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A NEW FORMULATION FOR THE TURBULENT ENERGY SPECTRUM IN THE UNIVERSAL EQUILIBRIUM RANGE

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A NEW FORMULATION FOR THE TURBULENT ENERGY SPECTRUM IN THE UNIVERSAL EQUILIBRIUM RANGE

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ABSTRACT

The three-dimensional turbulent energy spectrum function is derived for nondecaying homogeneous and isotropic turbulence of an incompressible fluid in the universal equilibrium range. The Ellison's hypothesis is used in order to obtain a formal mathematical solution of the energy spectrum dynamic equation, which has been compared with the energy spectra proposed in literature.

INTRODUCTION

In the study of physical and chemical processes in turbulent media, the analysis of the fluid velocity field is fundamental. In many applications the turbulent effects on the physical and chemical processes have to be taken into account and therefore a description of the turbulent motion is required.

Among the different approaches to the problem of the description of the turbulent velocity fluctuations, the spectral analysis is very powerful, because it shows the influence of the various scales on the turbulent phenomena explicitly. It is well known that, in order to obtain the energy spectrum function $E(k,t)$ in closed form, both mathematical difficulties and closure problems have to be overcome.

The aim of the present work is to obtain a simplified formulation for the turbulent energy spectrum valid in the universal equilibrium range; our final goal is the investigation of the interaction between turbulence and chemical processes, in special way the influence of turbulent phenomena on yield and selectivity of very fast reactions (micromixing), and on the characteristics of a solid product (precipitation and agglomeration).

Several closure hypotheses for the energy transfer mechanism have been proposed in the last decades, but the validity of the obtained correlations for $E(k,t)$ is restricted to limited wavenumber subranges.

A simplified solution has been obtained from Kolmogoroff postulates by neglecting the energy dissipation: this assumption is valid only in the inertial subrange, where the energy spectrum function is given by:

$$E(k,t) = A\varepsilon^{2/3}k^{-5/3} \tag{1}$$

The hypotheses formulated for the energy transfer in the universal equilibrium range (originally proposed by Obukhoff, Kovasznay, Heisenberg and later modified) can describe this behaviour at low k values. On the other hand, it has been demonstrated that some of them (Obukhoff's, Kovasznay's and Heisenberg's hypotheses) do not correctly describe the energy transfer at high k values, where they predict an algebraic fall-off of energy at increasing wavenumbers. Some other transfer models (by Pao and Corrsin, Dugstad, Saffman) lead to an exponential decrease of $E(k,t)$ at high k values, but it must be remarked that Pao and Corrsin's assumption is still not valid for very high wavenumbers while Saffman obtained a solution valid only at very high k values. Panchev and Kesich showed that the hypotheses by Pao and Corrsin and by Saffman are a simplified form of the hypothesis of Ellison (1961), and proposed an interpolation formula for the universal equilibrium range using the results of Pao and Corrsin (at low wavenumbers) and Saffman (at high wavenumbers).

An empirical equation has been proposed by Sato *et al.* (1984) for the triple velocity-correlation function, equivalent to the energy transfer function in wavenumber space: comparing calculations by the previous hypotheses and the empirical equation proposed, it was noted that the differences were very small at small elapsed time, but became remarkable at longer time.

It has to be pointed out that, up to now, no unique formulation has been found. In particular, there is not even a generally accepted expression for the spectrum in the universal equilibrium range, the one in which the turbulence is no longer affected by the external conditions. Recently, Qian (1984) applied the methods of non-equilibrium statistical mechanics combined with a perturbation-variation approach to solve closure problems of turbulence theory ant to calculate the velocity spectrum, but no definitive results have been found up today.

The validity of the "global scaling invariance" itself, postulated by Kolmogoroff (1941) theory is questionable in the high frequency dissipation range. Kolmogoroff (1962) himself had proposed a modification of his theory, assuming that spatial fluctuations of the energy dissipation are intermittent; this means that the high wavenumber turbulent activity comes in bursts separated by quiescent periods.

The intermittency phenomena have been discussed deeply by Frisch (1991): the multifractal model of turbulence recently proposed can model intermittency but implies the absence of a universal energy spectrum in the Kolmogoroff sense assuming only "local scaling invariance". But it must be evidenced that recently She and Jackson (1993) reexamined available experimental data, concluding that "current experimental spectral data support the Kolmogoroff's universal theory, as well as the multifractal-type universality.

The energy transfer hypothesis of Ellison will be adopted in this work. The theoretical discussion about the physical basis of this closure overcomes the aims of the present work, which is the derivation of a formulation applicable in a wider range of the wavenumber space. In what follows, isotropic homogeneous nondecaying turbulence and incompressible fluid will be considered.

THE DYNAMIC EQUATION FOR THE ENERGY SPECTRUM AND THE ELLISON'S CLOSURE HYPOTHESIS

The behaviour of the energy spectrum as a function of time and wavenumber co-ordinates is described by the dynamic equation:

$$\frac{\partial E(k,t)}{\partial t} = F(k,t) - D(k,t) \tag{2}$$

where $F(k,t)$ is the energy-transfer-spectrum function and $D(k,t) = 2\nu k^2 E(k,t)$ represents the dissipation spectrum function. $F(k,t)$ is the Fourier transform of the third-order correlation tensor $\mathbf{S}(r,t)$ and represents the convective term originated by the interactions among eddies of different size.

Equation (2) describes the behaviour of the energy spectrum in the whole wavenumber space, and therefore it is surely valid in all the universal equilibrium range. This range has been identified by the Kolmogoroff first hypothesis, stating that at sufficiently high Reynolds numbers there must be a range of high wavenumbers in which turbulence is statistically in equilibrium, determined by the parameters ε and ν only. It is defined "universal" because independent of external conditions; the inertial subrange is included in the universal equilibrium range as its lower edge. A characteristic wavenumber k_d relative to the range where the viscous effects becomes predominant, can be introduced; it is usually defined as the reciprocal of the Kolmogoroff microscale λ_K:

$$k_d = \left(\frac{\varepsilon}{\nu^3}\right)^{1/4} \tag{3}$$

Equation (2) cannot be solved, because the transfer function is unknown: a closure equation for $F(k,t)$ is needed.

A classical approach to the problem consists in the assumption of some relationship between the energy transfer spectrum $F(k,t)$, the energy spectrum $E(k,t)$ and the wavenumber k.

Various assumptions have been suggested in literature and have been critically reviewed by Pao (1965), Hinze (1975) and Sato *et al.* (1984); very promising is the use of the Ellison's assumption, which is a modification of the Obukhoff's hypothesis and, as it has been shown by Panchev and Kesich (1969), can lead to the solutions obtained by Saffman (1963), Corrsin (1964) and Pao (1965) in specified subranges.

Let us consider the dynamic equation (2) in its integrated form:

$$\frac{\partial}{\partial t}\int_0^k E(k',t)\mathrm{d}k' = \int_0^k F(k',t)\mathrm{d}k' - 2\nu\int_0^k k'^2 E(k',t)\mathrm{d}k' \tag{4}$$

and introduce the integral transfer function:

$$G(k,t) = \int_0^k F(k',t)\mathrm{d}k' . \tag{5}$$

$G(k,t)$ represents the interaction of eddies associated to different wavenumbers transferring energy to or from the eddies in the region 0-k by inertial effects.

Ellison (1961) proposed for $G(k,t)$ the expression:

$$G(k,t) = -\alpha k E(k,t)\left[2\int_0^k k'^2 E(k',t)\mathrm{d}k'\right]^{1/2} \tag{6}$$

in which α is a proportionality constant.

It is possible to give a physical interpretation of the members on the right-hand side of equation (6): Obukhoff considered the energy transfer in the wavenumber space analogous to the energy transfer from the main motion to the turbulent velocity fluctuations and posed the integral transfer function as proportional to the Reynolds stress. The Reynolds stress was assumed proportional to the product of the kinetic energy of the eddies associated to wavenumbers in the range k-∞ and the mean rate of strain of the eddies associated to wavenumbers in the range 0-k. Ellison wrote the kinetic energy in the form $kE(k,t)$, implying in this way that only the part of Reynolds stress associated with the wavenumber k interacts with the mean rate of shear of the eddies of wavenumber smaller than k (Hinze, 1975).

By substituting equations (5) and (6) in equation (4), in the universal equilibrium range it is possible to write the dynamic equation for $E(k,t)$ in the form (Hinze, 1975):

$$kE(k,t)\left[2\int_0^k k'^2E(k',t)\mathrm{d}k'\right]^{1/2} = \tfrac{2\nu}{\alpha}\int_k^\infty k'^2E(k',t)\mathrm{d}k' \tag{7}$$

As said above, solutions of equation (7) have been found by Panchev and Kesich (1969) in the simplified cases of very low and very high k values: they proposed an interpolated solution valid in the whole universal equilibrium range, but no complete and formally derived solution has been found up today.

THE PROPOSED ENERGY SPECTRUM FUNCTION

By introducing the quantity:

$$\Theta = \Theta(k) = \left[\tfrac{2\nu}{\varepsilon}\int_0^k k'^2E(k')\mathrm{d}k'\right]^{1/2} \tag{8}$$

we can rewrite the members of equation (7), obtaining for non-decaying turbulence

$$E(k) = \frac{\varepsilon\Theta}{\nu k^2}\frac{\mathrm{d}\Theta}{\mathrm{d}k} \tag{9}$$

and

$$\tfrac{2\nu}{\alpha}\int_k^\infty k'^2E(k')\mathrm{d}k' = \tfrac{\varepsilon}{\alpha}\left(1-\Theta^2\right) \tag{10}$$

Substituting equations (9) and (10) in equation (7) and rearranging, the energy spectrum dynamic equation may thus be written in the form:

$$\frac{\mathrm{d}\Theta}{\mathrm{d}k} = \frac{\left(1-\Theta^2\right)}{\alpha\Theta^2}\frac{k}{k_d^2} \tag{11}$$

which, for the boundary condition $\Theta = 0$ at $k = 0$, admits the implicit formal solution:

$$\frac{1+\Theta}{1-\Theta} = \exp\left(\frac{k^2}{\alpha k_d^2} + 2\Theta\right) \tag{12}$$

Analysing the two members of equation (12), useful informations about the behaviour of Θ may be obtained.

At high values of k/k_d, $\Theta = 1$ and, in the right-hand side of equation (12), the term

2Θ becomes negligible. At very low values of k/k_d, Θ goes to zero more slowly than $(k/k_d)^2$ and the first term in the argument of the exponential function becomes negligible. It is thus possible to substitute for Θ in the right-hand side of equation (12) the solution of equation (11) obtained at low wavenumbers.

In this region, the energy spectrum is described by the Kolmogoroff spectrum law (1): substitution in equation (6) gives the modified integral transfer function $G'(k)$ valid for low wavenumber values:

$$G'(k) = \alpha\left(\tfrac{3A}{2}\right)^{1/2} \varepsilon^{1/3} k^{5/3} E(k) \tag{13}$$

which is the well-known integral transfer spectrum proposed by Corrsin and Pao.

By substituting in equation (4), the expression for Θ valid in the lower wavenumber range can be obtained:

$$\Theta^* = \left[1 - \exp\left(-\frac{1}{(\alpha A)^2}\frac{k^{4/3}}{k_d^{4/3}}\right)\right]^{1/2} \tag{14}$$

in which we introduce the notation Θ^* in order to outline that this is only a particular integral of equation (11).

The right-hand side of equation (12) thus becomes:

$$\exp\left(\frac{k^2}{\alpha k_d^2} + 2\left[1 - \exp\left(-\frac{1}{(\alpha A)^2}\frac{k^{4/3}}{k_d^{4/3}}\right)\right]^{1/2}\right) = \exp m \tag{15}$$

if, for sake of simplicity, we indicate as m the argument of the exponential term; from equation (12):

$$\Theta = \frac{\exp m - 1}{\exp m + 1}$$

and finally from equation (9):

$$E(k) = 4(\varepsilon\nu^5)^{1/4}\left(\frac{1}{\alpha}\left(\frac{k}{k_d}\right)^{-1} + \frac{2}{3(A\alpha)^2}\left(\frac{k}{k_d}\right)^{-5/3}\frac{\exp\left[-\frac{1}{(\alpha A)^2}\left(\frac{k}{k_d}\right)^{4/3}\right]}{\left\{1 - \exp\left[-\frac{1}{(\alpha A)^2}\left(\frac{k}{k_d}\right)^{4/3}\right]\right\}^{1/2}}\right)\frac{\exp m\,(\exp m - 1)}{(\exp m + 1)^3} \tag{16}$$

Equation (16) gives the three-dimensional turbulent energy spectrum function in the universal equilibrium range.

Value of the constants

The value of the constants α and A must be either theoretically calculated or experimentally derived: a great importance has been given in literature to this problem (Hinze, 1975). In particular, Kraichnan's Lagrangian-History-Direct-Interaction predicts for the Kolmogoroff constant A an upper limit of 1.77. An experimental investigation, carried out by Pao (1965), gave the value A = 1.70 which is in good accordance with the theoretical

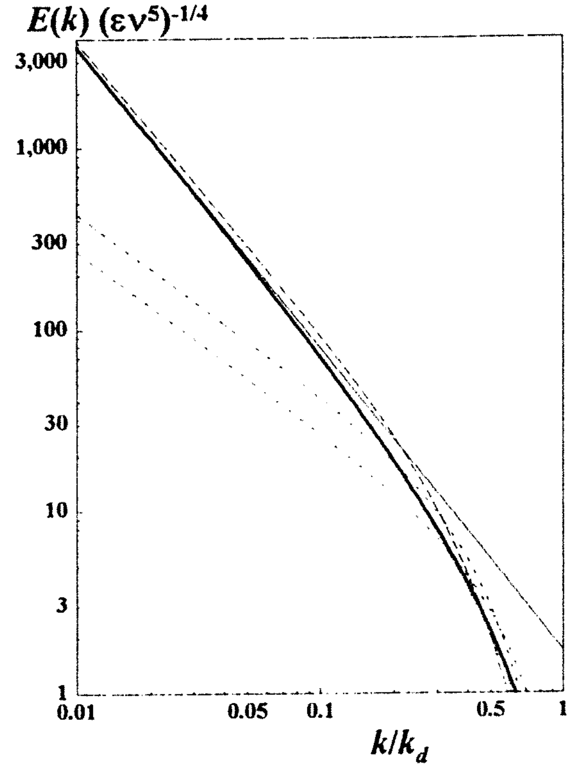

Figure 1. Comparison among the proposed spectrum and the simplified solutions derived from Ellison's hypothesis in the lower wavenumber range: (——) this work; (——) Kolmogoroff (eq. 1); (········) Pao and Corrsin (eq. 19); (– ·· – ·· –) Saffman calculated with $\alpha = 0.23$ and (- - - - -) with $\alpha = 0.37$ (eq. 20); (– – –) Panchev and Kesich, interpolated (eq. 24).

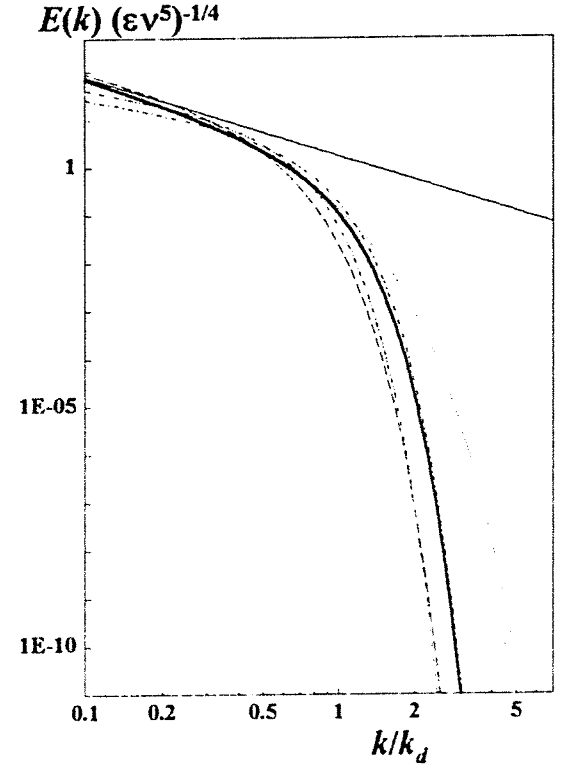

Figure 2. Comparison among the proposed spectrum and the simplified solutions derived from Ellison's hypothesis in the higher wavenumber range: (——) this work; (——) Kolmogoroff (eq. 1); (········) Pao and Corrsin (eq. 19); (– ·· – ·· –) Saffman calculated with $\alpha = 0.23$ and (- - - - -) with $\alpha = 0.37$ (eq. 20); (– – –) Panchev and Kesich, interpolated (eq. 24).

predictions of Kraichnan. It is also necessary to explicate the relationship between the Kolmogoroff constant and the proportionality constant α. Particular attention has been reserved to the α-value by the Heisenberg's theory (which will be shortly discussed later): it has been shown that, according to the previous theory, it is possible to obtain the relationship between α and A as:

$$A = \left(\frac{8}{9\alpha}\right)^{2/3} \tag{17}$$

In this case the theoretical lower value obtained by Kraichnan is $\alpha = 0.38$, while the experimental analysis of Pao gives $\alpha = 0.40$. As regards Ellison's assumption, the simplified solution in the case of no energy dissipation brings to the Kolmogoroff spectrum law with the relationship:

$$A = \left(\frac{2}{3\alpha^2}\right)^{1/3} \tag{18}$$

which, using Pao's data, gives for the proportionality constant the value $\alpha = 0.37$.

Saffman (1963) gives for the value of his constant (that in our model is equivalent to α^{-1}), a range 4.4 ÷ 2.7, corresponding to values for α varying from 0.225 to 0.37.

In the following the value of the Kolmogoroff constant obtained by Pao (A=1.7) and the relative value of α as given by equation (18) ($\alpha = 0.37$) will be used.

COMPARISON WITH THE ENERGY SPECTRA PROPOSED BY PREVIOUS AUTHORS

As a first step, it is interesting to compare the behaviour of the proposed spectrum with the simplified solutions derived from Ellison's theory and obtained by Pao (1965) and Corrsin (1964), by Saffman (1963) and by Panchev and Kesich (1969).

Figure 1 shows the comparison at low wavenumbers; the Kolmogoroff spectrum is also reported. In this region the solution proposed in this work is practically coincident with the spectrum proposed by Pao and Corrsin:

$$E(k) = A(\varepsilon \nu^5)^{1/4}(k/k_d)^{-5/3}\exp\left[-1.5A(k/k_d)^{4/3}\right] \tag{19}$$

It must be noted that the latter cannot be applied in the high-wavenumbers region but it has been validated by experimental data in the region of lower wavenumbers.

In the high wavenumber range the proposed spectrum closely follows the solution obtained originally by Saffman (1963) [if calculated using the $\alpha = 0.37$ constant] and later on proposed by Panchev and Kesich (1969):

$$E(k) = \alpha^{-1}(\varepsilon \nu^5)^{1/4}(k/k_d)^{-1}\exp\left[-\alpha^{-1}(k/k_d)^2\right] \tag{20}$$

while the difference from the Pao and Corrsin's spectrum is evident (see Figure 2).

As said above, Saffman gave a range for his constant: the spectra corresponding to the higher and lower value are plotted. As expected, the spectrum proposed by Saffman fails at low k-values.

In Figure 3 the comparison among the proposed spectrum and some spectra obtained in literature starting from different hypotheses is shown; as said before, all the spectra more or less closely follow the Kolmogoroff spectrum law for the lower k-values, so the comparison will be carried out in the higher wavenumber range.

The Heisenberg (1948) solution has been derived using the integral transfer function:

$$G(k)_H = -2\alpha\int_k^\infty k'^{-3/2}E(k')^{1/2}\,\mathrm{d}k'\int_0^k k'^2E(k')\,\mathrm{d}k' \tag{21}$$

in which the first integral term represents a sort of kinematic turbulent viscosity taking into account the effect of eddies associated with high k-values. It has been demonstrated, as said earlier, that this hypothesis is not valid especially in the range of high wavenumbers, where the

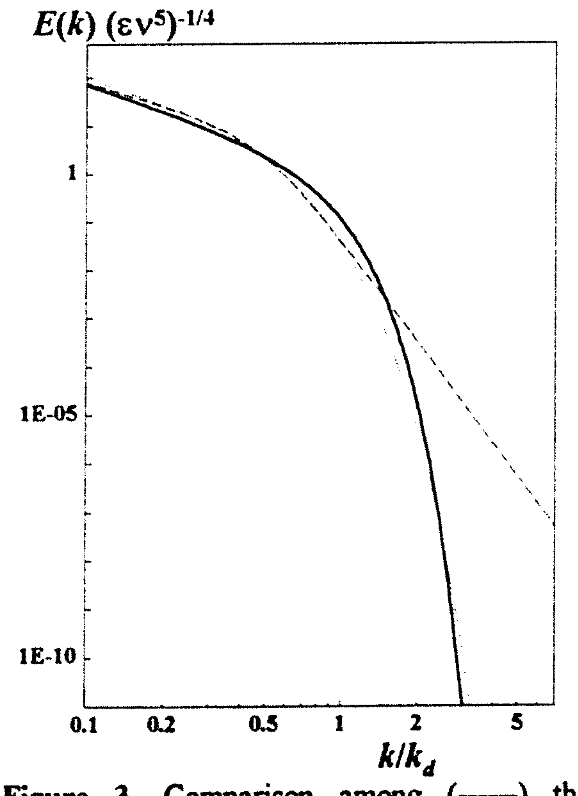

Figure 3. Comparison among (——) the proposed spectrum and the results obtained by (– – –) Heisenberg (eq. 22) and (········) Qian (eq. 23).

Heisenberg spectrum:

$$E(k) = A\,(\varepsilon\nu^5)^{1/4} \frac{(k/k_d)^{-5/3}}{\left[1+(8/3\alpha^2)(k/k_d)^4\right]^{4/3}} \tag{22}$$

fails.

It is remarkable that, at high k-values, the Heisenberg spectrum shows an algebraic fall-off of energy while most of the solutions proposed later, and the one obtained in this work, show an exponential behaviour. A pseudo-algebraic law has been recently predicted also by Frisch and Vergassola (1991) in the intermediate dissipative range, using a multifractal model; but a much faster decrease is expected beyond the bottom of this intermediate range.

Kraichnan (1967) pointed out that a stronger-than-algebraic decrease implies an increase of intermittency of the turbulent fluctuations; but up today no conclusive experimental data showing the exact shape of the fall-off of energy at high wavenumbers have been found and the final cut-off shape is a point of discussion (Frisch and Morf, 1981), even if Sreenivasan (1985), discussing currently available data, observed that in the limit of k large $E(k)$ is best fit by an exponential.

Recently Foias *et al.* (1990) gave support to the prediction of an exponential fall-off establishing a theoretical lower bound on the power to which the wavenumber k is raised in the exponential decay of the dissipation range spectrum, showing that available data are consistent with a simple exponential decay (Manley, 1992).

It can be remembered, on the other side, that, as demonstrated by Stewart and Townsend (1951), every correlation for $G(k,t)$ given as the product of two integral terms dealing respectively with the regions 0-k and k-∞ leads to an algebraic behaviour of the spectrum irrespective of the exact form of the involved functions.

The previous spectra (Pao and Corrsin, Saffman and Panchev & Kesich) and the spectrum proposed in this work derive from Ellison's closure which is not expressed as a product of two integral terms.

A spectrum function for the universal equilibrium range has been recently derived by Qian (1984) on the basis of the statistical mechanics theory of turbulence: he obtained the relationship:

$$E(k) = 1.19(\varepsilon\nu^5)^{1/4}(k/k_d)^{-5/3}\left[1+5.3(k/k_d)^{2/3}\right]\exp\left[-5.4(k/k_d)^{4/3}\right] \tag{23}$$

deriving an exponential fall-off of energy for higher wavenumbers (see Figure 3).

Panchev and Kesich (1969), on the base of their work concerning Ellison's assumption, proposed an interpolation formula, having Saffman's spectrum and Pao and Corrsin's spectrum as limiting solutions, thus covering the whole universal equilibrium range:

$$E(k) = (\varepsilon\nu^5)^{1/4}\left[A(k/k_d)^{-5/3}+\alpha^{-1}(k/k_d)^{-1}\right]\exp\left[-1.5A(k/k_d)^{4/3}-\alpha^{-1}(k/k_d)^2\right] \tag{24}$$

The spectrum described by equation (24) is quite similar to the spectrum proposed by the authors (see Figures 1 and 2), because the limit solutions have been obtained adopting Ellison's closure. But it must be pointed out that the former is just an interpolation, while the latter has been obtained by the solution of the dynamic equation for the energy spectrum. The difference is highlighted if the dissipation spectra are compared.

In Figure 4 the dissipation spectrum obtained by our calculation is shown; a maximum is observed at $k/k_d = 0.19$. The quantity Θ defined in equation (8) can be interpreted as a non-dimensional cumulative shear stress of the eddies associated to the wavenumber range 0-k. Θ^2 is the (dimensionless) integral of the dissipation spectrum.

In Figure 5 the dissipation spectra derived from the energy spectra mentioned earlier are compared. It can be seen that the maximum is generally located in correspondence of a value k_{max} in the range $0.17k_d - 0.25k_d$. The different values are reported in Table 1.

CONCLUSIONS

A formal mathematical derivation for the three-dimensional turbulent energy spectrum function in nondecaying homogeneous and isotropic turbulence has been proposed. To this aim, the Ellison's closure has been adopted.

The energy spectrum in the whole universal range has been obtained and a comparison with the spectra in literature has been carried out; it must be noted that only a few of them were derived for the entire universal equilibrium range. The comparison shows that the proposed spectrum follows the solutions of Pao and Corrsin and of Saffman at low and high wavenumbers respectively. It is similar, but significantly different, from the interpolation spectrum proposed by Panchev and Kesich.

All the spectra are roughly coincident in the region of low wavenumbers, while the

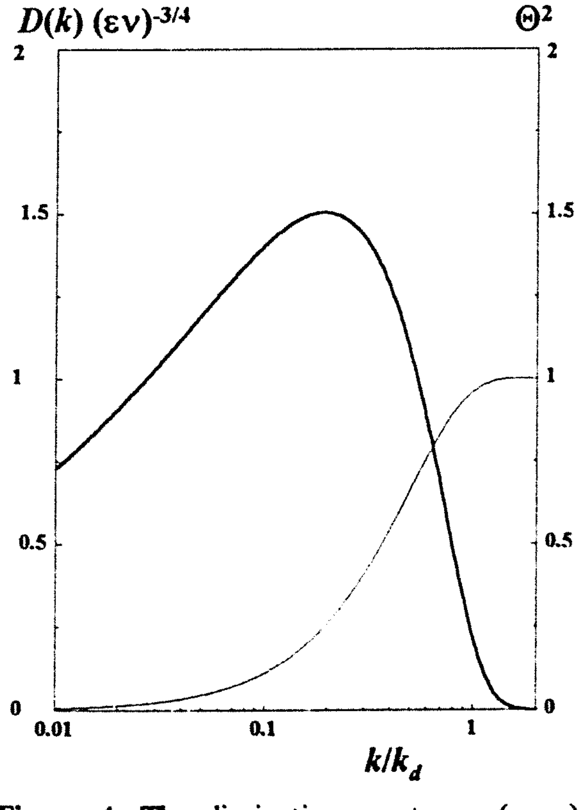

Figure 4. The dissipation spectrum: (——) differential and (——) integrated form.

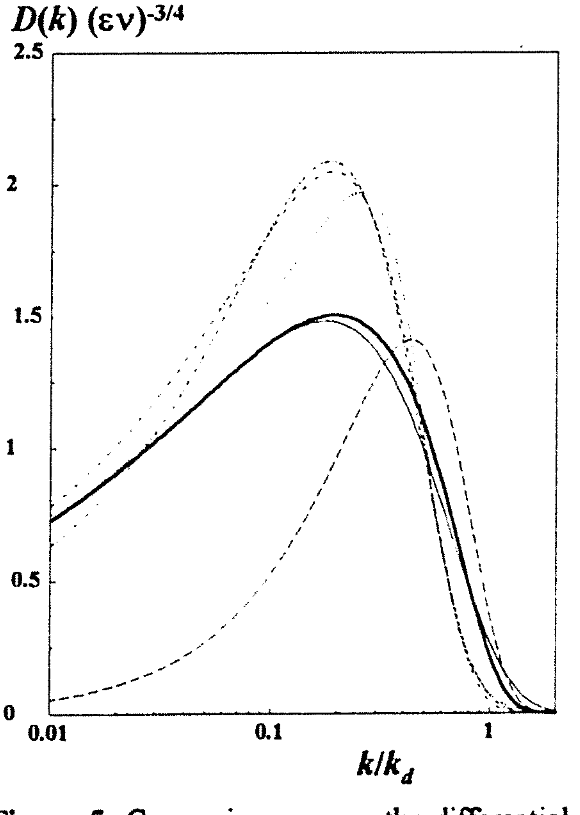

Figure 5. Comparison among the differential dissipation spectra: (——) this work; (——) Pao and Corrsin; (·······) Heisenberg; (– – –) Saffman, α=0.37; (– ·· – ·· –) Qian; (- - - - -) Panchev and Kesich (interpolation formula).

Table 1. Co-ordinates of the maximum of the dissipation spectrum calculated using different correlations from literature

Spectrum	eq.	k_{max}/k_d	$D(k_{max})/(\varepsilon\nu)^{3/4}$
This work	(16)	0.19	1.50
Pao (1965), Corrsin (1964)	(19)	0.17	1.48
Saffman (1963): α=0.23	(20)	0.34	1.79
Saffman (1963): α=0.37	(20)	0.43	1.41
Heisenberg (1948)	(22)	0.25	1.96
Qian (1984)	(23)	0.18	2.09
Panchev and Kesich (1969) (interpolation formula)	(24)	0.19	2.05

behaviour in the dissipative range is clearly a function of the type of closure that has been used and, therefore, of the proposed energy transfer mechanism. The differences are highlighted if the dissipation spectra are compared.

This work is a preliminary result of a project whose final goal is the investigation and description of the turbulence effects on chemical processes.

Work is currently in progress in order to describe the energy spectrum in case of decaying turbulence as a function of the local turbulent Reynolds, and to deal with non-homogeneous turbulence. The next step will be the investigation of the scalar spectrum.

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NOTATION

A	Kolmogoroff constant
$D(k,t)$	dissipation spectrum function
$E(k,t)$	energy spectrum function
$F(k,t)$	energy-transfer spectrum function
$G(k,t)$	integral transfer function
k	wavenumber
k_d	wavenumber range of main dissipation
k_{max}	k value at which the maximum in the differential dissipation spectrum is observed
m	defined in eq. (15)
t	time

Greek symbols

α	constant in eq. (6)
ε	turbulent energy dissipation rate
Θ	nondimensional cumulative shear stress of the eddies associated to wavenumber range 0-k; defined in eq. (8)
λ_K	Kolmogoroff microscale
ν	kinematic viscosity

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