Evaluation of the Dependence on System Parameters of Non-Linear Interference Accumulation in Multi-Span Links

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Abstract *Non-linear effects in uncompensated links generate a noise-like additive interference: we study in detail the accumulation law vs. number of spans as a function of main system parameters (fiber types and WDM total bandwidth). In all practical cases, we analytically found a slightly super-linear accumulation confirmed by numerical simulations.*

Introduction

Non-linear propagation in uncompensated links has been extensively studied and substantial evidence shows that the disturbance can be approximately modeled as additive Gaussian noise[1,2]. Under this assumption, system performance depends on a modified OSNR which includes both ASE (P_{ASE}) and non-linear interference (P_{NLI}) contributions:

$$OSNR_{NL}=P_{TX}/(P_{ASE}+P_{NLI}).$$

Several analytical models can be found in the literature: based on different approximations and approaches, they all provide expressions for P_{NLI} as a function of system parameters. A complete literature review of available models is summarized in[3]. Results presented in this work stems from the model presented in[3].

While the dependence of P_{NLI} on fiber and system parameters is evident from the model, the accumulation law along the transmission link is not straightforward. In fact, after developing the model for propagation over a single-span, two different assumptions could be made for applying it to multi-span systems: incoherent or coherent accumulation of P_{NLI} generated in each span. A first analysis[3] carried out through system simulations did not reach conclusive results, also because the predicted differences between the two assumptions, in terms of resulting system maximum reach, were relatively small. More detailed analyses targeted at specifically evaluating P_{NLI} accumulation showed experimental evidence[2,4] of super-linear growth with the number of spans (N_{span}), which is indicative of coherent accumulation. However, the different experiments[2,4] provide somewhat contrasting estimates of P_{NLI} accumulation and only some of such estimates seem to be compatible with the coherent accumulation analytical model[3].

In order to deepen the insight on this topic, in this work we present a comprehensive analytical study of the dependence of P_{NLI} accumulation on system parameters. Then, we validate model predictions in practical cases by comparison with accurate simulation results directly aiming at P_{NLI} evaluation.

Theory and accumulation exponent ρ

To study non-linear interference accumulation along the link, we first evaluate its power spectral density $G_{NLI}(f)$ using Eq. (18) in[3]. Then assuming it to be approximately constant over a channel bandwidth, we get the non-linear noise power as:

$$P_{NLI}=G_{NLI}(0)\cdot B_N$$

where B_N is the OSNR reference bandwidth. In this work we assume B_N=0.1 nm, for both P_{NLI} and P_{ASE}.

Analyzing the P_{NLI} accumulation in different system conditions, it appears that P_{NLI} dependence on N_{span} can be fitted with high accuracy by the following expression:

$$P_{NLI}=P_{NLI}^{(1)}\cdot N_{span}^{\rho}$$

where $P_{NLI}^{(1)}$ is the amount of non-linear interference generated in the first span.

Normalizing P_{NLI} with respect to $P_{NLI}^{(1)}$, we can estimate the accumulation rate using a single parameter, ρ, referred in the following as the “accumulation exponent”. Throughout the paper we compare systems under different conditions in terms of this parameter. Analyzing the expression of $G_{NLI}(f)$, it can be clearly seen that non-linear interference coherent accumulation is driven by the so called “phased-array factor”, typical of FWM in multi-span systems[5]: it is a function of chromatic dispersion and span length (L_{span}). These parameters, together with the spectrum of the launched WDM comb, have an impact on non-linear interference components propagation altering the coherent interaction among G_{NLI} generated in different spans.

Analyzing the “phased-array factor”, lower and upper bounds for ρ can be derived. The minimum achievable value is equal to 1, corresponding to incoherent accumulation: it happens when coherency is completely canceled. On the other hand, a maximum value equal to 2 can be achieved if propagation is fully

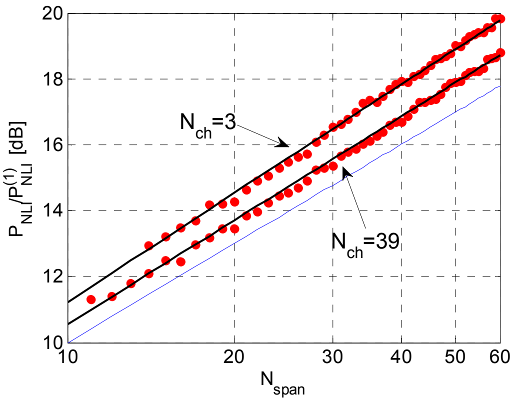

Fig. 1: P_{NLI} accumulation as a function of N_{span} for a system with Δf=33.6 GHz (SMF fiber; L_{span}=80 km). Black solid lines: model. Red dots: simulations. Blue solid line: incoherent accumulation. N_{ch}: number of channels in the WDM comb.

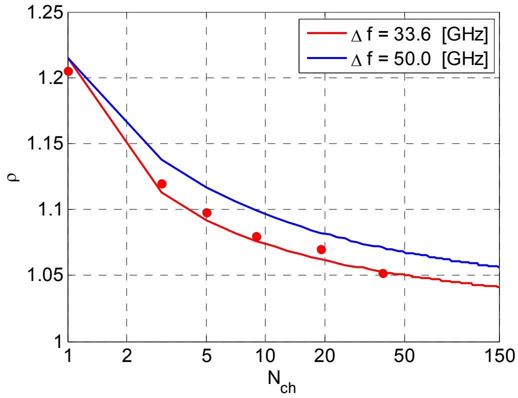

Fig. 2: Solid lines: calculated accumulation exponent ρ as a function of N_{ch} from model, for systems with Δf=33.6 GHz (red) and Δf=50 GHz (blue). Link made up of 60 spans of SMF fiber with L_{span}=80 km. Dots: simulation results for the case Δf=33.6 GHz.

coherent, i.e. when accumulated phase-shifts are negligible and do not break coherency.
In general, the larger is the accumulated phase-shift, the lower is the amount of coherency, i.e. the lower is the exponent ρ. This happens increasing chromatic dispersion, enlarging span length and widening the overall WDM spectrum.

System and simulation setup

We consider a Nyquist-WDM system based on PM-16QAM modulation with coherent detection. At the transmitter digital pre-filtering is applied to obtain a square-root-raised-cosine spectrum with roll-off equal to 0.02 for the four quadratures. Then, four ideal DACs generates electrical signals driving two nested Mach-Zehnder modulators operated in the linear trans-characteristic range. Symbol rate (R_S) is set to 32 Gbaud. The channel spacings (Δf) is 33.6 GHz and the launch power (P_{TX}) is equal to -1 dBm per channel.
The uncompensated link is composed of 60 spans of SMF fiber (D=16.7 ps/nm/km; α=0.22 dB/km; γ=1.3 1/W/km), with lumped EDFA amplification exactly recovering the fiber loss. Amplifier noise figure is F=5 dB. Optical filtering is not applied, neither at Tx nor at Rx side. Channel selection is performed at Rx by properly tuning the local oscillator (laser phase noise is neglected). The overall electrical Rx bandwidth is $B_{elt}=0.5\cdot R_S$. Then, electronic dispersion compensation, polarization de-multiplexing, carrier and phase recovery are applied in the digital domain. BER evaluation is performed on the WDM comb center channel through direct error counting over 2^{16} symbols (2^{20} bits). In order to exactly match model assumptions[3] (Gaussian-distributed signal at the system input and noise added at the Rx), we introduced dispersion pre-compensation (D_{pre}=16700 ps/nm) and performed ASE noise loading before entering the receiver. However, at 32 Gbaud the signal becomes Gaussian already after 2 spans of SMF. Therefore over a large number of spans D_{pre} has little or no impact. Simulations are all based on full band split-step method and carried out using OptSim™.
In order to estimate P_{NLI} from BER, we also accurately evaluated by simulation the back-to-back BER vs. OSNR curve, determining the exact relationship:

$$\mathrm{BER}=\Phi(\mathrm{OSNR})$$

for the Tx-Rx pair we are using. Since non-linear propagation adds a disturbance modeled as an additive Gaussian noise, the BER performance at the Rx after non-linear propagation is BER=$\Phi(\mathrm{OSNR}_{NL})$, taking into account P_{NLI} in OSNR_{NL}. Inverting the BER vs. OSNR function, we can estimate the non-linear interference as:

$$P_{NLI}=(P_{TX}/\Phi^{-1}(\mathrm{BER}))-P_{ASE}$$

with $P_{ASE}=hf_0F(G-1)N_{span}B_N$, where h is Planck's constant, f_0 is the channel center frequency, and G is the EDFA gain. We measured BER, and consequently estimated P_{NLI}, after each span in order to evaluate ρ and have the full picture of the non-linear interference accumulation along the link. Note that, according to the model, the value of ρ is independent of the value of P_{TX} used in simulations.
In Fig.1, P_{NLI} accumulation is shown for a number of WDM channels equal to N_{ch}=3 and N_{ch}=39 (Δf=33.6 GHz) when L_{span}=80 km. Red dots are values derived from BER obtained in simulations. Black solid line is derived using the model with coherent accumulation: a very good agreement is observed. The blue solid line is obtained assuming incoherent accumulation (ρ=1), testifying the super-linear P_{NLI} growth (ρ=1.12 and 1.05 for N_{ch}=3 and 39 respectively).
To highlight the dependence of ρ on total WDM bandwidth, we swept the number of channels

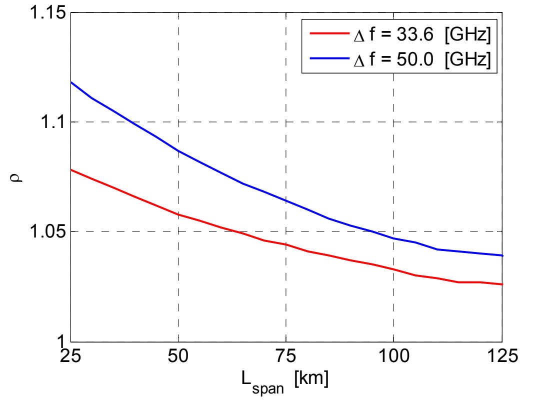

Fig. 3: Accumulation exponent ρ as a function of L_{span} for systems with Δf=33.6 GHz (red) and Δf=50 GHz (blue). SMF fiber, 60 spans. C-band fully loaded.

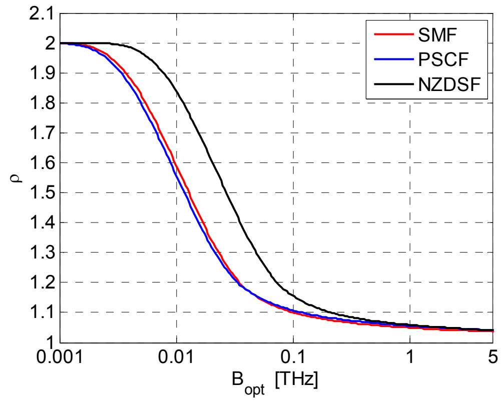

Fig. 4: Accumulation exponent ρ as a function of B_{opt} for a system at Nyquist limit (Δf=R_S, roll-off=0.0). Link made of 60 spans with L_{span}=80 km.

from 1 to 150 (full C-band at Δf=33.6 GHz): results are shown in Fig. 2. Simulations have been carried out in selected points up to a maximum of 39 channels (limited by CPU time requirements). Solid lines refer to the model and dots to simulations: a good agreement can be observed. As expected, an increase in the total WDM bandwidth, obtained enlarging N_{ch}, generates a reduction in ρ values due to larger phase-shifts accumulated along the link by non-linear interference components. The analytical results for system with Δf=50 GHz show a slightly higher ρ: this can be ascribed to intra-channel non-linear contribution, which are more "coherent", being generated on a smaller band, and whose weight on the overall disturbance is larger in Δf=50 GHz systems.

In Fig. 3 we report the dependence of ρ on span length, evaluated using the analytical model: for both Δf we have considered the proper value of N_{ch} that fully load the C-band. Even though we swept L_{span} over a wide range, the variation of ρ is limited to less than 5%. Results confirm that enlarging L_{span} the phase-shift is increased, thus coherency is reduced. Finally, we point out that according to the model these results are independent of the modulation format.

Model results at the Nyquist limit

To further increase the comprehension of the phenomenon and better understand the mechanism behind non-linear interference accumulation, we studied a system at the Nyquist limit. In this case channel spacing is exactly equal to the symbol rate and we assume an ideal rectangular shaped spectrum. Using the analytical model[3], we can study P_{NLI} accumulation as a function of the overall optical system bandwidth (B_{opt}) only. Under this conditions, the P_{NLI} depends on total occupied bandwidth $B_{opt}=N_{ch}\cdot R_S$, independently of N_{ch} and R_S. Besides the SMF fiber studied in previous section, we consider two further fiber types: PSCF (D=20.1 ps/nm/km; α=0.18 dB/km; γ=0.9 1/W/km) and NZDSF (D=3.8 ps/nm/km; α=0.22 dB/km; γ=1.5 1/W/km). Results are shown in Fig. 4 with B_{opt} spanning from 1 GHz to 5 THz (full C-band). It can be observed that, for small values of B_{opt}, ρ reaches its upper bound value equal to 2. For all fibers, when B_{opt} is very small, phase-shifts become negligible and coherency is maintained along the whole link. Such a situation happens for unrealistic scenarios, for example B_{opt}=1 GHz that can be obtained with N_{ch}=1 and R_S= 1 Gbaud. In all practical conditions (B_{opt}>100 GHz), for all three fibers, ρ is always smaller than 1.2. When the full C-band is loaded, again all fibers behave in the same way, showing a very small accumulation exponent around 1.04.

In Fig. 4 the impact of chromatic dispersion can also be clearly observed: the NZDSF fiber, due to the lower dispersion value, preserves an higher level of coherency over larger B_{opt}, corresponding to a higher ρ.

Conclusions

Our analysis showed that, in all practical conditions, the accumulation exponent of non-linear interference over multi-span links is only slightly higher than 1, meaning a mild super-linear growth. Theoretical results have been successfully validated through simulations of a Nyquist-WDM PM-16QAM system.

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