Information densities for block-fading MIMO channels

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G. Alfano* C.-F.Chiasserini† A. Nordio‡ S. Zhou§

Abstract — **This paper provides a characterization of the output and of the transition probability density function (pdf) of a MIMO block-fading channel, paving the way to its information-theoretic analysis. The information density, whose average in ergodic conditions gives the mutual information conveyed over the channel, is needed to characterize the channels with arbitrary behavior, as one in this case can neither resort to ergodicity nor to consequences of any law of large numbers. The information density expression is then provided under several assumptions on the channel model and also in the presence of multiple-antenna equipped interferers.**

1 INTRODUCTION

Information density methods allow for the study of arbitrary channels, as they do not take recourse to ergodic results like, e.g. the Asymptotic Equipartition Property (AEP). Relying on the survey in [1], in this work we aim at characterizing the information density for some MIMO channel of interest, characterized by block-independent fading. The crucial steps toward the derivation of a formula for the information density are the characterization of the probability density function (pdf) of the channel output, as well as of the transition probability, i.e. the conditional law of the channel output given the input. A recent analysis in [2], whose authors were mainly interested in proposing a computationally efficient way for evaluating the mutual information conveyed on a MIMO block fading channel with no Channel State Information (CSI) at either ends of the link, paved the way to our derivation. Indeed, the strategy in [2] for evaluating the output pdf can be extended from block-Rayleigh MIMO channels to channels with different fading laws, exploiting some results of finite-dimensional random matrix theory. The transition probability follows in many cases from simple observations on the fading law. Our focus is on MIMO channel fed by i.i.d. Gaussian inputs, which exhibit the nice feature of being invariant (in law) with respect to left and/or right multiplication times a unitary matrix, and serve as a reference with respect to otherwise optimized inputs.

The paper is organized as follows: next Section describes the System Model, while in Section 3 the analytical derivation of the information density for each channel model of interest is reported.

Throughout the paper, matrices are denoted by uppercase boldface letters, vectors by lowercase boldface. The pdf of a random matrix $\mathbf{Z}$, $p_{\mathbf{Z}}(\mathbf{Z})$, is simply denoted by $p(\mathbf{Z})$. The complex, matrix-variate Gaussian law with mean matrix $\mathbf{\Delta}$ and covariance matrix $\mathbf{\Xi}$ is denoted by $\mathcal{CN}(\mathbf{\Delta}, \mathbf{\Xi})$. The complex matrix-variate F law with p and n degrees of freedom is denoted by $F_m(p, n)$, with m the size of the matrix and, in order to avoid rank deficiencies, $p \geq m$ and $n \geq m$. $(\cdot)^\dagger$ indicates the conjugate transpose operator, $|\cdot|$ and $\mathsf{Tr}(\cdot)$ denote, respectively, the determinant and the trace of a square matrix, and $||\cdot||$ stands for the Euclidean norm[1]. $\Gamma_p(q)$, with $p \leq q$, is the complex multivariate Gamma function [3]

$$\Gamma_p(q) = \pi^{\frac{p(p-1)}{2}} \prod_{\ell=1}^{p} (q-\ell)!$$

We denote by $\mathbf{I}_m$ the $m \times m$ identity matrix and, letting $\mathbf{A}$ be an $n \times n$ Hermitian matrix with ordered eigenvalues[2] $\lambda_1, \ldots, \lambda_n$, we denote by $\mathcal{V}(\mathbf{A})$ the Vandermonde determinant of $\mathbf{A}$ [4], i.e.,

$$\mathcal{V}(\mathbf{A}) = \prod_{1 \leq i < \ell \leq n} (\lambda_i - \lambda_\ell).$$

2 SYSTEM MODEL

We consider a single-user multiple-antenna communication, with n_r and n_t denoting the number of receive and, respectively, of transmit antennas. Assuming block-fading with block length n_b, the channel output can be described by the following linear

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[1] As applied to a matrix, we mean $||\mathbf{A}||^2 = \mathsf{Tr}(\mathbf{A}^\dagger\mathbf{A})$

[2] In the following, all the joint eigenvalue distributions assume the eigenvalues to be ordered.

relationship:

$$\mathbf{Y} = \sqrt{\gamma}\mathbf{H}\,\mathbf{X} + \mathbf{N}\,. \quad (1)$$

In (1), $\mathbf{Y}$ is the $n_r \times n_b$ output, $\mathbf{X}$ is the i.i.d. zero-mean, unit variance complex Gaussian $n_t \times n_b$ input matrix, and $\mathbf{N}$ is the $n_r \times n_b$ matrix of additive complex circularly symmetric Gaussian noise. $\mathbf{H}$ is the $n_r \times n_t$ complex channel matrix, whose entries represent the fading coefficients between each transmit and each receive antenna, and whose singular value decomposition reads as[3] $\mathbf{H} = \mathbf{U}\mathbf{\Sigma}^{1/2}\mathbf{V}^\dagger$. Finally, $\gamma = \text{SNR}/n_t$ represents the normalized per-transmit antenna signal-to-noise ratio (SNR).

We apply the well known definition of the information density to the channel in (1), and write

$$i_{\mathbf{X},\mathbf{Y}}(\mathbf{X},\mathbf{Y}) = \log \frac{p(\mathbf{Y}|\mathbf{X})}{p(\mathbf{Y})}\,. \quad (2)$$

As largely discussed in [5, and references therein], this quantity plays an essential role in the information theoretic analysis of finite block-length communications. Moreover, information density methods are crucial in the characterization of non-ergodic channels at large. In the remaining of the paper, we will then provide analytical results for the output and the transition pdf of a block-fading channel, since they constitute the building blocks of (2).

3 ANALYTICAL RESULTS

The characterization we propose works for arbitrary number of transmit and receive antennas and arbitrary fading duration, however the analytical details of the derivation depend on the relative values of n_t and n_r (as in [2, and references therein]). Due to the lack of space, we focus on the case of $n_t \geq n_r$.

Given a channel as in (1), with i.i.d. Gaussian $\mathbf{X}$, the output pdf when $n_t \geq n_r$ can be conveniently written [2, Formulae (40) and (41)] as

$$p(\mathbf{Y}) = T \int_{\mathbf{\Sigma}>0} \frac{p(\mathbf{\Sigma})}{\mathcal{V}(\mathbf{\Sigma})} |\mathbf{E}| \prod_{\ell=1}^{n_r} (1+\gamma\sigma_\ell)^{n_r-n_b-1}\, \mathrm{d}\mathbf{\Sigma}\,, \quad (3)$$

with

$$T = \frac{\prod_{\ell=1}^{n_r}(\ell-1)!}{\gamma^{n_r(n_r-1)/2}\pi^{n_r n_b}} \frac{\mathrm{e}^{-||\mathbf{Y}||^2}}{\mathcal{V}(\mathbf{Y}^\dagger\mathbf{Y})}\,,$$

σ_i the i-th eigenvalue of $\mathbf{\Sigma}$, y_j^2 the j-th non-zero eigenvalue of $\mathbf{Y}\mathbf{Y}^\dagger$ and $(\mathbf{E})_{i,j} = \mathrm{e}^{y_i^2\gamma\sigma_j/(1+\gamma\sigma_j)}$. By particularizing this integral to the channel model in force, in the following we will evaluate and list the output pdf in closed form for some scenarios of interest.

3.1 Ricean channel

In this case, the channel is traditionally modeled as a superposition of a scattered plus a Line-Of-Sight (LOS) component, i.e.

$$\mathbf{H} = \sqrt{\frac{\kappa}{\kappa+1}}\,\bar{\mathbf{H}} + \sqrt{\frac{1}{\kappa+1}}\,\mathbf{H}_w \quad (4)$$

where κ is the Ricean factor, representing the ratio between the average power of the unfaded and faded channel components, $\bar{\mathbf{H}}$ is deterministic independent zero-mean unit-variance complex Gaussian. The joint density of the unordered squared singular values of the channel matrix reads in this case as [6, Formula (34)]

$$\begin{aligned} p(\mathbf{\Sigma}) = &\tfrac{(1+\kappa)^{n_r(n_t-n_r+1)}\mathrm{e}^{-n_r n_t \kappa}}{n_r!\mathcal{V}(\bar{\mathbf{H}}\bar{\mathbf{H}}^\dagger)} \\ &\cdot \frac{\prod_{\ell=1}^{n_r-1}(n_r-\ell)^\ell(n_t-\ell)^\ell}{\prod_{\ell=1}^{n_r}(n_r-\ell)!(n_t-\ell)!} \\ &\cdot \mathrm{e}^{-(1+\kappa)\mathsf{Tr}(\mathbf{\Sigma})}|\mathbf{F}||\mathbf{\Sigma}|^{n_t-n_r}\mathcal{V}(\mathbf{\Sigma})\,, \end{aligned} \quad (5)$$

with $(\mathbf{F})_{i,j} = {}_0F_1(n_t-n_r+1,(1+\kappa)^2\mu_i\sigma_j)$, μ_i being the i-th squared singular value of $\bar{\mathbf{H}}$. As anticipated earlier in this section, the output pdf can be evaluated by replacing (5) in (3) and performing the average over $\mathbf{\Sigma}$, by exploiting [7, Corollary I]. This way, we obtain for a MIMO channel affected by block-Ricean fading:

$$\begin{aligned} p(\mathbf{Y}) = &\frac{\prod_{\ell=1}^{n_r}(\ell-1)!}{\gamma^{n_r(n_r-1)/2}\pi^{n_r n_b}} \frac{\mathrm{e}^{-||\mathbf{Y}||^2}}{\mathcal{V}(\mathbf{Y}^\dagger\mathbf{Y})}\,\cdot \\ &\cdot \frac{(1+\kappa)^{n_r(n_t-n_r+1)}\mathrm{e}^{-n_r n_t\kappa}}{n_r!\mathcal{V}(\bar{\mathbf{H}}\bar{\mathbf{H}}^\dagger)} \\ &\cdot \frac{\prod_{\ell=1}^{n_r-1}(n_r-\ell)^\ell(n_t-\ell)^\ell}{\prod_{\ell=1}^{n_r}(n_r-\ell)!(n_t-\ell)!}|\mathbf{Z}|\,, \end{aligned} \quad (6)$$

where

$$\begin{aligned} (\mathbf{Z})_{i,j} = &\int_0^{+\infty} \mathrm{e}^{y_i^2\frac{\sigma\gamma}{1+\sigma\gamma}-(1+\kappa)\sigma} \\ &\cdot \frac{\sigma^{n_t-n_r}{}_0F_1(n_t-n_r+1,(1+\kappa)^2\mu_j\sigma)}{(1+\gamma\sigma)^{n_b-n_r+1}}\,\mathrm{d}\sigma\,, \end{aligned}$$

and ${}_0F_1(\cdot,\cdot)$ is the Bessel hypergeometric function [8]. Notice that, as already stressed in [9], the LOS matrix $\bar{\mathbf{H}}\bar{\mathbf{H}}^\dagger$ is assumed to be full-rank in this derivation. If this assumption is broken, then

[3] Herein, $\mathbf{U}$ and $\mathbf{V}$ are unitary matrices of size n_r, and, respectively, n_t, and $\mathbf{\Sigma}^{1/2}$ has non-negative entries. In the following, by $\mathbf{\Sigma}$ we'll mean the *square* matrix of size $s = \min\{n_r, n_t\}$ of the non-zero squared singular values of $\mathbf{H}$. Due to the further assumptions of next Section, $s = n_r$ necessarily.

one needs to resort to the limiting procedure in [9, Corollary I, and references therein].

As for the transition probability of the channel, in this case $\mathbf{Y}|\mathbf{X} \sim \mathcal{CN}\left(\sqrt{\frac{\gamma\kappa}{\kappa+1}}\,\bar{\mathbf{H}}\mathbf{X}, \mathbf{I} + \frac{\gamma}{\kappa+1}\mathbf{X}\mathbf{X}^\dagger\right)$. Gathering together this pdf and (6), (2) can be conveniently evaluated by taking the logarithm.

3.2 Interference-limited

In this case the AWGN is neglected, and the actual impairment is generated by the co-channel interferers, each seen from the direct link receiver under its own random channel, which we assume to be affected by Rayleigh fading with block-length n_b, the same as for the useful signal. Assuming to have L active interferers in the network, following [10, Sec. IV], it turns out that channel matrix Gramian can be modeled as an $F_{n_r}(n_t, Ln_t)^4$-distributed matrix variate, whose unordered joint eigenvalues law can be written as [3, Formula (98)]

$$p(\mathbf{\Sigma}) = \frac{|\mathbf{\Omega}|^{-n_t}}{\mathcal{V}(-\mathbf{\Omega}^{-1})} \prod_{\ell=1}^{n_r} \frac{((L+1)n-\ell)!}{(Ln_t-\ell)!(n_t-\ell)!(n_r-\ell)!} \cdot \prod_{\ell=1}^{n_r-1} \frac{\ell^{n_r-\ell}}{(n_t(L+1)+\ell)^{n_r-\ell}} |\mathbf{\Sigma}|^{n_t-n_r}\mathcal{V}(\mathbf{\Sigma})|\widetilde{\mathbf{F}}|\,, \tag{7}$$

with $\mathbf{\Omega}$ the compound receive spatial correlation matrix[5]. By replacing (7) into (3), it turns out that the output is distributed according to

$$p(\mathbf{Y}) = T\frac{|\mathbf{\Omega}|^{-n_t}}{\mathcal{V}(-\mathbf{\Omega}^{-1})} \prod_{\ell=1}^{n_r} \frac{((L+1)n_t-\ell)!}{(Ln_t-\ell)!(n_t-\ell)!(n_r-\ell)!} \cdot \prod_{\ell=1}^{n_r-1} \frac{\ell^{n_r-\ell}}{(n_t(L+1)+\ell)^{n_r-\ell}} |\mathbf{Z}|\,,$$

where

$$(\mathbf{Z})_{i,j} = \int_0^{+\infty} \exp\left(y_i^2 \frac{\sigma\gamma}{1+\sigma\gamma}\right) \cdot \frac{\sigma^{n_t-n_r}}{(1+\gamma\sigma)^{n_b-n_r+1}(1+\sigma/\omega_j)^{n_t+n_b-n_r+1}}\,\mathrm{d}\sigma.$$

If $\mathbf{\Omega} = \mathbf{I}$, then $p(\mathbf{\Sigma})$ strongly simplifies, namely

$$p(\mathbf{\Sigma}) = \prod_{\ell=1}^{n_r} \frac{((L+1)n_t-\ell)!}{(Ln_t-\ell)!(n_t-\ell)!(n_r-\ell)!} \cdot \frac{|\mathbf{\Sigma}|^{n_t-n_r}}{|\mathbf{I}+\mathbf{\Sigma}|^{(L+1)n_t}} \mathcal{V}^2(\mathbf{\Sigma})\,,$$

so that in this case

$$p(\mathbf{Y}) = T\prod_{\ell=1}^{n_r} \frac{((L+1)n_t-\ell)!}{(Ln_t-\ell)!(n_t-\ell)!(n_r-\ell)!}|\mathbf{Z}|\,,$$

where

$$(\mathbf{Z})_{i,j} = \int_0^{+\infty} \mathrm{e}^{y_i^2 \frac{\sigma\gamma}{1+\sigma\gamma}} \frac{\sigma^{n_t-n_r+j-1}}{(1+\gamma\sigma)^{n_b-n_r+1}}\,\mathrm{d}\sigma\,.$$

Notice that, as we are neglecting the thermal noise, the law of $\mathbf{Y}|\mathbf{X}$ can be characterized through its characteristic function, exploiting tools in [11, and references therein].

3.3 Land Mobile Satellite communication

The Land Mobile Satellite (LMS) MIMO channel can be viewed as a non-central channel with random mean, so that the channel matrix model is akin to that in (4), but with random $\bar{\mathbf{H}}$. Then the Gramian $\bar{\mathbf{H}}^\dagger\bar{\mathbf{H}}$ follows a matrix-variate $\Gamma(\alpha, \mathbf{\Phi})$ distribution [13, 12]. Here, α plays the role of a shape parameter (indeed, for integer values α coincides with the number of degrees of freedom of the Γ-distributed matrix-variate), while $\mathbf{\Phi}$ is a scale parameter. From a physical point of view, they both refer to the average power of the random LOS component, as shown in detail in [12]. For this scenario, the channel eigenvalues law reads as

$$p(\mathbf{\Sigma}) = \frac{\prod_{\ell=1}^{n_r-1}\left[\frac{(\alpha-n_r+\ell)}{\ell(n_t-n_r+\ell)}\right]^{\ell-n_r}}{\mathcal{V}\left((\mathbf{I}+\mathbf{\Phi})^{-1}\right)}|\mathbf{F}|\mathcal{V}(\mathbf{\Sigma}) \cdot \mathrm{e}^{-\mathsf{Tr}(\mathbf{\Sigma})}|\mathbf{\Sigma}|^{n_t-n_r}\prod_{\ell=1}^{n_r}\frac{\phi_\ell^\alpha(1+\phi_\ell)^{-\alpha}}{(n_t-\ell)!(n_r-\ell)!}$$

with $(\mathbf{F})_{i,j} = {}_1F_1\left(\alpha-n_r+1; n_t-n_r+1; \frac{\sigma_j}{1+\phi_i}\right)$, and ϕ_i the i-th ordered eigenvalue of $\mathbf{\Phi}$. Recall here that ${}_1F_1(a;b;x)$ is the confluent hypergeometric function of scalar argument [8, Ch. 13].

Following the lines of the previous Subsections, it turns out that

$$p(\mathbf{Y}) = T\frac{\prod_{\ell=1}^{n_r-1}\left[\frac{\alpha-n_r+\ell}{\ell(n_t-n_r+\ell)}\right]^{\ell-n_r}}{\mathcal{V}\left((\mathbf{I}+\mathbf{\Phi})^{-1}\right)} \cdot |\mathbf{Z}|\prod_{\ell=1}^{n_r}\frac{\phi_\ell^\alpha(1+\phi_\ell)^{-\alpha}}{(n_t-\ell)!(n_r-\ell)!}\,,$$

[4] The parameters here exploited for the definition of the F matrix descend from our assumption $n_r < n_t$, and refer to a scenario where the number of transmit antennas n_t is the same at each transmitter.

[5] comprehensive of both the correlation among the receive antennas as well as of the correlation arising from interferer's transmission, for a detailed explanation on how this correlation structure arises the interested reader is referred to [10, and refs. teherein].

where

$$(\mathbf{Z})_{i,j} = \int_0^{+\infty} \mathrm{e}^{y_i^2 \frac{\sigma\gamma}{1+\sigma\gamma} - \sigma} \frac{\sigma^{n_t - n_r}}{(1+\gamma\sigma)^{n_b - n_r + 1}} \cdot {}_1F_1\left(\alpha - n_r + 1; n_t - n_r + 1; \frac{\sigma}{1+\phi_j}\right) \mathrm{d}\sigma.$$

Also in this case, when[6] $\mathbf{\Phi} = \mathbf{I}$ the channel eigenvalues pdf simplifies, and reduces indeed to

$$p(\mathbf{\Sigma}) = \frac{\mathrm{e}^{-\mathsf{Tr}(\mathbf{\Sigma})}|\mathbf{\Sigma}|^{n_t - n_r}}{\Gamma_{n_r}(n_t)\Gamma_{n_r}(n_r)} |\widetilde{\mathbf{F}}| \mathcal{V}(\mathbf{\Sigma}),$$

with $(\widetilde{\mathbf{F}})_{i,j} = \sigma_i^{n_r - j} {}_1F_1(\alpha - j + 1; n_t - j + 1; \sigma_i)$, so that the final result is

$$p(\mathbf{Y}) = T \frac{|\widetilde{\mathbf{Z}}|}{\Gamma_{n_r}(n_t)\Gamma_{n_r}(n_r)},$$

where

$$(\mathbf{Z})_{i,j} = \int_0^{+\infty} \mathrm{e}^{y_i^2 \frac{\sigma\gamma}{1+\sigma\gamma} - \sigma} \frac{\sigma^{n_t - n_r}}{(1+\gamma\sigma)^{n_b - n_r + 1}} \cdot {}_1F_1(\alpha - j + 1; n_t - j + 1; \sigma)\, \mathrm{d}\sigma.$$

In the LMS MIMO case, the conditional law of $\mathbf{Y}|\mathbf{X}$ can be evaluated at high-SNR resorting to the Cholesky decomposition of the channel matrix Gramian, instance of which is reported in [12, Sec. II].

4 Conclusion

The output pdf and the conditional law of the output given the i.i.d. Gaussian input has been evaluated for block-fading channels. The results are exploited to characterize the information density of the analyzed channel models, which is needed in the information-theoretic characterization of arbitrary non-ergodic channels. Extensions of the results to other input/channel structures are currently under investigation.

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[6] This case is of particular interest because under this assumption the channel turns out to be *unitarily invariant*; for the definition and the practical impact of this statistical feature on the signalling strategy see e.g. [14, and refs. therein].

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