular pillsized food-grade tracer particles to be inserted directly into grain during harvest. These tracers have printed with food-grade ink a miniaturized data-matrix code carrying identity information related to product origins, and are composed of materials that can be safely eaten such as sugar or cellulose. Anyway, these solutions remain principally an off-line approach, suitable for modelling and validation purposes, since collecting and identifying the tracers would usually still require interrupting the production.

The problem of the traceability of fluid products in the case of continuous processing has been, to our best knowledge, addressed first by Skoglund and Dejmek (2007), who used dynamic models and simulations to identify the changeover of lots of liquid product in a pipe. The presence of portions of product deriving from the partial mixing of two subsequent lots led to the introduction of the concept of *fuzzy traceability*. An interesting approach has been also proposed by Bollen et al. (2007) and by Riden and Bollen (2007), who considered the case of apples processed in a packhouse. Apples, supplied to the packhouse in bulk bins, are moved in a bulk flow (water dump) up to the grader that handles individual fruits and directs them into packaging lines. At the end of these lines the fruits are placed into homogeneous packs (in terms of colour or size). During their flow in the water dump and then in the packaging lines, some mixing among different lots of apples occurs. Even if apples are discrete items, their fluidized flow can be assimilated to the flow of small particles. In their first paper, Bollen et al. (2007) developed and validated a set of statistical models using the measured arrival sequence of 100 blue marker balls. The proposed models are able to assign a probability of bin origin to any individual fruit in the final packs.

The performances of a traceability system can be identified with the skill of limiting the quantity of final product to be recalled in the averted case of food safety crisis (Dabbene & Gay, 2011). However, at present methods to precisely estimate the amount of product that has to be discarded in the case of a recall are available only for the case in which discrete lots of product are processed (Dabbene, Gay, & Tortia, 2013; Dabbene & Gay, 2011; Dupuy, Botta-Genoulaz, & Guinet, 2005). The quantity of product to be recalled, to which a *recall cost* is associated, may depend on many factors, among which the size of the batches that have been individually tracked and managed by the traceability system (and hence the skill of the firm in managing and maintaining segregated different batches of product), and the way the batches of different components have been mixed to obtain the final product.

These methods have been applied to many different supply chains, e.g. for fruits (Bollen et al., 2007; Riden & Bollen, 2007), meat (Barge P., Gay P., Merlino V., & Tortia C., 2013;

Donnelly, Karlsen, & Olsen, 2009; Dupuy et al., 2005), fish (Karlsen, Donnelly, & Olsen, 2011; Randrup et al., 2008), grains (Thakur & Donnelly, 2010; Thakur & Hurburgh, 2009; Thakur, Wang, & Hurburgh, 2010), chocolate (Saltini & Akkerman, 2012), perishable products (Li, Kehoe, & Drake, 2005; Rong & Grunow, 2010; Wang X., Li D., & O'Brien C., 2009) etc.

To allow traceability of bulk products, a convenient model of the production plant is needed. This model should provide a description of the production process in terms of mass transfer and storage at a lot level of detail, in order to enable an accurate prediction of the dynamics of each supply-lot that can therefore be conveniently tracked.

The paper is structured as follows: in Section 2 a thorough theoretical analysis is carried out and a modelling framework based on compartmental models is derived. Section 3 addresses the problem of the determination of specific models of the two basic cases of uniform-mixing and FIFO tanks. A simulation case study, showing the effectiveness of the proposed methodology, is proposed in Section 4. Finally, conclusions are drawn in Section 5.

## 2. Definitions and problem formulation

The first step for developing the framework introduced in this work consists in providing a formal definition of lots and of lot homogeneity.

**Definition 2.1** (Lot) *A* lot *is defined as a set of units of a product that are homogeneous in terms of composition and processing history*.

This definition is coherent with the one reported in ISO 22005 (2008), where a lot is defined as *"set of units of a product which have been produced and/or processed or packaged under similar circumstances",* and it extends to some degree the concept of traceable unit (TRU) introduced by Kim et al., 1999. It should be noted that at this point the notions of *homogeneity* and *composition* considered in Definition 2.1 are still rather vague, and need a rigorous formalization to be of practical value. To this end, the concept of S-lot (supply lot) is explicitly defined next

**Definition 2.2** (S-lot) *An* S-lot *is defined as a set of units of homogeneous raw materials that enter the system from outside.*

More specifically, S-lots represent raw-materials or semi-processed products fed into the system by a supplier as a unique lot. At each instant, the traceability system should be able to determine the relative composition, in terms of S-lots, of the material present in the different intermediate production stages, with specific attention to the composition of the final products leaving the production chain.

To exemplify, consider the case in which two different raw-materials are fed into the system and are labelled for simplicity as 'A' and 'B'. Then, the relative composition of a final product $X$ leaving the chain is given by the percentages $\gamma^{\mathrm{A}}(X)$ and $\gamma^{\mathrm{B}}(X)$ of materials 'A' and 'B' present[1] in $X$. More generally, the composition of a product can be defined as follows

**Definition 2.3** (Composition) *Let* $\mathcal{L} = \{\text{'A'}, \text{'B'}, \text{'C'}, \ldots\}$ *denote the (ordered) list of possible S-lots entering the system. Then, the (relative)* composition *of a product X is defined as the vector of percentages of the different S-lots composing X, that is*,

$$c(X) \doteq [\gamma^{\mathrm{A}}(X)\ \gamma^{\mathrm{B}}(X)\ \gamma^{\mathrm{C}}(X)\ \cdots]^T. \tag{1}$$

The above definition is instrumental to a rigorous definition of homogeneous materials, in terms of composition, which in turn represents a fundamental step towards a rigorous treatment of the traceability problem for the case of bulk materials. To this end, the *composition-distance* between two products $X$ and $Y$ is introduced as follows

$$d(X,Y) \doteq ||c(X) - c(Y)||_{\infty}, \tag{2}$$

where $||x||_{\infty} \doteq \max_{L=1,\ldots} x_L$ denotes the $\ell_{\infty}$-norm of vector $x$. Note that composition-distances different than (2) can be introduced: for instance a *weighted-norm* version, with $||x||_{\infty}^{W} \doteq \max_{L=1,\ldots} w_L\, x_L$ , can be considered in order to take into account the different risks associated with the different S-lots. In this case, the larger is the *risk-factor* $w_L$, the more importance is given to S-lot $L$. The concept of composition distance $d(X,Y)$ allows the following rigorous formalization of homogeneity.

**Definition 2.4** (Homogeneous products) *Given a threshold level* $\delta$*, two products X and Y are said to be* homogeneous in composition *(up to accuracy* $\rho$*) if their composition-distance is less than* $\delta$*, i.e.*

$$d(X,Y) < \delta. \tag{3}$$

Note that this definition does not take into account processing history. Clearly, a homogeneous-in-composition lot processed in $m > 1$ sessions splits in $m$ 'production' lots

[1] Here $\gamma^{\mathrm{A}}(X)$ has to be interpreted as the percentage of product coming from S-lot 'A' present in *X*. A formal definition is given in Section 3.

characterized by the same composition vector. The handling of these production lots can be performed in a completely analogous way to the one discussed in this paper, and it is not considered in the present work for sake of simplicity.

Note also that the introduction of the quantization level $\delta$ is absolutely necessary when dealing with bulk products, since in principle the relative composition of the materials can vary with continuity. This approach, based on a threshold level, reflects what proposed in the EC Regulation No 1829/2003 (European Commission, 2003) for genetic modified (GM) and non-GM grains labelling. In this case, for the consumer information, these regulations guarantee that any food containing material that contains more than 0.9% of GM would be labelled as "contains GM".

It immediately follows from Definition 2.4 that two materials whose composition-distance is greater than $\delta$ cannot belong to the same lot (according to Definition 2.1). Consequently, every time two products in the supply chain assume a composition-distance greater than the considered threshold, the traceability system should be able to detect this event and keep trace of the two products (and their specific composition) separately. Hence, this framework provides a direct and natural way of discriminating final products and, possibly, to divide them into homogeneous lots.

Like the already mentioned case of GM and non-GM grains, there are other situations related to ethical, organic, low carbon footprint, issued or subject to disciplinary, as well as to religious constraints, where lots should be maintained as much as possible separated and facilities and logistics have to be designed and planned accordingly. Different management strategies have been proposed to cope with this problem and these are typically based on separation of products *in space*, allocating specific collecting units (e.g. silos) for any different lot, or on separation *in time*, where different lots are processed in successive sessions, separated by suitable cleaning cycles (see e.g. Coléno, 2008; Maier, 2006).

In this work, to derive accurate methods for tracing the composition of the product in terms of S-lots are derived using specific *compartmental models*. Compartmental models are mathematical models widely used to describe the way in which materials and/or energies are transferred among (and stored within) the different parts of a physical system (Godfrey, 1983). Although compartmental models have been primarily developed in biomedical engineering (the interested reader can refer to Rescigno (2001) for a short overview and a critical analysis of their use), they have been also used recently by Comba et al. (2011) to describe heat-transfer phenomena in food plants characterized by mixed continuous/discontinuous flow food plants of materials.

Indeed, in principle, a food production plant can be modelled as a set of storage compartments, each one corresponding to a storage container or to a batch processing station. Examples of compartment are tanks, vats, silos but also grain dryers, mixers, chocolate conching machine, cheese-vats etc. Material is transferred from a compartment to another either by flows, that in most cases are discontinuous (in time), or in batches. The description of these phenomena is usually simple and quite precise, since flows between compartments and masses of batches are known with good precision, and mass transfer equations are accurate. This information can be easily acquired from the plant itself, by monitoring the states of valves, pumps, conveyors, and, in general, any device that controls the flow of the material in the plant. Then, assuming that the relative composition of flows and batches in terms of S-lots is properly known, also the dynamics of such lots, in connection with the mass transfers among tanks, can be accurately determined (see, for instance, Skoglund and Dejmek, 2007 for the case of liquid products).

The crucial point is indeed to know such relative composition, which is not always an easy task. In order to better understand this point, the behaviour of the compartments used to describe the production plant should be analysed, since any product flow or product batch transferring masses from compartment to compartment can be regarded as the output of a specific compartment. Only the inflow into the system of S-lots cannot be regarded as the output of a compartment, but the composition of such flow (or batch delivery) in terms of S-lots is indeed well known.

Any compartment, whether it represents a storage unit, like a silo, or a processing station, like a mixer, a concentrator, a heater, etc., is itself a dynamic system. As a matter of fact it can store some amount of mass delivered to it over time trough one or more inputs and each one of its output flows is a suitable combination of the masses stored in it.

Assuming that the relative composition of input flows in the compartment (or batch deliveries to it) in terms of S-lots is perfectly known, then the relative composition of the outputs can be accurately computed only if the storage mechanism in the compartment is accurately known together with the laws supervising the way in which output flows are formed from the stored material.

There are at least two important and representative cases in which this happens. The first case is when all the material delivered to a compartment is instantaneously and uniformly mixed. Under this condition, referred to as *uniform-mixing* (UM) *compartment*, the relative composition of the material in the compartment in terms of S-lots is perfectly known at any time from the knowledge of the composition of the input flows (or batch deliveries). Hence, the relative composition of the output flow at any time is the same of the material in the tank at the same time.

The second case is when a single-input-single-output compartment behaves as a first-in-first-out (FIFO) buffer in which, however, input and output mass flows do not need to share the same intensity-time profiles. This second condition is referred to as *FIFO compartment.*

Remark that if a plant can be fully described using only UM or FIFO compartments, then the relative composition of any lot in the plant can be accurately derived, as detailed in Sections 3 and 4, and thus lot traceability can be conveniently implemented.

## 3. Modelling uniform-mixing and FIFO compartments

In this section, the two important cases of UM and FIFO compartments, schematizing storage units or processing stations in food processing plants, are analysed, and specific models are derived.

In the following, it is assumed that a total of $\ell$ different S-lots are available, belonging to the set of labels $\mathcal{L} = \{\text{'A'}, \text{'B'}, \text{'C'}, \dots\}$, with $\text{card}(\mathcal{L}) = \ell$. Moreover, for the sake of simplicity and without loss of generality, it is assumed that any mass that is fed to the production chain belongs to one and only one S-lot at the time it enters the system.

The case of $n$ interconnected tanks is considered, with material flowing from the outside and between them. Considering a generic compartment $i$, it follows that there are possibly up to $n$ different mass inflows $q_{ij}(t)$, $j = 0, \dots, n, j \neq i$ entering compartment $i$ from other $n - 1$ compartments, or from outside the system. So, $q_{ij}(t)$ represents the mass flow leaving compartment $j$ and entering compartment $i$, while $q_{i0}(t)$ represents the flow entering the $i$ -th compartment from outside the system, and $q_{0j}(t)$, represents the flow leaving the system from the $i$ -th compartment. Remark that the flows $q_{ij}(t)$ are bounded to be positive or zero, and can never assume negative values. In particular, if no flow exists from compartment $j$ to compartment $i$, then we assume $q_{ij}(t) = 0$. Hence, we can define the following *flow matrix*

$$Q \doteq \left[q_{ij}\right]_{i,j=0,\dots,n}. \tag{4}$$

Formally, the matrix $Q \in \mathbb{R}^{n+1,n+1}$ coincides with the adjacency matrix of the weighted graph representing the interconnections between compartments; see for instance (Godsil & Royle, 2001). Note that, by construction, the matrix $Q$ is square with zero diagonal elements.

3.1. Compartments ensuring uniform mixing

Hereafter the case in which compartments schematizing a storage container or a processing station ensure uniform (instantaneous) mixing of their content is considered first. Note that this kind of assumption is rather common for several modelling problems, in particular when compartmental models are used (Godfrey, 1983). Moreover, the assumption of uniform and instantaneous mixing appears quite reasonable in several processes typically encountered in the food processing industry. Indeed, inside the different compartments in which the process stages are carried on, the processed material is usually mixed in a continuous way in order to avoid settling phenomena, and to suppress possible thermal or concentration gradients. This is sometimes also the case of many storage devices, for instance whenever the processed material is liquid, so that diffusion and convection motions lead over time to a uniform mixing. Clearly, in real systems the mixing is never really instantaneous. However, it is in general rather fast, and the mixing time-constants are usually faster than those governing the process itself. On top of this, it should be noted that a non-uniform mixing would mainly induce errors only in the relative composition of the outflow from the compartment. Hence, whenever inflows and outflows are discontinuous and do not occur at the same time, truly uniform mixing actually occurs also in a real plant.

In order to describe the dynamics governing the different lots, a compartmental model is introduced, where each compartment coincides with a tank in the system. First, to describe the dynamic behaviour of a generic compartment $i$ a set of suitable *state variables* that fully account for its status at any time is chosen.

To this regard, denote by $m_i(t)$ the total mass available in compartment $i$ at time $t$. This mass can be divided into $\ell$ different sub-masses $m_i^L(t)$, one for every $L \in \mathcal{L}$, representing the fraction of the mass $m_i(t)$ containing material from S-lot $L$. The masses $m_i^L(t)$, $L \in \mathcal{L}$ are the *state variables* that fully describe the dynamics of compartment $i$.

Then, the following quantities are defined

$$\gamma_i^L(t) \doteq \frac{m_i^L(t)}{m_i(t)}\ , \text{ for } i = 1, \dots, n \text{ and } L \in \mathcal{L}, \tag{5}$$

denoting the fraction of S-lot $L$ contained in compartment $i$ at time $t$. Obviously, by definition, it holds that $\sum_{L \in \mathcal{L}} \gamma_i^L(t) = 1$. Notice also that, again by definition, the quantity

$$\gamma_i(t) \doteq \left[\gamma_i^{\mathrm{A}}(t)\ \ \gamma_i^{\mathrm{B}}(t)\ \ \gamma_i^{\mathrm{C}}(t) \cdots\right]^T, \tag{6}$$

coincides with the *instantaneous composition* of the material present in compartment $i$ at time $t$.

At any time, the mass flow $q_{ij}(t)$ is composed by masses belonging to different S-lots. In particular, it can be easily seen that the relative fraction of $q_{ij}(t)$ which is constituted by a mass-flow belonging to the S-lot $L$ is given by $\gamma_j^L(t) q_{ij}(t)$.

The quantities previously defined allow to compactly write the state equations of the mass exchange in the $i$ -th compartment as follows

$$\dot{m}_i^L(t) = \sum_{j=0}^{n} \gamma_j^L(t)\, q_{ij}(t) - \gamma_i^L(t) \sum_{i=0}^{n} q_{ji}(t)\, , \text{ for } L \in \mathcal{L}, \tag{7}$$

where $\dot{m}_i^L(t) = \frac{\mathrm{d}m_i^L(t)}{\mathrm{d}t}$ denotes the time variation of mass $m_i^L(t)$. The first summation on the right-hand side of equation (7) represents the total inflow of material belonging to S-lot $L$ entering compartment $i$, while the second term is the total outflow of material belonging to S-lot $L$ leaving the compartment. Under the assumption that a uniform and instantaneous mixing takes place in all compartments of the production chain, then the whole system can be easily described by means of $n$ different sets of equations (7), one for each compartment.

To show the behaviour of the introduced model in this case of completely uniform mixing, an illustrative example is introduced next.

*Example 1 (Completely uniform mixing).* In order to clarify the concepts previously presented, a simple system depicted in Fig. 1 is introduced. Focusing on the first part of the plant, constituted by the cascade of two storage compartments (Tank 1 and Tank 2) characterized by uniform mixing, we consider the following situation: At initial time $t_0 = 0$ s, Tank 1 is filled with 100 *kg* of mass belonging to S-lot 'A'. Then, at time $t_1 = 10$ s a flow of 1 $\mathrm{kgs}^{-1}$ is transferred into Tank 2 for a duration of 60 seconds. Subsequently, at time $t_2 = 80$ s, an outflow of 0,5 $\mathrm{kgs}^{-1}$ starts from Tank 2. At $t_3 = 90$ s, additional 70 *kg* of mass belonging to S-lot ‘B’ are added to Tank 1. Finally, at time $t_4 = 100$ s, a flow of 1 kgs$^{-1}$ is again transferred into Tank 2 for other 100 seconds. Values of the mass flows between the three tanks over the time interval [0,300] $s$ are plotted in Fig. 2.

Assuming that the material is uniformly mixed in the first two compartments, the masses $m_1^{\mathrm{A}}(t)$, $m_1^{\mathrm{B}}(t)$, $m_2^{\mathrm{A}}(t)$, $m_2^{\mathrm{B}}(t)$ of material belonging to S-lots ‘A’ and ‘B’ in Tank 1 and Tank 2 are reported in Fig. 3 and Fig. 4, respectively, over the interval [0,300] $s$. Fig. 5 and 6 report the fractions $\gamma_1^{\mathrm{A}}(t)$, $\gamma_1^{\mathrm{B}}(t)$, $\gamma_2^{\mathrm{A}}(t)$, and $\gamma_2^{\mathrm{B}}(t)$, describing the relative composition in terms of S-lots ‘A’ and ‘B’ of the two flows $q_{21}(t)$ and $q_{32}(t)$, respectively. In particular, in Fig. 6 it can be seen that the composition of the flow from Tank 2 to Tank 3 is continuously varying, with the percentage of material belonging to S-lot ‘B’ increasing and the one from S-lot ‘A’ decreasing. The blue vertical lines in Fig. 6 refer to the introduction of quantization levels, which is discussed in the next section.

3.2. Compartments behaving as FIFO buffer

The case in which a generic $i$-th compartment behaves like a first-in-first-out buffer is surely more complex, and is discussed hereafter. Note that the FIFO model can represent several practical situations encountered in real production lines when dealing with bulk solids and powders. Indeed, there is a growing research pursued by the industrial technology in designing specific devices and tank configurations that ensure plug-flow. Plug flow (referred also as *mass flow*) silos are frequently used in industrial processing because of some beneficial properties. Plug flow is the most productive flow, it eliminates problems like channelling, hang-ups, flooding of powders, prevents stagnant regions formation, while caking, degrading and segregation phenomena are minimized. In silos and hoppers filled with a densely packed product, upon opening of the outlet, a narrow plug-type zone of flowing material establishes and propagates upward. Except in the proximity of the outlet, the boundaries of the plug-flow zone are nearly vertical, and the zone widens laterally and may reach eventually the walls (Waters & Drescher, 2000). The main disadvantage in designing plug-flow silos is that a steep hopper angle is required, making the silo relatively tall. Moreover, flowability characteristics of granular solids and powders depends on many factors, among which moisture content, temperature, particle size, compacting pressure, relative humidity of the interstitial and head space air and the addition of flow conditioners and anticaking agents that can vary (Ganesan, Rosentrater, & Muthukumarappan, 2008).

Some general solutions to facilitate plug flow in grain handling and drying are, for example, the use of inserts to improve material flow patterns (Wójcik, Tejchman, & Enstad, 2012), the adoption of revolving extracting screws (see e.g. Borghi, 2012; Mulmix, (2012)) and blade extractors for homogeneous bin empting and powered grain spreaders to evenly fill the silos. Different techniques are nowadays available to measure and verify if flow conditions corresponds to manufacturer's claims. See, for instance, the case of the application of RDIF tags (Chen, Rotter, Ooi, & Zhong, 2007) or of specific tracers (Job, Dardenne, & Pirard, 2009), directly introduced at the top of the silo.

A FIFO compartment can be schematically represented as a vertical cylinder of constant cross-section $S_i$, in which the outflow is at the bottom, i.e. at height $h = 0$, while the material inflowing the compartment enters the silo or tank from above and it is uniformly deposited at height $H_i(t)$ on top of the material that is already stored. Notice that the total level $H_i(t)$ of material stored in the pipe is in general time-varying: if the total inflow is larger than the total outflow it increases in time, while it decreases if the outflow is larger than the inflow. Obviously, it results that $H_i(t) \geq 0$ for all $t$ and the mass stored in this $i$ -th compartment at any time $i$ is equal to $m_i(t) = \rho S_i H_i(t)$, where $\rho$ is the density of the material contained in the FIFO compartment. In order to ensure a purely FIFO behaviour for compartment $i$, it is assumed that all the material stored in the compartment strictly moves only downwards and at the same speed, which is equal to $q_{OUT,i}(t)/(\rho S_i)$, where the total inflow to compartment $i$ is defined as follows $q_{OUT,i}(t) \doteq \sum_{j=0}^{n} q_{ji}(t)$. Similarly, the total inflow to compartment $i$ is defined as $q_{IN,i}(t) \doteq \sum_{j=0}^{n} q_{ij}(t)$.

Then, relative fraction of flow *entering* compartment $i$ at time $t$ that is constituted of material belonging to S-lot $L$ only can be written as follows

$$\gamma_{IN,i}^{L}(t) \doteq \frac{q_{IN,i}^{L}(t)}{q_{IN,i}(t)} = \frac{\sum_{j=0}^{n} \gamma_j^L(t)\, q_{ij}(t)}{\sum_{j=0}^{n} q_{ij}(t)}, \text{ for } L \in \mathcal{L}. \tag{8}$$

Obviously, it holds that $\sum_{L \in \mathcal{L}} \gamma_{IN,i}^{L}(t) = 1$. Also, the following vector can be introduced

$$\gamma_{IN,i}(t) \doteq \left[\gamma_{IN,i}^{\mathrm{A}}(t)\ \gamma_{IN,i}^{\mathrm{B}}(t)\ \gamma_{IN,i}^{\mathrm{C}}(t) \cdots\right]^T, \tag{9}$$

which can be interpreted as the instantaneous composition of the inflow into compartment $i$ at time $t$.

It follows then that also for the material stored in this compartment it is possible to derive $\ell$ functions $\gamma_i^L(h,t)$ that provide, at any cross-section at height $h$ in the pipe, the relative fraction of material belonging to each S-lot $L$ at time $t$. Note that these functions vary continuously with respect to the height $h$.The total percentage of S-lot $L$ contained in tank $i$ at time $t$ can be computed integrating $\gamma_i^L(h,t)$ in the interval $[0,h]$, that is

$$\gamma_i^L(t) = \int_0^{H_i(t)} \gamma_i^L(h,t)\mathrm{d}h \ , \text{ for } \ L \in \mathcal{L}. \tag{10}$$

Similarly, the total mass of material belonging to S-lot $L$ contained in tank $i$ at time $t$ can be obtained as $m_i^L(t) = \gamma_i^L(t)m_i(t)$, for $L \in \mathcal{L}$.

Notice that the functions $\gamma_i^L(h,t)$, $L \in \mathcal{L}$, fully describe the state of the tank $i$ with FIFO behaviour, which turns out being a dynamic system with an *infinite dimensional* state vector. The dynamics of the tank can therefore be precisely represented only by partial differential equations. The integration of such equations, however, is usually performed numerically by approximating the system with discrete or finite elements techniques, which provide approximating models with a finite dimensional state vector (González-Montellano, Gallego, Ramírez-Gómez, & Ayuga, 2012; Ketterhagen et al., 2007).

In our case this task can be easily done directly approximating the functions $\gamma_{IN,i}^L(t)$, $L \in \mathcal{L}$, by quantizing them over a given number of levels. It means that the inflow relative composition is assumed to be constant over time as long as its composition does not vary more than given thresholds. Obviously the same holds also for the outgoing flow leaving the tank.

In the sequel, adopting a compartmental models terminology, the amount of material with a homogeneous composition (up to quantization level $\delta$), in terms of share of S-lots, that enters or leaves a compartment is called a *cohort*. The status of the $i$-th compartment with first-in-first-out behaviour is then fully described by the ordered list of the cohorts that are stored in it. Formally, the $i$-th compartment is hence completely described by the list

$$\begin{bmatrix} TOP \\ queued_v \\ \vdots \\ queued_1 \\ BOTTOM \end{bmatrix}_i \tag{10}$$

of its contained cohorts. To each of these cohorts is associated the information relative to its total mass and its composition.

For what concerns the $i$-th compartment, if at time $t$ the composition-distance between the inflow $IN, i$ entering the compartment at time $t$ and the material already present in the top cohort of the compartment $TOP, i$ is greater than the given quantization level, so that $d\left(\gamma_{IN,i}(t), \gamma_{TOP,i}(t)\right) > \delta$, then a new cohort is created. This new generated cohort, with all the information that fully describes its composition, is then piled in the FIFO array. For the sake of clarity, the algorithm is schematized in Fig. 7. In particular, the differential equations in (7) are simulated (step 4) until a new event, such as a valve opening/closing or a pump start/stop, occurs.

In order to clarify the impact of using cohorts, the dynamics of the scheme introduced in Example 1 is now analysed focusing on the third tank, schematized as a FIFO container.

*Example 2.* The analysis is carried out twice, using two different quantization levels, $\delta_1 = 0.1$ and $\delta_2 = 0.02$, so that the influence of quantization levels can be considered as well. In the time instant $t_1 = 80$ s the valve on the connection between Tank 2 and Tank 3 is opened and the flow of product $q_{32}(t)$ that is established is equal to 0.5 kgs$^{-1}$. Tank 3 starts to release product out of the system at $t_2 = 110$ s, with a flow rate $q_{03}(t) = 0.2$ kgs$^{-1}$, as shown in Fig. 2. The threshold $\delta_1$ to generate new cohorts in Tank 3 is applied on the composition of flow $q_{32}(t)$, whose relative amount of S-Lot 'A' and S-Lot 'B' is represented in Fig. 6, using $\gamma_{IN,2}^{\mathrm{A}}(t)$ and $\gamma_{IN,2}^{\mathrm{B}}(t)$ indexes. The time instants in which one of the $\gamma_{IN,2}^{\mathrm{L}}(t)$ crosses a quantization levels, with a threshold set of $\delta_1 = 0.1$, are reported in Fig. 6 with vertical lines. Masses $m_3^{\mathrm{A}}(t)$ and $m_3^{\mathrm{B}}(t)$ of material belonging to S-Lot '*A*' and to S-Lot '*B*', and the overall mass $m_3(t)$, in Tank 3 are shown in Fig. 9, that however lacks of information about the cohorts that have been generated during the filling phase with $q_{32}(t)$. For this reason, Fig. 10 is reported, in which mass content of Tank 3 is represented in six different time instants. Each cohort is characterized by a different colour, related to the relative composition in terms of S-Lot 'A' and 'B'. The influence of product quantization in Tank 3 on the outflow $q_{03}(t)$ can be seen in Fig. 8, where $\gamma_{OUT,2}^{\mathrm{A}}(t)$ and $\gamma_{OUT,2}^{\mathrm{B}}(t)$ indexes are plotted over the time. Results obtained setting a threshold $\delta_2$ equal to 0.02, are reported in Fig. 11 and 12. Note that the generated cohorts are in this case smaller and more homogeneous. The movie of this simulation example is recorded in MPEG files S1 and S2, for thresholds $\delta_1$ and $\delta_2$, respectively.

## 4. A case study: plant with both UM and FIFO tanks

In order to clarify the concepts and the procedures introduced in previous sections, a case study, consisting in seven interconnected tanks depicted in Fig. 13, is now presented. In this example, all compartments behave as FIFO buffers, with the exception of Tank 6, where an agitator ensures a uniform mixing of processed products. At time $t$=0 s, Tanks 1 to 4 are filled with homogeneous raw material. More in detail, 100 kg of S-lot ‘A’ and 200 kg of ‘B’ are stored into Tank 1, Tank 2 is filled with 50 kg of S-lot ‘C’ and 200 kg of ‘D’, Tank 3 with 200 kg of S-lot ‘E’, and finally 300 kg of S-lot ‘F’ and 100 kg of ‘G’ are stored in Tank 4. Valves opening at time $t_1$=60 s allow product flows $q_{51}(t) = 0.32$ kgs$^{-1}$ and $q_{52}(t) = 0.2$ kgs$^{-1}$ from Tanks 1 and 2 to Tank 5. At time $t_2$=120 s, flows $q_{63}(t)$=0.18 kgs$^{-1}$ and $q_{64}(t)$=0.28 kgs$^{-1}$ start from Tanks 3 and 4 to Tank 6, where the incoming products are continuously mixed. Then, at time $t_3$=300 s the product in Tanks 5 and 6 starts flowing into Tank 7 with a rate of $q_{75}(t)$ and $q_{76}(t)$ equal to 0.3 kgs$^{-1}$. Fig. 14 shows the evolution of the flows between storage units and processing stations over time. Adopting a quantization level $\delta$ equal to 0.05, six cohorts of final product, characterized by different percentages of S-Lots ‘A’ to ‘G’, are generated. The simulation movie of the working plant is reported in MPEG file S3. The way in which the S-lots spread into the plant and mix to produce the six cohorts of final product in Tank 7 is schematized in the graph of Fig. 15 where the composition of each cohort is directly reported in the node. Note that this dispersion graph can be directly used to measure (and possibly to optimize) the performances of the traceability system as proposed in (Dabbene & Gay, 2011). As already remarked, the level of detail of the traceability, and therefore the number of generated cohorts, depends on the choice of the threshold $\delta$. Simulations were performed at different values of $\delta$ ranging from $10^{-3}$ to $10^{-1}$. Fig. 16 (left) shows how the number of generated cohorts considerably increases for decreasing values of threshold $\delta$. As expected, at increasing number of cohorts, it correspond smaller average cohort sizes (Fig. 16, right) and more homogeneous compositions. Figure 16 shows also masses of the largest and smallest cohort generated in each simulation of the set. These figures show that there exists a clear trade-off between the quantization level $\delta$ and the number of different lots generated. This trade-off should be taken in due consideration by the supply chain manager in designing and optimizing the traceability system.

## 5. Conclusions and future directions

The paper proposed a methodology for efficiently tackling the problem of traceability when continuously processing and storing bulk materials. In particular, the introduced framework is particularly suitable for the management of the *internal traceability*, i.e. during the production processes within a company. According to the key advantages provided by internal traceability, as discussed in Moe (1998), this methodology makes it possible monitoring (and avoiding) uneconomic mixing of high and low-quality raw materials and ingredients, and gives the basis for the adoption of efficient recall procedures to minimizes losses, at present available only for the processing of discrete lots of products. In particular, this method allows the proper identification and definition of batches of homogeneous product, without resorting to the nowadays often-adopted operation of oversizing the lots. In particular, the availability of precise information about the composition, in terms of lots of raw ingredients, introduces the possibility to correlate product data with raw materials and then to optimize the recipes for each final product type. In the same way, the availability of this information can be exploited to design improved process control strategies.

The present work analysed two representative cases of product containers, namely the uniform-mixing and the first-in-first-out compartments. It is however important to notice that the approach introduced in the paper can be extended to the more general case of storage compartments that do not show a UM or FIFO behaviour. Fundamental in this case is the availability of an accurate description of the dynamics governing the way the material delivered to the compartment is stored within its volume, and of the laws by which such material is recombined into the output flow. The problem of experimentally determining such laws has been the subject of growing interest in the literature. See, for instance, the recent works of Ganesan et al. (2008), González-Montellano et al. (2011), Mellmann et al. (2011), Sielamowicz & Czech (2010), and Sielamowicz et al., (2011), which applied finite/discrete elements techniques to describe tank filling/emptying dynamics. Indeed, once the laws governing the storing and mixing phenomena taking place in the tanks are adequately modelled, these mathematical models can be directly integrated in the framework discussed so far, since compartmental models are well-suited to cope with such situations. Specific cases are currently under study, and will be the subject of further works.

Finally, in the context of the present work, the fraction of the inflow allocated to each S-lots has been considered exactly known. However, it appears possible to consider instead the case when such fraction is subject to uncertainty. For instance, this could account for situations in which the UM or FIFO models are not sufficiently accurate in describing the real behaviour of the processes or some uncertainties affect flow dynamics (for example in the case in which the flow is dependent on some product conditions like temperature, moisture content etc.). In such case, the knowledge of the real composition of the outflow is not precise, and it can be determined only up to a given tolerance. Hence, it could be important to develop a framework able to determine the maximum amount of each S-lot that could be present in each compartment as well as in each flow.

**Acknowledgements**

This work was partially supported by the grants of the projects Namatech-Converging Technologies (CIPE2007), Regione Piemonte, Italy and PRIN 2009 (prot. 2009FXN7HW_002), MIUR, Italy.

## References


Barge P., Gay P., Merlino V., & Tortia C. (2013). RFID technologies for livestock management and meat supply chain traceability. *Canadian Journal of Meat Science*, *93*(1), 23–33. doi:10.4141/cjas2012-029

Bollen, A. F., Riden, C. P., & Cox, N. R. (2007). Agricultural supply system traceability, Part I: Role of packing procedures and effects of fruit mixing. *Biosystems Engineering*, *98*(4), 391–400. doi:10.1016/j.biosystemseng.2007.07.011

Borghi. (2012). Automatic unloaders and revolving extracting screws. www.borghigroup.it (web site visited July 2012).

Chen, J. F., Rotter, J. M., Ooi, J. Y., & Zhong, Z. (2007). Correlation between the flow pattern and wall pressures in a full scale experimental silo. *Engineering Structures*, *29*(9), 2308–2320. doi:10.1016/j.engstruct.2006.11.011

Coléno, F. C. (2008). Simulation and evaluation of GM and non-GM segregation management strategies among European grain merchants. *Journal of Food Engineering*, *88*(3), 306–314. doi:10.1016/j.jfoodeng.2008.02.013

Comba, L., Belforte, G., & Gay, P. (2011). Modelling techniques for the control of thermal exchanges in mixed continuous–discontinuous flow food plants. *Journal of Food Engineering*, *106*(3), 177–187. doi:10.1016/j.jfoodeng.2011.04.015

Dabbene, F., & Gay, P. (2011). Food traceability systems: Performance evaluation and optimization. *Computers and Electronics in Agriculture*, *75*(1), 139–146. doi:10.1016/j.compag.2010.10.009

Dabbene, F., Gay, P., & Tortia, C. (2013). Traceability oriented supply chain management and optimization: A review. *Submitted*.

Donnelly, K. A.-M., Karlsen, K. M., & Olsen, P. (2009). The importance of transformations for traceability – A case study of lamb and lamb products. *Meat Science*, *83*(1), 68–73. doi:10.1016/j.meatsci.2009.04.006

Dupuy, C., Botta-Genoulaz, V., & Guinet, A. (2005). Batch dispersion model to optimise traceability in food industry. *Journal of Food Engineering*, *70*(3), 333–339. doi:10.1016/j.jfoodeng.2004.05.074

European Commission. (2003). Regulation (EC) No 1829/2003 of the European Parliament and of the Council of 22 September 2003 concerning the traceability and labeling of genetically modified organisms and the traceability of food and feed products produced from genetically modified organisms and amending directive 2001/18/EC. *Official Journal of the European Union L268*, 1–23.

Ganesan, V., Rosentrater, K. A., & Muthukumarappan, K. (2008). Flowability and handling characteristics of bulk solids and powders – a review with implications for DDGS. *Biosystems Engineering*, *101*(4), 425–435. doi:10.1016/j.biosystemseng.2008.09.008

Godfrey, K. (1983). Compartmental Models and Their Applications. *Academic Press, London and New York*.

Godsil, C. D., & Royle, G. (2001). *Algebraic Graph Theory*. Springer.

González-Montellano, C., Gallego, E., Ramírez-Gómez, á., & Ayuga, F. (2012). Three dimensional discrete element models for simulating the filling and emptying of silos: Analysis of numerical results. *Computers & Chemical Engineering*, *40*, 22–32.

doi:10.1016/j.compchemeng.2012.02.007

González-Montellano, C., Ramírez, á., Gallego, E., & Ayuga, F. (2011). Validation and experimental calibration of 3D discrete element models for the simulation of the discharge flow in silos. *Chemical Engineering Science*, *66*(21), 5116–5126. doi:10.1016/j.ces.2011.07.009

ISO 22005. (2008). Traceability in the feed and food chain: general principles and basic requirements for system design and implementation.

Job, N., Dardenne, A., & Pirard, J.-P. (2009). Silo flow-pattern diagnosis using the tracer method. *Journal of Food Engineering*, *91*(1), 118–125. doi:10.1016/j.jfoodeng.2008.08.010

Karlsen, K. M., Donnelly, K. A.-M., & Olsen, P. (2011). Granularity and its importance for traceability in a farmed salmon supply chain. *Journal of Food Engineering*, *102*(1), 1–8. doi:10.1016/j.jfoodeng.2010.06.022

Ketterhagen, W. R., Curtis, J. S., Wassgren, C. R., Kong, A., Narayan, P. J., & Hancock, B. C. (2007). Granular segregation in discharging cylindrical hoppers: A discrete element and experimental study. *Chemical Engineering Science*, *62*(22), 6423–6439. doi:10.1016/j.ces.2007.07.052

Kim, H. M., Fox, M. S., & Grüninger, M. (1999). An ontology for quality management—enabling quality problem identification and tracing. *BT Technology Journal*, *17*(4), 131–140.

Kvarnström, B., Bergquist, B., & Vännman, K. (2011). RFID to Improve Traceability in Continuous Granular Flows—An Experimental Case Study. *Quality Engineering*, *23*(4), 343–357. doi:10.1080/08982112.2011.602278

Kvarnström, B., & Oghazi, P. (2008). Methods for traceability in continuous processes–Experience from an iron ore refinement process. *Minerals Engineering*, *21*(10), 720–730. doi:10.1016/j.mineng.2008.02.002

Lee, K.-M., Armstrong, P. R., Thomasson, J. A., Sui, R., Casada, M., & Herrman, T. J. (2010). Development and Characterization of Food-Grade Tracers for the Global Grain Tracing and Recall System. *Journal of Agricultural and Food Chemistry*, *58*(20), 10945–10957. doi:10.1021/jf101370k

Li, D., Kehoe, D., & Drake, P. (2005). Dynamic planning with a wireless product identification technology in food supply chains. *The International Journal of Advanced Manufacturing Technology*, *30*(9-10), 938–944. doi:10.1007/s00170-005-0066-1

Liang, K., Thomasson, J. A., Lee, K.-M., Shen, M., Ge, Y., & Herrman, T. J. (2012). Printing data matrix code on food-grade tracers for grain traceability. *Biosystems Engineering*, *113*(4), 395–401. doi:10.1016/j.biosystemseng.2012.09.012

Maier, D. E. (2006). Engineering design and operation of equipment to assure grain quality and puritypdf.pdf. In *9th International Working Conference on Stored Product Protection, San Paulo, Brazil* (pp. 1316–1326).

Mellmann, J., Iroba, K. L., Metzger, T., Tsotsas, E., Mészáros, C., & Farkas, I. (2011). Moisture content and residence time distributions in mixed-flow grain dryers. *Biosystems Engineering*, *109*(4), 297–307. doi:10.1016/j.biosystemseng.2011.04.010

Moe, T. (1998). Perspectives on traceability in food manufacture. *Trends in Food Science & Technology*, *9*(5), 211–214. doi:10.1016/S0924-2244(98)00037-5

Mulmix. (2012). Fixed rotating extractors. www.mulmix.it (web site visited July 2012).

Randrup, M., Storøy, J., Lievonen, S., Margeirsson, S., Árnason, S. V., Ólavsstovu, D. í, … Frederiksen, M. T. (2008). Simulated recalls of fish products in five Nordic countries. *Food Control*, *19*(11), 1064–1069. doi:10.1016/j.foodcont.2007.11.005

Rescigno, A. (2001). The rise and fall of compartmental analysis. *Pharmacological Research*, *44*(4), 337–342. doi:10.1006/phrs.2001.0873

Riden, C. P., & Bollen, A. F. (2007). Agricultural supply system traceability, Part II: Implications of packhouse processing transformations. *Biosystems Engineering*, *98*(4), 401–410. doi:10.1016/j.biosystemseng.2007.07.004

Rong, A., & Grunow, M. (2010). A methodology for controlling dispersion in food production and distribution. *OR Spectrum*, *32*(4), 957–978. doi:10.1007/s00291-010-0210-7

Saltini, R., & Akkerman, R. (2012). Testing improvements in the chocolate traceability system: Impact on product recalls and production efficiency. *Food Control*, *23*(1), 221–226. doi:10.1016/j.foodcont.2011.07.015

Sielamowicz, I., & Czech, M. (2010). Analysis of the radial flow assumption in a converging model silo. *Biosystems Engineering*, *106*(4), 412–422. doi:10.1016/j.biosystemseng.2010.05.004

Sielamowicz, I., Czech, M., & Kowalewski, T. A. (2011). Empirical description of granular flow inside a model silo with vertical walls. *Biosystems Engineering*, *108*(4), 334–344. doi:10.1016/j.biosystemseng.2011.01.004

Skoglund, T., & Dejmek, P. (2007). Fuzzy Traceability: A Process Simulation Derived Extension of the Traceability Concept in Continuous Food Processing. *Food and Bioproducts Processing*, *85*(4), 354–359. doi:10.1205/fbp07044

Thakur, M., & Donnelly, K. A.-M. (2010). Modeling traceability information in soybean value chains. *Journal of Food Engineering*, *99*(1), 98–105. doi:10.1016/j.jfoodeng.2010.02.004

Thakur, M., & Hurburgh, C. R. (2009). Framework for implementing traceability system in the bulk grain supply chain. *Journal of Food Engineering*, *95*(4), 617–626. doi:10.1016/j.jfoodeng.2009.06.028

Thakur, M., Wang, L., & Hurburgh, C. R. (2010). A multi-objective optimization approach to balancing cost and traceability in bulk grain handling. *Journal of Food Engineering*, *101*(2), 193–200. doi:10.1016/j.jfoodeng.2010.07.001

US Food and Drugs Administration. (2013). Recalls, Market Withdrawals, & Safety Alerts. *http://www.fda.gov/safety/recalls/*.

Wang X., Li D., & O’Brien C. (2009). Optimisation of traceability and operations planning: an integrated model for perishable food production. *Internationa Journal of Production Research*, *47*(11), 2865–2886. doi:10.1080/00207540701725075

Waters, A. J., & Drescher, A. (2000). Modeling plug flow in bins/hoppers. *Powder Technology*, *113*, 168–175.

Wójcik, M., Tejchman, J., & Enstad, G. G. (2012). Confined granular flow in silos with inserts — Full-scale experiments. *Powder Technology*, *222*, 15–36. doi:10.1016/j.powtec.2012.01.031

## Nomenclature

| Parameter | Meaning |
|---|---|
| $\mathcal{L} = \{'A', 'B', 'C', \ldots\}$ | Ordered list of possible S-lots entering the system |
| $\ell$ | Cardinality of $\mathcal{L}$ (number of S-lot) |
| $n$ | Number of compartments |
| $t$ | Time variable [s] |
| $c(X)$ | Composition of product $X$ |
| $\gamma^{\mathrm{A}}(X)$ | Percentage of product coming from S-lot A present in product $X$ |
| $d(X, Y)$ | Composition-distance between products $X$ and $Y$ |
| $\delta$ | Threshold level of homogeneity |
| $\|\|x\|\|_{\infty}$ | $\ell_{\infty}$-norm of vector $x$ |
| $\|\|x\|\|_{\infty}^{W}$ | Weighted $\ell_{\infty}$-norm of vector $x$ |
| $w_L$ | Risk-factor |
| $m_i(t)$ | Mass in the $i$-th compartment at time instant $t$ [kg] |
| $m_i^L(t)$ | Fraction of the mass $m_i(t)$ containing material from S-lot $L$ [kg] |
| $\dot{m}_i^L(t)$ | Time derivative of the mass fraction $m_i^L(t)$ [$\mathrm{kgs^{-1}}$] |
| $q_{ij}(t)$ | Mass flow from compartment $j$ to compartment $i$ at time $t$ [$\mathrm{kgs^{-1}}$] |
| $q_{i0}(t)$ | Mass flow entering the $i$-th compartment at time $t$ [$\mathrm{kgs^{-1}}$] |
| $q_{0j}(t)$ | Mass flow leaving the $j$-th compartment at time $t$ [$\mathrm{kgs^{-1}}$] |
| $q_{IN,i}(t)$ | Sum of incoming mass flow $q_{ij}(t)$ in compartment $i$ at time $t$ |
| $q_{OUT,i}(t)$ | Total outflow from the $i$-th compartment |
| $Q$ | Flow matrix collecting the $q_{ij}$'s |
| $S_i$ | Cross-section of compartment $i$ |
| $H_i(t)$ | Height of material in compartment $i$ at time $t$ |
| $\rho$ | Product density [$\mathrm{kgm^{-3}}$] |
| $\gamma_i^L(t)$ | Percentage of S-lot $L$ contained in compartment $i$ at time $t$ |
| $\gamma_i\ (t)$ | Instantaneous composition of the material present in compartment $i$ at time $t$ |
| $\gamma_{IN,i}^L(t)$ | Relative fraction of flow entering compartment $i$ at time $t$ that is constituted of material belonging to S-lot $L$ only, at time $t$ |

| | |
|---|---|
| $\gamma_i^L(h,t)$ | Relative fraction of material in compartment $i$ belonging to S-lot $L$ at a cross-section at height $h$, at time $t$ |
| $t_{next\_event}$ | Time of the occurrence of the next event in the algorithm for creation of homogeneous cohorts in FIFO compartments |
| $t_{end}$ | Simulation end time of algorithm for homogeneous cohorts creation in FIFO compartments |
| $\Delta t$ | Simulation time interval for the algorithm for homogeneous cohorts creation in FIFO compartments |

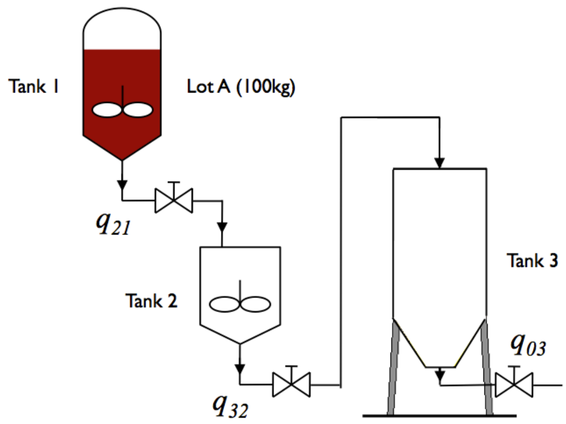


**Figure 1.** Scheme of the plant in examples 1 and 2

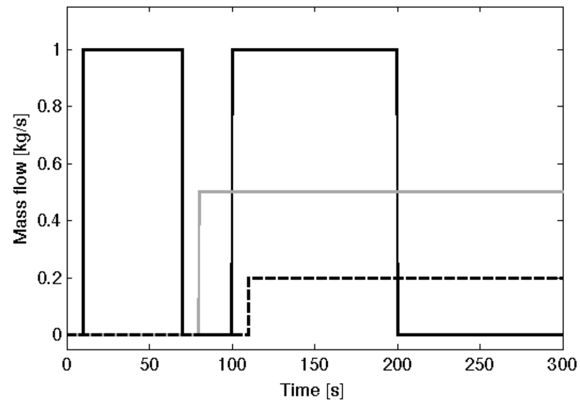


**Figure 2.** Mass flow $q_{21}$ (black solid) from Tank 1 to Tank 2, $q_{32}$ (gray solid) from Tank 2 to Tank 3, and $q_{03}$ (black dashed) from Tank 3 out of the system

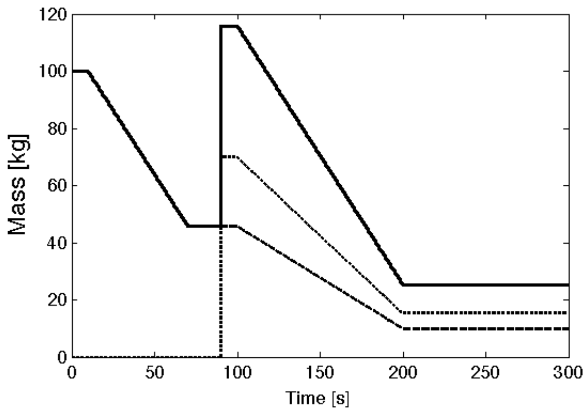


**Figure 3.** Mass of product belonging to S-lot A (black dashed), S-lot B (black dotted), and overall mass amount in Tank 1 (black solid).

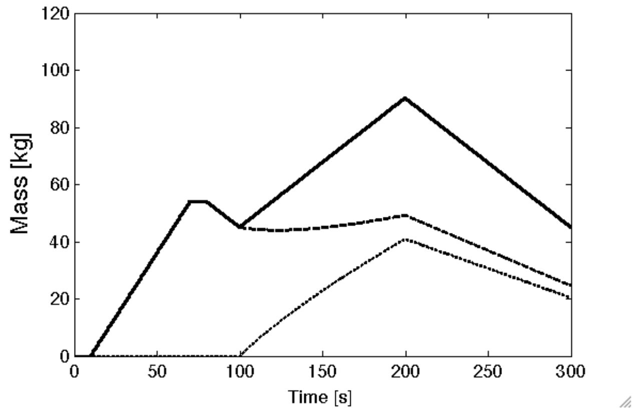


**Figure 4.** Mass of product belonging to S-lot A (black dashed), S-lot B (black dotted), and overall mass amount in Tank 2 (black solid).

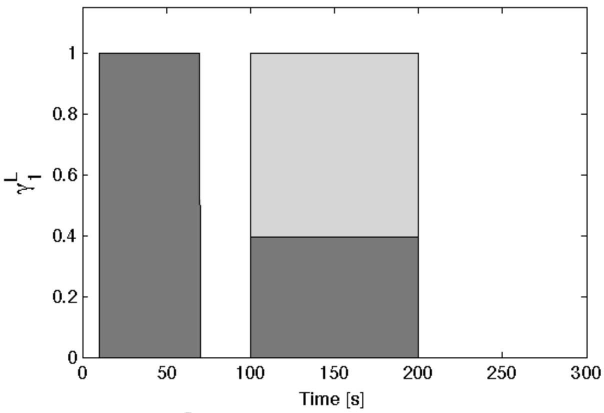


**Figure 5.** Relative fractions $\gamma_1^{\mathrm{A}}(t)$ (dark gray) and $\gamma_1^{\mathrm{B}}(t)$ (light gray) of $q_{21}$ flow constituted by mass belonging to lot of product A and B respectively. The sum of $\gamma_1^{\mathrm{A}}(t)$ and $\gamma_1^{\mathrm{B}}(t)$is always equal to 1.

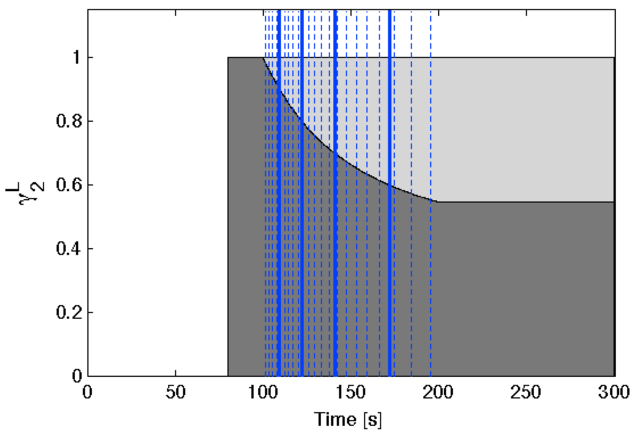


**Figure 6.** Relative fractions $\gamma_2^{\mathrm{A}}(t)$ (dark gray) and $\gamma_2^{\mathrm{B}}(t)$ (light gray) of flow $q_{02}$ constituted of mass belonging to S-lot A and B respectively. The sum of $\gamma_2^{\mathrm{A}}(t)$ and $\gamma_2^{\mathrm{B}}(t)$ is always equal to 1. Time instants in which a new cohort is generated inside Tank 3 are represented by vertical solid and dashed lines, for the two cases of quantization level $\delta$ equal to 0.1 and 0.02, respectively.

1: $j \leftarrow 0$
2: Do
3: $\quad t_{j+1} \leftarrow min(t_j + \Delta t, t_{next\_event})$
4: $\quad$ Simulate $\gamma_{IN,i}(t)$ for $t \in [t_j, t_{j+1}]$
5: $\quad$ For $t = t_j$ to $t_{j+1}$ do
6: $\quad\quad$ If $d\left(\gamma_{IN,i}(t), \gamma_{IN,i}(t_j)\right) > \delta$ then
7: $\quad\quad\quad j \leftarrow j + 1$
8: $\quad\quad\quad t_j \leftarrow t$
9: $\quad\quad\quad v \leftarrow v + 1$
10: $\quad\quad\quad queued_v \leftarrow TOP$
10: $\quad\quad\quad$ Create new $TOP$ cohort
11: $\quad\quad$ Goto 3
12: $\quad$ End
13: $\quad j \leftarrow j + 1$
14: While $t < t_{end}$
15: End

**Figure 7.** Algorithm for the creation of homogeneous cohorts in a FIFO compartment. Simulation parameters: $t_{next\_event}$ - time of the occurrence of the next event after $t$; $t_{end}$ – end time of the simulation; $\Delta t$ arbitrary time interval.

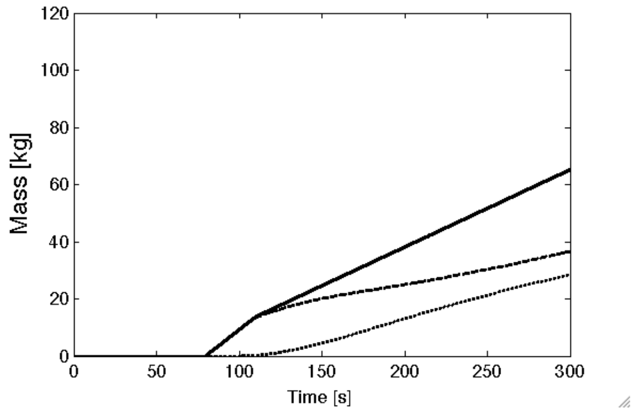


**Figure 8.** Mass of product belonging to S-lot A (black dashed), S-lot B (black dotted), and overall mass amount in Tank 3 (black solid).

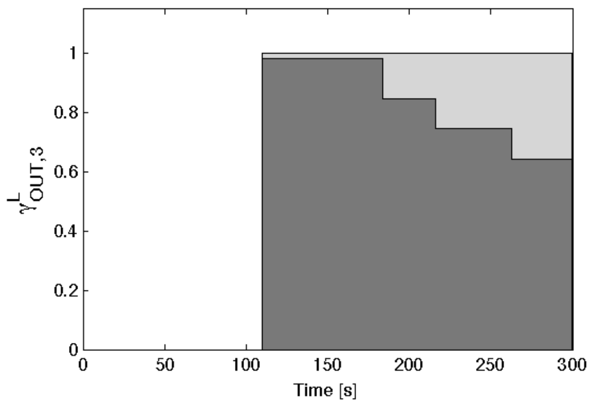


**Figure 9.** Relative fractions $\gamma^{A}_{OUT,3}(t)$ (dark gray) and $\gamma^{B}_{OUT,3}(t)$ (light gray) of flow $q_{03}$ constituted of mass belonging to S-lot A and B respectively, in the case of quantization level $\delta$ equal to 0.1

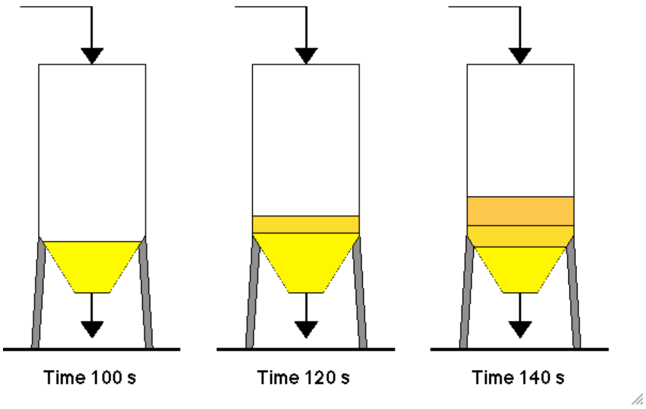


**Figure 10**. Tank 3 content at t=100, 120 and 140 seconds, in the case of quantization level $\delta$ equal to 0.1. Different cohorts are represented with color hues proportional to the % of product belonging to S-Lot A and S-Lob B.

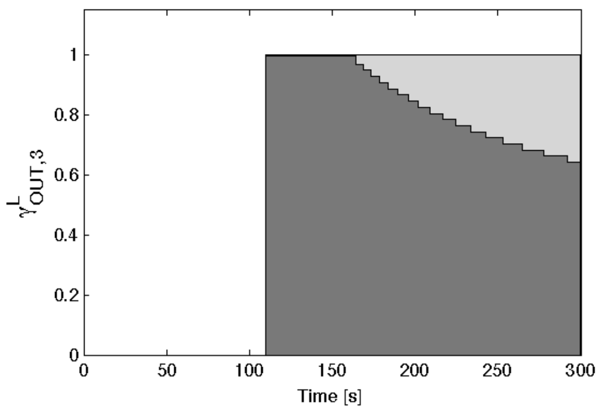


**Figure 11.** Relative fractions $\gamma^{A}_{OUT,3}(t)$ (dark gray) and $\gamma^{B}_{OUT,3}(t)$ (light gray) of flow $q_{03}$ constituted of mass belonging to S-lot A and B respectively, in the case of quantization level $\delta$ equal to 0.02.

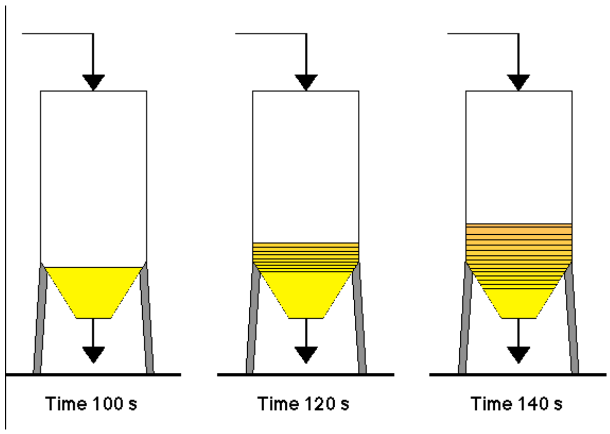


**Figure 12**. Tank 3 content at t=100, 120 and 140 seconds, in the case of quantization level $\delta$ equal to 0.02. Different cohorts are represented with color hues proportional to the % of product belonging to S-Lot A and S-Lob B.

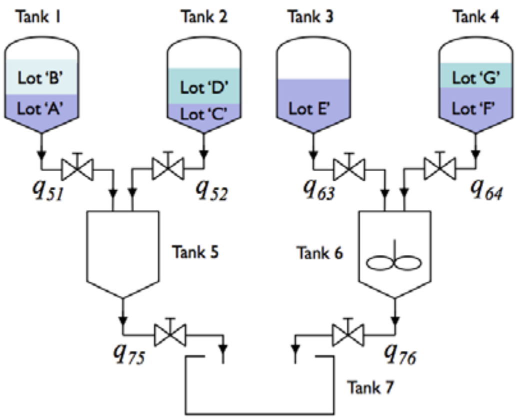


**Figure 13.** Scheme of the plant in the case study at time $t = 0$

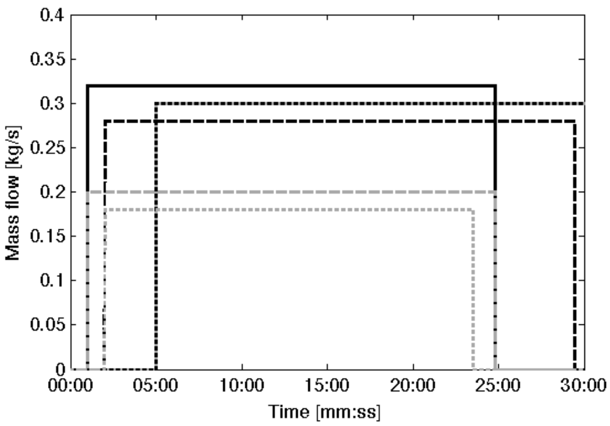


**Figure 14.** Mass flows $q_{51}(t)$ (black solid) from Tank 1 to 5, $q_{52}(t)$ (gray dashed) from Tank 2 to 5, $q_{63}(t)$ (gray dotted) from Tank 3 to 6, $q_{64}(t)$ (black dashed) from Tank 4 to 6, $q_{75}(t)$ (gray solid) from Tank 5 to 7, and $q_{76}(t)$ (black dotted) from Tank 6 to 7.

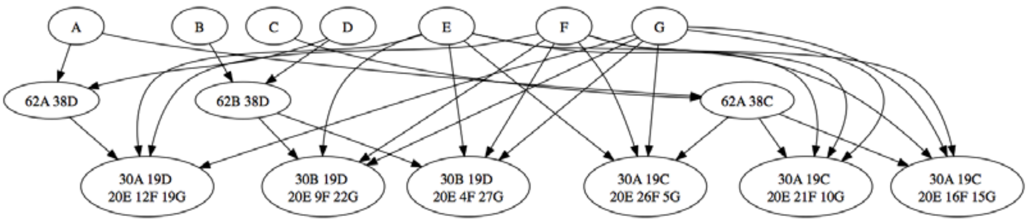


**Figure 15.** Graph of the composition of the six cohorts. The label of each node in the graph reports the composition of the cohort, where the numbers express the percentage of the different S-lots (A to G).

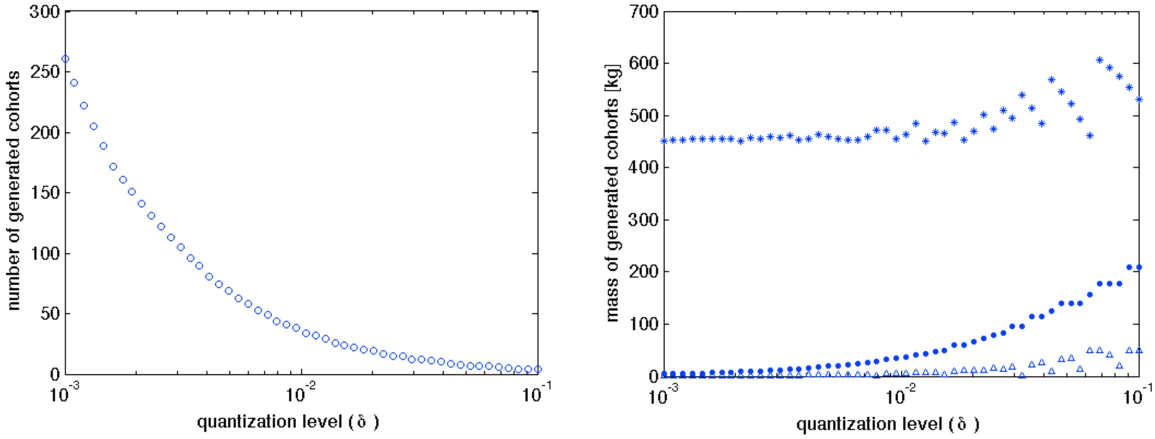


**Figure 16.** Number (on the left) and average mass (dotted, on the right) of generated cohorts obtained with different quantization levels $\delta$ ranging from $10^{-3}$ to $10^{-1}$ in Example 3. On the right, masses of the biggest ($*$) and smallest ($\Delta$) cohort are also reported.